DELPHI Collaboration DELPHI 2000-118 CONF 417
23 June, 2000

More About the Multiplicity of Symmetric Three Jet Events

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Abstract

The measurement of the hadron multiplicity in mirror symmetric three jet events is compared to recent theoretical calculations. Jets are defined with the Cambridge algorithm. From this data in comparison to the hadronic multiplicity in e^+e^-–annihilation a determination of the gluon to quark colour factor ratio yields:

$$\frac{C_A}{C_F} = 2.221 \pm 0.032(\mathrm{stat}) \pm 0.047(\mathrm{exp}) \pm 0.058(\mathrm{hadc}) \pm 0.075(\mathrm{theo}) \quad .$$

A measurement of the hadron multiplicity in equivalent gluon gluon events as function of the energy scale is presented. The increase with energy scale of the hadron multiplicity in gluon gluon events is observed to be about twice as strong as in quark antiquark events. This presents very direct evidence for the triple gluon vertex and the higher colour charge of the gluon.

Contributed paper 640 to ICHEP 2000, Osaka, Japan

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1 Introduction

In a recent paper [1] the DELPHI–Collaboration presented a first measurement of the multiplicity of symmetric three jet events in dependence of the opening angle between the low energy jets. From a comparison of this data with the multiplicity of hadronic events in e^+e^-annihilation a precise value of the colour factor ratio C_A/C_F has been measured as well as the energy scale dependence of the multiplicity of gluon gluon events. Both of these measurements rely on the introduction of a non–perturbative parameter N_0, assumed to be constant, to account for differences in the fragmentation of gluon and quark jets.

The interpretation of N_0 is tightly coupled to the behaviour of the fragmentation functions of gluons and quarks. The quark fragmentation function outreaches that of gluons at large x which can be explained by the leading particle effect. Alternatively, this behaviour can be explained by energy conservation [2]. In consequence the multiplicity ratio of gluon to quark jets must fall below the naïve colour factor expectation even if the ratio of the gluon to quark fragmentation functions is equal to the colour factor ratio at small scaled momenta. Lower limits for N_0 were deduced from fragmentation function data [3] to be 0.61 ± 0.02 from so–called Y– and 0.58 ± 0.05 from "Mercedes–events".

As the value of N_0 has been determined from real data it unavoidably accounts for finite energy effects. Thus care has to be taken when combining a non–perturbative offset with calculations which explicitly account for finite energy effects which are often also called recoil effects. In [1] the prediction [4] therefore has not been applied as it did not reproduce the behaviour shown by the fragmentation models.

In parallel with and shortly after the publishing process of our experimental paper several theoretical papers appeared which discuss the hadron multiplicity in gluon and quark jets and in three jet events in detail [5–7]. In [5] it has been recommended to employ the Cambridge jet algorithm [8]. Therefore this method is used in this note and the relevant data is presented. This high statistics data allow to access the multiplicity in equivalent gluon gluon events over a wide range of the underlaying energy scale enabling detailed tests of the calculations mentioned above. Beyond classical measurements at fixed scale [3, 9, 10] it becomes especially possible to measure energy slopes directly and to distinguish dynamical and non–perturbative terms.

This note is organised as follows: Sect. 2 presents the data and gives the references of the data analysis. In Sect. 3 we give a brief overview on the relevant theoretical predictions. In Sect. 4 we perform fits of predictions to the data. In Sect. 5 we use the theoretical predictions to derive the multiplicity of a gg–event from the measured three jet multiplicity and calculate the ratio of the multiplicities r found in $q\bar{q}$– and the extracted gg–events as well as the ratio of the derivatives of these quantities. Finally in Sect. 6 we summarise and conclude.

2 The Data

The charged multiplicity is measured in mirror symmetric three jet events as function of the opening angle, θ_1, between the low energy jets. As the centre of mass energy is equal to the Z mass this angle fully specifies the event kinematic. Only a small correction has to be applied for the cases where the gluon leads to the formation of the most energetic jet.

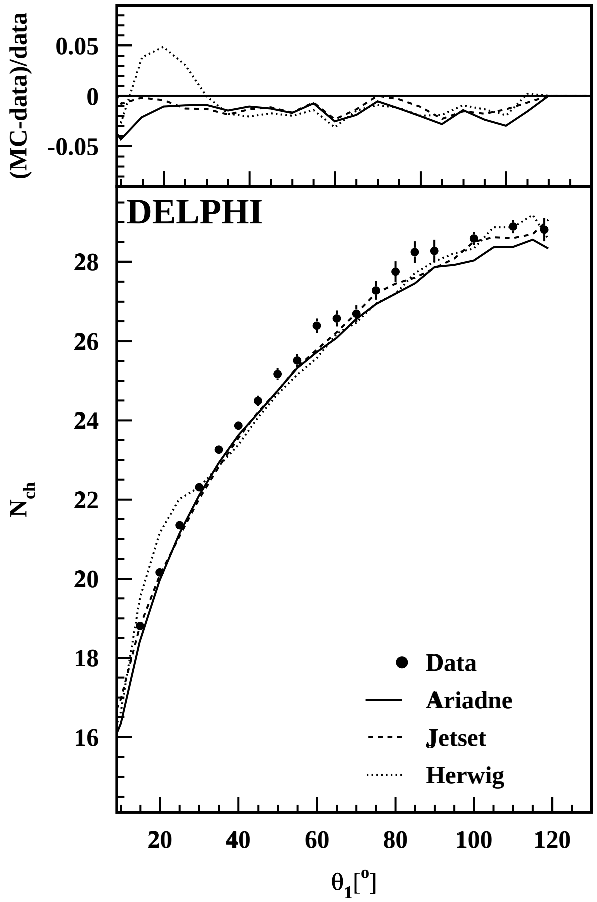

Figure 1: The dependence of the event multiplicity on the opening angle θ_1 in comparison with Monte–Carlo models

As proposed in [5] the jet axes were reconstructed using the Cambridge jet algorithm [8] demanding exactly three reconstructed jets. To obtain symmetric events it was required that the angles between the most energetic jet and both low energy jets are the same within small tolerances, i.e. $2.5°$ or $5°$ for large angles θ_1.

