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$\mu^-$  MESON CATALYSIS OF NUCLEAR FUSION  
IN A MIXTURE OF DEUTERIUM AND TRITIUM

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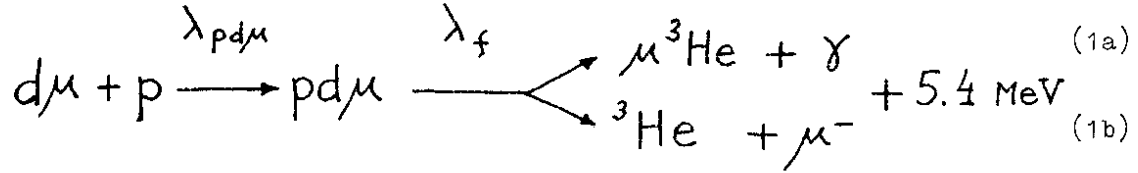
ABSTRACT

Due to the existence of a weakly bound state of mesic molecule  $dt\mu$ , one  $\mu^-$  meson in a mixture of deuterium and tritium can catalyze  $\sim 10^2$  of the fusion reactions  $dt\mu \rightarrow {}^4\text{He} + n + \mu^- + 17.6 \text{ MeV}$  and release  $\sim 2 \text{ GeV}$  of energy.

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1. The synthesis reaction of the hydrogen and deuterium nuclei proceeds through the intermediate state of  $\mu$  molecule formation in the chain



This process is now extensively studied both experimentally and theoretically. [The complete list of references is presented in <sup>1),2)</sup>.]

The reaction chain (1) was suggested by Frank <sup>3)</sup> in 1947, just after the discovery of pions <sup>4)</sup> for the explanation of the observed monochromatic muons produced by  $\pi \rightarrow \mu$  decay at rest. Ten years later this process was observed by Alvarez et al. <sup>5)</sup>.

The process (1) is known as catalysis of the nuclear synthesis reaction in "cold" hydrogen by  $\mu^-$  mesons <sup>6)</sup>. This process is particularly effective if one muon can cause several reactions of the synthesis repeatedly. For this purpose it is necessary to satisfy at least three conditions:

$$\lambda_{pd\mu} \gg \lambda_0, \quad \lambda_f \gg \lambda_0, \quad w_s \ll 1, \quad (2)$$

where  $\lambda_0 = 0.455 \times 10^6 \text{ s}^{-1}$  is the decay rate of free muons,  $\lambda_{pd\mu}$  is the formation rate of  $pd\mu$  molecules,  $\lambda_f$  is the nuclear synthesis reaction rate in the mesic molecule and  $w_s$  is the probability of muons "sticking" to  ${}^3\text{He}$ , which is equal to the ratio of the reaction probabilities (1a) and (1b).

For the reactions (1) none of these conditions is fulfilled because  $\lambda_{pd\mu} \approx 6 \times 10^6 \text{ s}^{-1}$ ,  $\lambda_f \approx 0.3 \times 10^6 \text{ s}^{-1}$ , and  $w_s \approx 1$ . The latest measurements <sup>7)</sup> and calculations <sup>8)</sup> confirm the presented value of  $\lambda_{pd\mu}$ . It was supposed in the calculations that  $\mu$  molecules are formed by the dipole E1 transition (or monopole E0-transition) with conversion of the valence electron of the molecule.

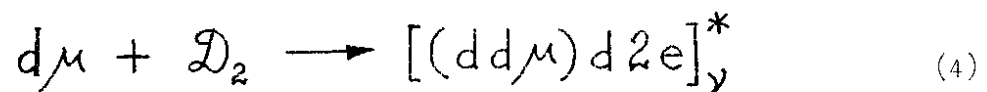
2. The analogous calculations <sup>8)</sup> of the formation rate of the  $dd\mu$  molecule according to the scheme



give the value  $\lambda_{dd\mu} \approx 0.5 \times 10^5 \text{ s}^{-1}$  which significantly differs from the one measured by Dzhelepov's group<sup>9)</sup>:  $\lambda_{dd\mu} = (0.73 \pm 0.07) \times 10^6 \text{ s}^{-1}$ .

