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MIXING OF PSEUDOSCALAR MESONS AND M1 RADIATIVE DECAYS

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A B S T R A C T

We discuss the mixing of pseudoscalar mesons, in particular SU(3) breaking effects, based on ideas abstracted from QCD. The admixtures of η and η' in η_c are calculated and utilized to estimate the M1 radiative decays $\psi \to \eta\gamma$, $\eta'\gamma$. A reasonable consistent picture emerges for these decays as well as others involving only the light quarks.

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1. - INTRODUCTION

Thus far the most successful description of the new particles and their spectroscopy is the one based on a new quark flavour with electric charge 2/3, conventionally described as charm in a broken SU(4) symmetry [1]. Much of the spectroscopy of the observed states and the transitions between them can be understood in relatively simple terms [2], but there are some puzzles. Most of these involve the pseudoscalar mesons and concern transitions within the "hidden charm" ($\bar{c}c$) group of states and also to the ordinary hadrons. The first puzzle concerns the width for the decay $\psi \to \gamma \eta_c$, where $\eta_c = X(2.82)$. The experimental upper limit is $\Gamma(\psi \to \gamma \eta_c) <$ < 3 keV [3], while theoretical expectations for this allowed M1 transition are of the order of 20-30 keV [2]. A second puzzle is that the radiative decays $\psi \to \gamma \eta$, $\psi \to \gamma \eta'$ $[\Gamma(\psi \to \gamma \eta) \simeq 76$ eV, $\Gamma(\psi \to \gamma \eta') \simeq 166$ eV$]$ seem stronger than naively expected. From the experimental result $\Gamma(\psi \to \rho^0 \pi^0) \simeq$ 260 eV and ρ dominance, one estimates $\Gamma(\psi \to \gamma \pi^0) \sim 1$ eV, for example. Besides the question of radiative transitions, there are puzzles of a purely hadronic character - the rather large splitting among the p states and apparently between singlet and triplet s states $(\psi - \eta_c, \psi' - \eta_c')$; the surprisingly large branching ratio (~4%) for $\psi' \to \psi \eta$ [4], a decay that is inhibited by p wave phase space and SU(3).

In this paper we show that a reasonably consistent picture of the pseudoscalar meson system and the various radiative decays mentioned above, except the decay $\psi \to \gamma X(2.82)$, can be developed within the framework of the quark-gluon theory of hadrons (QCD). We do not consider here the purely hadronic puzzles. First the mixing of the light neutral pseudoscalars is discussed via a quadratic mass matrix, with the mixing caused by $\bar{q}q$ annihilation into gluons. With the assumption of two-gluon dominance the renormalization group (asymptotic freedom) and factorization can be used to deduce the structure of the (broken) SU(4) mass matrix and determine the admixture of η and η' in η_c. The various radiative (M1) decays of the vector and pseudoscalar mesons are examined, using the language of a bound state model with flip of the quark spins. Where necessary, the Zweig rule violating admixtures in the vector mesons are deduced from arguments analogous to those for the pseudoscalars or from empirical evidence.

2. - η-η' MIXING

It is well known that the pseudoscalar mesons η and η' show a special mixing pattern, which distinguishes them from the vector and tensor mesons. While the latter segregate according to quark flavour, the neutral mass eigenstates being in a good approximation $(1/\sqrt{2})(\bar{u}u \pm \bar{d}d)$ and $\bar{s}s$, the pseudoscalar mesons show strong mixing between non-strange and strange flavours. Applying a quadratic SU(3) mass formula, one finds an octet-singlet mixing angle of $\theta = 11^{\circ}$ (essentially the same angle results if one calculates the decay rate for the decay $\eta \to \gamma\gamma$ within the PCAC framework and compares the result to the experimental value). It is amusing that an 11° octet-singlet mixing angle corresponds to the case where the probabilities to find a $\bar{s}s$ pair or a $\bar{u}u/\bar{d}d$ pair in the η, η' meson are approximately equal [5]. If we write

$$\eta = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)\cos\alpha - \bar{s}s\sin\alpha ,$$

$$\eta' = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)\sin\alpha + \bar{s}s\cos\alpha , \qquad (1)$$

then $\theta = 11^{\circ}$ corresponds to $\alpha = 43.7^{\circ}$. Exact equality $(\alpha = 45^{\circ})$ corresponds to $\theta = 9.74^{\circ}$, with the simple wave functions $\frac{1}{2}(\bar{u}u + \bar{d}d \mp \sqrt{2}\,\bar{s}s)$.

