HIGH ENERGY BEHAVIOUR IN A NON-ABELIAN GAUGE FIELD THEORY

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A B S T R A C T

We investigate the high energy behaviour of a massive non-Abelian gauge theory by deriving a Reggeon field theory. Analysing the leading $\ln s$ coefficient in arbitrary order perturbation we find that due to extensive cancellations between different Feynman diagrams the transverse momentum is always strongly cut off. This leads to the existence of moving Regge poles as leading singularities in all quantum number configurations. We give a general description of the leading $\ln s$ piece in arbitrary order perturbation theory and compute, in lowest order, Reggeon trajectory function and triple Reggeon vertices. We find that the Reggeon belonging to the highest multiplet has a negative mass (intercept above one), and we discuss the possible implication of this result.

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Non-Abelian gauge field theories (NAGFT) have become a promising candidate for real physics, not only weak and electromagnetic but also strong interactions. Much interest has been given to asymptotic freedom and the possibility of quark confinement in these theories, but so far rather little is known about high energy scattering in the Regge limit. The property of asymptotic freedom makes it very attractive to use renormalization group arguments, but all attempts to overcome the difficulties connected with exceptional momenta in the Regge limit have failed. On the other hand, the appropriate language to describe this kinematic region seems to be the j plane formalism. Gribov's Reggeon calculus, together with the more recent development of Reggeon field theory (RFT) provides us with a powerful tool to study the rather complex j plane structure. It is therefore suggestive to describe the high energy behaviour of a given relativistic quantum field theory (QFT) by means of a RFT which has to be derived from the underlying theory.

It is important to explain why such a program has not been carried out in the case of φ^3 theory or QED. There are two different reasons. In order to derive the RFT from the underlying QFT, one calculates Reggeon trajectory functions, which give Reggeon mass and slope parameter for the RFT, and Reggeon interaction terms. In the absence of more powerful techniques, these parameters are given in a power series expansion in the underlying coupling constant, and only the lowest non-trivial order can be calculated (weak coupling limit). It is a particular feature of φ^3 theory, being a scalar QFT, that in the weak coupling limit the high energy behaviour is trivial. This is why the program of deriving a RFT from φ^3 theory has either to go beyond weak coupling limit calculations or will be useless. In the case of QED, the leading j plane singularities are, at least in the weak coupling limit, fixed cuts and not moving Regge poles. This is closely related to the fact that the photon in QED does not Reggeize, and the Gribov-Pomeranchuk singularities are not shielded by Regge cut singularities. It is not known whether - and if so, how - Reggeon calculus and RFT can be used for such fixed cut singularities.

In this letter we wish to report an analysis of the high energy behaviour in a massive NAGFT model. None of the reasons which in φ^3 theory and QED made the idea of deriving a RFT rather unattractive applies to NAGFT, and we have computed Reggeon trajectories and triple Reggeon interaction terms. Our starting point is the analysis of high energy

behaviour in perturbation theory. Several groups [1)-4)] have calculated the leading $\ell n\ s$ pieces up to sixth order perturbation theory [one paper in Ref. 4) goes up to 8th order], and generalizing from these calculations, attempts have been made [5),6)] to extend the analysis up to infinite order, but only within a particular subset of Feynman diagrams. The important result of these studies is that the vector particle Reggeizes. The situation of the vacuum channel (the Pomeron), however, remains unclear. In all these papers the NAGFT model had $SU(2)$ gauge symmetry, and the gauge mesons were made massive via the Higgs mechanism. In the unitary gauge, such a theory des-cribes the interaction of three massive vector particles, forming an iso-triplet $(I = 1)$, and one massive isosinglet, the Higgs scalar. In order to make contact with these calculations, we choose the same model, but we shall restrict ourselves to the meson sector and disregard fermions. As to the method of calculating leading $\ell n\ s$ pieces in a given order of pertur-bation theory, we adopt Lipatov's [2)] idea of using dispersion relations, but have to generalize it in a non-trivial way. The other authors have used momentum space techniques and computed the leading pieces of individual Feynman diagrams. However, because of extensive cancellations between se-veral diagrams of the same order perturbation theory, only the sum of quite a large number of these leading pieces yields a meaningful result free of ultraviolet divergences. In higher order perturbation theory, these can-cellations become immense, and that is why we prefer Lipatov's method.

