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Measuring $B \to \rho\pi$ decays and the unitarity angle α

A. Deandrea,[1] R. Gatto,[2] M. Ladisa,[1,3] G. Nardulli,[1,3] and P. Santorelli[4]
[1]Theory Division, CERN, CH-1211 Genève 23, Switzerland
[2]Département de Physique Théorique, Université de Genève, 24 quai E.-Ansermet, CH-1211 Genève 4, Switzerland
[3]Dipartimento di Fisica, Università di Bari and INFN Bari, via Amendola 173, I-70126 Bari, Italy
[4]Dipartimento di Scienze Fisiche, Università di Napoli ''Federico II'' and INFN Napoli, Mostra d'Oltremare 20, I-80125 Napoli, Italy
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The decay mode $B \to \rho\pi$ is currently studied as a channel allowing us, in principle, to measure without ambiguities the angle α of the unitarity triangle. It is also investigated by the CLEO Collaboration where a branching ratio larger than expected for the decay mode $B^{\pm} \to \rho^0 \pi^{\pm}$ has been found. We investigate the role that the B^* and $B_0(0^+)$ resonances might play in these analyses.

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I. INTRODUCTION

The measurement of the angle α in the unitarity triangle will be one of the paramount tasks of the future b factories, such as the dedicated e^+e^- machines for the BaBar experiment at SLAC [1] and the BELLE experiment at KEK [2], or hadron machines such as the Large Hadron Collider (LHC) at CERN, with its program for B physics.[1] Differently from the investigation of the β angle, for which the $B \to J/\psi K_S$ channel has been pinned up [3] and ambiguities can be resolved [4], the task of determining the angle α is complicated by the problem of separating two different weak hadronic matrix elements, each carrying its own weak phase. The evaluation of these contributions, referred to in the literature as the *tree* (T) and the *penguin* (P) contributions, suffers from the common theoretical uncertainties related to the estimate of composite four-quark operators between hadronic states. For these estimates, only approximate schemes, such as the factorization approximation, exist at the moment, and for this reason several ingenuous schemes have been devised, trying to disentangle T and P contributions. In general one tries to exploit the fact that in the P amplitudes only the isospin-1/2 part[2] of the nonleptonic Hamiltonian is active [5]; by a complex measurement involving several different isospin amplitudes, one should be able to separate the two amplitudes and to get rid of the ambiguities arising from the ill-known penguin matrix elements.

One of the favorite proposals involves the study of the reaction $B \to \rho\pi$, i.e., six channels arising from the neutral B decay:

$$\overline{B^0} \to \rho^+ \pi^-, \tag{1}$$

$$\overline{B^0} \to \rho^- \pi^+, \tag{2}$$

$$\overline{B^0} \to \rho^0 \pi^0, \tag{3}$$

together with the three charge-conjugate channels, and the charged decay modes:

$$B^- \to \rho^- \pi^0, \tag{4}$$

$$B^- \to \rho^0 \pi^-, \tag{5}$$

with two other charge-conjugate channels. Different strategies have been proposed to extract the angle α, either involving all the decay modes of a B into a $\rho\pi$ pair as well as three time-asymmetric quantities measurable in the three channels for neutral B decays [6–8], or attempting to measure only the neutral B decay modes by looking at the time-dependent asymmetries in different regions of the Dalitz plot.[3]

Preliminary to these analyses is the assumption that, using cuts in the three invariant masses for the pion pairs, one can extract the ρ contribution without significant background contamination. The ρ has spin 1, the π spin 0 as well as the initial B, and therefore the ρ has angular distribution $\cos^2\theta$ (θ is the angle of one of the ρ decay products with the other π in the ρ rest frame). This means that the Dalitz plot is mainly populated at the border, especially the corners, by this decay. Only very few events should be lost by excluding the interior of the Dalitz plot, which is considered a good way to exclude or at least reduce backgrounds. Analyses following these hypotheses were performed by the BaBar working groups [1]; Monte Carlo simulations, including the background from the f_0 resonance, show that, with cuts at $m_{\pi\pi} = m_\rho \pm 300$ MeV, no significant contributions from other sources are obtained. Also the role of excited resonances such as the ρ' and the nonresonant background has been discussed [9].

A signal of possible difficulties for this strategy arises from new results from the CLEO Collaboration recently reported at the DPF99 and APS99 Conferences [10]:

[1]Opportunities for B physics at the LHC have been recently discussed at the workshop on Standard Model Physics (and More) at the LHC, 1999; copies of transparencies can be found at the site http://home.cern.ch/ mlm/lhc99/oct14ag.html

[2]If one neglects electroweak penguin amplitudes.

[3]In this way the measurement of a decay mode with two neutral pions in the final state, Eq. (4), can be avoided.