It has been emphasized by various authors [6)-9)] that the reason for the abnormal pseudoscalar mixing pattern may be found within QCD, where a new term corresponding to the annihilation of the $\bar{q}q$ system into gluons has to be added to the mass matrix of isoscalar mesons. The annihilation term must be small for vector and tensor mesons but relatively big for the pseudoscalars. (In lowest order perturbation theory the annihilation term for the pseudoscalars consists of a two-gluon term ; for the vector mesons it consists of a three-gluon term.)

The mass (squared) matrix for the pseudoscalar mesons can be written as follows, including the annihilation term, denoted by λ :

$$M^2_{\bar{q}q} = \begin{pmatrix} M_u^2 + \lambda_{uu} & \lambda_{ud} & \lambda_{us} & \lambda_{uc} \\ \lambda_{ud} & M_d^2 + \lambda_{dd} & \lambda_{ds} & \lambda_{dc} \\ \lambda_{us} & \lambda_{ds} & M_s^2 + \lambda_{ss} & \lambda_{sc} \\ \lambda_{uc} & \lambda_{dc} & \lambda_{sc} & M_c^2 + \lambda_{cc} \end{pmatrix} \qquad (2)$$

Here $\lambda_{q_1q_2}$ denotes the contribution of the annihilation term to the matrix elements describing the transitions $\bar{q}_1q_1 \rightleftarrows \bar{q}_2q_2$. Isospin symmetry gives $\lambda_{uu} = \lambda_{dd} = \lambda_{ud}$. (Note : $M_u^2 = M_d^2 = m_\pi^2$, $M_s^2 = 2m_K^2 - m_\pi^2$, $M_c^2 = M_{\eta_c}^2$, we use $M_{\eta_c} \simeq 3$ GeV.) Since $\lambda/M_c^2 \ll 1$, we can neglect the charm contributions in a good approximation, in which case and after using isospin symmetry, the matrix (2) reduces to

$$M^2_{\bar{q}q} = \begin{pmatrix} M_u^2 + 2\lambda_{uu} & \sqrt{2}\,\lambda_{us} \\ \sqrt{2}\,\lambda_{us} & M_s^2 + \lambda_{ss} \end{pmatrix}. \qquad (3)$$

[isospin symmetry requires one of the eigenvectors of (2) to be $\pi_o = (1/\sqrt{2})(\bar{u}u - \bar{d}d)$].

It was shown in Ref. 6) that an SU(3) symmetric annihilation term ($\lambda_{uu} = \lambda_{us} = \lambda_{ss}$) describes the $\eta - \eta'$ mixing pattern qualitatively correctly, but fails quantitatively. In fact, the observed masses and mixing angles can only be reproduced if one allows a relatively large SU(3) breaking in the annihilation terms. As is familiar from degenerate state perturbation theory, a mixing angle of $\alpha = 45^\circ$ occurs when the diagonal elements in Eq. (3) are exactly equal. With this constraint and the experimental masses m_π, m_K, m_η, $m_{\eta'}$ the annihilation parameters in Eq. (3) are found to be (in GeV^2) : $\lambda_{uu} = 0.29(5)$, $\lambda_{us} = 0.21(9)$, $\lambda_{ss} = 0.13(6)$. Actually, in what follows we make use of the annihilation elements that give a mixing angle in Eq. (1) corresponding exactly to $\Theta = 11^\circ$. The diagonal elements in Eq. (3) are still almost degenerate, differing by 5% ; the λ values are

$$\lambda_{uu} = 0.30(2), \quad \lambda_{us} = 0.21(8), \quad \lambda_{ss} = 0.12(3)$$
$$\lambda_{us} / \lambda_{ss} = 1.77.$$

There is substantial SU(3) symmetry breaking evident in the annihilation terms. It is interesting to note that this pattern of SU(3) breaking is the one expected in QCD. In lowest order perturbation theory the annihilation terms $\lambda_{qq'}$ are proportional to $\varkappa(M_q^2)\,\varkappa(M_{q'}^2)$ where $\varkappa = g^2/4\pi$ [$g(M_q^2)$: quark-gluon coupling constant, renormalized at M_q^2]. It is a universal feature of asymptotically free gauge theories [10)] such as QCD that $\varkappa(\mu^2)$ decreases with increasing renormalization mass μ, presumably rather rapidly at first, and then more gradually in the weak coupling region, following the renormalization group equation

$$\kappa(\mu^2) = \frac{\kappa(\mu_0^2)}{1 + \frac{25}{12\pi}\,\kappa(\mu_0^2)\,\ln\left(\frac{\mu^2}{\mu_0^2}\right)} . \tag{4}$$