This method can briefly be described as follows. In the first step one calculates the tree approximation for $T_{2 \to 2}$ (in the Regge limit), $T_{2 \to 3}$ (in the double Regge limit), $T_{2 \to 4}$.... etc. This has to be done by inspecting all Feynman diagrams which contribute to that order, and one explicitly sees the cancellations that we have mentioned. But the results are sufficiently simple. Loop corrections to these amplitudes are calcu-lated via dispersion relations and unitary equations. For $T_{2 \to 2}$ the dispersion relation reads :

$$T_{2 \to 2}(s,t) = c_0 + c_1 s + \frac{s^2}{\pi^2} \int_{s}^{\infty} ds' \frac{disc\ T_{2 \to 2}(s',t)}{s'^2\,(s - s')} \qquad (1)$$

For $disc\ T_{2 \to 2}\ (s,t)$ one uses :

$$(2)$$

The treatment of inelastic amplitudes is illustrated by the simplest case $T_{2 \to 3}$. In the double Regge limit, this amplitude splits into two pieces :

$$T_{2 \to 3}(s, s_{ab}, s_{bc}) \to T_{ab}(s, s_{ab}) + T_{bc}(s, s_{bc}) \tag{3}$$

The first part has simultaneous energy discontinuities in s and s_{ab}, the second one in s and s_{bc}, as it is required by the Steinmann relations. For large energies $(s, s_{ab}, s_{bc} \to \infty, s_{ab}/s, s_{bc}/s \to 0)$ each of the two terms in (3) satisfies a double dispersion relation, e.g.,

$$T_{bc}(s, s_{bc}) = a_0 + \frac{s_{bc}}{\pi} \int_{\bar{s}_{bc}}^{\infty} ds_{bc}' \frac{disc_{s_{bc}} T_{bc}(s_0, s_{bc}')}{s_{bc}'(s_{bc}' - s_{bc})}$$

$$+ a_1 s + \frac{s \, s_{bc}}{\pi} \int_{\bar{s}_{bc}}^{\infty} ds_{bc}' \frac{disc_{s_{bc}} \frac{\partial T_{bc}(s_0, s_{bc}')}{\partial s}}{s_{bc}'(s_{bc}' - s_{bc})} \tag{4}$$

$$+ \frac{s^2}{\pi} \int_{\bar{s}}^{\infty} ds' \frac{disc_s T_{bc}(s', s_{bc0})}{s'^2(s' - s)} + \frac{s^2 s_{bc}}{\pi^2} \int_{\bar{s}}^{\infty} ds' \int_{\bar{s}_{bc}}^{\infty} ds_{bc}' \frac{disc_s \, disc_{s_{bc}} T_{bc}(s', s_{bc}')}{s'^2(s' - s) s_{bc}'(s_{bc}' - s_{bc})}$$

For the energy discontinuities which appear in (4) we use the unitarity equations

$$\tag{5}$$

$$\tag{6}$$

The existence of the decomposition (3) and the double dispersion relation (4) (both in the double Regge limit) can be justified on rather general grounds [7]. Using these dispersion relations (1) and (4), one now computes seccessively higher and higher order perturbation theory. For example, $T_{2 \to 2}$ in fourth order is obtained by using (1) and employing $T_{2 \to 2}$ in second order (which is the tree approximation) for the right-hand side of (2). For $T_{2 \to 2}$ in sixth order, the three-particle intermediate state in (2) is needed, but $T_{2 \to 3}$ in third order (which is the tree approximation)

is known already. In the next step, we need $T_{2 \to 3}$ in fifth order. This will be calculated by using (3), (4) and (5), (6) for s_{ab} and s_{bc}. Continuing this procedure, one can go to higher and higher order, and at each step one makes use of results that have been obtained in one of the previous steps.

Besides the existence of dispersion relations there is another assumption which goes into these calculations. The striking consequence of this method is that Faddeev-Popov ghosts never appear. This is because we work in the unitary gauge (R_ξ gauge with $\xi \to 0$). In general, one is not allowed to take the limit $\xi \to 0$ before doing Feynman loop integrations. The infinite momentum technique calculations of Refs. 3), 4) have shown that, as far as the leading $\ln s$ pieces of Feynman diagrams are concerned, ghosts are not needed and, hence, one may take the limit $\xi \to 0$ before doing the integrals. This is plausible because the exchange of ghosts being spin $\frac{1}{2}$ objects is down by a power of s compared to the exchange of vector particles. But it is nevertheless an assumption that we make about the allowance of interchanging the order of taking $\xi \to 0$ and performing the loop integrations in higher order perturbation theory.