Further experiments<sup>10)</sup> have confirmed this result and established the temperature dependence of the formation rate  $\lambda_{dd\mu}(T)$ : it turned out when the temperature of the deuterium changes from  $-160^\circ\text{C}$  to  $+100^\circ\text{C}$  (the collision energy interval corresponds to  $\sim 0.04 \text{ eV}$ ) the rate  $\lambda_{dd\mu}$  increases by a factor of four<sup>10)</sup>.

For the explanation of this discrepancy Wesman<sup>11)</sup> considered in 1967 the mechanism of resonance formation of  $dd\mu$  molecules according to the scheme



The essence of the resonant mechanism is as follows: in the reaction (4) the electron emission does not occur and the energy released in the formation of  $dd\mu$  molecule is transferred by the excitation of the vibrational states of the peculiar molecule  $[(dd\mu)d2e]$  which includes  $(dd\mu)^+$  molecule produced as a nucleus. Such a resonance process is possible if the difference between the energy  $E_v$  of the  $v$ th vibrational state of the excited molecule  $[(dd\mu)d2e]_v^*$  in the rotational state  $K=1$  and the energy  $\bar{E}_C$  of the ground state of the molecule  $D_2$  is equal to the sum of the binding energy  $\epsilon_{11}$  of the  $dd\mu$  molecule and the kinetic energy  $\epsilon_0$  of the colliding  $d\mu$  atom (see Fig. 1). However, this mechanism is possible if  $dd\mu$  molecule has the weakly bounded state with the binding energy  $\epsilon_{Jv} < 3 \text{ eV}$ , i.e., less than the ionization electron energy of  $D_2$  molecule ( $\sim 15 \text{ eV}$ ) and the dissociation energy of  $D_2$  molecule ( $\sim 4.5 \text{ eV}$ ).

3. Recently in the adiabatic representation of the three-body problem the effective scheme of calculations of the mesic molecule binding energies have been developed<sup>12)</sup>. In this scheme, the three-body wave function is expanded over the complete set of the solutions of the two-centre problem. The calculations of all of the energy levels for muonic molecules<sup>13)</sup> have proved the effectiveness of this scheme.

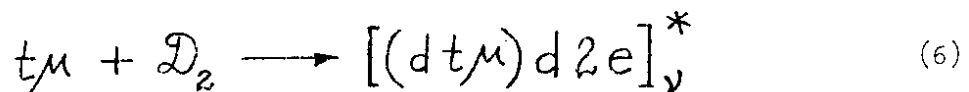
In particular the developed method has allowed us to establish reliably the existence of the weakly bound excited state of  $dd\mu$  molecule with the quantum numbers  $J=1$  (rotational),  $v=1$  (vibrational) and the binding energy  $\epsilon_{Jv} = -2.2 \text{ eV}$ . The calculated dependence  $\lambda_{dd\mu}(T)$  as a function of the temperature  $T$  is given by the formula<sup>14)</sup>:

$$\lambda_{dd\mu}(T) = \beta \frac{2\pi^2}{3} \left(\frac{m_e}{m_\mu}\right)^5 \left(\frac{m_\mu}{m_a}\right)^2 |d_{fi}|^2 I_\nu^2 \gamma(\epsilon_0, \epsilon_T) \frac{m_e e^4}{\hbar^3} s^{-1} \quad (5)$$