The behaviour of $\kappa(\mu^2)$ with μ^2, together with the assumed quark masses, implies a definite pattern of symmetry breaking for the λ_{ij}, namely $\lambda_{uu} > \lambda_{us} > \lambda_{ss}$ as found empirically *). Furthermore, one expects the elements of (λ_{ij}) to satisfy the factorization relation $\lambda_{ii}\lambda_{jj} = \lambda_{ij}^2$. We find $\sqrt{\lambda_{uu}\lambda_{ss}} = 0.19(3)$, in quite good agreement with $\lambda_{us} = 0.21(8)$.

To discuss the properties of the $\bar{c}c$ pseudoscalar η_c it is entirely adequate to use lowest order perturbation theory on Eq. (2). It is necessary, of course, to know the values of λ_{uc}, λ_{sc} and λ_{cc}. We use Eq. (4) to scale the λ values, assuming two-gluon exchange (note that these are consistent hypotheses). The charmonium value, $\kappa(M_c^2) = 0.19$ [2], gives $\kappa(M_s^2)/\kappa(M_c^2) = 1.60$. Thus we obtain $\lambda_{sc} = \lambda_{ss}\cdot\kappa(M_s^2)/\kappa(M_c^2) = 0.077$ and $\lambda_{cc} = \lambda_{ss}\cdot[\kappa(M_s^2)/\kappa(M_c^2)]^2 = 0.048$. The mass value for the η_c is negligibly different from M_c in Eq. (2) ; the quark content of the η_c is given in lowest order by

$$\eta_c = \bar{c}c + (\lambda_{uc}\,\bar{u}u + \lambda_{dc}\,\bar{d}d + \lambda_{sc}\,\bar{s}s)/M^2$$

where $M^2 = M_c^2 - \frac{1}{2}(m_\eta^2 + m_{\eta'}^2)$. This can be written variously as

$$\begin{aligned} \eta_c &= \bar{c}c + [1.77(\bar{u}u + \bar{d}d) + \bar{s}s]\cdot\lambda_{sc}/M^2 \\ &= \bar{c}c + \varepsilon\cdot\eta + \varepsilon'\cdot\eta' , \end{aligned} \tag{5}$$

where η and η' are given by Eq. (1) with $\alpha = 43.7^\circ$ ($\theta = 11^\circ$) and $\varepsilon = 1.0(2).10^{-2}$, $\varepsilon' = 2.2(4).10^{-2}$.

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*) We do not attempt to use Eq. (4) to give quantitative expectations of the magnitude of the symmetry breaking in the η-η' sector where $M^2 \lesssim 1\ \mathrm{GeV}^2$. The weak coupling regime surely does not extend to such low masses. We do use Eq. (4), with some trepidation, for the extrapolation $M_s^2 \to M_c^2$.

Note the substantial breaking of SU(3) : η_c tends to mix much more with non-strange than strange $\bar{q}q$ pairs. Qualitatively the same behaviour is obtained within a similar approach to the vector mesons *).

Harari [11] has suggested that the η and η' mesons have sizeable $\bar{c}c$ contributions in their wave functions. His values of ϵ and ϵ' (0.1, 0.3, respectively) seem unreasonably large. In particular, with such large values of ϵ and ϵ' it is difficult to understand the magnitudes of the decays $\psi \to \eta\gamma$, $\psi \to \eta'\gamma$, as they are presently known (see below).

