NEUTRINO MASSES, MIXING AND OSCILLATIONS

S.M. Bilenky
Joint Institute for Nuclear Research, Dubna, Russia, and Dipartimento di Fisica, Università di Roma 'Tor Vergata' Via della Ricerca Scientifica 1, 00133 Roma, Italy

1. INTRODUCTION

For many years neutrino physics has been a very important branch of elementary particle physics. In the last few years the interest in neutrinos has increased. This is connected, first of all, with the success of the Super-Kamiokande experiment in which very convincing evidence in favour of oscillations of atmospheric neutrinos was obtained.

It is plausible that tiny neutrino masses and neutrino mixing are connected with the new large scale in physics. This scale determines the smallness of neutrino masses with respect to the masses of charged leptons and quarks. In such a scenario neutrinos with definite masses are truly neutral Majorana particles (quarks and leptons have charges and are Dirac particles). It is evident, however, that many new experiments are necessary to reveal the real origin of neutrino masses and mixing.

Experimental neutrino physics is a very difficult and exciting field of research, and many new ideas and methods are being proposed. At CERN and other laboratories projects of new neutrino experiments are developing. The possibility of a new neutrino facility, a neutrino factory, is being investigated in different laboratories. It is therefore a very appropriate time to discuss neutrino physics at the CERN–JINR School.

I will consider different possibilities of neutrino mixing. Then I will discuss, in some detail, neutrino oscillations in vacuum and in matter. In the last part of the lectures I will consider the present experimental situation.

I have tried to give some important results and details of the derivation of some results. I hope that they will be useful for those who wish to study the physics of massive neutrinos. More results and details can be found in Refs. [1]–[5] (books) and [6]–[17] (reviews).

Most references to original papers can be found in Ref. [17].

2. NEUTRINO MIXING

According to the Standard Model of electroweak interaction the Lagrangian of the interaction of neutrinos with other particles is given by the Charged Current (CC) and the Neutral Current (NC) Lagrangians:

$$\mathcal{L}_I^{\mathrm{CC}} = -\frac{g}{2\sqrt{2}}\, j_\alpha^{\mathrm{CC}}\, W^\alpha + \mathrm{h.c.}\ , \tag{1}$$

$$\mathcal{L}_I^{\mathrm{NC}} = -\frac{g}{2\cos\theta_W}\, j_\alpha^{\mathrm{NC}}\, Z^\alpha\,. \tag{2}$$

Here g is the electroweak interaction constant, θ_W is the weak (Weinberg) angle, and W^α and Z^α are the fields of the W^{+-} and Z^0 vector bosons. If neutrino masses are equal to zero, in this case the CC and NC interactions conserve electron L_e, muon L_μ, and tauon L_τ lepton numbers

$$\sum L_e = \mathrm{const},\ \ \sum L_\mu = \mathrm{const},\ \ \sum L_\tau = \mathrm{const}\ . \tag{3}$$

The values of the lepton numbers of charged leptons, neutrinos, and other particles are given in Table 1.

Table 1: Lepton numbers of neutrinos and charged leptons.
Lepton numbers of all other particles are equal to zero.

	L_e	L_μ	L_τ
$(\nu_e\,,\ e^-)$	$+1$	0	0
$(\nu_\mu\,,\ \mu^-)$	0	$+1$	0
$(\nu_\tau\,,\ \tau^-)$	0	0	$+1$

According to the *neutrino mixing hypothesis*, masses of neutrinos are different from zero, and the *neutrino mass term* does not conserve lepton numbers. For the fields of ν_{lL} that enter into CC and NC Lagrangians (1) and (2) we have, in this case,

$$\nu_{lL} = \sum_i U_{li}\,\nu_{iL} \ , \tag{4}$$

where ν_i is the field of the neutrino with mass m_i and U is the unitary mixing matrix.

The relation (4) leads to a violation of the lepton numbers due to small neutrino mass differences and neutrino mixing. To reveal such effects special experiments (neutrino oscillation experiments, neutrinoless double β-decay experiments, and others) are necessary. We will discuss these experiments later. Now we shall consider *different possibilities of neutrino mixing*.

Let us note first of all that relation (4) is similar to the analogous relation in the quark case. The standard CC current of quarks have the form

$$j_\alpha^{CC} = 2(\overline{u_L}\gamma_\alpha d'_L + \overline{c}_L\gamma_\alpha s'_L + \overline{t}_L\gamma_\alpha b'_L)\ . \tag{5}$$

Here

$$d'_L = \sum_{q=d,s,b} V_{uq} q_L\ , \qquad s'_L = \sum_{q=d,s,b} V_{cq} q_L\ , \qquad b'_L = \sum_{q=d,s,b} V_{tq} q_L\ , \tag{6}$$

where V is the Cabibbo–Kobayashi–Maskawa quark mixing matrix. There can be, however, a fundamental difference between the mixing of quarks and neutrino mixing. Quarks are charged four-component Dirac particles: quarks and antiquarks have different charges.

For neutrinos with definite masses there are two possibilities:

1. If the total lepton number $L = L_e + L_\mu + L_\tau$ is conserved, neutrinos with definite masses ν_i are four-component *Dirac particles* (neutrinos and antineutrinos differ by the sign of L);
2. If there are no conserved lepton numbers, neutrinos with definite masses ν_i are two-component *Majorana particles* (there are no quantum numbers in this case that allow us to distinguish neutrinos from antineutrinos).

The nature of the neutrino masses and the character of the neutrino mixing is determined by the *neutrino mass term*.

2.1 Dirac neutrinos

If the neutrino mass term is generated by the same standard Higgs mechanism, which is responsible for the mass generation of quarks and charged leptons, then for the neutrino mass term we have

$$\mathcal{L}^{\rm D} = -\sum_{l,l'} \overline{\nu_{l'R}}\, M^{\rm D}_{l'l}\, \nu_{lL} + {\rm h.c.} \tag{7}$$

where M^D is the complex 3×3 matrix and ν_{lR} is the right-handed singlet. In the case of the mass term (7) the total Lagrangian is invariant under global gauge invariance

$$\nu_{lL} \to e^{i\alpha}\nu_{lL}\,, \qquad \nu_{lR} \to e^{i\alpha}\nu_{lR}\,, \qquad l \to e^{i\alpha} l\,, \tag{8}$$

where α is a constant that does not depend on the flavour index l. The invariance under the transformation (8) means that the total lepton number $L = L_e + L_\mu + L_\tau$ is conserved

$$\sum L = \mathrm{const}\,. \tag{9}$$

Now let us diagonalize the mass term (7). The complex matrix M^{D} can be diagonalized by biunitary transformation

$$M^{\mathrm{D}} = V m U^\dagger\,, \tag{10}$$

where $V^\dagger V = 1$, $U^\dagger U = 1$, and $m_{ik} = m_i \delta_{ik}\,, \quad m_i > 0$.

With the help of (10), from (7) for the neutrino mass term we obtain the standard expression

$$\mathcal{L}^{\mathrm{D}} = -\sum_{l',l,i} \overline{\nu_{l'R}}\, V_{l'i}\, m_i (U^\dagger)_{il}\, \nu_{lL} + \mathrm{h.c.} = -\sum_{i=1}^{3} m_i \overline{\nu_i} \nu_i\,. \tag{11}$$

Here

$$\nu_i = \nu_{iL} + \nu_{iR} \qquad (i = 1, 2, 3)$$

and

$$\nu_{iL} = \Sigma_l (U^\dagger)_{il} \nu_{lL}$$
$$\nu_{iR} = \Sigma_l (V^\dagger)_{il} \nu_{lR}\,.$$

For the neutrino mixing we have

$$\nu_{lL} = \sum_i U_{li}\, \nu_{iL}\,. \tag{12}$$

Processes in which the total lepton number is conserved, like $\mu \to e + \gamma$ and others, are, in principle, allowed in the case of mixing Dirac massive neutrinos. It can be shown, however, that the probabilities of such processes are much smaller than the experimental upper bounds.

The neutrinoless double β–decay,

$$(A, Z) \to (A, Z + 2) + e^- + e^-\,,$$

due to the conservation of the total lepton number, is forbidden in the case of Dirac massive neutrinos.

2.2 Majorana neutrinos

Neutrino mass terms that are generated in the framework of the models beyond the Standard Model, like the Grand Unified SO(10) Model, do not conserve lepton numbers L_e, L_μ and L_τ. Let us build the most general neutrino mass term that does not conserve L_e, L_μ and L_τ.

The neutrino mass term is a linear combination of the products of left-handed and right-handed components of neutrino fields. Notice that $(\nu_L)^C = C(\overline{\nu}_L)^T$ is the right-handed component and $(\nu_R)^C = C(\overline{\nu}_R)^T$ is the left-handed component. Here C is the charge conjugation matrix that satisfies the relations $C\gamma_\alpha^T C^{-1} = -\gamma_\alpha$, $C^T = -C$, and $C^\dagger C = 1$. [1]

[1] In fact, L and R components satisfy the relations

$$\frac{1+\gamma_5}{2}\nu_L = 0 \qquad \frac{1-\gamma_5}{2}\nu_R = 0\,.$$

From the first of these relations we have $\overline{\nu}_L(1-\gamma_5)/2 = 0$. Furthermore, from this last relation we obtain $[(1-\gamma_5)/2]^T \overline{\nu}_L^T = 0$. Multiplying this relation by the matrix C from the left, and taking into account that $C\gamma_5^T C^{-1} = \gamma^5$, we have $[(1-\gamma_5)/2](\nu_L)^C = 0$. Thus, $(\nu_L)^C$ is the right-handed component. Analogously we can show that $(\nu_R)^C$ is the left-handed component.