Except for using the Cambridge algorithm the data analysis is identical to that described in [1]. Furthermore a cut applied to the data presented in [1] has been removed thus the available statistics is increased. It has been verified that the cut removal did not significantly alter the multiplicity presented in [1] for the Durham algorithm in the angular range used for fits to the data.

The measured charged hadron multiplicity in symmetric three jet events as function of the opening angle is given in Tab. 1 and compared to several fragmentation models in Fig. 1. The models and their parameter settings are described in detail in [11]. Jetset and Ariadne describe the data well in the whole range of θ_1. The deviation of about 2% visible in the upper part of Fig. 1 is well within the expected precision of this measurement and the model tunings. For $\theta_1 > 40°$ the agreement of Herwig is similarly good. At small angles, however, a significant overshoot of the model is visible. It turned out that to large extend this deviation is due to events with primary b quarks.

$\theta_1 [^\circ]$	$N_{q\bar{q}g}$	$\theta_1 [^\circ]$	$N_{q\bar{q}g}$	$\theta_1 [^\circ]$	$N_{q\bar{q}g}$
14.83	18.80 ± 0.03	49.96	25.17 ± 0.15	84.95	28.25 ± 0.27
19.86	20.16 ± 0.05	54.98	25.51 ± 0.17	89.95	28.27 ± 0.29
24.92	21.35 ± 0.07	59.97	26.39 ± 0.18	99.95	28.59 ± 0.16
29.93	22.31 ± 0.08	64.98	26.57 ± 0.20	109.95	28.89 ± 0.17
34.96	23.26 ± 0.10	69.97	26.69 ± 0.21	117.93	28.81 ± 0.29
39.95	23.87 ± 0.11	74.97	27.28 ± 0.24		
44.96	24.49 ± 0.13	79.97	27.75 ± 0.26		

Table 1: $N_{q\bar{q}g}$ measured in symmetric three jet events as function of the opening angle θ_1. Errors are statistical.

3 Theoretical Predictions

Important results for the comparison of theoretical results to data are included in [5]. Here it is shown that in the Dipole formulation (i.e. resumming all leading logarithmic terms) the evolution of the gluon multiplicity with scale is given by the differential equation:

$$\left.\frac{dN_{gg}(L')}{dL'}\right|_{L'=L+c_g-c_q} = \frac{N_C}{C_F}\left(1-\frac{\alpha_0 c_r}{L}\right)\frac{d}{dL}N^h_{q\bar{q}}(L) \tag{1}$$

with

$$L = \ln\left(\frac{s}{\Lambda^2}\right) \quad , \quad c_g = \frac{11}{6} \quad , \quad c_q = \frac{3}{2} \quad , \quad c_r = \frac{10}{27}\pi^2 - \frac{3}{2}$$

Λ is the QCD scale parameter. The solution of this differential equation implies a constant of integration. Extrapolating the solution of Eqn. 1 to small scales neglecting the constant of integration would imply that the multiplicity in a gg–system would still be significantly larger than in a q$\bar{\text{q}}$–system. At very small scales, however, the hadronic multiplicity of both systems should mainly be determined by hadronic phase space and thus should become almost equal [5]:

$$N^h_{gg}(L_0) \approx N^h_{q\bar{q}}(L_0) = N(L_0) \tag{2}$$

Thus a non–perturbative constant term appears in the solution for the gluon multiplicity as was expected from the behaviour of the fragmentation functions [1]. In [5] it is suggested to determine $N(L_0)$ from data on charmonium or bottonium states. The multiplicity of three jet event then is given by the two alternative formulations [6]:

$$N_{q\bar{q}g} = N_{q\bar{q}}(L_{q\bar{q}}, \kappa_{\mathrm{Lu}}) + \frac{1}{2}N_{gg}(\kappa_{\mathrm{Le}}) \quad , \tag{3a}$$

$$N_{q\bar{q}g} = N_{q\bar{q}}(L, \kappa_{\mathrm{Lu}}) + \frac{1}{2}N_{gg}(\kappa_{\mathrm{Lu}}) \tag{3b}$$

with

$$L = \ln\left(\frac{s}{\Lambda^2}\right) \quad , \quad L_{q\bar{q}} = \ln\left(\frac{s_{q\bar{q}}}{\Lambda^2}\right) \quad , \quad \kappa_{\mathrm{Lu}} = \ln\left(\frac{p^2_{\perp\mathrm{Lu}}}{\Lambda^2}\right) \quad , \quad \kappa_{\mathrm{Le}} = \ln\left(\frac{p^2_{\perp\mathrm{Le}}}{\Lambda^2}\right)$$

and

$$p^2_{\perp\mathrm{Lu}} = \frac{s_{qg}s_{\bar{q}g}}{s} \quad , \quad p^2_{\perp\mathrm{Le}} = \frac{s_{qg}s_{\bar{q}g}}{s_{q\bar{q}}} \quad , \quad s_{ij} = (p_i + p_j)^2 \quad .$$

Both predictions 3a and 3b differ in the definition of the gluon contribution to the event multiplicity. While in equation 3a the q$\bar{\mathrm{q}}$–contribution is given mainly by the invariant mass of the q$\bar{\mathrm{q}}$–system which is also the relevant scale in an q$\bar{\mathrm{q}}$–event of the same topology with the gluon replaced by a hard photon, the q$\bar{\mathrm{q}}$–contribution in Eqn. 3b is given by the centre of mass energy of the whole event.

The expression $N_{q\bar{q}}(L,\kappa)$ for the quark contribution to the three jet multiplicity takes into account that the resolution of a gluon jet at a given p_t implies restrictions on the phase space of the quark system. This restricted multiplicity is linked to the multiplicity of an unrestricted q$\bar{\mathrm{q}}$–system $N_{q\bar{q}}(L)$ via [5]:

$$N_{q\bar{q}}(L,\kappa_{\mathrm{cut}}) = N_{q\bar{q}}(\kappa_{\mathrm{cut}} + c_q) + (L - \kappa_{\mathrm{cut}} - c_q)\left.\frac{dN_{q\bar{q}}(L')}{dL'}\right|_{L'=\kappa_{\mathrm{cut}}+c_q} . \qquad (4)$$

Both predictions Eqn. 3a and Eqn. 3b use different scales for this effect, the topology dependence of the q$\bar{\mathrm{q}}$–term in Eqn. 3b enters only due to this phase space restriction.