We then have carried out several steps in the successive calculation of $T_{2 \to 2}$, $T_{2 \to 3}$ in the order g^2, g^3,... Some of the low order results are already contained in Ref. 2), and they all agree with the results obtained by other methods in Refs. 3) and 4). We went beyond these existing calculations, but we rather soon found a general structure of the leading $\ln s$ pieces, which will persist in all orders of perturbation theory. The first important observation is that, as a consequence of the extensive cancellation, transverse momentum is always strongly cut off. This, together with the Reggeization of the elementary vector particle, implies the existence of moving Regge poles (as opposed to fixed cut singularities) in all quantum number configurations. For the channel with the quantum number of the vector particle ($I = 1$ in our case), this moving pole is, of course, the Reggeizing vector particle. In the other two channels ($I = 0$, $I = 2$) the moving Regge poles belong to bound states of two vector particles. The second observation concerns the t dependence of the leading $\ln s$ coefficient. In any order perturbation theory, the leading term for $T_{2 \to 2}$ is of the form

$$s \, (\ln s)^p \cdot \mathcal{F}(t) \cdot g^n \tag{7}$$

where the power p and the function F(t) depend on the order of perturbation theory and the set of Feynman diagrams which is considered. F(t) is always given as an integral over two-dimensional transverse momentum and can be constructed out of the pieces shown in Fig. 1. The horizontal lines denote the scattering particles, and vertical lines stand for propagator $[K_\perp^2 + M^2]^{-1}$ depending on the two-dimensional transverse momentum $\vec{k}_\perp$. There are vertices with arbitrarily many transverse momentum lines coupling to the external particles (upper and lower part of Fig. 1a), and vertices with n transverse lines going into m transverse lines (n,m arbitrary) (shown in the central part of Fig. 1a). Taking any of these vertices and connecting them via transverse momentum lines gives the most general form of F(t) in (7). Some examples are shown in Fig. 1b-d. It is important to note that these transverse momentum diagrams do not directly come out as the high energy behaviour of a few Feynman diagrams. For example, the bubble string in Fig. 1b is not directly the high energy behaviour of ladder Feynman diagrams : they alone are highly ultra-violet divergent, and one has to include many more diagrams before one obtains a finite result. However, once the cancellation of divergent parts has taken place, one is left with simple structures like Fig. 1b-d.

The form (7) together with the structure of F(t) is highly reminiscent of a super-renormalizable theory such as φ^3 theory. This similarity can be used to find the explicit form for the vertices of Fig. 1a, and, for a given structure F(t), to determine the power p of $\ell n\ s$. We do not want here to present these rules in all detail, but only mention the general idea. For a given structure of F(t), one essentially looks for φ^3 Feynman diagrams which in the high energy limit lead to such a coefficient function F(t), and makes certain charges at the vertices of these Feynman graphs. One then calculates their high energy behaviour, which in φ^3 theory is not too difficult, and in the final results changes the power of s to s^1. For details we refer to Ref. 8).

Once we have found a method how to compute leading $\ell n\ s$ pieces in any order perturbation theory, we are faced with the problem of how to sum them up. We suggest to use RFT as the most suitable way of doing this summation. Reggeons are defined as the sum of bubble strings (Fig. 2a), and the higher order pieces of Fig. 1a are used to build Reggeon interaction terms (Fig. 2b). Allowing for Reggeon interactions of arbitrarily high order (Fig. 2c), we thus translate the problem of summing all structures in

Fig. 1b into the task of solving a RFT with infinitely many couplings. Reggeon propagators and Reggeon couplings have to be computed from inspecting the appropriate parts of $F(t)$. We wish to point out the difference between this way of summing leading ℓn s pieces and others that have been used so far. In Refs. 1)-4), the authors sum over all contributions for a given order perturbation theory. Except for the t channel with the quantum number of the vector particle, where the contributions sum up to make the vector particle Reggeize, no simple result emerges, and nothing can be learned about the vacuum channel (the Pomeron). In Ref. 5), an integral equation is used to determine the j plane singularities in both the $I = 1$ and $I = 0$ t channels. Such integral equations can be understood as taking the sum of a particular subset of diagrams like Fig. 1b. Apart from the question why one may sort out a particular subset and neglect others, we find that the vertices that go into the equation for the $I = 0$ channel do not agree with our results. For the $I = 1$ channel we find a large number of cancellations among the diagrams which represent the integral equation, and this explains why the solution to this integral equation is just the sum over diagrams of Fig. 1b.