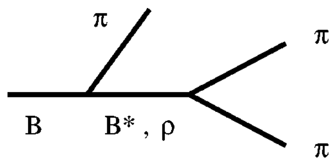

FIG. 1. The polar diagram. For the B resonances ($B^*=1^-$, 0^+) the strong coupling is on the left and the weak coupling on the right; the situation is reversed for the ρ production.

$$\mathcal{B}(B^\pm\to\rho^0\pi^\pm)=(1.5\pm0.5\pm0.4)\times10^{-5}, \tag{6}$$

$$\mathcal{B}(B\to\rho^\mp\pi^\pm)=(3.5^{+1.1}_{-1.0}\pm0.5)\times10^{-5}, \tag{7}$$

with a ratio

$$R=\frac{\mathcal{B}(B\to\rho^\mp\pi^\pm)}{\mathcal{B}(B^\pm\to\rho^0\pi^\pm)}=2.3\pm1.3. \tag{8}$$

As discussed in [10], this ratio looks rather small; as a matter of fact, when computed in simple approximation schemes, including factorization with no penguin contributions, one gets, from the Deandrea–Di Bartolomeo–Gatto–Nardulli (DDGN) model of Ref. [11], $R\simeq13$, admittedly with a large uncertainty; another popular approach, i.e., the Wirbel-Bauer-Stech (WBS) model [12], gives $R\simeq6$ (in both cases we use $a_1=1.02$, $a_2=0.14$). The aim of the present study is to show that a new contribution, not discussed before, is indeed relevant to decay (5) and to a lesser extent to decay (3). It arises from the virtual resonant production depicted in Fig. 1, where the intermediate particle is the B^* meson resonance or other excited states. The B^* resonance, because of phase-space limitations, cannot be produced on the mass shell. Nonetheless the B^* contribution might be important, owing to its almost degeneracy in mass with the B meson; therefore its tail may produce sizable effects in some of the decays of B into light particles, also because it is known theoretically that the strong coupling constant between B, B^* and a pion is large [13]. Concerning other states, we expect their role to decrease with their mass, since there is no enhancement from the virtual particle propagator; we shall only consider the 0^+ state B_0 with $J^P=0^+$ because its coupling to a pion and the meson B is known theoretically to be uniformly (in momenta) large [13]. The plan of the paper is as follows. In Sec. II we list the hadronic quantities that are needed for the computation of the widths; in Sec. III we present the results and finally, in Sec. IV, we give our conclusions.

II. MATRIX ELEMENTS

The effective weak nonleptonic Hamiltonian for the $|\Delta B|=1$ transition is[4]

$$H=\frac{G_F}{\sqrt{2}}\left\{V_{ub}^*V_{ud}\sum_{k=1}^{2}C_k(\mu)Q_k-V_{tb}^*V_{td}\sum_{k=3}^{6}C_k(\mu)Q_k\right\}. \tag{9}$$

The operators relevant to the present analysis are the so-called current-current operators:

$$Q_1=(\overline{d}_\alpha u_\beta)_{V-A}(\overline{u}_\beta b_\alpha)_{V-A},$$
$$Q_2=(\overline{d}_\alpha u_\alpha)_{V-A}(\overline{u}_\beta b_\beta)_{V-A}, \tag{10}$$

and the QCD penguin operators:

$$Q_3=(\overline{d}_\alpha b_\alpha)_{V-A}\sum_{q'=u,d,s,c,b}(\overline{q}'_\beta q'_\beta)_{V-A},$$
$$Q_4=(\overline{d}_\alpha b_\beta)_{V-A}\sum_{q'=u,d,s,c,b}(\overline{q}'_\beta q'_\alpha)_{V-A},$$
$$Q_5=(\overline{d}_\alpha b_\alpha)_{V-A}\sum_{q'=u,d,s,c,b}(\overline{q}'_\beta q'_\beta)_{V+A},$$
$$Q_6=(\overline{d}_\alpha b_\beta)_{V-A}\sum_{q'=u,d,s,c,b}(\overline{q}'_\beta q'_\alpha)_{V+A}, \tag{11}$$

We use the following values of the Wilson coefficients: $C_1=-0.226$, $C_2=1.100$, $C_3=0.012$, $C_4=-0.029$, $C_5=0.009$, $C_6=-0.033$; they are obtained in the 't Hooft–Veltmann (HV) scheme [14], with $\Lambda^{(5)}_{\overline{MS}}=225$ MeV, $\mu=\overline{m}_b(m_b)=4.40$ GeV, and $m_t=170$ GeV. For the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [15] we use the Wolfenstein parametrization [16] with $\rho=0.05$, $\eta=0.36$, and $A=0.806$ in the approximation accurate to order λ^3 in the real part and λ^5 in the imaginary part, i.e., $V_{ud}=1-\lambda^2/2$, $V_{ub}=A\lambda^3[\rho-i\eta(1-\lambda^2/2)]$, $V_{td}=A\lambda^3(1-\rho-i\eta)$, and $V_{tb}=1$.