The most general Lorentz-invariant neutrino mass term in which flavour neutrino fields ν_{lL} and right-handed singlet fields ν_{lR} enter has the following form

$$\mathcal{L}^{\mathrm{D-M}} = -\frac{1}{2}\overline{(n_L)^C}\, M n_L + \mathrm{h.c.} \tag{13}$$

Here

$$n_L = \begin{pmatrix} \nu'_L \\ (\nu'_R)^C \end{pmatrix} \quad \text{with} \quad \nu'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} \quad \text{and} \quad \nu'_R = \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix}, \tag{14}$$

M is a complex 6×6 matrix. Taking into account that $\overline{(\nu_L)^C} = -\nu_L^T C^{-1}$ we have

$$\mathcal{L}^{\mathrm{D-M}} = \frac{1}{2} n_L^T\, \mathcal{C}^{-1}\, M\, n_L + \mathrm{h.c.}\ . \tag{15}$$

From this expression it is obvious that there is no global gauge invariance in the case of the mass term (13), i.e. the mass term (13) does not conserve lepton numbers.

The matrix M is symmetric. In fact, taking into account the commutation properties of fermion fields, we have

$$n_L^T\, \mathcal{C}^{-1}\, M\, n_L = -n_L^T\, (\mathcal{C}^T)^{-1}\, M^T\, n_L = n_L^T\, \mathcal{C}^{-1}\, M^T\, n_L\,. \tag{16}$$

From this relation it follows that

$$M^T = M\ .$$

The symmetric 6×6 matrix can be presented in the form

$$M = \begin{pmatrix} M_L & (M_{\mathrm{D}})^T \\ M_{\mathrm{D}} & M_R \end{pmatrix} \tag{17}$$

where $M_L = M_L^T$, $M_R = M_R^T$, and M^D are 3×3 matrices. With the help of (17) for the mass term (15) we have

$$\mathcal{L}^{\mathrm{D-M}} = \mathcal{L}_L^{\mathrm{M}} + \mathcal{L}^{\mathrm{D}} + \mathcal{L}_R^{\mathrm{M}}\,. \tag{18}$$

Here $\mathcal{L}^D$ is the Dirac mass term that we considered before, and the new terms

$$\mathcal{L}_L^{\mathrm{M}} = -\frac{1}{2}\sum_{l',l} \overline{(\nu_{l'L})^c}\, M_{l'l}^L\, \nu_{lL} + \mathrm{h.c.}\,, \tag{19}$$

$$\mathcal{L}_R^{\mathrm{M}} = -\frac{1}{2}\sum_{l',l} \overline{(\nu_{l'R})^c}\, M_{l'l}^R\, \nu_{lR} + \mathrm{h.c.}\,, \tag{20}$$

which do not conserve lepton numbers are called left-handed and right-handed Majorana mass terms, respectively. The mass term (13) is called the Dirac–Majorana mass term.

A symmetrical matrix can be diagonalized with the help of unitary transformation

$$M = (U^\dagger)^T m U^\dagger\,.$$

Here U is the unitary matrix and $m_{ik} = m_i \delta_{ik}$, $m_i > 0$. Using the relation (11) we can write the mass term (15) in the standard form

$$\mathcal{L}^{\mathrm{D-M}} = -\frac{1}{2}\overline{(U^\dagger n_L)}^C\, m U^+ n_L + \mathrm{h.c.} = -\frac{1}{2}\overline{\nu} m \nu = -\frac{1}{2}\sum_{i=1}^{6} m_i \overline{\nu}_i \nu_i\ , \tag{21}$$

where

$$\nu = U^{+} n_L + (U^{+} n_L)^C = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \vdots \\ \nu_6 \end{pmatrix} . \tag{22}$$

Thus the fields ν_i (i=1,2...6) are the fields of neutrinos with mass m_i. From (22) it follows that the fields ν_i satisfy the *Majorana condition*

$$\nu_i^C = \nu_i \,, \tag{23}$$

Let us now obtain the relation that connects the left-handed flavour fields ν_{lL} with the massive fields ν_{iL}. From (22) for the left-handed components we have

$$n_L = U \nu_L \,. \tag{24}$$

From this relation for the flavour field ν_{lL} it follows that

$$\nu_{lL} = \sum_{i=1}^{6} U_{li} \nu_{iL} \qquad (l = e, \mu, \tau) \,. \tag{25}$$

Thus, in the case of the Dirac–Majorana mass term, the flavour fields are linear combinations of left-handed components of six massive Majorana fields. From (25) it follows that the fields ν_{lR}^C are orthogonal linear combinations of the same massive Majorana fields

$$(\nu_{lR})^C = \sum_{i=1}^{6} U_{\bar{l}i} \nu_{iL} \,. \tag{26}$$

In the case of Majorana field particles and antiparticles, the quanta of the field are identical. In fact, for fermion fields $\nu(x)$ we have, in general

$$\nu(x) = \int \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2p^0}} \left[c_r(p) u^r(p) e^{-ipx} + d_r^{\dagger}(p) C \left[\overline{u}^r(p) \right]^T e^{ipx} \right] d^3p \,, \tag{27}$$

where $c_r(p)$ $\left(d_r^{\dagger}(p)\right)$ is the operator of the absorption of the particle (creation of antiparticle) with momentum p and helicity r. If the field $\nu(x)$ satisfies the Majorana condition (23), then we have

$$c_r(p) = d_r(p) \,. \tag{28}$$

Let us stress that it is natural for the neutrinos with definite masses in the case of the Dirac–Majorana mass term to be Majorana neutrinos: in fact, there are no conserved quantum numbers that could allow us to distinguish particles from antiparticles.

2.3 The simplest case of one generation (Majorana neutrinos)

It is instructive to consider in detail the Dirac–Majorana mass term in the simplest case of one generation. We have

$$\begin{aligned} \mathcal{L}^{\mathrm{D-M}} &= -\frac{1}{2} m_L \, (\overline{\nu}_L)^c \, \nu_L - m_{\mathrm{D}} \, \overline{\nu}_R \, \nu_L - \frac{1}{2} m_R \, \overline{\nu}_R \, (\nu_R)^c + \text{h.c.} \\ &= -\frac{1}{2} \, (\overline{n}_L)^c \, M \, n_L + \text{h.c.} \,, \end{aligned} \tag{29}$$

where

$$n_L \equiv \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix} , \qquad M \equiv \begin{pmatrix} m_L & m_{\mathrm{D}} \\ m_{\mathrm{D}} & m_R \end{pmatrix} . \tag{30}$$

Let us assume that the parameters m_L, m_R, and m_D are real (the case of CP invariance). In order to diagonalize the mass term (29) let us write the matrix M in the form

$$M = \frac{1}{2}\,\mathrm{Tr}M + \underline{M}\ , \tag{31}$$

where Tr $M = m_L + m_D$ and

$$\underline{M} = \begin{pmatrix} -\frac{1}{2}(m_R - m_L) & m_\mathrm{D} \\ m_\mathrm{D} & \frac{1}{2}(m_R - m_L) \end{pmatrix} . \tag{32}$$

For the symmetrical real matrix we have

$$\underline{M} = \mathcal{O}\,\underline{m}\mathcal{O}^T . \tag{33}$$

Here

$$\mathcal{O} = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix} \tag{34}$$

is an orthogonal matrix, and $\underline{m}_{ik} = \underline{m}_i\delta_{ik}$, where

$$\underline{m}_{1,2} = \mp\frac{1}{2}\sqrt{(m_R - m_L)^2 + 4m_D^2} \tag{35}$$

are eigenvalues of the matrix $\underline{M}$.

From (33), (34), and (35) for the parameters $\cos\vartheta$ and $\sin\vartheta$ we easily find the following expressions

$$\cos 2\vartheta = \frac{m_R - m_L}{\sqrt{(m_R - m_L)^2 + 4\,m_D^2}}\,, \quad \tan 2\vartheta = \frac{2m_D}{(m_R - m_L)}\ . \tag{36}$$

For the matrix M from (33) and (35) we have

$$M = Om'O^T\ ,$$

where

$$m'_{1,2} = \frac{1}{2}\,(m_R + m_L) \mp \sqrt{(m_R - m_L)^2 + 4\,m_D^2}\ . \tag{37}$$

The eigenvalues m'_i can be positive or negative. Let us write

$$m'_i = m_i\eta_i\ , \tag{38}$$

where $m_i = |m_i|$ and η_i is the sign of the i-eigenvalue. With the help of (33) and (38) we have

$$M = (U^\dagger)^T mU^\dagger\ .$$

Here

$$U^\dagger = \sqrt{\eta}O^T\ ,$$

where $\sqrt{\eta}$ takes the values 1 and i.

Now using the general formulas (21) and (22) for the mass term we have

$$\mathcal{L}^{D-M} = -\frac{1}{2}\sum_{i=1,2} m_i\overline{\nu}_i\nu_i\ . \tag{39}$$

Here $\nu_i = \nu_i^C$ is the field of the Majorana particles with mass m_i. The fields ν_L and $(\nu_R)^C$ are connected with massive fields by the relation

$$\begin{pmatrix} \nu_L \\ (\nu_R)^C \end{pmatrix} = U \begin{pmatrix} \nu_{1L} \\ (\nu_{2L} \end{pmatrix}\,, \tag{40}$$

where $U = O(\sqrt{\eta})^*$ is a 2×2 mixing matrix.

Let us consider now three special cases.