Since we consider the formulation and use of RFT as the most convenient way to perform the summation of all leading ℓn s pieces, we have calculated Reggeon trajectories and triple-Reggeon vertices. For the three t channel configurations $I = 0, 1, 2$ we find the trajectories :

$$I = 0 : \quad \alpha_0(t) = 1 + \left(2t - \tfrac{5}{2}M^2\right)\beta(t) \, , \quad \text{signature} \ \tau_0 = + \tag{8}$$

$$I = 1 : \quad \alpha_1(t) = 1 + (t - M^2)\beta(t) \, , \quad \text{signature} \ \tau_1 = - \tag{9}$$

$$I = 2 : \quad \alpha_2(t) = 1 - (t - 2M^2)\beta(t) \, , \quad \text{signature} \ \tau_2 = + \tag{10}$$

$$\beta(t) = \frac{g^2}{(2\pi)^3} \int d^2k \, \frac{1}{k^2 + M^2} \cdot \frac{1}{(q-k)^2 + M^2} \, , \quad t = -\vec{q}^{\,2} \tag{11}$$

(M is the mass of the vector particle and g the coupling in the NAGFT). Our RFT thus contains three multiplets of Reggeon fields : a singlet φ for $I = 0$, a triplet φ_i with $I = 1$, and a quintet φ_{ij} for $I = 2$

(φ_{ij} is a symmetric and traceless tensor and has five independent components). Reggeon masses and slopes are given as :

$$\Delta_I = 1 - \alpha_I(0) \tag{12}$$

$$\alpha_I' = \frac{\partial \alpha_I(t)}{\partial t}\bigg|_{t=0} \tag{13}$$

From (8)-(10) we obtain for the Reggeon masses and slopes

$$\Delta_0 = \frac{5}{2} \cdot \frac{g^2}{8\pi^2} \quad , \qquad \alpha_0' = \frac{19}{12} \cdot \frac{g^2}{8\pi^2 M^2} \tag{14}$$

$$\Delta_1 = \frac{g^2}{8\pi^2} \quad , \qquad \alpha_1' = \frac{5}{6} \cdot \frac{g^2}{8\pi^2 M^2} \tag{15}$$

$$\Delta_2 = -2\frac{g^2}{8\pi^2} \quad , \qquad \alpha_2' = -\frac{2}{3}\frac{g^2}{8\pi^2 M^2} \tag{16}$$

With these parameters we have the following RFT :

$$\mathcal{L}(\varphi, \varphi_i, \varphi_{ij}) = \mathcal{L}_{kin} + \mathcal{L}_{int} \tag{17}$$

$$\begin{aligned}
\mathcal{L}_{kin} = &\frac{i}{2}\varphi^+ \overleftrightarrow{\partial_t}\varphi - \alpha_0' \nabla\varphi^+ \cdot \nabla\varphi \\
&+ \frac{i}{2}\varphi_i^+ \overleftrightarrow{\partial_t}\varphi_i - \alpha_1' \nabla\varphi_i^+ \cdot \nabla\varphi_i \\
&+ \frac{i}{2}\varphi_{ij}^+ \overleftrightarrow{\partial_t}\varphi_{ij} - \alpha_2' \nabla\varphi_{ij}^+ \cdot \nabla\varphi_{ij}
\end{aligned} \tag{18}$$

$$\mathcal{L}_{int} = - \Delta_o \, \varphi \varphi^+ - \Delta_1 \, \varphi_i \, \varphi_i^+ - \Delta_2 \, \varphi_{ij} \, \varphi_{ij}^+$$

$$+ i \, \frac{\lambda_{oo}^o}{2!} \, (\varphi^+ \varphi \varphi + h.c.) \; + \; i \, \frac{\lambda_{22}^o}{2!} \, (\varphi^+ \varphi_{ij} \, \varphi_{ij} + h.c.)$$

$$+ i \, \frac{\lambda_{22}^2}{2!} \, (\varphi_{ij}^+ \, P_2(ij|k\ell,mn) \varphi_{k\ell} \, \varphi_{mn} + h.c.)$$