Here:  $\beta$  - the statistical factor which is equal  $1/3$  for  $dd\mu$  molecule,  $m_e$  and  $m_\mu$  - the masses of electron and muon;  $m_a^{-1} = m_e^{-1} + M_a^{-1}$  and  $M_a$  is the nuclear mass;  $d_{fi}$  - the dipole matrix element calculated between the functions in the initial state (i) of the system  $d\mu + d$  and final state (f) ( $J=1, v=1$ ) of  $dd\mu$  molecule;  $I_\nu$  - the matrix element of the dipole interaction, corresponding to the transition from the ground state ( $K=0, v=0$ ) of  $D_2$  molecule to the excited state ( $K=1, v=8$ ) of the molecule  $[(dd\mu) d 2e]$ ;  $\gamma(\epsilon_0, \epsilon_T) = (27\epsilon_0/2\pi)^{1/2} \epsilon_T^{-3/2} \exp\{-3\epsilon_0/2\epsilon_T\}$  - the Maxwell distribution,  $\epsilon_T = \frac{3}{2}kT$  - the average kinetic energy of mesonic atoms  $d\mu$  at the temperature  $T$ .

The function (5) represents in all details the characteristics and the peculiarities of the measured temperature dependence  $\lambda_{dd\mu}(T)$  ((see Fig. 2). The value  $\lambda_{dd\mu}^{\max}$  calculated according to the formula (5) at the resonance energy  $\epsilon_0 = \epsilon_T = 0.053$  eV equals  $0.8 \times 10^6$  s $^{-1}$  in good agreement with the measured one  $\lambda_{dd\mu}^{\max} = (0.807 \pm 0.056) \times 10^6$  s $^{-1}$  (14), (15). (All the rates are recalculated to the density of liquid hydrogen  $N_0 = 4.25 \times 10^{22}$  cm $^{-3}$ .)

4. In the analogous calculations of the binding energies of  $dt\mu$  molecule states, the existence of the weakly bound state with quantum numbers  $J=1$  and  $v=1$  and binding energy  $\epsilon_{Jv} = -1.1$  eV has been established (14). The existence of this level as in the case of  $dd\mu$  molecule opens the channel of  $dt\mu$  molecule resonance formation:

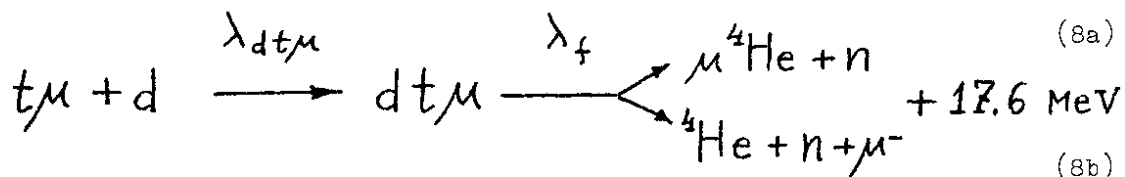


The detailed calculation performed recently (14), show that in reaction (6) the molecule  $[(dt\mu) d 2e]$  forms in the rotational state  $K=1$  and vibrational state  $v=4$ , and the dependence  $\lambda_{dt\mu}(T)$  is described by formula (5) with  $\beta=1$  and  $M_a = M_t$ . The resonance value  $\lambda_{dt\mu}^{\max}$  corresponding to this transition equals

$$\lambda_{dt\mu} \approx 10^8 \text{ s}^{-1} \quad (7)$$

and exceeds all known rates of the mesic molecule formation by at least one order of magnitude.

In the chain of reactions



the rate  $\lambda_f$  of the nuclear fusion reaction equals  $\sim 10^{11} \text{ s}^{-1}$  and the probability of the "sticking" of muon to  ${}^4\text{He}$  in the reaction (8a) equals  $\sim 10^{-2}$  1), 2). It means that every  $\mu^-$  meson in the mixture deuterium and tritium can produce  $\sim 10^2$  of the nuclear fusion reaction and release  $\sim 2 \text{ GeV}$  which is 20 times the rest mass of muon.

5. Leaving the question open about the possibilities of practical applications of the phenomenon discovered [e.g. the production of energy in fusion reactors, the breeding of tritium by ejected neutrons in the mixture  ${}^6\text{Li}$  and  ${}^7\text{Li}$ , the ignition of the thermonuclear fusion reaction provided by lasers etc., see in this connection the paper by Tan <sup>16)</sup>]. Nevertheless, we would like to call attention to it.