In [1] a theoretical prediction of the ratio r of the multiplicities of a gg– and a q$\bar{\mathrm{q}}$–system [12] has been used, where r in $\mathcal{O}(\alpha_s^2)$ i.e. NNLO is given as

$$r(y) = r_0(1 - r_1\gamma_0 - r_2\gamma_0^2) \qquad (5)$$

with

$$r_0 = \frac{N_C}{C_F} \quad , \quad r_1 = \frac{1}{6}\left(1 + \frac{n_f}{C_A} - \frac{2n_fC_F}{C_A^2}\right) \quad , \quad r_2 = \frac{r_1}{6}\left(\frac{25}{8} - \frac{3}{4}\frac{n_f}{C_A} - \frac{n_fC_F}{C_A^2}\right)$$

$$\gamma_0 = \sqrt{\frac{2N_C\alpha_s}{\pi}} = \sqrt{\frac{4N_C}{\beta_0 y}\left(1 - \frac{\beta_1 \ln 2y}{\beta_0^2 y}\right)}$$

$$\beta_0 = \frac{1}{3}(11N_C - 2n_f) \quad , \quad \beta_1 = \frac{1}{3}(51N_C - 19n_f) \quad .$$

A more recent 3NLO-calculation including energy conservation [7] gives the ratio r as

$$r(y) = r_0(1 - r_1\gamma_0 - r_2\gamma_0^2 - r_3\gamma_0^3) \qquad (6)$$

with

$$y = \ln\left(\frac{p\theta}{Q_0}\right) \quad , \quad r_0 = \frac{N_C}{C_F}$$

The coefficients $r_{1,2,3}$ are different from those in Eqn. 5. They can be found in [13] and are calculated to be $r_1 = 0.185$, $r_2 = 0.426$, $r_3 = 0.189$ for $N_f = 3$. Q_0 is a cutoff which defines the limit of perturbative evolution.

Following the original proposal to measure the ratio of the slopes of the multiplicity of a gg– and a q$\bar{\mathrm{q}}$–system [14] rather than the ratio of the multiplicities themselves also this quantity is calculated in [7]:

$$\frac{dN_{gg}/dy}{dN_{q\bar{q}}/dy} = r^{(1)} \simeq \frac{r}{\rho_1} \qquad (7)$$

where ρ_1 is given by

$$\rho_1 = 1 - 0.0694\cdot\gamma_0^2\cdot\left[1 + 5.070\cdot\gamma_0 + 5.714\cdot\gamma_0^2\right] \qquad (8)$$

for $N_f = 3$ and r from Eqn. 6.

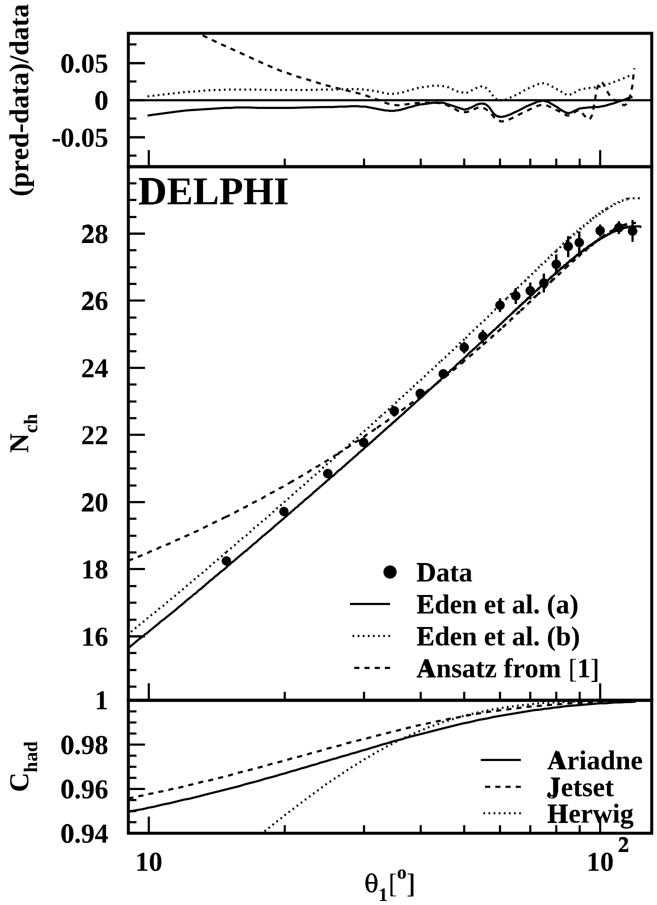

Figure 2: Analytic predictions for the event multiplicity as a function of θ_1. In the lower part the correction for the effect of the fragmentation on the angle θ_1 as predicted by Monte–Carlo is shown.

4 The fit of C_A/C_F

In symmetric events as analysed here, there are only two free parameters to describe the event topology, e.g. $\sqrt{s}$ and θ_1, the angle opposite to the leading jet, where in this measurement $\sqrt{s} = m_Z$ is fixed. So all scales can be expressed as functions of θ_1 only, assuming that the leading jet is not the gluon jet which is true for most events. The fraction of events in which the gluon jet is the leading jet is taken from Monte–Carlo simulations [1]. The multiplicity predictions are then calculated as a weighted mean of the predictions for topologies in which the gluon jet is leading or subleading respectively. Because of the symmetry of the chosen events, no care has to be taken which of the subleading jets is the gluon jet.