1. No mixing
   Assume m_D = 0. In this case $\theta = 0$, $m_1 = m_L$, $m_2 = m_R$, and $\eta = 1$ (assuming that m_L and m_R are positive). From (40) we have

$$\nu_L = \nu_{1L} \qquad (\nu_R)^C = \nu_{2L} \ . \tag{41}$$

   Thus, if $m_{\rm D} = 0$ there is no mixing. For the Majorana fields ν_1 and ν_2 we have

$$\nu_1 = \nu_L + (\nu_L)^C \tag{42}$$

$$\nu_2 = \nu_R + (\nu_R)^C \, . \tag{43}$$

2. Maximal mixing
   Let us assume that $m_R = m_L$, $m_D \neq 0$. From Eqs. (36), (37), and (40) we have

$$\theta = \frac{\pi}{4} \, , \qquad m_{1,2} = m_L \mp m_D \, , \tag{44}$$

   (assuming $|m_D| < m_L$) and

$$\nu_L = \frac{1}{\sqrt{2}}\nu_{1L} + \frac{1}{\sqrt{2}}\nu_{2L} \, ; \qquad (\nu_R)^C = -\frac{1}{\sqrt{2}}\nu_{1L} + \frac{1}{\sqrt{2}}\nu_{2L} \ . \tag{45}$$

   Thus if the diagonal elements of the mass matrix M are equal, then we have maximal mixing.

3. See-saw mechanism of neutrino mass generation
   Let us assume that $m_L = 0$ and

$$m_D \ll m_R \, . \tag{46}$$

   From (35) and (37) we have, in this case,

$$m_1 \simeq \frac{m_D^2}{m_R} \, , \quad m_2 \simeq m_R \, , \quad \theta \simeq \frac{m_D}{m_R} \quad (\eta_1 = -1, \eta_2 = 1) \, . \tag{47}$$

   Neglecting terms linear in $m_D/m_R \ll 1$, from (40) we have

$$\nu_L \simeq -i\nu_{1L} \, , \qquad (\nu_R)^C \simeq \nu_{2L} \, . \tag{48}$$

   For the Majorana fields we have

$$\nu_1 \simeq i\nu_L - i(\nu_L)^C, \qquad \nu_2 = \nu_R + (\nu_R)^C \, . \tag{49}$$

   Thus if the condition (46) is satisfied, in the spectrum of masses of Majorana particles there is one light particle with the mass $m_1 \ << \ m_D$, and one heavy particle with the mass $m_1 \ >> \ m_D$. The condition $m_L \ = \ 0$ means that the lepton number is violated only by the right-handed term $-\frac{1}{2}m_R\overline{\nu}_R(\nu_R)^C$ which is characterized by the large mass m_R. It is natural to assume that the parameter m_D which characterizes the Dirac term $-m_D\overline{\nu}_R\nu_L$ is of the order of lepton or quark masses. The mass of the light Majorana neutrino m_1 will in this case be much smaller than the mass of the lepton or the quark. This is the famous *see-saw mechanism*. This mechanism connects the smallness of the neutrino masses with respect to the masses of other fundamental fermions with the violation of the lepton numbers at very large scale (usually $m_D \simeq M_{GUT} \simeq 10^{16}$ GeV.
   With the see-saw for three families in the spectrum of masses of Majorana particles there are three light masses m_1, m_2, m_3 (masses of neutrinos) and three very heavy masses M_1, M_2, M_3. Masses of neutrinos are connected with the masses of heavy Majorana particles by the see-saw relation

$$m_i \simeq \frac{(m_{\rm f}^i)^2}{M_i} \ll m_{\rm f}^i \qquad (i = 1, 2, 3) \, , \tag{50}$$

   where $m_{\rm f}^i$ is the mass of the lepton or quark in the i-family. The see-saw mechanism is a plausible explanation of the experimentally observed smallness of neutrino masses. Let us stress that if neutrino masses are of the see-saw origin then
   a. neutrinos with definite masses are Majorana particles;
   b. there are three massive neutrinos;
   c. there must be a hierarchy of neutrino masses $m_1 \ll m_2 \ll m_3$.

3. NEUTRINO OSCILLATIONS

The most important consequences of neutrino mixing are so-called *neutrino oscillations*. Neutrino oscillations were first considered by B. Pontecorvo many years ago in 1957–58. Only one type of neutrino was known at that time and there was a general belief that the neutrino is a massless two-component particle. B. Pontecorvo drew attention to the fact that there is no known principle that requires the neutrino to be massless (like gauge invariance for the photon) and that the investigation of neutrino oscillations is a very sensitive method for searching for effects of small neutrino masses. We will consider here in detail the phenomenon of neutrino oscillations.

Let us assume that there is neutrino mixing

$$\nu_{\alpha L} = \sum_i U_{\alpha i}\, \nu_{iL} \,, \tag{51}$$

where $U^\dagger U = 1$ and ν_i is the field of the neutrino (Dirac or Majorana) with the mass m_i. The fields $\nu_{\alpha L}$ in (51) are flavour fields ($\alpha = e, \mu, \tau$) and in general also sterile ones ($\alpha = s_1, ...$). Let us assume that neutrino mass differences are small and different neutrino masses cannot be resolved in neutrino production and detection processes.

For the state of the neutrino with momentum $\vec{p}$ we have

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}\, |\nu_i\rangle \,, \tag{52}$$

where $|\nu_i\rangle$ is the vector of the state of the neutrino with momentum $\vec{p}$, energy

$$E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p} \qquad (p \gg m_i) \,, \tag{53}$$

and (up to the terms m_i^2/p^2) helicity is equal to -1. If at the initial time $t = 0$ the state of the neutrino is $|\nu_\alpha\rangle$, at the time t for the neutrino state we have

$$|\nu_\alpha\rangle_t = \sum_i U^*_{\alpha i}\, e^{-iE_i t}\, |\nu_i\rangle \ . \tag{54}$$

The vector $|\nu_\alpha\rangle$ is the superposition of the states of all types of neutrino. In fact, from (52), using unitarity of the mixing matrix, we have

$$|\nu_i\rangle = \sum_{\alpha'} |\nu_{\alpha'}\rangle U_{\alpha' i} \ . \tag{55}$$

From (54) and (55) we have

$$|\nu_\alpha\rangle_t = \sum_{\alpha'} |\nu_{\alpha'}\rangle \mathcal{A}_{\nu'_\alpha;\nu_\alpha}(t) \ , \tag{56}$$

where

$$\mathcal{A}_{\nu'_\alpha;\nu_\alpha}(t) = \sum_i U_{\alpha' i} e^{-iE_i t} U^*_{\alpha i} \ , \tag{57}$$

is the amplitude of the transition $\nu_\alpha \to \nu_{\alpha'}$ at the time t. The transition amplitude $\mathcal{A}_{\alpha';\alpha}(t)$ has a simple meaning: the term $U^*_{\alpha i}$ is the amplitude of the transition from the state $|\nu_\alpha\rangle$ to the state $|\nu_i\rangle$; the term $e^{-iE_i t}$ describes the evolution in the state with energy E_i; the term $U_{\alpha' i}$ is the transition amplitude from the state $|\nu_i\rangle$ to the state $|\nu'_\alpha\rangle$.

The different $|\nu_i\rangle$ gives a coherent contribution to the amplitude $\mathcal{A}_{\nu'_\alpha;\nu_\alpha}(t)$. From (57) it follows that the transitions between the different states can take place only if: i) at least two neutrino masses are different; ii) the mixing matrix is non-diagonal. In fact, if all neutrino masses are equal we have $a(t) = e^{-iEt} \sum U_{\alpha' i} U^*_{\alpha i} = e^{-iEt} \delta_{\alpha' \alpha}$. If the mixing matrix is diagonal (no mixing), we have $\mathcal{A}_{\nu'_\alpha;\nu_\alpha}(t) = e^{-iE_\alpha t} \delta_{\alpha' \alpha}$.

Let us numerate the neutrino masses in such a way that $m_1 < m_2 < ... < m_n$. For the transition probability, from (57), we have the following expression:

$$P_{\nu_\alpha \to \nu_{\alpha'}} = \left| \sum_i U_{\alpha' i} \, \left[\left(e^{-i(E_i - E_1)t} - 1 \right) + 1 \right] \, U^*_{\alpha i} \right|^2 \tag{58}$$

$$= \left| \delta_{\alpha\alpha'} + \sum_i U_{\alpha' i} \, U^*_{\alpha i} \left(e^{-i \, \Delta m^2_{i1} \, \frac{L}{2p}} - 1 \right) \right|^2 ,$$

where $\Delta m^2_{i1} = m_i^2 - m_1^2$ and $L \simeq t$ is the distance between the neutrino source and the neutrino detector. Thus the neutrino transition probability depends on the ratio L/E, the range of values of which is determined by the conditions of an experiment.

It follows from Eq. (58) that the transition probability usually depends on $(n-1)$ the neutrino mass squared differences, and parameters that characterize the mixing matrix U. The $n \times n$ matrix U is characterized by $n_\theta = n(n-1)/2$ angles. The number of phases for Dirac and Majorana cases is different. If the neutrino with definite masses ν_i are Dirac particles the number of phases is equal to $n_\phi^D = (n-1)(n-2)/2$. If the ν_i are Majorana particles the number of phases is equal to $n_\phi^{M_j} = n(n-1)/2$.

It should be noted that from (58) it follows that the transition probability is invariant under the transformation

$$U_{\alpha i} \to e^{-i\beta_\alpha} \, U_{\alpha i} \, e^{i\alpha_i} \, , \tag{59}$$

where β_α and α_i are arbitrary real phases. From (59) it follows that the number of phases that enter into the transition probability is equal to $n_\phi = (n-1)(n-2)/2$ in both the Dirac and Majorana cases. We come to the conclusion that additional Majorana phases do not enter into the transition probability. Thus, by investigating neutrino oscillations it is impossible to distinguish between Dirac neutrinos and Majorana neutrinos.