The diagram of Fig. 1 describes two processes. For the B^* intermediate state there is an emission of a pion by strong interactions, followed by the weak decay of the virtual B^* into two pions; for the ρ intermediate state there is a weak decay of $B\to\rho\pi$ followed by the strong decay of the ρ resonance. We compute these diagrams as Feynman graphs of an effective theory within the factorization approximation, using information from the effective Lagrangian for heavy and light mesons and form factors for the couplings to the weak currents.[5]

To start with we consider the strong coupling constants. They are defined as

[4]We omit, as usual in these analyses, the electroweak operators Q_k ($k=7$, 8, 9, 10); they are in general small, but for Q_9, whose role might be sizable; its inclusion in the present calculations would be straightforward.

[5]In the second reference of [4] a similar approach has been used to describe the decay mode $B^0\to D^+D^-\pi^0$; the main difference is that for $B\to3\pi$ we cannot use soft pion theorems and chiral perturbation theory, because the pions are in general hard; therefore we have to use information embodied in the semileptonic beauty meson form factors. This is also the main difference with respect to [8].

$$\langle \overline{B}^0(p')\pi^-(q)|B^{*-}(p,\epsilon)\rangle = g^{B^*B\pi}\epsilon\cdot q,$$

$$\langle B^-(p')\pi^+(q)|\overline{B}_0^0(p)\rangle = G^{B_0B\pi}(p^2), \tag{12}$$

$$\langle \pi^0(q')\pi^-(q)|\rho^-(p,\epsilon)\rangle = g_\rho\epsilon\cdot(q'-q).$$

In the heavy quark mass limit one has

$$g^{B^*B\pi} = \frac{2m_B g}{f_\pi}, \tag{13}$$

$$G^{B_0B\pi}(s) = -\sqrt{\frac{m_{B_0}m_B}{2}}\,\frac{s-m_B^2}{m_{B_0}}\,\frac{h}{f_\pi}. \tag{14}$$

For g and h we have limited experimental information and we have to use some theoretical inputs. For g and h reasonable ranges of values are $g=0.3$–0.6, $-h=0.4$–0.7 [17]. These numerical estimates encompass results obtained by different methods: QCD sum rules [13], potential models [18], effective Lagrangian [19], Nambu–Jona-Lasinio–(NJL-) inspired models [20]. Moreover $g_\rho=5.8$ and $f_\pi\sim 130$ MeV. This value of g_ρ is commonly used in the chiral effective theories including the light vector meson resonances and corresponds to $\Gamma_\rho\simeq 150$ MeV; see, for instance [17], where a review of different methods for the determination of g is also given.

For the matrix elements of quark bilinears between hadronic states, we use the following matrix elements:

$$\langle \pi^-|\overline{d}\gamma_5 u|0\rangle = \frac{i f_\pi m_\pi^2}{2m_q},$$

$$\langle \pi^0(q)|\overline{u}\gamma_5 b|B^{*-}(p)\rangle = i\epsilon^\mu(q-p)_\mu\frac{2m_{B^*}A_0^\pi}{m_b+m_q},$$

$$\langle \rho^+(q,\epsilon)|\overline{u}\gamma_5 b|\overline{B}^0(p)\rangle = i\epsilon^{*\mu}(p-q)_\mu\frac{2m_\rho A_0}{m_b+m_q},$$

$$\langle \pi^+(q)|\overline{u}\gamma_\mu b|\overline{B}^0(p)\rangle = F_1\left[(p+q)^\mu - \frac{m_B^2-m_\pi^2}{(p-q)^2}(p-q)^\mu\right] + F_0\frac{m_B^2-m_\pi^2}{(p-q)^2}(p-q)^\mu,$$

$$\langle \pi^+(q)|\overline{u}\gamma^\mu(1-\gamma_5)b|\overline{B}_0^0(p)\rangle = i\left\{\tilde{F}_1\left[(p+q)^\mu - \frac{m_{B_0}^2-m_\pi^2}{(p-q)^2}(p-q)^\mu\right] + \tilde{F}_0\frac{m_{B_0}^2-m_\pi^2}{(p-q)^2}(p-q)^\mu\right\},$$

$$\langle 0|\overline{u}\gamma^\mu d|\rho^-(q,\epsilon)\rangle = f_\rho\epsilon^\mu, \tag{15}$$

where $f_\rho=0.15\ \ \mathrm{GeV}^2$ [22] and

$$A_0^\pi = A_0^\pi(0) = 0.16,\quad A_0 = A_0(0) = 0.29, \tag{16}$$

$$F_1 = F_1(0) = F_0(0) = 0.37,\quad F_0^\pi = \tilde{F}_1(0) = \tilde{F}_0(0) = -0.19. \tag{17}$$

The first three numerical inputs have been obtained by the relativistic potential model; A_0 and F_1 can be found in [21], while A_0^π has been obtained here for the first time, using the same methods. The last figure in (17) concerns F_0^π, for which such an information is not available; for it we used the methods of [17] and the strong coupling $BB_0\pi$ computed in [23].

III. AMPLITUDES AND NUMERICAL RESULTS

For all the channels we consider three different contributions A_ρ, A_{B^*}, A_{B_0}, due respectively to the ρ resonance, the B^* pole, and the B_0 positive parity 0^+ resonance, whose mass we take[6] to be 5697 MeV.