The measurement of $N_{q\bar{q}}(L)$ which enters the predictions 3a and 3b via Eqn. 4 can be taken from several measurements of the multiplicity $N_{\mathrm{e^+e^-}}$ in $\mathrm{e^+e^-} \rightarrow q\bar{q}$ as function of $\sqrt{s}$ [1,15]. To be able to perform the evaluation of Eqn. 1 and Eqn. 4 the parameterisations of $N_{\mathrm{e^+e^-}}$ [16,17] given in [1] have been approximated by a polynomial in L of order three.

The hadronic multiplicity of the decay $\chi_b'(J = 2) \rightarrow gg$ at $E_{cm} =$ 9.9132GeV has been measured precisely by the CLEO–collaboration to be

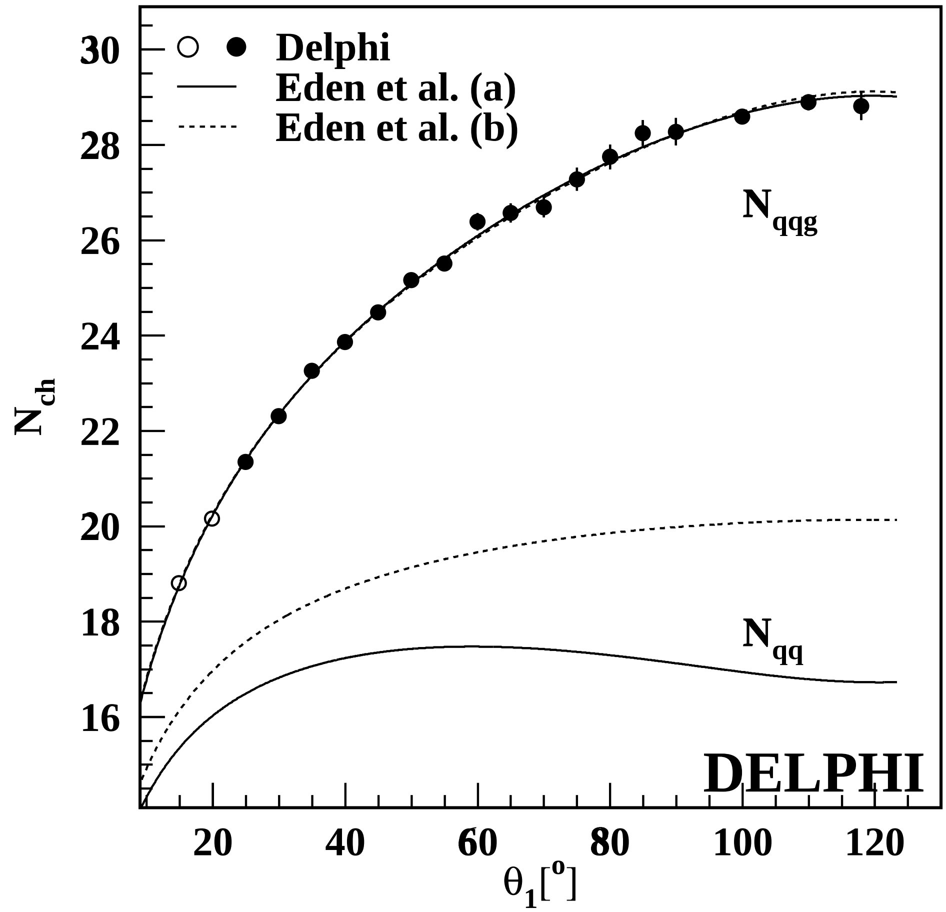

Figure 3: Fits of equations 3a,3b to the data. The lower curves display the contribution of the q$\bar{\mathrm{q}}$–system to the event multiplicity. The full points depict the range of the fit.

$N_{gg}(9.9132\mathrm{GeV}) = 9.339 \pm 0.090 \pm 0.045$ [18]. This measurement is used to fix the parameters L_0 of equations 3a and 3b assuming $\Lambda = 250\mathrm{MeV}$ with the result $L_0 = 5.30$. Therefore above equations give explicit predictions for the dependence of the event multiplicity $N_{q\bar{q}g}(\theta_1)$ on the event topology.

It is well known that the fragmentation process tends to pull near by jets even closer together. In order to correct for this effect, which is related to the so–called string effect, the Monte–Carlo models are used. To get a correction for this effect, the hadronic multiplicities are sorted according to the angles between the jets of hadrons or the angles between the jets of partons before fragmentation, respectively. The ratio of both is used as a correction to the predictions. As Ariadne and Jetset describe the data well, these models are taken for this correction with Ariadne being used for the central result and Jetset entering the systematic uncertainties. In the lower part of Fig. 2 the corrections by which the predictions are divided are shown. They are below 5% even for opening angles θ_1 down to 10°.

In the central part of Fig. 2 the predictions Eqn. 3a and Eqn. 3b are compared to the multiplicity measured in b–anti–tagged events. Here b–anti–tagged events are chosen as the parameterisation of $N_{\mathrm{e^+e^-}}$ taken from [1] is corrected for the contribution of b events which varies with the centre of mass energy. As can be seen, prediction 3a is in excellent