Let us now consider oscillations of antineutrinos. For the vector of the state of the antineutrino with momentum $\vec{p}$ from (51) we have

$$|\overline{\nu}_\alpha\rangle = \sum_i U_{\alpha i} |\overline{\nu}_i\rangle \qquad (\text{Dirac case}) \, , \tag{60}$$

$$|\overline{\nu}_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle \qquad (\text{Majorana case}) \, , \tag{61}$$

where $|\overline{\nu}_i\rangle$ ($|\nu_i\rangle$) is the state of the antineutrino (neutrino) with momentum $\vec{p}$, energy $E_i = \sqrt{p^2 + m_i^2} \simeq p + m_i^2/2p$, and helicity equal to +1 (up to m_i^2/p^2 terms).

In analogy with (57), for the amplitude of the transition $\overline{\nu}_\alpha \to \overline{\nu}_{\alpha'}$ in both the Dirac and Majorana cases we have

$$\mathcal{A}_{\overline{\nu}_{\alpha'};\overline{\nu}_\alpha}(t) = \sum_i U^*_{\alpha' i} e^{-iE_i t} U_{\alpha i} \; . \tag{62}$$

If we compare (57) and (62) we come to the conclusion that

$$\mathcal{A}_{\overline{\nu}_{\alpha'};\overline{\nu}_\alpha}(t) = \mathcal{A}_{\nu_\alpha;\nu_{\alpha'}}(t) \, . \tag{63}$$

Thus for the transition probabilities we have the following relation

$$\mathrm{P}(\nu_\alpha \to \nu_{\alpha'}) = \mathrm{P}(\overline{\nu}_{\alpha'} \to \overline{\nu}_\alpha) \, . \tag{64}$$

This relation is the consequence of CPT invariance. If CP invariance in the lepton sector takes place, then for Dirac neutrinos we have

$$U^*_{\alpha i} = U_{\alpha i} \, , \tag{65}$$

whilst for Majorana neutrinos, from CP invariance, we have

$$U_{\alpha i}\eta_i = U^*_{\alpha i}\ , \tag{66}$$

where $\eta_i = \pm i$ is the CP parity of the Majorana neutrino with mass m_i. From (57), (63), (65), and (66) it follows that in the case of CP invariance we have

$$\mathrm{P}(\nu_\alpha \to \nu'_\alpha) = \mathrm{P}(\overline{\nu}_\alpha \to \overline{\nu}'_\alpha)\,. \tag{67}$$

Let us go back to Eq. (58). It is obvious from this equation that if the conditions of an experiment are such that $\Delta m^2_{i1}\frac{L}{p} \ll 1$ for all i, then neutrino oscillations cannot be observed. To observe neutrino oscillations it is necessary for at least one neutrino mass squared difference the condition $\Delta m^2\frac{L}{p} \gtrsim 1$ to be satisfied. We will discuss this condition later.

3.1 Two neutrino oscillations

Let us consider in detail the simplest case of oscillations between two neutrinos $\nu_\alpha \leftrightarrows \nu_{\alpha'}$ ($\alpha' \neq \alpha; \alpha, \alpha'$ are equal to μ, e or τ, μ,...). The index i in Eq. (58) takes the values 1 and 2, and for the transition probability we have

$$\mathrm{P}(\nu_\alpha \to \nu'_\alpha) = |\delta_{\alpha'\alpha} + U_{\alpha'_2}U^*_{\alpha 2}(e^{-i\Delta m^2_{21}\frac{L}{2p}} - 1)|^2\ . \tag{68}$$

For $\alpha' \neq \alpha$ we have from (68)

$$\mathrm{P}(\nu_\alpha \to \nu'_\alpha) = \mathrm{P}(\nu_{\alpha'} \to \nu_\alpha) = \frac{1}{2}\mathrm{A}_{\alpha'\alpha}(1 - \cos\Delta m^2\frac{L}{2p})\ . \tag{69}$$

Here the amplitude of oscillations is equal to

$$\mathrm{A}_{\alpha';\alpha} = 4|U_{\alpha'2}|^2|U_{\alpha 2}|^2 \tag{70}$$

and $\Delta m^2 = m_2^2 - m_1^2$. Owing to the unitarity of the mixing matrix

$$|U_{\alpha 2}|^2 + |U_{\alpha' 2}|^2 = 1 \qquad (\alpha' \neq \alpha)\ . \tag{71}$$

Let us introduce the mixing angle θ

$$|U_{\alpha 2}|^2 = \sin^2\theta \qquad |U_{\alpha' 2}|^2 = \cos^2\theta\ . \tag{72}$$

Thus the oscillation amplitude $\mathrm{A}_{\alpha';\alpha}$ is equal to

$$\mathrm{A}_{\alpha';\alpha} = \sin^2 2\theta\ . \tag{73}$$

The survival probabilities $\mathrm{P}(\nu_\alpha \to \nu_\alpha)$ and $\mathrm{P}(\nu_{\alpha'} \to \nu_{\alpha'})$ can be obtained from (68) or from (the condition of) the conservation of the total probability $\mathrm{P}(\nu_\alpha \to \nu_\alpha) + \mathrm{P}(\nu_\alpha \to \nu_{\alpha'}) = 1$. We have

$$\mathrm{P}(\nu_\alpha \to \nu_\alpha) = \mathrm{P}(\nu_{\alpha'} \to \nu_{\alpha'}) = 1 - \frac{1}{2}\sin^2 2\theta(1 - \cos\frac{\Delta m^2 L}{2p})\ . \tag{74}$$

Thus in the case of two neutrinos the transition probabilities are characterized by two parameters, $\sin^2 2\theta$ and Δm^2.

It should be noted that in the case of transitions between two neutrinos only moduli of the elements of the mixing matrix enter into the expressions for the transition probabilities. This means that in this case the CP relation (64) is satisfied automatically. Thus, in order to observe effects of CP violation in the lepton sector transitions between three neutrinos must take place (this is similar to the quark case: for two families of quarks CP is conserved because of unitarity of the mixing matrix).

We also note that the expression (69) for the transition probability can be written in the form

$$\mathrm{P}(\nu_\alpha \to \nu_{\alpha'}) = \frac{1}{2}\sin^2 2\theta \left(1 - \cos 2\pi \frac{L}{L_0}\right) \,, \tag{75}$$

where

$$L_0 = 4\pi \frac{E}{\Delta m} \tag{76}$$

is the oscillation length. The expression (69) is written in the units $\hbar = c = 1$. We can write it in the form

$$\mathrm{P}(\nu_\alpha \to \nu_{\alpha'}) = \frac{1}{2}\sin^2 2\theta \left(1 - \cos\, 2.54 \Delta m^2 \frac{E}{L}\right) \,, \tag{77}$$

where Δm^2 is the neutrino mass squared difference in eV^2, L is the distance in m (km), and E is the neutrino energy in MeV (GeV). For the oscillation length we have

$$L_0 = 2.47 \frac{E(\mathrm{MeV})}{\Delta m^2(\mathrm{eV}^2)}\,\mathrm{m}\,. \tag{78}$$

Equations (69) and (74) describe periodical transitions (oscillations) between different types of neutrinos due to the difference of neutrino masses and neutrino mixing. The transition probability depends periodically on L/E. At the values of L/E at which the condition $2.54\,\Delta m^2(L/E) = \pi(2n+1) \quad (n = 0, 1, ...)$ is satisfied, the transition probability is equal to the maximal value $\sin^2 2\theta$. If the condition $2.54\,\Delta m^2(L/E) = 2\pi n$ is satisfied, the transition probability is equal to zero.

In order to see neutrino oscillations the parameter Δm^2 must be large enough for the condition $\Delta m^2(L/E) \geq 1$ to be satisfied. This condition allows us to estimate the minimal value of the parameter Δm^2 which can be revealed in an experiment when searching for neutrino oscillations. For short and long baseline experiments with accelerator (reactor) neutrinos for $\Delta m^2_{\mathrm{min}}$ we have, respectively, 10^{-1} eV^2, 10^{-2}–10^{-3} eV^2 (10^{-1}–10^{-2} eV^2, 10^{-2}–10^{-3} eV^2). For atmospheric and solar neutrinos for $\Delta m^2_{\mathrm{min}}$ we have 10^{-2}–10^{-3} eV^2 and 10^{-10}–10^{-11} eV^2, respectively. It should be noted that in the case of $\Delta m^2(L/E) \ll 1$, due to averaging over the neutrino spectrum and over distances between neutrino production and detection points, the term $\cos \Delta m^2(L/2p)$ in the transition probability disappears, and the averaged transition probabilities are given by $\overline{\mathrm{P}}(\nu_\alpha \to \nu_{\alpha'}) = \frac{1}{2}\sin^2 2\theta$ and $\overline{\mathrm{P}}(\nu_\alpha \to \nu_\alpha) = 1 - \frac{1}{2}\sin^2 2\theta$.

3.2 Three neutrino oscillations in the case of neutrino mass hierarchy

Two neutrino transition probabilities (69) and (74) are usually used for the analysis of experimental data. Let us now consider the case of the transitions between three flavour neutrinos.

General expressions for transition probabilities between three neutrino types are characterized by six parameters and have a rather complicated form. We will consider the case of the hierarchy of neutrino masses

$$m_1 \ll m_2 \ll m_3 \,,$$

which corresponds to the oscillations of solar and atmospheric neutrinos [bearing in mind that Δm^2_{21} can be relevant for oscillations of solar neutrinos and Δm^2_{31} can be relevant for oscillations of atmospheric neutrinos, from the analysis of the experimental data it follows that $\Delta m^2_{\mathrm{solar}} \simeq 10^{-5}$ eV^2 (or 10^{-10} eV^2) and $\Delta m^2_{\mathrm{atm}} \simeq 10^{-3}$ eV^2 (described later)]. We will see that transition probabilities have, in this case, the rather simple two-neutrino form.