For each of the amplitudes

$$A^{--+} = A(B^-\to\pi^-\pi^-\pi^+), \tag{18}$$

$$A^{-00} = A(B^-\to\pi^-\pi^0\pi^0), \tag{19}$$

$$A^{+-0} = A(\overline{B}^0\to\pi^+\pi^-\pi^0), \tag{20}$$

we write the general formula[7] $A^{ijk}=A_\rho^{ijk}+A_{B^*}^{ijk}+A_{B_0}^{ijk}$. We get, for the process (18),

$$A_\rho^{--+} = \overline{\eta}^0\left[\frac{t'-u}{t-m_\rho^2+i\Gamma_\rho m_\rho} + \frac{t-u}{t'-m_\rho^2+i\Gamma_\rho m_\rho}\right],$$

$$A_{B^*}^{--+} = K\left[\frac{\Pi(t,u)}{t-m_{B^*}^2+i\Gamma_{B^*}m_{B^*}} + \frac{\Pi(t',u)}{t'-m_{B^*}^2+i\Gamma_{B^*}m_{B^*}}\right],$$

[6]We identify the 0^+ state mass with the average mass of the B^{**} states given in [24].

[7]We add coherently the three contributions; the relative sign the B resonances on one side and the ρ contribution on the other is irrelevant, as the former are dominantly real and the latter is dominantly imaginary. The relative sign between B^* and B_0 is fixed by the effective Lagrangian for heavy mesons.

$$A_{B0}^{--+}=\tilde{K}^0\,(m_{B_0}^2-m_\pi^2)\left[\frac{1}{t-m_{B_0}^2+i\Gamma_{B_0}m_{B_0}}+\frac{1}{t'-m_{B_0}^2+i\Gamma_{B_0}m_{B_0}}\right], \tag{21}$$

where, if p_{π^-} is the momentum of one of the two negatively charged pions $t=(p_{\pi^-}+p_{\pi^+})^2$, t' is obtained by exchanging the two identical pions and u is the invariant mass of the two identical negatively charged pions. Clearly one has $u+t+t'=m_B^2+3m_\pi^2$. The expressions entering in the previous formulas are

$$\overline{\eta}^0=\frac{G_F}{\sqrt{2}}V_{ub}V_{ud}^*\frac{g_\rho}{\sqrt{2}}\left[f_\rho F_1\left(c_1+\frac{c_2}{3}\right)+m_\rho A_0 f_\pi\left(c_2+\frac{c_1}{3}\right)\right]+\frac{G_F}{\sqrt{2}}V_{tb}V_{td}^*\frac{g_\rho}{\sqrt{2}}\left[\left(c_4+\frac{c_3}{3}\right)(f_\rho F_1-m_\rho A_0 f_\pi)+2\left(c_6+\frac{c_5}{3}\right)m_\rho A_0 f_\pi\frac{m_\pi^2}{(m_b+m_q)2m_q}\right],$$

$$K=-4\sqrt{2}gm_B^2A_0^\pi\frac{G_F}{\sqrt{2}}\left\{V_{ub}V_{ud}^*\left(c_2+\frac{c_1}{3}\right)-V_{tb}V_{td}^*\left[c_4+\frac{c_3}{3}-2\left(c_6+\frac{c_5}{3}\right)\frac{m_\pi^2}{(m_b+m_q)2m_q}\right]\right\},$$

$$\tilde{K}^0=h\sqrt{\frac{m_B}{m_{B0}}}F_0^\pi\frac{G_F}{\sqrt{2}}(m_{B0}^2-m_B^2)\left\{V_{ub}V_{ud}^*\left(c_2+\frac{c_1}{3}\right)-V_{tb}V_{td}^*\left[c_4+\frac{c_3}{3}-2\left(c_6+\frac{c_5}{3}\right)\frac{m_\pi^2}{(m_b+m_q)2m_q}\right]\right\}, \tag{22}$$

with $m_b=4.6$ GeV, $m_q\sim m_u\sim m_d\simeq 6$ MeV, $\Gamma_{B*}=0.2$ keV, $\Gamma_{B_0}=0.36$ GeV [17]. Moreover, for the process (19)

$$A_\rho^{-00}=\overline{\eta}^-\left[\frac{s'-u}{s-m_\rho^2+i\Gamma_\rho m_\rho}+\frac{s-u}{s'-m_\rho^2+i\Gamma_\rho m_\rho}\right],$$

$$A_{B*}^{-00}=\frac{1}{\sqrt{2}}\left\{K_1\frac{s+s'-4m_\pi^2}{2m_{B*}^2}+\frac{K\,\Pi(s',s)+K_1\,\Pi(s',u)}{s'-m_{B*}^2+i\Gamma_{B*}m_{B*}}+\frac{K\,\Pi(s,s')+K_1\,\Pi(s,u)}{s-m_{B*}^2+i\Gamma_{B*}m_{B*}}\right\},$$

$$A_{B_0}^{-00}=\left(\frac{\tilde{K}^0+\tilde{K}^{cc}}{s-m_{B_0}^2+i\Gamma_{B_0}m_{B_0}}+\frac{\tilde{K}^0+\tilde{K}^{cc}}{s'-m_{B_0}^2+i\Gamma_{B_0}m_{B_0}}+\frac{\tilde{K}^0}{u-m_{B_0}^2+i\Gamma_{B_0}m_{B_0}}\right)\frac{(m_{B_0}^2-m_\pi^2)}{2}. \tag{23}$$