$$+ i \lambda_{02}^2 \, (\varphi_{ij}^+ \, \varphi_{ij} \, \varphi + h.c.) \; + \; i \lambda_{01}' \, (\varphi_i^+ \varphi_i \, \varphi + h.c.)$$

$$+ i \lambda_{21}' \, (\varphi_i^+ \varphi_j \, \varphi_{ij} + h.c.) \; + \; \frac{\lambda_{11}^o}{2!} \, (\varphi^+ \varphi_i \varphi_i + h.c.)$$

$$+ \frac{\lambda_{11}^2}{2!} \, (\varphi_{ij}^+ \varphi_i \varphi_j + h.c.) \; + \; \text{higher order interact.} \qquad (19)$$

The labels of Δ, λ refer to the isospin of the Reggeons ($I = 0,1,2$), and $P_2(ij|k\ell,mn)$ is the tensor which couples two $I = 2$ Reggeons to another $I = 2$ field (Fig. 3). The form of (19) is easily understood : certain couplings, such as λ_{11}^1, cannot exist because of signature conservation, and the question whether a triple vertex is real or imaginary follows from $Re(\xi_{\alpha_1} \xi_{\alpha_2})$ being positive or negative. Sign and size of the λ_{jk}^i will be discussed in Ref. 8). Here we only mention that λ_{11}^o and $\lambda_{11}^2 \sim 0(g^2)$, and $\lambda_{jk}^i \sim 0(g^4)$ otherwise.

The important feature of this RFT is the negative sign of Δ_2 in (16). It says that the Reggeon with highest isospin develops a negative mass. In relativistic quantum field theory with a continuous symmetry [e.g., $\lambda \Phi^4$ with $U(n)$], the appearance of a negative mass square leads (provided the dimensions are large enough) to a spontaneous breakdown of the symmetry, and massless particles (Goldstone particles) appear. If this argument applies to our RFT, we have a massless Reggeon, i.e., a Regge singularity with intercept 1 which might lead to a constant (up to powers of ℓn s) total cross-section. This idea of the Pomeron being a reflection of spontaneous breakdown of a continuous symmetry in Reggeon space has been expressed before [9], but the result in (16) for the first time gives a concrete reason where such a symmetry breaking might come from.

Because of the possible importance of the negative mass Reggeon we have extended our calculation to more general gauge groups SU(N). The way in which a massive SU(2) NAGFT can be generalized to massive SU(N) is described in Ref. 10), and we present only the results of calculating the Regge trajectories. For general SU(N), the vector mesons belong to the (N^2-1) dimensional adjoint representation, and quantum number assignment of the different Reggeon multiplets is found by decomposing the product $(N^2-1) \times (N^2-1)$ into its irreducible representations. One finds that the SU(3) result :

$$8 \times 8 = 1 + 8_A + 8_S + 10 + \overline{10} + 27 \tag{20}$$

can be generalized to

$$(N^2-1) \times (N^2-1) = 1 + (N^2-1)_A + (N^2-1)_S + \Sigma_A + \Sigma_S \tag{21}$$

Here Σ_A denotes the sum of all antisymmetric representations other than $(N^2-1)_A$ [this is the analogue of $10+\overline{10}$ in SU(3) and $45+\overline{45}$ in SU(4)], and Σ_S the sum of symmetric representations other than 1 and $(N^2-1)_S$ [this corresponds to 27 in SU(3) and 81+20 in SU(4)]. For the Reggeons belonging to the first three representations we find intercept below one (that means positive Reggeon mass). Reggeons belonging to Σ_A do not exist (in lowest order perturbation theory), and all Reggeons in Σ_S have the same intercept above one (negative Reggeon masses). In formulas :

$$\alpha_1(t) = 1 + N\left(t - \frac{N^2+1}{N^2} M^2\right)\beta(t) \qquad \tau_1 = + \tag{22}$$

$$\alpha_{(N^2-1)_A}(t) = 1 + \frac{N}{2}(t - M^2)\beta(t) \qquad \tau_{(N^2-1)_A} = - \tag{23}$$

$$\alpha_{(N^2-1)_S}(t) = 1 + \frac{N}{2}\left(t - \frac{N^2+4}{N^2} M^2\right)\beta(t) \qquad \tau_{(N^2-1)_S} = + \tag{24}$$

$$\alpha_{\Sigma_S}(t) = 1 - \frac{N^3}{\frac{1}{2}(N^2-1)(N^2-2)-1}(t - 2M^2)\beta(t) \qquad \tau_{\Sigma_S} = + \tag{25}$$

(These formulas can only be used for $N \geq 3$.) This result indicates that the existence of negative mass Reggeons in the highest multiplets may be a rather general property of NAGFT.