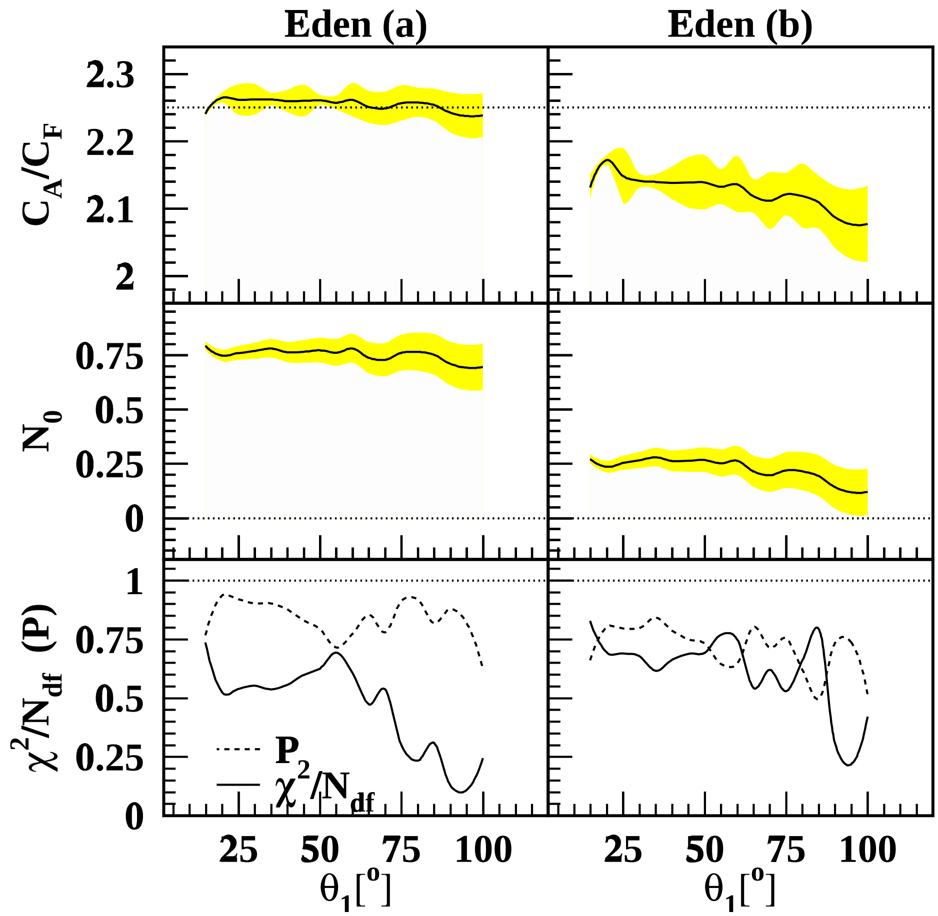

Figure 4: Stability of the fits shown in Fig. 3 In order to obtain the stability of each parameter uncorellated to the other, the other parameter has been set to its central value and not been varied.

agreement with the data, while prediction 3b overestimates the multiplicity a little, but describes the overall shape of the data, especially for low values of θ_1, quite reasonably. As the Monte Carlo models agree very well with the data this also implies that prediction 3a agrees slightly better with the models than prediction 3b.

Additionally in Fig. 2 the parameterisation of $N_{q\bar{q}g}$ used in [1] is shown. As this ansatz uses a phenomenologically motivated offset, which is not theoretically determined, this parameter has been fitted to the data. As can be seen, this prediction is in good agreement with the data as well as with the prediction 3a for $\theta_1 > 30°$, i.e. in the fit range used in [1]. As the phenomenological offset is replaced by a theoretical well motivated prescription in 3a and 3b and these predictions allow to extend the usable θ_1 range, this elder ansatz will not be used further on in this note.

The predictions of $N_{q\bar{q}g}$ can be used to test the group structure of QCD by fitting the colour factor ratio C_A/C_F. The colour factors C_A and C_F govern the radiation of a gluon by another gluon or a quark, respectively. Therefore this ratio plays a crucial role in any calculation of N_{gg} from $N_{q\bar{q}}$ as can be seen in equations 1, 5 and 6.

To avoid any bias introduced by an anti–b–tagging procedure, data including also events with initial b–quarks are used. To account for the additional multiplicity introduced

	C_A/C_F	N_0
Eqn. 3a	2.262 ± 0.032	0.760 ± 0.047
Eqn. 3b	2.148 ± 0.043	0.252 ± 0.035

Table 2: Results of the fits. Errors are statistical only.

by decays of b–hadrons, an offset N_0 is added to equations 3a and 3b and fitted to the data. N_0 also accounts for small differences in the multiplicity normalisation of this measurement and the overall charged hadron multiplicity measurements in e^+e^-–annihilations. The introduction of N_0 also assures that the value obtained for the colour factor ratio C_A/C_F is not affected by the overall normalisation but only by the slope of the measured multiplicity.

Equations 3a and 3b are used with L_0 pinpointed by the measurement of N_{gg} by CLEO. The values for L_0 now depend on the values for C_A/C_F. The fits include all data points with $\theta_1 \geq 25°$ where the hadronisation correction is below 3%.

Fig. 3 shows the resulting curves. Both fits are in good agreement with the data. The lower curves show the contributions of the $q\bar{q}$–system to the event multiplicity. The shape of the curve of $N_{q\bar{q}}$ according to equation 3b is only influenced by the phase space restriction due to the angle to the gluon jet which is strongest at small angles θ_1. At large angles this curve is almost constant. On the other hand the curve for $N_{q\bar{q}}$ according to equation 3a decreases again at large angles θ_1 as the invariant mass of the $q\bar{q}$–system $\sqrt{s_{q\bar{q}}}$ decreases with growing opening angle θ_1 i.e. growing energy of the gluon jet. Such a behaviour would also be expected for a $q\bar{q}$–system where the emitted gluon would have been replaced by an equivalent photon.

The results of the fits are given in Tab. 2. Fitting equation 3a to the data yields $N_0 = 0.760 \pm 0.047$. This value is in good agreement with the value expected from the multiplicity difference of anti–b–tagged and overall hadronic Z decays [19–22] $N_0^{exp.} = N_{ch} - N_{ch}^{udsc} = 0.67 \pm 0.08$. Consequently the small value of $N_0 = 0.211 \pm 0.052$ obtained for the anti–b–tagged sample is fully consistent with zero within the systematic precision of the general multiplicity measurements $\mathcal{O}(1\%)$ [15]. The result obtained when fitting Eqn. 3b, $N_0 = 0.252 \pm 0.035$ for the full sample ($N_0 = -0.304 \pm 0.040$ for the anti–b–tagged sample), agrees less good with this expectation, however the limited precision of the experimental data does not allow to draw further conclusions.

In Fig. 4 the variation of the fit parameters and the χ^2 and probability with the fit range is shown. Each parameter is individually allowed to float while setting the other parameter to its central value (see Tab. 2). The grey bands indicate the uncorrelated errors. The abscissa shows the angle θ_1 at which the fit starts. Both fits are stable and have good χ^2 and probabilities.