In this case we define $s=(p_{\pi^-}+p_{\pi^0})^2$, if p_{π^0} is the momentum of one of the two identical neutral pions, s' is obtained by exchanging the two neutral pions and u is their invariant mass (again we have a relation among the different Mandelstam variables: $s+s'+u=m_B^2+3m_\pi^2$). Then $\overline{\eta}^-$, K_1, and $\tilde{K}^{cc}$ are given by

$$\overline{\eta}^-=\frac{G_F}{\sqrt{2}}V_{ub}V_{ud}^*\frac{g_\rho}{\sqrt{2}}\left[f_\rho F_1\left(c_2+\frac{c_1}{3}\right)+m_\rho A_0 f_\pi\left(c_1+\frac{c_2}{3}\right)\right]+\frac{G_F}{\sqrt{2}}V_{tb}V_{td}^*\frac{g_\rho}{\sqrt{2}}\left[\left(c_4+\frac{c_3}{3}\right)(-f_\rho F_1+m_\rho A_0 f_\pi)-2\left(c_6+\frac{c_5}{3}\right)m_\rho A_0 f_\pi\frac{m_\pi^2}{(m_b+m_q)2m_q}\right],$$

$$K_1=-4gm_B^2A_0^\pi\frac{G_F}{\sqrt{2}}\left\{V_{ub}V_{ud}^*\left(c_1+\frac{c_2}{3}\right)+V_{tb}V_{td}^*\left[c_4+\frac{c_3}{3}-2\left(c_6+\frac{c_5}{3}\right)\frac{m_\pi^2}{(m_b+m_q)2m_q}\right]\right\},$$

$$\tilde{K}^{cc}=h\sqrt{\frac{m_B}{m_{B0}}}F_0^\pi\frac{G_F}{\sqrt{2}}(m_{B0}^2-m_B^2)\left\{V_{ub}V_{ud}^*\left(c_1+\frac{c_2}{3}\right)+V_{tb}V_{td}^*\left[c_4+\frac{c_3}{3}-2\left(c_6+\frac{c_5}{3}\right)\frac{m_\pi^2}{(m_b+m_q)2m_q}\right]\right\}, \tag{24}$$

and

$$\Pi(x,y)=m_\pi^2-\frac{y}{2}+\frac{x(m_B^2-m_\pi^2-x)}{4m_{B*}^2}, \tag{25}$$

while $\widetilde{K}^0$ was given above in (22).

Finally, for the neutral B decay (20), we have

$$A_\rho^{+-0}=\eta^0\frac{u-s}{t-m_\rho^2+i\Gamma_\rho m_\rho}+\eta^+\frac{s-t}{u-m_\rho^2+i\Gamma_\rho m_\rho}+\eta^-\frac{t-u}{s-m_\rho^2+i\Gamma_\rho m_\rho},$$

$$A_{B^*}^{+-0}=\frac{K\,\Pi(s,t)+K_1\,\Pi(s,u)}{s-m_{B^*}^2+i\Gamma_{B^*}m_{B^*}}-\frac{K\,\Pi(t,s)}{t-m_{B^*}^2+i\Gamma_{B^*}m_{B^*}},$$

$$A_{B_0}^{+-0}=\left(\frac{\widetilde{K}^0+\widetilde{K}^{cc}}{s-m_{B_0}^2+i\Gamma_{B_0}m_{B_0}}+\frac{\widetilde{K}^0}{t-m_{B_0}^2+i\Gamma_{B_0}m_{B_0}}\right)(m_{B_0}^2-m_\pi^2), \tag{26}$$

where $s=(p_{\pi^-}+p_{\pi^0})^2$, $t=(p_{\pi^-}+p_{\pi^+})^2$, $u=(p_{\pi^+}+p_{\pi^0})^2$, and $s+t+u=m_B^2+3m_\pi^2$. The constants appearing in these equations are

$$\eta^0=-\frac{g_\rho}{2}(f_\rho F_1+m_\rho A_0 f_\pi)\frac{G_F}{\sqrt{2}}\left[V_{ub}V_{ud}^*\left(c_1+\frac{c_2}{3}\right)+V_{tb}V_{td}^*\left(c_4+\frac{c_3}{3}\right)\right]+g_\rho m_\rho A_0 f_\pi\frac{G_F}{\sqrt{2}}V_{tb}V_{td}^*\left(c_6+\frac{c_5}{3}\right)\frac{m_\pi^2}{(m_b+m_q)2m_q},$$