3. - $V \to P\gamma$, $P \to V\gamma$ TRANSITIONS AMONG ORDINARY HADRONS

Before considering the $V \to P\gamma$ transitions involving $\bar{c}c$ states, we examine the magnetic dipole transitions among the ordinary vector and pseudoscalar mesons. We use the language of the naive non-relativistic quark model in which the transition probability is $\Gamma(V \to P\gamma) = (4\alpha/3)k^3(e_q/m_q)^2\Omega^2$ and $\Gamma(P \to V\gamma)$ is three times larger. Here e_q/m_q is the transition magnetic moment, k is the photon energy, and Ω is an overlap integral. A comparison with experiment is shown in the Table. The transition moments are calculated with the effective masses $m_u = 0.34$ GeV, $m_s = 0.46$ GeV, values that give the experimental static magnetic moments of the baryons. For $\psi \to \eta\gamma$, the 11° mixing angle is assumed for the η wave function. The overlap integral is reasonable in magnitude, except perhaps for $\rho \to \pi\gamma$. The comparison can

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*) We cannot apply the same method (based on the mass matrix) to vector mesons since the relevant mass difference $m_\omega - m_\rho$ is not known precisely enough. However, we can calculate the relative amounts of mixing in terms of $\varkappa$. One finds $\psi = \bar{c}c + \epsilon[\alpha(\bar{u}u+\bar{d}d) + \bar{s}s]$ where ϵ_V is a small number and $\alpha = [\varkappa(m_\omega^2)/\varkappa(m_\phi^2)]^{\frac{3}{2}} > 1$. SU(3) symmetry would imply $\alpha = 1$. For $\varkappa(m_\phi^2) = 0.6$ we find from the (in this regime unreliable) Eq. (4) that $\alpha = 1.5$. In particular one obtains, taking into account phase space effects $\Gamma(\psi \to K^*_+ K_-)/\Gamma(\psi \to \rho_+ \pi_-) = 0.85(1/4 + 1/2\alpha + 1/4\alpha^2)$. Experimentally the ratio is 0.4 ± 0.1, which gives $\alpha = 2.5^{+2.5}_{-0.6}$, slightly larger than our perturbative estimation above.

be taken as satisfactory *).

The last entry in the Table, $\eta' \to \rho\gamma$, is a special case because only an upper limit (1 MeV) is known for the η' total width. With the reasonable value, $\Omega = 0.84$ (taken from $\varphi \to \eta\gamma$), we expect $\Gamma(\eta' \to \rho\gamma) \simeq 100$ keV. The branching ratio of 30% for this decay then implies $\Gamma_t(\eta') \simeq 330$ keV, comfortably within the present upper limit. A similar value for the total width emerges from application of the PCAC formula that works well for $\pi^0 \to \gamma\gamma$ and $\eta \to \gamma\gamma$ to the transition $\eta' \to \gamma\gamma$. The rate is given by

$$\frac{\Gamma(\eta' \to \gamma\gamma)}{\Gamma(\pi_0 \to \gamma\gamma)} = 18 \left(\frac{m_\eta}{m_{\pi_0}}\right)^3 \left(\frac{F_\pi}{F_{\eta'}}\right)^2 \left[-\sin 11^\circ \cdot \frac{e_u^2+e_d^2-2e_s^2}{\sqrt{6}} + \cos 11^\circ \cdot \frac{e_u^2+e_d^2+e_s^2}{\sqrt{3}}\right]^2$$

where $F_{\eta'}$ is the η' decay constant. Setting $F_{\eta'} = F_\pi$, one obtains $\Gamma(\eta' \to \gamma\gamma)/\Gamma(\pi_0 \to \gamma\gamma) = 2.23(m_{\eta'}/m_{\pi_0})^3$ and $\Gamma(\eta' \to \gamma\gamma) \simeq 6.3$ keV **). With a branching ratio of $2.0 \pm 0.3\%$, this implies $\Gamma_t(\eta') \simeq 315$ keV just as before. We conclude that the quark model estimate for $\eta' \to \rho\gamma$ and the PCAC prediction for $\eta' \to \gamma\gamma$ are consistent and plausible ***) and that $\Gamma_t(\eta') \simeq 0.3$ MeV.

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*) Numerous discussions have been made of these radiative transitions with vector meson dominance models [12)-14)] usually with attention to detailed agreements and disagreements. For our purpose the fine points are not essential ; rough agreement is sufficient. For the decay $V \to P\gamma$, the connection between the VMD and the bound state approach is

$$\sum_j f_j^{-1} g(V V_j P) \leftrightarrow \sqrt{2}\,(e_q/m_q)\,\Omega\,\sqrt{1+(m_P/m_V)^2}\,.$$

The empirical dependence of the $V \rightleftarrows \gamma$ coupling constant e/f on the mass as m_V^{-1} is understood in the bound state picture in terms of the dependence of the transitional magnetic moment on quark mass as m_q^{-1}. The explicit quark charges convey the SU(3) character of the operative part of the electromagnetic current.