To estimate the systematic error of this fit some variations of the procedure have been made and lead to the specified relative deviations in C_A/C_F:

1. All systematics due to cuts and data handling are the same as in [1] and are taken over, deviation 0.61%

2. The dataset used to parameterise $N_{q\bar{q}}$ has been varied. Instead of the multiplicity from several e^+e^-–annihilation experiments only data from events with ISR measured by DELPHI have been used, see [1], deviation 0.92%

3. The fit has been performed on anti–b–tagged data, deviation 0.34%

cut variations	0.61%		
e^+e^-–datasets	0.92%		
b–tagging	0.34%	2.12%	
fit start	0.51%		
$N_0 \leftrightarrow K_0$	1.7%		
JETSET ↔ARIADNE	1.0%	2.6%	4.0%
30% of had.–corr.	2.4%		
Variation of L_0	0.0%		
Variation of Λ	1.0%	2.2%	
Variation of c_r	2.0%		

Table 3: Systematics to C_A/C_F

4. The starting point of the fit in θ_1 has been varied between 20° and 30°, deviation 0.51%

5. Instead of an additional offset N_0 a factorial normalisation K_0 has been used to compensate the multiplicity due to b–decays, deviation 1.7%. Results for K_0 are 1.02 for Eqn. 3a and 1.01 for Eqn. 3b with very small uncertainties.

6. JETSET instead of ARIADNE has been used to calculate the influence of the fragmentation on the angles of an event, deviation 1.0%

7. Conservatively 30% of the ARIADNE correction factor is regarded as uncertainty. This variation is large enough to also cover the corrections given by HERWIG, deviation 2.4%

8. L_0 has been varied within the limits given by the error of the CLEO measurement, which had no influence on C_A/C_F thus stressing the meaning of L_0 as a constant of integration

9. $\Lambda = 250$MeV has been varied from 200MeVto 300MeV, deviation 1.0%

10. The parameter c_r in Eqn. 1 has been varied by 10% as this is given as a theoretical uncertainty of the prediction, deviation 2.0%

Tab. 3 gives an overview over the uncertainties grouped in experimental uncertainties (1–5), uncertainties due to the hadronisation correction (6,7) and theoretical uncertainties (8–10) of the predictions.

Although equation 3b is disfavoured in comparison to equation 3a because of the implausible choice of scales and the inferior agreement with the data, the result for C_A/C_F is averaged over the values obtained with both equations in a conservative manner with half the difference entering the theoretical uncertainty:

$$\frac{C_A}{C_F} = 2.221 \pm 0.032(\mathrm{stat}) \pm 0.047(\mathrm{exp}) \pm 0.058(\mathrm{hadc}) \pm 0.075(\mathrm{theo}) \quad . \tag{9}$$

Using the Durham algorithm to reconstruct the jets and the ansatz described in [1] the fit to the multiplicities results in $C_A/C_F = 2.256 \pm 0.039(\mathrm{stat})$ in excellent agreement with the result presented therein.

5 The extraction of N_{gg}

Instead of using the predictions Eqn. 3a and Eqn. 3b to measure the colour factor ratio C_A/C_F, they can be used to extract the multiplicity of an equivalent gg–system from the measured three jet multiplicity. For this purpose C_A/C_F is set to its theoretical value of 9/4. Although the two predictions define different parts of the three jet event as the gluon contribution, the application of both would result in a consistent N_{gg} within the limits of the consistency of the predictions themselves. Due to the choice of scales, the values for N_{gg} gained by applying equation Eqn. 3b would be at lower equivalent centre of mass energies as the results provided by Eqn. 3a. But as only prediction Eqn. 3a is found to be in good agreement with the measurement only this prediction is used for the extraction. Again, data including events with initial b–quarks and an offset N_0 to compensate the additional multiplicity are used to avoid systematics due to b–tagging.

The multiplicity of an equivalent gg–system is then gained by inverting Eqn. 3a:

$$N_{gg}(\kappa_{\mathrm{Le}}) = 2[N_{q\bar{q}g}(\theta_1) - N_{q\bar{q}}(L_{q\bar{q}}, \kappa_{\mathrm{L}u}) - N_0] \tag{10}$$

The correct value of $\kappa_{\mathrm{L}e}$ is gained from the scale–values for the cases where the gluon is in the leading jet and where it is not with a properly weighted arithmetic mean. As $\kappa_{\mathrm{L}e}$ is a logarithmic scale, this arithmetic mean corresponds to a geometric mean for the scale $p_{\perp\mathrm{L}e}$ which has been used in [1]. The result of this procedure is shown in Fig. 5 and tabulated in Tab. 4. The lower curve shows the parameterisation of $N_{q\bar{q}}$ and the data it has been fitted to. The upper curve is the prediction for N_{gg} as given by Eqn. 1 with L_0 fixed by the CLEO measurement of N_{gg} at ~ 10GeV denoted by an open star. The solid dots denote the values extracted from the three jet multiplicity according to Eqn. 10, the triangles represent further measurements by the CLEO–collaboration and are without systematic errors [23]. The quadrate marker represents a measurement of N_{gg} by the OPAL collaboration where only gluon jets being the leading jet of a three jet event have been investigated [10]. The agreement between the different measurements is good and it can be clearly seen that the multiplicity of a gg–system increases roughly twice as fast with energy than the multiplicity of a q$\bar{\mathrm{q}}$–system. This stronger increase presents very direct evidence for the triple gluon vertex and the higher colour charge of the gluon [1,14].