$$\eta^+=g_\rho m_\rho A_0 f_\pi\frac{G_F}{\sqrt{2}}\left\{V_{ub}V_{ud}^*\left(c_2+\frac{c_1}{3}\right)-V_{tb}V_{td}^*\left[c_4+\frac{c_3}{3}-2\left(c_6+\frac{c_5}{3}\right)\frac{m_\pi^2}{(m_b+m_q)2m_q}\right]\right\},$$

$$\eta^-=g_\rho f_\rho F_1\frac{G_F}{\sqrt{2}}\left\{V_{ub}V_{ud}^*\left(c_2+\frac{c_1}{3}\right)-V_{tb}V_{td}^*\left(c_4+\frac{c_3}{3}\right)\right\}. \tag{27}$$

For the charged B decays we obtain the results in Tables I and II. In order to show the dependence of the results on the numerical values of the different input parameters, we consider in Table I results obtained with $g=0.40$ and $h=-0.54$, which lie in the middle of the allowed ranges, while in Table II we present the results obtained with $g=0.60$ and $h=-0.70$, which represent in a sense an extreme case (we do not consider the dependence on other numerical inputs, e.g., form factors, which can introduce further theoretical uncertainty). In both cases the branching ratios are obtained with $\tau_B=1.6$ psec and, by integration over a limited section of the Dalitz plot, defined as $m_\rho-\delta\leqslant(\sqrt{t},\sqrt{t'})\leqslant m_\rho+\delta$ for $B^-\to\pi^-\pi^-\pi^+$ and $m_\rho-\delta\leqslant(\sqrt{s},\sqrt{s'})\leqslant m_\rho+\delta$ for $B^-\to\pi^-\pi^0\pi^0$. For δ we take 300 MeV. This amounts to require that two of the three pions (those corresponding to the charge of the ρ) reconstruct the ρ mass within an interval of 2δ. Numerical uncertainty due to the integration procedure is $\pm5\%$.

We can notice that the inclusion of the new diagrams (B resonances in Fig. 1) produces practically no effect for the $B^-\to\pi^-\pi^0\pi^0$ decay mode, while for $B^-\to\pi^+\pi^-\pi^-$ the effect is significant. For the choice of parameters in Table I the overall effect is an increase of 50% of the branching ratio as compared to the result obtained by the ρ resonance alone. In the case of Table II we obtain an even larger result, i.e., a total branching ratio $\mathcal{B}(B^-\to\pi^+\pi^-\pi^-)$ of 0.82×10^{-5}, in reasonable agreement with the experimental result (6) (the contribution of the ρ alone would produce a result smaller by a factor of 2). It should be observed that the events arising from the B resonances diagrams represent an irreducible background, as one can see from the sample Dalitz plot depicted in Fig. 2 for the $B^-\to\pi^+\pi^-\pi^-$ (on the axis the two $m_{\pi^+\pi^-}^2$ squared invariant masses). The contributions from the B resonances populate the whole Dalitz plot and, therefore, cutting around $t\sim t'\sim m_\rho$ significantly reduces them. Nevertheless their effect can survive the experimental cuts, since there will be enough data at the corners, where the contribution from the ρ dominates. Integrating on the whole Dalitz plot, with no cuts and including all contributions, gives

TABLE I. Effective branching ratios for the charged B decay channels into three pions for the choice of the strong coupling constants $g=0.40$ and $h=-0.54$. Cuts as indicated in the text.

Channels	ρ	$\rho+B^*$	$\rho+B^*+B_0$
$B^-\to\pi^-\pi^0\pi^0$	1.0×10^{-5}	1.0×10^{-5}	1.0×10^{-5}
$B^-\to\pi^+\pi^-\pi^-$	0.41×10^{-5}	0.58×10^{-5}	0.63×10^{-5}

TABLE II. Effective branching ratios for the charged B decay channels into three pions for the choice of the strong coupling constants $g=0.60$ and $h=-0.70$. Cuts as indicated in the text.

Channels	ρ	$\rho+B^*$	$\rho+B^*+B_0$
$B^-\to\pi^-\pi^0\pi^0$	1.1×10^{-5}	1.0×10^{-5}	1.1×10^{-5}
$B^-\to\pi^+\pi^-\pi^-$	0.41×10^{-5}	0.74×10^{-5}	0.82×10^{-5}