We feel that these results raise sufficient interest to continue our program of describing the high energy behaviour in NAGFT via RFT. The next thing to do is then to investigate the RFT (14)-(19), in particular to study the question of spontaneous symmetry breaking in RFT. The recent work on RFT beyond the critical point indicates that results of relativistic QFT cannot be applied to RFT without further investigations. Several groups [11] have studied RFT with a single Pomeron field beyond the critical point. In the absence of a continuous symmetry, such a theory has only the discrete symmetry $\psi \rightleftarrows \psi^{+}$. The important result of these investigations is that, when $\alpha_{0} > \alpha_{oc}$, there is no spontaneous breakdown of this symmetry (in fact, there is an instanton solution to RFT). It is, therefore, possible that in the presence of a continuous symmetry the situation might be different from what one expects on the basis of results of relativistic QFT.

Despite this uncertainty we would like to mention two intriguing consequences which might arise if spontaneous symmetry breaking exists and results in zero mass objects in RFT. As we have pointed out earlier, an adequate description of the high energy behaviour of NAGFT can be given only by a RFT with infinitely many Reggeon interactions. In general, such a RFT cannot be solved, but we know that near the critical point the theory is governed by its lowest order interaction, the triple Reggeon vertex. So if we have a symmetry breaking in our theory, then for small values of g we might be close to the phase transition point, and we do not need to consider higher order Reggeon interactions. Our second comment concerns the validity of our work coupling limit calculation. Since we have been working in the leading $\ell n \, s$ approximation, all our Reggeon field theory parameters $(\Delta, \lambda_{jk}^{i})$ are computed in lowest non-trivial order perturbation theory. Next to leading $\ell n \, s$ pieces would, presumably, lead to higher order correction to these quantities. However, if one finds that our RFT, with the parameters computed in lowest order, describes a phase in which the symmetry is broken and massless Reggeons do exist, then this phase depends primarily on the sign of the Δ's as opposed to its actual size. Higher order corrections to Δ will not change the sign within a finite, non-zero range of the coupling constant g. This then leads to the expectation that

the behaviour of σ_{tot}, etc., will remain unchanged for a non-zero range of values g, and our weak coupling limit results are sufficient to describe the high energy behaviour outside the limit $g \to 0$.

Details of calculations presented here will be published elsewhere [8].

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FIGURE CAPTIONS

Figure 1   Elements of the leading $\ln s$ coefficient $F(t)$:

(a) vertices ;

(b)-(d) a few examples of diagrams.