$p_{\perp\mathrm{L}e}$[GeV]	N_{gg}	$p_{\perp\mathrm{L}e}$[GeV]	N_{gg}	$p_{\perp\mathrm{L}e}$[GeV]	N_{gg}
5.95	6.68 ± 0.39	21.05	15.28 ± 0.34	39.46	21.96 ± 0.54
8.00	8.03 ± 0.37	23.46	15.91 ± 0.36	42.19	22.21 ± 0.58
10.08	9.46 ± 0.34	25.95	17.66 ± 0.39	47.24	23.22 ± 0.32
12.16	10.72 ± 0.32	28.54	18.08 ± 0.41	51.08	24.13 ± 0.33
14.30	12.14 ± 0.32	31.20	18.41 ± 0.44	52.57	24.10 ± 0.58
16.48	13.04 ± 0.31	33.93	19.71 ± 0.48		
18.73	14.06 ± 0.32	36.70	20.80 ± 0.53		

Table 4: N_{gg} derived from $N_{q\bar{q}g}$ using equation 3a. Errors are statistical.

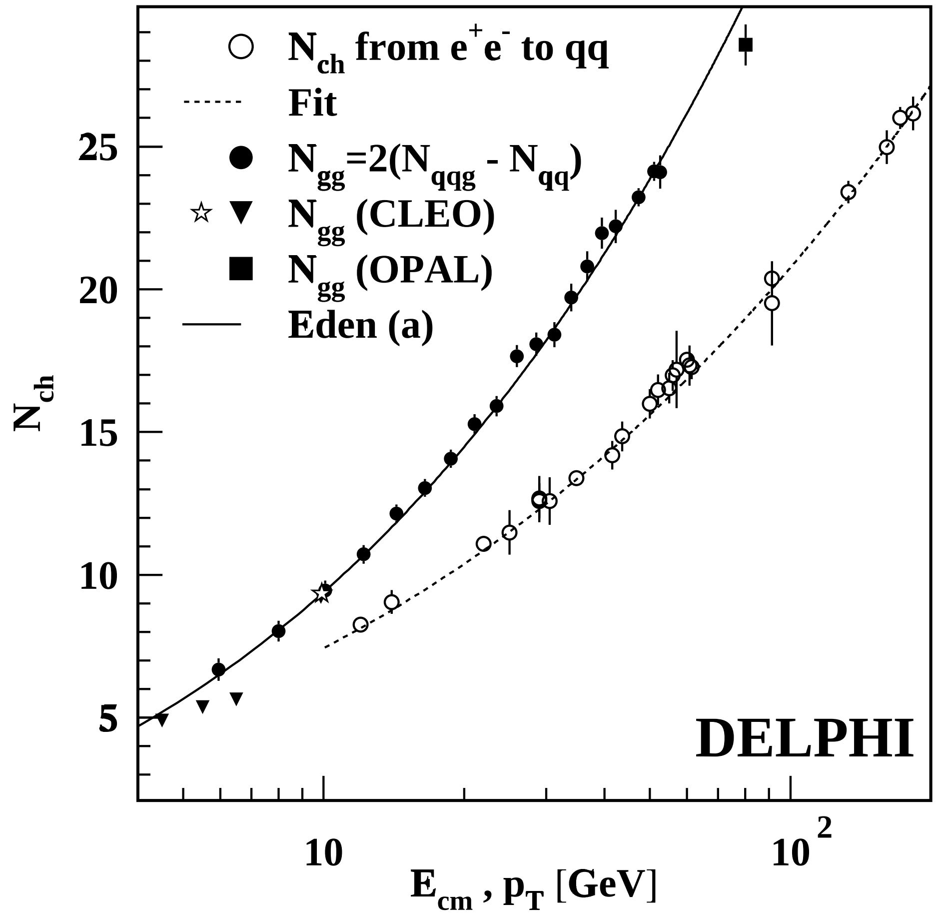

Figure 5: $N_{q\bar{q}}$ and N_{gg} as function of $\sqrt{s}$ and $p_{\perp Le}$, N_{gg} derived with Eqn. 3a.

5.1 The ratios of the multiplicities and their derivatives

With the extracted N_{gg} as a measurement of the multiplicity of a gg–system over a wide variation of the energy–scale, the ratio r of the multiplicities in q$\bar{\text{q}}$– and gg–events can be calculated directly. For this purpose the extracted multiplicities of equivalent gg–events are divided by the multiplicity of a q$\bar{\text{q}}$–system of the same energy as given by the parameterisation shown as dashed line in Fig. 5. These measured ratios are shown as dots in the upper half of Fig. 6 with the error bars indicating statistical errors only. The solid line represents the prediction by Eden and Gustafsson [5] which has been calculated as the ratio of the prediction for the multiplicity of a gg–system (i.e. the solution of Eqn. 1 shown as solid line in Fig. 5) and the parameterisation of the q$\bar{\text{q}}$–multiplicity. This prediction for r is in very good agreement with the measurement as expected because both, the prediction for N_{gg} and the parameterisation of $N_{\text{e}^+\text{e}^-}$ follow the data well. The measurement of Opal [10] at $\sim$ 80GeV lies slightly below the prediction for r as is evident from Fig. 5. On the other hand the predictions Eqn. 5 and Eqn. 6, shown as a dotted and a dashed line respectively, overestimate r with the NNLO calculation Eqn. 5 giving even higher results than the 3NLO calculation of Eqn. 6 which deviates from the measurement by $\sim$ 10%. Note that neither Eqn. 5 nor Eqn. 6 account for possible non–perturbative contributions like the prediction by Eden *et al.*