Let us consider neutrino oscillations in experiments for which the largest neutrino mass squared difference Δm^2_{31} is relevant. For such experiments

$$\Delta m^2_{12} \frac{L}{2p} \ll 1 \,, \tag{79}$$

and for the probability of the transition $\nu_\alpha \to \nu_{\alpha'}$, from (58) we obtain the following expression

$$\mathrm{P}(\nu_\alpha \to \nu_{\alpha'}) = \left|\delta_{\alpha'\alpha} + U_{\alpha' 3} U^*_{\alpha 3} \left(e^{-i\Delta m^2_{31}\,\frac{L}{2p}} - 1\right)\right|^2 . \tag{80}$$

For the transition probability $\nu_\alpha \to \nu_{\alpha'}$ $(\alpha' \neq \alpha)$ from (80) we have

$$\mathrm{P}(\nu_\alpha \to \nu'_\alpha) = \frac{1}{2}\mathrm{A}_{\alpha';\alpha}\left(1 - \cos\Delta m^2_{31}\,\frac{L}{2p}\right) , \tag{81}$$

where the amplititude of oscillations is given by

$$\mathrm{A}_{\alpha';\alpha} = 4|U_{\alpha' 3}|^2 |U_{\alpha 3}|^2 . \tag{82}$$

Using unitarity of the mixing matrix, for the survival probability we obtain, from (81) and (82),

$$\mathrm{P}(\nu_\alpha \to \nu_\alpha) = 1 - \sum_{\alpha' \neq \alpha} \mathrm{P}(\nu_\alpha \to \nu_{\alpha'}) = 1 - \frac{1}{2}\mathrm{B}_{\alpha;\alpha}\left(1 - \cos\Delta m^2_{31}\frac{L}{2p}\right) , \tag{83}$$

where

$$\mathrm{B}_{\alpha;\alpha} = 4|U_{\alpha 3}|^2(1 - |U_{\alpha 3}|^2) . \tag{84}$$

It is natural that Eqs. (81) and (82) have the same dependence on the parameter L/E as the standard two-neutrino formulae (68) and (74): only the largest Δm^2 is relevant for the oscillations. The oscillation amplitudes $\mathrm{A}_{\alpha;\alpha}$ and $\mathrm{B}_{\alpha;\alpha}$ depend on the moduli squared of the mixing matrix elements that connect neutrino flavours with the heaviest neutrino ν_3. Furthermore, from the unitarity of the mixing matrix it follows that

$$|U_{e3}|^2 + |U_{\mu 3}|^2 + |U_{\tau 3}|^2 = 1 . \tag{85}$$

Thus, in the three-neutrino case with hierarchy of the neutrino masses, the transition probabilities in experiments for which Δm^2_{31} is relevant are described by three parameters: Δm^2_{31}, $|U_{e3}|^2$, and $|U_{\mu 3}|^2$ (remember that in the two-neutrino case there are two parameters, Δm^2 and $\sin^2 2\theta$).

Since only the moduli of the elements of the mixing matrix enter into the transition probabilities, the relation

$$\mathrm{P}(\nu_\alpha \to \nu_{\alpha'}) = \mathrm{P}(\overline{\nu}_\alpha \to \overline{\nu}_{\alpha'}) \tag{86}$$

holds (as in the two-neutrino case). Thus the violation of the CP invariance in the lepton sector cannot be revealed in the case of three neutrinos with mass hierarchy. Notice that the relation

$$\mathrm{P}(\nu_\alpha \to \nu_\alpha) = \mathrm{P}(\nu_{\alpha'} \to \nu_{\alpha'}) , \tag{87}$$

which takes place in the case of two-neutrino oscillations, is not valid in the three-neutrino case.

Let us now consider neutrino oscillations in the case of experiments for which Δm^2_{21} is relevant ($\Delta m^2_{21}\,\frac{L}{2p} \gtrsim 1$). From (57) for the survival probability we obtain, in this case, the following expression

$$\mathrm{P}(\nu_\alpha \to \nu_\alpha) = \left|\sum_{i=1,2} |U_{\alpha i}|^2 e^{-i\Delta m^2_{i1}\,\frac{L}{2p}} + |U_{\alpha 3}|^2 e^{-i\Delta m^2_{31}\,\frac{L}{2p}}\right|^2 . \tag{88}$$

Due to averaging over neutrino spectra and source–detector distances, the interference term $\cos\Delta m^2_{31}\,(L/2p)$ in Eq. (88) disappears and for the probability we have

$$\mathrm{P}(\nu_\alpha \to \nu_\alpha) = |\sum_{i=1,2} |U_{\alpha i}|^2 e^{-i\Delta m^2_{i1}\,\frac{L}{2p}}|^2 + |U_{\alpha 3}|^4 . \tag{89}$$

Furthermore, from the unitarity relation $\sum_{i=1}^{3} |U_{\alpha i}|^2 = 1$ we have

$$\sum_{i=1,2} |U_{\alpha i}|^4 = (1 - |U_{\alpha 3}|^2)^2 - 2|U_{\alpha 1}|^2 |U_{\alpha 2}|^2 \,. \tag{90}$$

Using (90) we can present the survival probability in the form

$$\mathrm{P}(\nu_\alpha \to \nu_\alpha) = (1 - |U_{\alpha 3}|^2)^2 \mathrm{P}^{(1,2)}(\nu_\alpha \to \nu_\alpha) + |U_{\alpha 3}|^4 \,. \tag{91}$$

Here

$$\mathrm{P}^{(1,2)}(\nu_\alpha \to \nu_\alpha) = 1 - \frac{1}{2} \sin^2 2\overline{\theta}_{12} (1 - \cos^2 \Delta m_{21}^2 \, \frac{L}{2p}) \tag{92}$$

and the angle $\overline{\theta}_{12}$ is determined by the relations

$$\cos^2 \overline{\theta}_{12} = \frac{|U_{\alpha 1}|^2}{\sum_{i=1,2} |U_{\alpha i}|^2} \,, \qquad \sin^2 \overline{\theta}_{12} = \frac{|U_{\alpha 2}|^2}{\sum_{i=1,2} |U_{\alpha i}|^2} \,. \tag{93}$$

The probability $\mathrm{P}^{(1,2)}(\nu_e \to \nu_e)$ has the two-neutrino form and it is characterized by two parameters: Δm_{31}^2 and $\sin^2 2\overline{\theta}_{12}$. We have derived the expression (92) for the case of oscillations in vacuum. It should be noted that a similar expression is valid for the case of neutrino transitions in matter.

The expressions (81), (83), and (92) can be used to describe neutrino oscillations in atmospheric and long baseline neutrino experiments (LBL), as well as in solar neutrino experiments. In the framework of neutrino mass hierarchy, the transition of atmospheric (LBL) and solar neutrinos are defined by different Δm^2s (Δm_{31}^2 and $\Delta m_{2,1}^2$, respectively) and the only element that connects oscillations of atmospheric (LBL) and solar neutrinos is $|U_{e3}|^2$. From the LBL reactor experiment CHOOZ and the Super-Kamiokande experiment it follows that this element is small (described later). This means that oscillations of atmospheric (LBL) and solar neutrinos are described by different elements of the neutrino mixing matrix.

4. NEUTRINO IN MATTER

So far we have considered oscillations of neutrinos in vacuum. If there is neutrino mixing the effects of the matter can significantly enhance the probability of the transitions between different types of neutrinos (MSW effect). We will consider here this effect in some detail.

Let us consider neutrinos with momentum $\vec{p}$. The equation of the motion for a free neutrino has the form

$$i \frac{\partial |\psi(t)\rangle}{\partial t} = H_0 |\psi(t)\rangle \,. \tag{94}$$

Let us develop the state $|\psi(t)\rangle$ over states of neutrinos with definite flavour $|\nu_\alpha\rangle$ ($\alpha = e, \mu, \tau$). We have

$$|\psi(t)\rangle = \sum_\alpha |\nu_\alpha\rangle a_\alpha(t) \,, \tag{95}$$

where $a_\alpha(t)$ is the wave function of the neutrino in the flavour representation. From (94) for $a_\alpha(t)$ we obtain the equation

$$i \frac{\partial a_\alpha(t)}{\partial t} = \sum_{\alpha'} \langle \nu_\alpha | H_0 | \nu_{\alpha'} \rangle a_{\alpha'}(t) \,. \tag{96}$$

Now we will develop the state $|\nu_\alpha\rangle$ over the eigenstates $|\nu_i\rangle$ of the free Hamiltonian H_0:

$$H_0 |\nu_i\rangle = E_i |\nu_i\rangle \,, \tag{97}$$

$$E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p} \,. \tag{98}$$

We have:

$$|\nu_\alpha\rangle = \sum_i |\nu_i\rangle\langle\nu_i|\nu_\alpha\rangle \ . \tag{99}$$

If we compare (99) and (52) we find

$$\langle\nu_i|\nu_\alpha\rangle = U^*_{\alpha i} \qquad \langle\nu_\alpha|\nu_i\rangle = U_{\alpha i} \ . \tag{100}$$

Furthermore, we have

$$\langle\nu_\alpha|H_0|\nu_{\alpha'}\rangle = \sum_i \langle\nu_\alpha|\nu_i\rangle\langle\nu_i|H_0|\nu_i\rangle\langle\nu_i|\nu_{\alpha'}\rangle = \sum_i U_{\alpha i}\frac{m_i^2}{2p}U^\dagger_{i\alpha'} + p\delta_{\alpha\alpha'} \ . \tag{101}$$