Figure 2   Definition of

(a) Reggeons, and

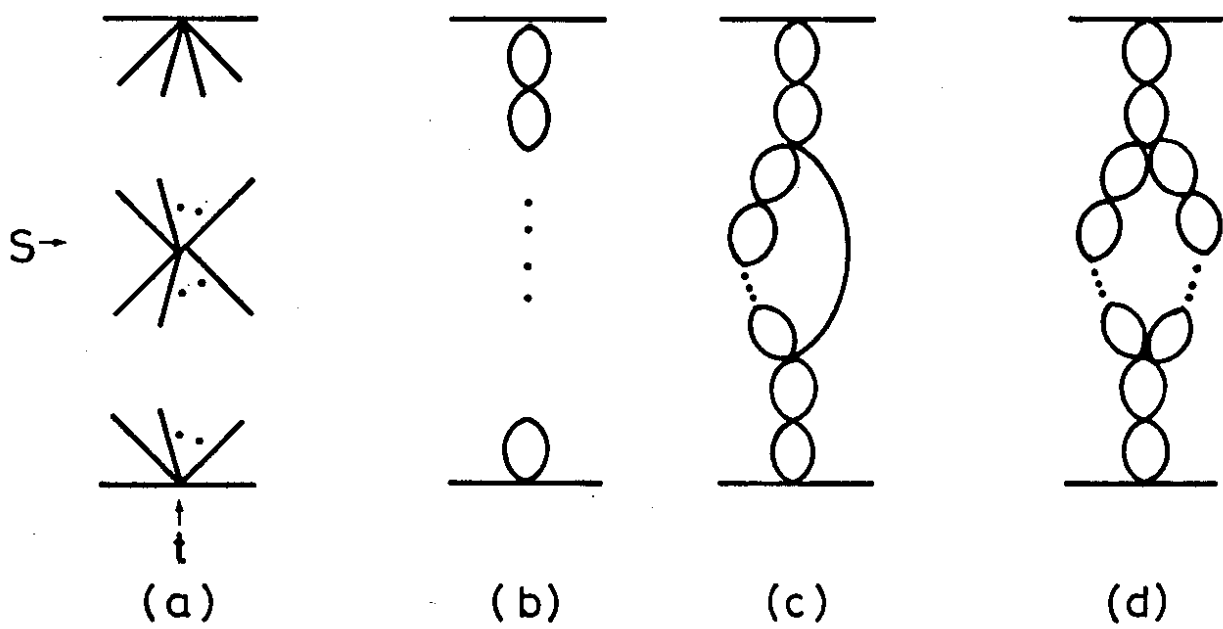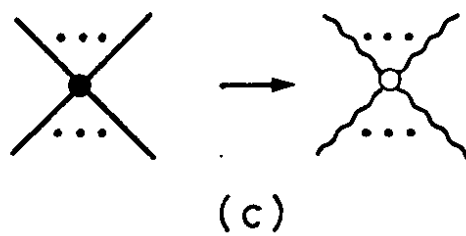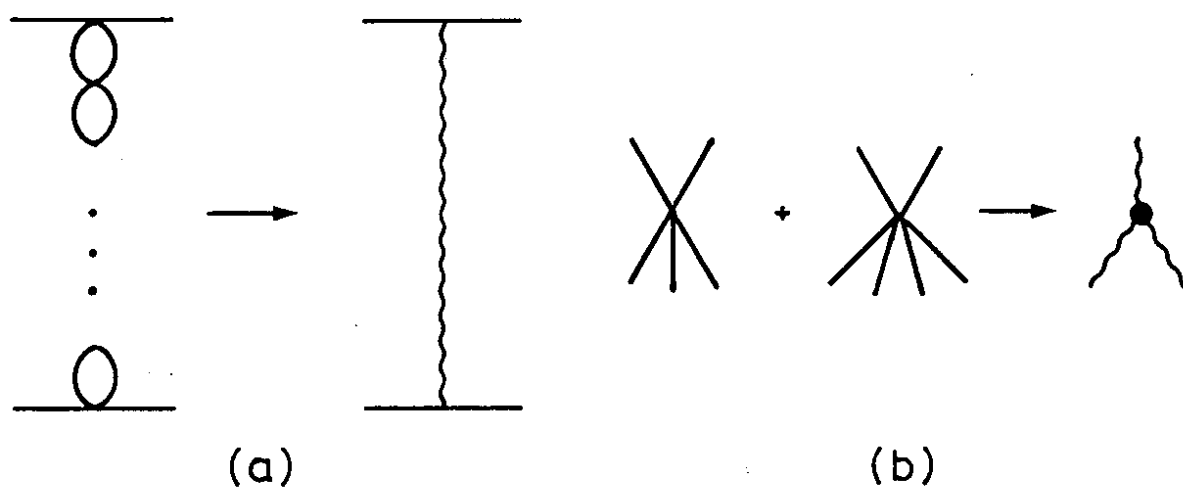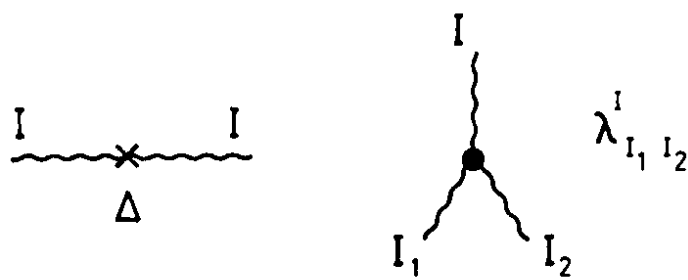(b)-(c) Reggeon couplings.

Figure 3   Elements of the RFT.

S→

t

(a)   (b)   (c)   (d)

FIG.1

(a)   (b)

(c)

FIG.2

I I

Δ

I

$\lambda^{I}_{I_1 I_2}$

I_1 I_2

FIG. 3