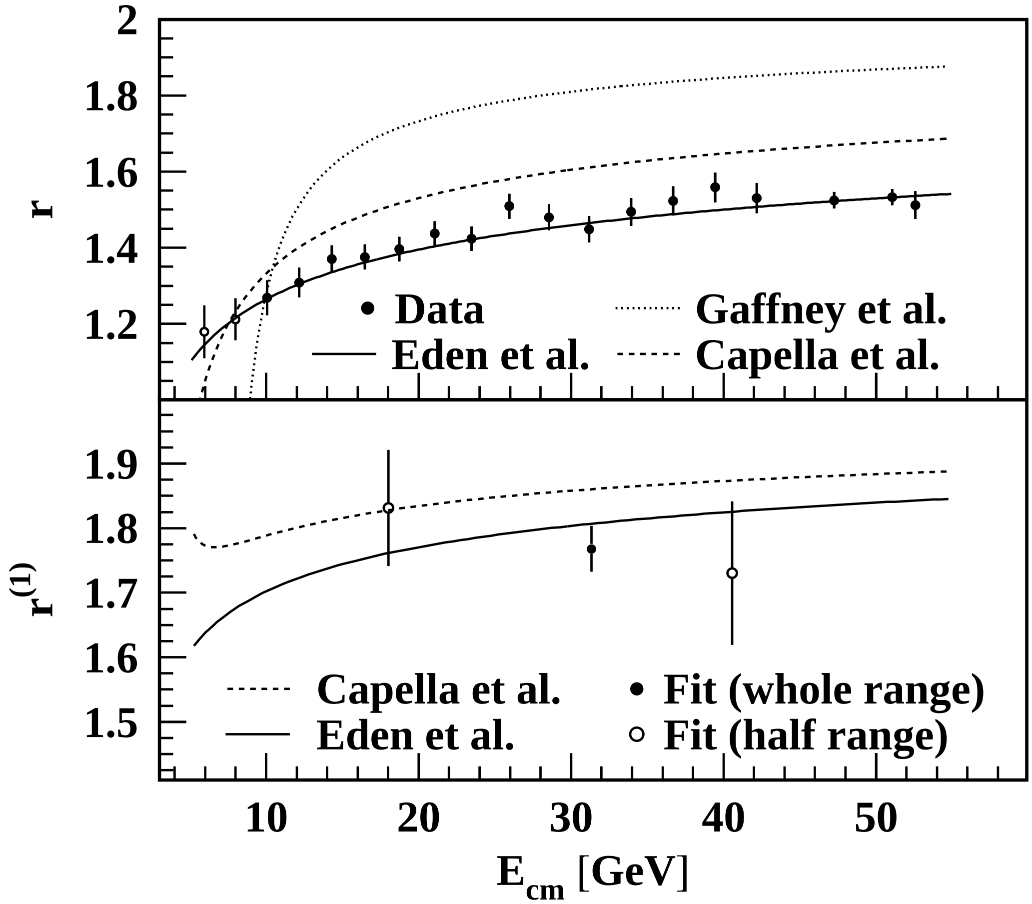

Figure 6: The predictions for the multiplicity ratio r and the ratio of the derivatives compared to the data.

In the lower half of Fig. 6 the ratio $r^{(1)}$ of the derivatives with respect to L of both multiplicities are shown. A parameterisation of $N_{\rm ch}$ [16] which has already been used to parameterise $N_{\rm e^+e^-}$ has been fitted to the extracted gluon multiplicity. In order to gain a good description of the extracted $N_{\rm gg}$ an additional offset had to be introduced. As, except for this constant offset, $N_{\rm e^+e^-}$ and $N_{\rm gg}$ are described by the same type of parameterisation, the measured ratio of the derivatives is by construction a constant. The measurement $r^{(1)} = 1.77 \pm 0.03(\rm stat)$ is indicated by the solid dot in Fig. 6. Additionally, the fit of $N_{\rm gg}$ has also been performed over only the lower and the upper half of the fit range, respectively. The results of this procedure are indicated by the open dots. As the three values for $r^{(1)}$ are in full agreement with each other, this measurement implies no sensitivity on the energy dependence of $r^{(1)}$. The measurement of a slope implies an average over a range of scales. Therefore there is an uncertainty on the exact abscissa of the $r^{(1)}$ measurements. In Fig. 6 the results are given at the centre of the fit intervals.

The predictions Eqn. 1 and Eqn. 7 for $r^{(1)}$ are indicated by the solid and the dashed line, respectively. Although the predictions for $r^{(1)}$ are obtained using different theoretical approaches they deviate from each other by only $\sim 3\%$ in contrary to the predictions for r. This strongly supports the presumption [14] that in order to investigate perturbative effects $r^{(1)}$ is much superior an observable than r. Within two standard deviations of the indicated statistical error both predictions agree with the measurement. Note also the uncertainty on the abscissa position of the measurement. The fact that the predictions for $r^{(1)}$ agree and the prediction Eqn. 1 which contains a non–perturbative contribution due to the constant of integration $N(L_0)$ gives a result for r consistent with the data while the purely perturbative calculations overestimate r clearly shows that non–perturbative effects strongly influence the multiplicity. This implies that measurements of the colour factor ratio C_A/C_F cannot be realised from measurements of the gluon to quark multiplicity ratio at only a single scale.

6 Conclusions

The charged hadronic multiplicity in symmetric three jet events has been investigated. It has been found in good agreement with a prediction based on the Dipole model Eqn. 3a and Eqn. 3b, with a preference for formulation 3a. A fit of these predictions to the data results in a measurement of the colour factor ratio:

$$\frac{C_A}{C_F} = 2.221 \pm 0.032(\mathrm{stat}) \pm 0.047(\mathrm{exp}) \pm 0.058(\mathrm{hadc}) \pm 0.075(\mathrm{theo}) \quad .$$

Alternatively, this prediction has been used to determine the multiplicity of a gg–system at various equivalent centre of mass energies out of the multiplicity of hadronic three jet events. The result has been found in good agreement with previous measurements of the gg–multiplicity. The about twice as fast increase of the hadron multiplicity in gg–events compared to $q\bar{q}$–events presents very direct evidence for the triple gluon vertex and the higher colour charge of the gluon.

The extracted N_{gg} has been used to calculate the ratio, r, of the multiplicity in a gg–system and a $q\bar{q}$–system as well as the ratio between the derivatives, $r^{(1)}$, of these multiplicities. A NLO and a 3NLO prediction have both been found to overestimate r while the Dipole calculation including a non–perturbative contribution agrees perfectly well with the data. The corresponding Dipole and 3NLO calculations of $r^{(1)}$ agree reasonable with each other as well as with the measurement. These findings imply that for measurements of the colour factor ratio C_A/C_F the slope ratio $r^{(1)}$ is an observable superior to the multiplicity ratio as was presumed in [14]. Furthermore measurements of the colour factor ratio C_A/C_F cannot be realised from measurements of the gluon to quark multiplicity ratio at only a single scale.

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