The last term of 101, which is proportional to the unit matrix, cannot change the flavour state of the neutrino. This term can be excluded from the equation of motion by redefining the phase of the function $a(t)$. We obtain:

$$i\frac{\partial a(t)}{\partial t} = U\frac{m^2}{2p}U^\dagger a(t) \ . \tag{102}$$

This equation can be easily solved. Let us multiply (102) by the matrix $U^\dagger$ on the left. Taking into account unitarity of the mixing matrix we have:

$$i\frac{\partial a'(t)}{\partial t} = \frac{m^2}{2p}a'(t) \ , \tag{103}$$

where $a'(t) = U^\dagger a(t)$. The solution of equation (103) has the form

$$a'(t) = e^{-i\frac{\Delta m^2}{2p}t}a'(0) \, . \tag{104}$$

For the function $a(t)$ in the flavour representation, from (103) and (104), we find

$$a(t) = Ue^{-i\frac{\Delta m^2}{2p}t}U^\dagger a(0) \ , \tag{105}$$

and for the amplitude of the $\nu_\alpha \to \nu_{\alpha'}$ transition in vacuum from (105) we obtain the expression

$$\mathcal{A}_{\nu_{\alpha'};\nu_\alpha}(t) = \sum_i U_{\alpha' i}e^{-i\frac{\Delta m_i^2}{2p}}U^*_{\alpha i} \ , \tag{106}$$

which (up to the irrelevant factor e^{-ipt}) coincides with (57).

Let us now introduce the effective Hamiltonian of the interaction of the flavour neutrino with matter. Due to coherent scattering of the neutrino in matter, the refraction index of the neutrino is given by the following classical expression:

$$n(x) = 1 + \frac{2\pi}{p^2}\mathrm{f}(0)\rho(x) \ . \tag{107}$$

Here $\mathrm{f}(0)$ is the amplitude of elastic neutrino scattering in the forward direction, and $\rho(x)$ is the number density of matter (the axis x is the direction of $\vec{p}$). The effective interaction of neutrinos with matter is determined by the second term of Eq. (107):

$$H_I(x) = p[n(x) - 1] = \frac{2\pi}{p}\mathrm{f}(0)\rho(x) \ . \tag{108}$$

NC scattering of neutrinos on electrons and nucleons (due to the Z-exchange) cannot change the flavour state of neutrinos. This is connected with $\nu_e, \ \nu_\mu, \ \nu_\tau$ universality of NC: the corresponding effective Hamiltonian is proportional to the unit matrix[2].

[2]It should be noted that if there are flavour and sterile neutrinos, NC interactions with matter must be taken into account.

CC interaction (due to the W-exchange) only contributes to the amplitude of the elastic ν_e -e scattering

$$\nu_e + e \to \nu_e + e\ . \tag{109}$$

For the corresponding effective Hamiltonian we have

$$\mathcal{H}_I(x) = \frac{G_F}{\sqrt{2}} 2\overline{\nu}_{eL}\gamma^{\alpha}\nu_{eL}\overline{e}\gamma_{\alpha}(1-\gamma_5)e + \text{h.c.} \tag{110}$$

The amplitude of process (109) is given by

$$\mathrm{f}_{\nu_e e} = \frac{1}{\sqrt{2}\pi} G_F p \tag{111}$$

and, from (108) and (111), for the effective Hamiltonian in flavour representation we have

$$H_I(x) = \sqrt{2} G_F \rho_e(x) \beta\ , \tag{112}$$

where $(\beta)_{\nu_e;\nu_e} = 1$, whilst all the other elements of the matrix β are equal to zero and $\rho_e(x)$ is the electron number density at the point x.

The effective Hamiltonian of the neutrino interaction with matter can also be obtained by calculating the average value of the Hamiltonian (110) in the state which describes matter and neutrino with momentum $\vec{p}$ and negative helicity. Taking into account that for non-polarized media

$$\langle \mathrm{mat}\,|\overline{e}(\vec{x})\gamma^{\alpha}e(\vec{x})|\,\mathrm{mat}\rangle = \rho_e(\vec{x})\delta_{\alpha 0}\ \ , \tag{113}$$

$$\langle \mathrm{mat}|\overline{e}(\vec{x})\gamma^{\alpha}\gamma_5 e(\vec{x})|\mathrm{mat}\rangle = 0\ \ , \tag{114}$$

from (110) we obtain (112).

The evolution equation of neutrino in matter can be written, from (102) and (112), in the following form ($t = x$):

$$i\frac{\partial a(x)}{\partial x} = (U\frac{m^2}{2p}U^{\dagger} + \sqrt{2}G_F\rho_e(x)\beta)a(x)\ . \tag{115}$$

Let consider in detail the simplest case of two-flavour neutrinos (say, ν_e and ν_μ). In this case we have

$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}\ , \tag{116}$$

where θ is the mixing angle. Furthermore it is convenient to write the Hamiltonian in the form

$$H = \frac{1}{2}\mathrm{Tr}H + H^m\ , \tag{117}$$

where Tr $H = \frac{1}{2p}(m_1^2 + m_2^2) + \sqrt{2}G_F\rho_e$. The first term of (117), which is proportional to the unit matrix, can be omitted. For the Hamiltonian we then have

$$H^m(x) = \frac{1}{4p}\begin{pmatrix} -\Delta m^2\cos 2\vartheta + A(x) & \Delta m^2 \sin 2\vartheta \\ \Delta m^2\sin 2\vartheta & \Delta m^2\cos 2\vartheta - A(x) \end{pmatrix}\ , \tag{118}$$

where $\Delta m^2 = m_2^2 - m_1^2$ and $A(x) = 2\sqrt{2}G_F\rho_e(x)p$. The effect of matter is described by the quantity $A(x)$. Notice that this quantity enters only into the diagonal elements of the Hamiltonian and has the dimensions of M^2.

Let us first consider the case of constant density. In order to solve the equation of motion we will diagonalize the Hamiltonian. We have:

$$H^m = U^m E^m U^{m\dagger}\ , \tag{119}$$

where E_i^m is the eigenvalue of the matrix H^m and

$$U^m = \begin{pmatrix} \cos\vartheta^m & \sin\vartheta^m \\ -\sin\vartheta^m & \cos\vartheta^m \end{pmatrix} . \tag{120}$$

It is easy to see that

$$E^m_{1,2} = \mp\frac{1}{4p}\sqrt{(\Delta m^2\cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2} . \tag{121}$$

Now, with the help of Eqs. (119)–(121), for the angle θ^m we have

$$\tan 2\theta^m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2\cos 2\theta - A} ; \qquad \cos 2\theta^m = \frac{\Delta m^2\cos 2\theta - A}{\sqrt{(\Delta m^2\cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}} . \tag{122}$$

The states of the flavour neutrinos are given by

$$|\nu_e\rangle = \cos\theta^m|\nu_{1m}\rangle + \sin\theta^m|\nu_{2m}\rangle; \qquad |\nu_\mu\rangle = -\sin\theta^m|\nu_{1m}\rangle + \cos\theta^m|\nu_{2m}\rangle , \tag{123}$$

where $|\nu_{im}\rangle$ ($i = 1, 2$) are eigenvectors of the Hamiltonian of the neutrino in matter and θ^m is the mixing angle of the neutrino in matter.

The solution of the evolution equation

$$i\frac{\partial a(x)}{\partial x} = H_m a(x) \tag{124}$$

can now easily be found. With the help of (119) we have

$$i\frac{\partial a'(x)}{\partial x} = E^m a'(x) , \tag{125}$$

where

$$a'(x) = (U^m)^\dagger a(x) . \tag{126}$$

Equation (125) has the following solution:

$$a'(x) = e^{-iE^m(x-x_0)}a'(x_0) , \tag{127}$$

where x_0 is the point where the neutrino was produced. Finally, from (126) and (127), we have

$$a(x) = U^m e^{-iE^m(x-x_0)}(U^m)^\dagger a(x_0) . \tag{128}$$

The amplitude of the $\nu_\alpha \to \nu_{\alpha'}$ transition in matter turns out to be

$$\mathcal{A}_{\nu_{\alpha'};\nu_\alpha} = \sum_{i=1,2} U^m_{\alpha' i} e^{-iE_i^m(x-x_0)} U^*_{\alpha i} \tag{129}$$

and, from (129) and (120), we obtain the following transition probabilities, in full analogy with the two-neutrino vacuum case:

$$P^m(\nu_e \to \nu_\mu) = P^m(\nu_\mu \to \nu_e) = \frac{1}{2}\sin^2 2\theta^m(1 - \cos\Delta E^m L) , \tag{130}$$

$$P^m(\nu_e \to \nu_e) = P^m(\nu_\mu \to \nu_\mu) = (1 - P^m(\nu_e \to \nu_\mu)) . \tag{131}$$

Here $\Delta E^m = E_2^m - E_1^m = \frac{1}{2p}\sqrt{(\Delta m^2\cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}$ and $L = x - x_0$ is the distance that the neutrino passes in matter.

For the oscillation length of the neutrino in matter with constant density we have

$$L_0^m = 4\pi\frac{p}{\sqrt{(\Delta m^2\cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}} . \tag{132}$$

The mixing angle and oscillation length in matter can differ significantly from the vacuum values. It follows from (122) that if the condition[3]

$$\Delta m^2 \cos 2\theta = A = 2\sqrt{2} G_F \rho_e p \tag{133}$$

is satisfied, the mixing in matter is maximal ($\theta^m = \pi/4$), independent of the value of the vacuum mixing angle θ. Notice also that if the condition (133) is satisfied, the distance between the energy levels of neutrinos in matter is minimal and the oscillation length in matter is maximal. We have

$$L_0^m = \frac{L_0}{\sin 2\theta}\ , \tag{134}$$

where $L_0 = 4\pi p/(\Delta m)$ is the oscillation length in vacuum. If the distance L in the transition probabilities (131) is large (as with the Sun), the effect of $\nu_e \to \nu_\mu$ transitions is large even in the case of a small vacuum mixing angle θ. The relation (133) is called the resonance condition.