The reactions (8) are not yet studied experimentally. We hope that such experiments will soon be performed despite the technical difficulties that are involved. No doubt the accurate calculations of the main parameters specifying the process: binding energy of  $dt\mu$  molecule, position and width of the  $dt\mu$  resonance formation rates, kinetics of the catalysis allowing for all other  $\mu$  molecular processes etc., requires more thorough calculations.

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FIGURE CAPTIONS

Fig. 1: The scheme of the resonance formation of the  $dd\mu$  molecule:  
 $\epsilon_0$  kinetic energy of  $d\mu$  atom,  $\epsilon_{11}$  binding energy of the  $dd\mu$  molecule in the state  $(J=1, v=1)$ . The released energy  $\epsilon_0 + \epsilon_{11}$  is transferred to the excitation of the  $[(dd\mu) d 2e]$  molecule in the rotational state  $K=1$  and vibrational state  $v=8$  from the ground state  $(K=0, v=0)$  of the  $D_2$  molecule.

Fig. 2: The dependence  $\lambda_{dd\mu}(T)$  calculated according to the formula (5) at the resonance energy  $\epsilon_0 = 0.053$  eV. The experimental values:

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- ⊕ Bystriisky et al. <sup>10)</sup>.



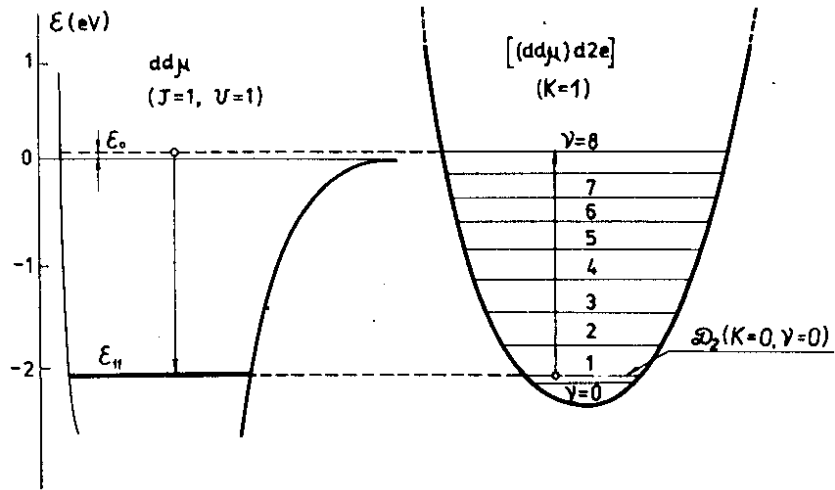


Fig. 1

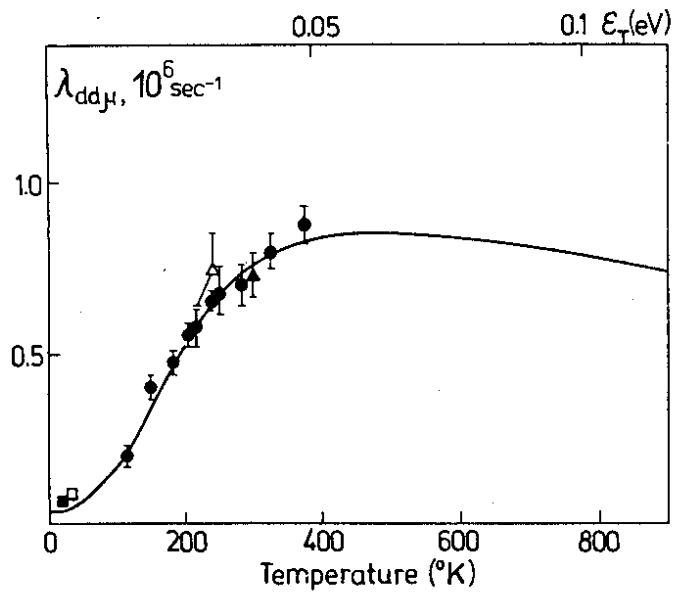


Fig. 2