**) The corresponding calculation for $\eta \to \gamma\gamma$ gives $\Gamma(\eta \to \gamma\gamma) \simeq 410$ eV, compared with the observed 323 ± 50 eV.

***) As emphasized in Ref. 15), a colour octet contribution to the electromagnetic current would influence the $\eta' \to \gamma\gamma$ width. Our results are in good agreement with the conventional colour singlet picture for the electric charges [see also Ref. 15)].

4. - $\psi \to \eta\gamma$, $\eta'\gamma$

These decays can either proceed via the mixing of ψ with $\bar{q}q$ vector states $(q:u,d,s)$, which then decay radiatively, or via photon emission and subsequent mixing of η_c with η and η'. According to the mixing scheme discussed above, the second mechanism will by far dominate *). Within our approach the decay widths can be calculated using the formula

$$\Gamma(\psi \to \eta\gamma) = \varepsilon^2 \cdot \frac{4\alpha}{3m_c^2} \cdot (2/3)^2 \cdot k^3 \Omega^2 ,$$

and an analogous formula for the η' case. Assuming the same overlap integrals, the ratio of the widths is given by

$$\Gamma(\psi \to \eta'\gamma) / \Gamma(\psi \to \eta\gamma) = (k_{\eta'}/k_\eta)^3 \cdot (\varepsilon'/\varepsilon)^2 .$$

With the numbers of Eq. (5) we obtain $\Gamma(\psi \to \eta'\gamma)/\Gamma(\psi \to \eta\gamma) \simeq 3.9$, in satisfactory agreement with the measured value of 2.2 ± 1.0 18) considering the experimental uncertainties. Note that a SU(3) singlet admixture of light quarks in the η_c gives ~22 for this ratio (for $\theta = 11^\circ$) **).

For the absolute widths we use $m_c = 1650$ MeV and find $\Gamma(\psi \to \eta\gamma) \simeq 5400(\varepsilon\Omega)^2$ keV $= 560\ \Omega^2$ eV. In contrast to the transitions of the Table, where the maximum photon energy is 380 MeV, here the energies are 1.5 and 1.4 GeV. The overlap integral is therefore expected to be considerably smaller than unity. Simple calculations based on harmonic oscillator

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*) Since the transitional magnetic moment is proportional to e_q/m_q, it might be argued, as in Ref. 11), that the M1 amplitude involving c quarks is small compared with one involving light quarks. We find that the quark charges and masses are such that, for these transitions, the first mechanism is comparable to the second only if the ψ-ω or ψ-φ mixing is equal to the η_c-η mixing. Since $|\varepsilon_{\psi\omega}/\varepsilon| \lesssim 10^{-1}$ (see Section 5), the second mechanism dominates. See also Ref. 16) (quark model approach) and Ref. 17) (equivalent VMD discussion).

**) See, for example, Ref. 17). These authors observe the sensitivity of this ratio to SU(3) breaking. In our approach, the amplification caused by SU(3) breaking and the QCD renormalization group arguments is such that the experimental ratio is reproduced with $\theta \simeq 13^\circ$.

wave functions (whose scale parameters are adjusted to give the $|\psi(0)|^2$ values inferred from $V \to e^+e^-$) suggest $\Omega^2 \simeq 0.1$ for a $\bar{c}c \to \bar{q}q$ transition with $k = 1.5$ GeV. This implies $\Gamma(\psi \to \eta\gamma) \simeq 60$ eV. The observed partial width is 76 ± 29 eV. In view of the uncertainties in the theoretical values of ϵ and Ω, this excellent agreement may be viewed as somewhat fortuitous, but satisfying nonetheless. For the $\psi \to \eta'\gamma$, the values are 220 eV calculated versus 166 ± 50 eV measured.