The density of electrons in the Sun is not constant. It is maximal in the centre of the Sun and decreases practically exponentially to its periphery. Studying the dependence of ρ_e on x made it possible to discover the effects that the transitions of solar ν_e's into other states had on matter (MSW effect).

Let us consider the evolution equation when the Hamiltonian depends on the distance x that the neutrino passes in matter

$$i\frac{\partial a(x)}{\partial x} = H^m(x) a(x)\ . \tag{135}$$

The Hermitian Hamiltonian $H^m(x)$ can be diagonalized by a unitary transformation

$$H^m(x) = U^m(x) E^m(x) U^{m\dagger}(x)\ , \tag{136}$$

where $U^m(x)U^{m\dagger}(x) = 1$ and E_i^m(x) are eigenvalues of $H^m(x)$. From (135) and (136) we have

$$U^{m\dagger}(x) i\frac{\partial a(x)}{\partial t} = E^m(x) a'(x)\ , \tag{137}$$

where

$$a'(x) = U^{m\dagger}(x) a(x)\ . \tag{138}$$

Furthermore, by taking into account that

$$U^{m\dagger}(x) i\frac{\partial a(x)}{\partial x} = i\frac{\partial a'(x)}{\partial x} + iU^{m\dagger}(x)\frac{\partial U^m(x)}{\partial x} a'(x)\,, \tag{139}$$

we have the following equation for $a'(x)$:

$$i\frac{\partial a'(x)}{\partial x} = \left(E^m(x) - iU^{m\dagger}(x)\frac{\partial U^m(x)}{\partial x}\right) a'(x)\,. \tag{140}$$

When ρ_e = const Eq. (140) coincides with (125).

Let us now assume that the function $\rho_e(x)$ depends weakly on x and the second term in Eq. (138) can be dropped (adiabatic approximation). It is evident that the solution of the equation

$$i\frac{\partial a_i'(x)}{\partial x} = E_i^m(x) a_i'(x) \tag{141}$$

has the form

$$a_i'(x) = e^{-i\int_{x_0}^{x} E_i^m(x)\,dx} a_i'(x_0) \tag{142}$$

[3] Equation (131) is the condition at which the diagonal elements of the Hamiltonian of neutrino in matter vanish. It is evident that in such a case the mixing is maximal.

(x_0 being the initial point).

It follows from (141) and (142) that, in the adiabatic approximation, a neutrino on the way from the point x_0 to the point x remains at the same energy level. From (138) and (142) we obtain the following solution of the evolution equation in the flavour representation:

$$a(x) = U^m(x) e^{-i\int_{x_0}^{x} E^m(x)\,dx} U^{m\dagger}(x_0) A(X_0)\,. \tag{143}$$

Moreover, the amplitude of the $\nu_\alpha \to \nu_{\alpha'}$ transition in the adiabatic approximation is given by

$$\mathcal{A}_{\nu_{\alpha'};\nu_\alpha} = \sum U^m_{\alpha' i}(x) e^{-i\int_{x_0}^{x} E_i^m(x)\,dx} U^{m*}_{\alpha i}(x_0)\,. \tag{144}$$

The latter is similar to expressions (106) and (129) for the amplitudes of transition in vacuum and in matter with ρ_e = const.

For the two-flavour neutrinos

$$U^m(x) = \begin{pmatrix} \cos\vartheta^m(x) & \sin\vartheta^m(x) \\ -\sin\vartheta^m(x) & \cos\vartheta^m(x) \end{pmatrix} \tag{145}$$

and $\tan 2\theta^m(x)$ and $\cos 2\theta^m(x)$ are given by Eq. (122) in which

$$\mathrm{A}(x) = 2\sqrt{2} G_F \rho_e(x) p\,. \tag{146}$$

The eigenvalues of the Hamiltonian $H^m(x)$ are given by Eq. (121). From (145) we have

$$U^{m\dagger}(x)\,\frac{\partial U^m(x)}{\partial x} = \begin{pmatrix} 0 & \dfrac{\partial\theta^m(x)}{\partial x} \\ -\dfrac{\partial\theta^m(x)}{\partial x} & 0 \end{pmatrix} \tag{147}$$

and the exact equation (140) takes the form

$$i\frac{\partial}{\partial x}\begin{pmatrix} a_1' \\ a_2' \end{pmatrix} = \begin{pmatrix} E_1^m & -i\dfrac{\partial\theta^m}{\partial x} \\ i\dfrac{\partial\theta^m}{\partial x} & E_2^m \end{pmatrix}\begin{pmatrix} a_1' \\ a_2' \end{pmatrix}\,. \tag{148}$$

The Hamiltonian H^m on the right-hand side of this equation can be written in the form

$$H_m = \frac{1}{2}(E_1^m + E_2^m) + \begin{pmatrix} -\frac{1}{2}\Delta E^m & -i\dfrac{\partial\theta^m}{\partial x} \\ i\dfrac{\partial\theta^m}{\partial x} & \frac{1}{2}\Delta E^m \end{pmatrix}\,, \tag{149}$$

where $\Delta E^m = E_2^m - E_1^m$. As we have already stressed several times, the term of the Hamiltonian which is proportional to the unit matrix is not important for flavour evolution.

From Eq. (149) it follows that the adiabatic approximation is valid if the condition

$$\left|\frac{\partial\theta^m}{\partial x}\right| \ll \frac{1}{2}\Delta E^m \tag{150}$$

is satisfied. With the help of (122) it is easy to show that (150) can be written in the form

$$4\sqrt{2} G_F p^2 \Delta m^2 \sin 2\theta \left|\frac{\partial\rho_e}{\partial x}\right| \ll \left[(\Delta m^2\cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2\right]^{3/2}. \tag{151}$$

If the resonance condition

$$\Delta m^2 \cos 2\theta = A(x_R) \tag{152}$$

is satisfied at the point $x = x_R$, the condition of validity of the adiabatic approximation can be written in the form

$$\frac{2p\cos 2\theta \left|\frac{\partial}{\partial x}\ln\rho_e(x_R)\right|}{\Delta m^2 \sin^2 2\theta} \ll 1\,. \tag{153}$$

From Eq. (144) we obtain the following probability for the $\nu_\alpha \to \nu_{\alpha'}$ transition in the adiabatic approximation:

$$\begin{aligned} P(\nu_\alpha \to \nu_{\alpha'}) &= \sum_i |U^m_{\alpha' i}(x)|^2 |U^m_{\alpha i}(x_0)|^2 + \\ &+2Re\sum_{i<k} U^m_{\alpha' i}(x) {U^m_{\alpha' k}}^* e^{-i\int_{x_0}^{x}(E^m_i - E^m_k)\,dx} U^{m*}_{\alpha i}(x_0) U^m_{\alpha k}(x_0)\,. \end{aligned} \tag{154}$$

For solar neutrinos the second term on the r.h.s. of this expression disappears due to averaging over the energy and the region in which the neutrinos are produced. Hence for the averaged transition probability we have

$$\overline{P}(\nu_\alpha \to \nu_{\alpha'}) = \sum_i |U^m_{\alpha' i}(x)|^2 |U^m_{\alpha i}(x_0)|^2\,. \tag{155}$$

Thus, in the adiabatic approximation, the averaged transition probability is determined by the elements of the mixing matrix in matter at the initial and final points. For two-neutrino flavours we have the following simple expression for the ν_e survival probability

$$\begin{aligned} \overline{P}(\nu_e \to \nu_e) &= \cos^2\theta^m(x)\cos^2\theta^m(x_0) + \sin^2\theta^m(x)\sin^2\theta^m(x_0) \\ &= \frac{1}{2}\left[1 + \cos 2\theta^m(x)\cos 2\theta^m(x_0)\right]\,. \end{aligned} \tag{156}$$

From Eq. (156) it is easy to see that if the neutrino passes the point $x = x_R$ where the resonance condition is satisfied, a large effect of the disappearance of ν_e will be observed. In fact, the condition (152) is fulfilled if $\cos 2\theta > 0$ (neutrino masses are labelled in such a way that $\Delta m^2 > 0$). At the production point x_0 the density is larger than at point x_R and $A(x_0) > \Delta m^2 \cos 2\theta$. From (122) it follows that $\cos 2\theta(x_0) < 0$. Thus, if the resonance condition is fulfilled, we see from Eq. (156) that $P(\nu_e \to \nu_e) < \frac{1}{2}$. If the condition

$$A(x_0) \gg \Delta m^2 \tag{157}$$

is satisfied for neutrinos produced in the centre of the Sun,then $\cos 2\theta^m(x_0) \simeq -1$ and, for neutrinos passing through the Sun, the survival probability is equal to:

$$\overline{P}(\nu_e \to \nu_e) \simeq \frac{1}{2}(1 - \cos 2\theta)\,. \tag{158}$$

It is obvious from this expression that the ν_e survival probability at small θ is close to zero: all ν_e's are transformed into ν_μ's.