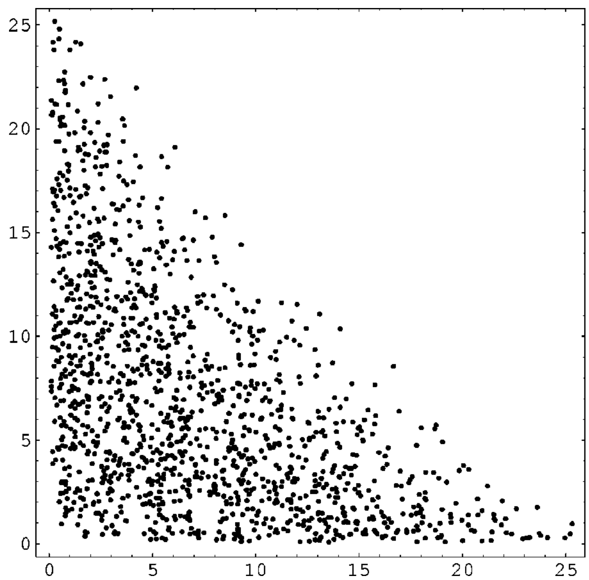

FIG. 2. Sample Dalitz plot for the decay $B^-\to\pi^+\pi^-\pi^-$. In order to show the mass distribution of the B resonance diagrams, only their contribution is taken into account for this plot.

$$X_{\rm Br}(B^-\to\pi^-\pi^0\pi^0)=1.5\times10^{-5},$$

$$X_{\rm Br}(B^-\to\pi^+\pi^-\pi^-)=1.4\times10^{-5}, \quad (28)$$

where the values of the coupling constants are as in Table I.

We now turn to the neutral B decay modes. We define effective width integrating the Dalitz plot only in a region around the ρ resonance:

$$\Gamma_{eff}(\overline{B^0}\to\rho^-\pi^+)=\Gamma(\overline{B^0}\to\pi^+\pi^-\pi^0)|_{m_\rho-\delta\le\sqrt{s}\le m_\rho+\delta}, \quad (29)$$

$$\Gamma_{eff}(\overline{B^0}\to\rho^+\pi^-)=\Gamma(\overline{B^0}\to\pi^+\pi^-\pi^0)|_{m_\rho-\delta\le\sqrt{u}\le m_\rho+\delta}, \quad (30)$$

$$\Gamma_{eff}(\overline{B^0}\to\rho^0\pi^0)=\Gamma(\overline{B^0}\to\pi^+\pi^-\pi^0)|_{m_\rho-\delta\le\sqrt{t}\le m_\rho+\delta}. \quad (31)$$

The Mandelstam variables have been defined above and again we use $\delta=300$ MeV [25]. Similar definitions hold for the B^0 decay modes. The results in Table III show basically no effect for the $\overline{B^0}\to\rho^\pm\pi^\mp$ decay channels and a moderate effect for the $\rho^0\pi^0$ decay channel. The effect in this channel is of the order of 20% (resp. 50%) for $\overline{B^0}$ (resp. B^0) decay, for the choice $g=0.60, h=-0.70$; for smaller values of the strong coupling constants the effect is reduced. Integration on the whole Dalitz plot, including all contributions, gives

$$X_{\rm Br}(\overline{B^0}\to\pi^+\pi^-\pi^0)=2.6\times10^{-5} \quad (32)$$

confirming again that most of the branching ratio is due to the ρ exchange (the first three lines of the ρ column in Table III sum up to 2.3×10^{-5}).

TABLE III. Effective branching ratios for the neutral B decay channels into $\rho\pi$ ($g=0.60$, $h=-0.70$). Cuts as indicated in the text.

Channels	ρ	$\rho+B^*$	$\rho+B^*+B_0$
$\overline{B^0}\to\rho^-\pi^+$	0.50×10^{-5}	0.52×10^{-5}	0.49×10^{-5}
$\overline{B^0}\to\rho^+\pi^-$	1.7×10^{-5}	1.7×10^{-5}	1.7×10^{-5}
$\overline{B^0}\to\rho^0\pi^0$	0.10×10^{-5}	0.15×10^{-5}	0.12×10^{-5}
$B^0\to\rho^+\pi^-$	0.49×10^{-5}	0.51×10^{-5}	0.48×10^{-5}
$B^0\to\rho^-\pi^+$	1.7×10^{-5}	1.7×10^{-5}	1.7×10^{-5}
$B^0\to\rho^0\pi^0$	0.11×10^{-5}	0.17×10^{-5}	0.15×10^{-5}

To allow the measurement of α, the experimental programs will consider the asymmetries arising from the time-dependent amplitude:

$$\mathcal{A}(t)=e^{-\Gamma/2t}\left(\cos\frac{\Delta mt}{2}A^{+-0}\pm i\sin\frac{\Delta mt}{2}\overline{A}^{+-0}\right), \quad (33)$$

where one chooses the $\pm$ sign according to the flavor of the B, and Δm is the mass difference between the two mass eigenstates in the neutral B system. Here $\overline{A}^{+-0}$ is the charge-conjugate amplitude. We have performed asymmetric integrations over the Dalitz plot for three variables: R_1, R_2, and R_3, which multiply, in the time-dependent asymmetry, respectively, 1, $\cos\Delta mt$, and $\sin\Delta mt$. We have found no significant effect due to the B^* or the B_0 resonance for R_1 and R_3. On the other hand these effects are present in R_2, but R_2 is likely to be too small to be accurately measurable.