5. - $\phi \to \pi^0\gamma$, $\psi \to \pi^0\gamma$

The decays $\phi \to \pi^0\gamma$ and $\psi \to \pi^0\gamma$ can only proceed in our picture via the mixing of the vector states, a mixing that is much smaller than for the pseudoscalars. Thus one expects the decay $\psi \to \pi^0\gamma$ to be much weaker than $\psi \to \eta\gamma$. Recently, the $\psi \to \pi^0\gamma$ decay mode has been observed with a partial width of the order of several electron volts [19], as compared with 76 eV for $\psi \to \eta\gamma$. A value of ~1 eV is expected from VMD applied to the observed hadronic rate of $\psi \to \pi^0\rho^0$, as has been mentioned in the Introduction. Here we see whether the mixing is plausible from our point of view. For $\phi \to \pi^0\gamma$ the mixing parameter $\epsilon_{\phi\omega}$, defined by $\phi = \bar{s}s + \epsilon_{\phi\omega}(\bar{u}u + \bar{d}d)/\sqrt{2}$, is determined from the ratio $\Gamma(\phi \to \pi^0\gamma_1)/\Gamma(\omega \to \pi^0\gamma_2).(k_2/k_1)^3 = \epsilon^2_{\phi\omega}$, assuming equal overlap integrals. Experimentally, $|\epsilon_{\phi\omega}| = (5.4 \pm 0.9).10^{-2}$ (mixing angle $\simeq 3^0$). If the $\psi \to \pi^0\gamma$ width is χ eV, a corresponding relation gives $|\epsilon_{\psi\omega}\Omega_{\psi\pi^0}| \simeq 1.3\sqrt{\chi}\,10^{-4}$ (we take $\Omega_{\omega\pi^0} \simeq 1$). If $\Omega_{\psi\pi^0} \simeq \Omega_{\psi\eta} \simeq 0.3$, we conclude that $|\epsilon_{\psi\omega}| \simeq 4\sqrt{\chi} \times 10^{-4}$ and $\epsilon_{\phi\omega}/\epsilon_{\psi\omega} \simeq 130/\sqrt{\chi}$. In the perturbative regime of the gluon exchange as the mechanism for mixing, we expect

$$\epsilon_{\phi\omega}/\epsilon_{\psi\omega} \simeq \left[\kappa(m_\phi^2)/\kappa(m_\psi^2)\right]^{3/2}(m_\psi^2 - m_\omega^2)/(m_\phi^2 - m_\omega^2).$$

With $\kappa(m_\psi^2) = 0.2$ in Eq. (4) to determine the ratio of the coupling constants one finds $\epsilon_{\phi\omega}/\epsilon_{\psi\omega} \simeq 35$; with the empirical value $\kappa(m_\phi^2) \simeq 0.5$ one obtains ~80. The order of magnitude of the ratio is in agreement with "experiment" ; more than this is not claimed or expected.

6. - CONCLUSIONS

In this paper we found that a globally consistent picture for the mixing of pseudoscalar mesons and the radiative decays of the vector and pseudoscalar mesons including $\bar{c}c$ states can be obtained within the QCD approach. In particular the decays $\psi \to \eta\gamma$, $\eta'\gamma$ are reasonably well understood. No understanding, however, for the slowness of the decay $\psi \to \eta_c\gamma$ ($\eta_c = X(2.82)$) is achieved. The fact that all radiative decays involving the light pseudoscalars η and η' are well understood deepens the puzzle connected with the $X(2.82)$ state. As mentioned in the Introduction, the allowed M1 transition is expected to have a partial width of 30 keV. With an overlap integral of $\Omega \simeq 0.8$ this is reduced to ~19 keV, still almost an order of magnitude larger than the experimental upper limit. It seems very implausible, even with the large singlet-triplet mass splitting between the ψ and the η_c, that the wave functions are so distorted as to give $\Omega^2 \lesssim 0.1$. We reach the conclusion that either the $X(2.82)$ is <u>not</u> the η_c meson, or the naive approach to $\bar{c}c$ states based on a simple non-relativistic dynamics is misleading.

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Decay process	Γ_{exp} (keV)	e_q/m_q (GeV^{-1})	Γ_{calc} for $\Omega = 1$ (keV)	Ω
$\omega \to \pi^o \gamma$	880 ± 50	1.47	1155	0.87
$\rho \to \pi \gamma$	36 ± 10	0.49	123	0.54
$\varphi \to \eta \gamma$	82 ± 15	0.50	115	0.84
$K^{o*} \to K^o \gamma$	75 ± 35	-0.85	208	0.60
$K^{+*} \to K^+ \gamma$	< 80	0.62	110	< 0.85
$\eta' \to \rho^o \gamma$	< 300	1.02	141	(< 1.5)

Table : Radiative decays, $V \to P\gamma$ and $P \to V\gamma$.

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