IV. CONCLUSIONS

In conclusion our analysis shows that the effect of including B resonance polar diagrams is significant for the $B^-\to\pi^-\pi^-\pi^+$ and negligible for the other charged B decay mode. This result is of some help in explaining the recent results from the CLEO Collaboration, since we obtain

$$R=3.5\pm0.8, \quad (34)$$

to be compared with the experimental result in Eq. (8). The ρ resonance alone would produce a result up to a factor of 2 higher. Therefore we conclude that the polar diagrams examined in this paper are certainly relevant in the study of the charged B decay into three pions.

In the case of neutral B decays we have found that, as far as the branching ratios are concerned, the only decay mode where the contribution from the fake ρ's (production of a pion and the B^* or the B_0 resonance) may be significant is the neutral $\rho^0\pi^0$ decay channel. As for the time-dependent asymmetry no significant effect is found. Therefore the $B\to\pi\pi\pi$ decay channel allows an unambiguous measurement of α, with two provisos: (1) only the neutral B decay modes are considered; (2) the $\rho^0\pi^0$ final state can be disregarded from the analysis.

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[1] The BaBar Physics Book, SLAC-R-504 (1998), http://www.slac.stanford.edu/pubs/slacreports/slac-r-504.html.

[2] BELLE Technical Design Report, KEK-Report 95-1, http://bsunsrv1.kek.jp/bdocs/tdr.html; F. Takasaki, 1999, hep-ex/9912004.

[3] Y. Azimov, Phys. Rev. D **42**, 3705 (1990); B. Kayser, hep-ph/9709382; A. Dighe, I. Dunietz, and R. Fleischer, Phys. Lett. B **433**, 147 (1998).

[4] J. Charles, A. Le Yaouanc, L. Oliver, O. Pène, and J. C. Raynal, Phys. Lett. B **425**, 375 (1998); **433**, 441(E) (1998); P. Colangelo, F. De Fazio, G. Nardulli, N. Paver, and Riazuddin, Phys. Rev. D **60**, 033002 (1999).

[5] M. Gronau and D. London, Phys. Rev. Lett. **65**, 3381 (1990).

[6] H. J. Lipkin, Y. Nir, H. R. Quinn, and A. E. Snyder, Phys. Rev. D **44**, 1454 (1991).

[7] A. E. Snyder and H. R. Quinn, Phys. Rev. D **48**, 2139 (1993); H. R. Quinn and J. P. Silva, hep-ph/0001290.

[8] N. G. Deshpande, G. Eilam, Xiao-Gang He, and J. Trampetic, Phys. Rev. D **52**, 5354 (1995); B. Bajc, S. Fajfer, R. J. Oakes, T. N. Pham, and S. Prelovsek, Phys. Lett. B **447**, 313 (1999).

[9] J. Charles, Ph.D. thesis (in French), http://qcd.th.u-psud.fr/preprints_labo/1999.ps/99-31.ps.gz; S. Versille, Ph.D. thesis (in French), http://www-lpnhep.in2p3.fr/babar/public/versille/Thesis/

[10] Y. Gao and F. Würthwein, hep-ex/9904008; CLEO Collaboration, CLEO CONF 99-13.

[11] A. Deandrea, N. Di Bartolomeo, R. Gatto, and G. Nardulli, Phys. Lett. B **318**, 549 (1993).

[12] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C **34**, 103 (1987).

[13] P. Colangelo, G. Nardulli, A. Deandrea, N. Di Bartolomeo, R. Gatto, and F. Feruglio, Phys. Lett. B **339**, 151 (1994); P. Colangelo, F. De Fazio, G. Nardulli, N. Di Bartolomeo, and R. Gatto, Phys. Rev. D **52**, 6422 (1995).

[14] A. J. Buras, in *Probing the Standard Model of Particle Interactions*, edited by F. David and R. Gupta (Elsevier Science, Amsterdam, in press).

[15] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[16] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).

[17] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Rep. **281**, 145 (1997).

[18] P. Colangelo, F. De Fazio, and G. Nardulli, Phys. Lett. B **334**, 175 (1994).

[19] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Lett. B **299**, 139 (1993).

[20] A. Deandrea, R. Gatto, G. Nardulli, and A. D. Polosa, J. High Energy Phys. **021**, 9902 (1999).

[21] P. Colangelo, F. De Fazio, M. Ladisa, G. Nardulli, P. Santorelli, and A. Tricarico, Eur. Phys. J. C **8**, 81 (1999); M. Ladisa, G. Nardulli, and P. Santorelli, Phys. Lett. B **455**, 283 (1999).

[22] P. Ball, J.-M. Frère, and M. Tytgat, Phys. Lett. B **365**, 367 (1996).

[23] A. D. Polosa, in Proceedings of 11th Rencontres de Blois: Frontiers of Matter, Chateau de Blois, France, hep-ph/9909371.

[24] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998), and 1999 partial update for edition 2000 (pdg.lbl.gov).

[25] See, for example, J. Libby, talk given at the workshop on Standard Model Physics (and More) at the LHC, 1999.