Let us consider the evolution of neutrino states in such a case. From Eq. (122) it follows that, at the production point, $\theta^m(x_0) \simeq \pi/2$. From (123) we then have

$$|\nu_e\rangle \simeq |\nu_{2m}\rangle\,; \qquad |\nu_\mu\rangle = -|\nu_{1m}\rangle \qquad (x = x_0)\,. \tag{159}$$

Thus at the production point the flavour states are states with definite energy. In the adiabatic approximation there are no transitions between energy levels. In the final point ρ_e = 0 and at small θ we have

$$|\nu_2\rangle \simeq |\nu_\mu\rangle\,, \qquad |\nu_1\rangle \simeq |\nu_e\rangle \qquad (x = x_0)\,. \tag{160}$$

Thus, all ν_e's transfer to ν_μ's. The resonance condition (152) was written in units $\hbar = c = 1$. We can rewrite it in the following form

$$\Delta m^2 \cos 2\theta \simeq 0.7 \times 10^{-7} E\rho\,\mathrm{eV}^2$$

where ρ is the density of matter in $\text{g} \cdot \text{cm}^{-3}$ and E is the neutrino energy in MeV. In the central region of the Sun $\rho \simeq 10^2 \ \text{g} \cdot \text{cm}^{-3}$ and the energy of the solar neutrinos is $\simeq 1$ MeV. Thus the resonance condition is satisfied at $\Delta m^2 \simeq 10^{-5}$ eV^2.

The expression (155) gives the averaged survival probability in the adiabatic approximation. In the general case we have

$$\overline{P}(\nu_\alpha \to \nu_{\alpha'}) = \sum |U^m_{\alpha' i}(x)|^2 P_{ik} |U^m_{\alpha k}(x_0)|^2 \ , \tag{161}$$

where P_{ik} is the probabilty of the transition from the state with energy E^m_k to the state with energy E^m_i. Let us consider the simplest case of the transition between two types of neutrinos. From the conservation of the total probability we have

$$P_{11} = 1 - P_{21} \ , \qquad P_{22} = 1 - P_{12} \ , \qquad P_{12} = P_{21} \ . \tag{162}$$

Thus in the case of two neutrinos all transition probabilities P_{ik} are expressed through P_{12}. With the help of (145), (161), and (162), for the ν_e survival probability we have:

$$\overline{P}(\nu_e \to \nu_e) = \frac{1}{2} + \left(\frac{1}{2} - P_{12}\right) \cos 2\theta^m(x) \cos 2\theta^m(x_0) \ . \tag{163}$$

In the literature different approximate expressions for the transition probability P_{12} exist. In the Landau–Zenner approximation, based on the assumption that the transition occurs mainly in the resonance region,

$$P_{12} = e^{-\frac{\pi}{2}\gamma_R F} \ , \tag{164}$$

where

$$\gamma_R = \frac{\frac{1}{2}\Delta E^m}{|\partial\theta^m/\partial x|} = \frac{\Delta m^2 \sin^2 2\theta}{2p \cos 2\theta |\frac{\partial}{\partial x} \ln \rho_e(x_R)|} \ . \tag{165}$$

In the above equation $F = 1$ for linear density and $F = 1 - \tan^2\theta$ for exponential density. The adiabatic approximation is valid if $\gamma_R \gg 1$ [see (150)]. In this case $P_{12} \simeq 0$.

This concludes the discussion on the phenomenological theory of neutrino mixing and on the theory of neutrino oscillations in vacuum and in matter. We shall now turn to the experimental data. There are three methods to search for the effects of neutrino masses and mixing:

I. The precise measurement of the high-energy part of the β–spectrum;

II. The search for the neutrinoless double β–decay;

III. The investigation of neutrino oscillations.

We shall now discuss the results obtained in some of the most recent experiments.

5. SEARCH FOR EFFECTS OF THE NEUTRINO MASS IN EXPERIMENTS ON THE MEASUREMENT OF THE β-SPECTRUM OF ^{3}H

We will discuss here briefly the results of searching for effects of neutrino masses in experiments on the measurement of the high-energy part of the β–spectrum in the decay

$$^3\text{H} \to {}^3\text{He} + e^- + \overline{\nu_e} \ . \tag{166}$$

The process (166) is a superallowed β–decay: the nuclear matrix element is constant and the β–spectrum is determined by the phase–space factor and the Coulomb interaction of the final e^- and ^{3}He. For the β–spectrum we have

$$\frac{dN}{dT} = C\, pE(Q-T)\sqrt{(Q-T)^2 - m_\nu^2} F(E) \ . \tag{167}$$

Here p is the electron momentum, $E = m_e + T$ is the total electron energy, $Q = m_{^3\text{H}} - m_{^3\text{He}} - m_e \simeq 18.6$ keV is the energy release, $C =$ const and $F(E)$ is the Fermi function which describes the

Coulomb interaction of the final particles. In Eq. (167) the term $(Q-T)$ is the neutrino energy (the recoil energy of ^{3}He can be neglected) and the neutrino mass enters through the neutrino momentum $p_\nu = \sqrt{(Q-T)^2 - m_\nu^2}$. Notice that in the derivation of Eq. (167) the simplest assumption was made that ν_e is the particle with mass m_ν.

The Kurie function is then determined as follows

$$K(T) = \sqrt{\frac{dN}{dt}\frac{1}{pEF(E)}} = \sqrt{C}\sqrt{(Q-T)\sqrt{(Q-T)^2 - m_\nu^2}}\,. \tag{168}$$

If m_ν = 0, the Kurie function is the straight line $K(T) = \sqrt{C}(Q-T)$, and $T_{\max} = 0$. If $m_\nu \neq 0$ then $T_{\max} = Q - m_\nu$ and at small m_ν the Kurie function deviates from the straight line in the region close to the maximum allowed energy. Thus, if $m_\nu \neq 0$ in the end-point part of the spectrum, a deficit of observed events must be measured (with respect to the number of events expected at $m_\nu = 0$).

In experiments on the search for effects of the neutrino mass by the ^{3}H–method, no positive indications in favour of $m_\nu \neq 0$ were found. In these experiments some anomalies were observed. Firstly, practically in all of the experiments the best-fit values of m_ν^2 are negative. This means that instead of a deficit of events, an excess is observed. Secondly, in the Troitsk experiment a peak in the electron spectrum is observed at the distance of a few eV from the end. The position of the peak changes periodically with time. There is no doubt that new, more precise experiments are necessary. The results of two running experiments are presented in Table 2.

Table 2: Neutrino masses from ^{3}H experiments.

Experiment	m_ν^2	m_ν
Troitsk	$-1.0 \pm 3.0 \pm 2.0$ eV^2	$<$ 2.5 eV
Mainz	$-0.1 \pm 3.8 \pm 1.8$ eV^2	$<$ 2.8 eV

6. NEUTRINOLESS DOUBLE β-DECAY

The decay

$$(A,Z) \to (A, Z+2) + e^- + e^- \tag{169}$$

is possible only if the total lepton number L is not conserved, i.e. if neutrinos with definite masses are Majorana particles. There are many experiments in which neutrinoless double β-decay [$(\beta\beta)_{0\nu}$–decay] of ^{76}Ge, ^{136}Xe, ^{130}Te, ^{82}Se, ^{100}Mo and other even–even nuclei are searched for.

Let consider the process (169) in the framework of neutrino mixing. The standard CC Hamiltonian of the weak interaction has the form

$$H_I = \frac{G_F}{\sqrt{2}} 2\,\overline{e}_L \gamma^\alpha \nu_{eL} j_\alpha + \text{h.c.} \tag{170}$$

Here j_α is the weak hadronic current and

$$\nu_{eL} = \sum U_{ei}\nu_{iL}\ , \tag{171}$$

where ν_i is the Majorana neutrino field with mass m_i.

The $(\beta\beta)_{0\nu}$ decay is a process of second order in G_F with an intermediate virtual neutrino. Neutrino masses and mixing enter into the neutrino propagator[4]

$$\begin{aligned}\nu_{eL}{}^{\bullet}(x_1)\nu_{eL}^{T\,\bullet}(x_2) &= \sum_i U_{ei}^2 \nu_{iL}{}^{\bullet}(x_1)\nu_{iL}^{T\,\bullet}(x_2) = -\sum U_{ei}^2 \frac{(1-\gamma_5)}{2}\nu_i{}^{\bullet}(x_1)\overline{\nu}_i{}^{\bullet}(x_2)\frac{(1-\gamma_5)}{2}C \\ &= -\sum U_{ei}^2 \frac{(1-\gamma_5)}{2}\frac{i}{(2\pi)^4}\int \frac{e^{-ip(x_1-x_2)}(\not{p}+m_i)}{p^2-m_i^2}\,d^4p\,\frac{(1-\gamma_5)}{2}C\,. \qquad (172)\end{aligned}$$

Taking into account that

$$\frac{(1-\gamma_5)}{2}\,(\not{p}+m_i)\,\frac{(1-\gamma_5)}{2} = m_i\frac{(1-\gamma_5)}{2}\,, \qquad (173)$$

we come to the conclusion that the matrix element of $(\beta\beta)_{0\nu}$–decay is proportional to[5]

$$\langle m\rangle = \sum U_{ei}^2 m_i\,. \qquad (174)$$

From (173) it is evident that the proportionality of the matrix element of $(\beta\beta)_{0\nu}$–decay to $\langle m\rangle$ is due to the fact that the standard CC interaction is the left-handed one. If neutrino masses are equal to zero $(\beta\beta)_{0\nu}$–decay is forbidden (conservation of helicity). Notice that, if there is some small admixture of right-handed currents in the interaction Hamiltonian, the $L-R$ interference gives a contribution proportional to the $\not{p}$ term in the neutrino propagator. Other mechanisms of $(\beta\beta)_{0\nu}$–decay are also possible (SUSY with violation of R-parity, etc.).

In the experiments on the search for $(\beta\beta)_{0\nu}$-decay, very strong bounds on the life-time of this process were obtained. The results of some of the latest experiments are presented in Table 3. From these data upper bounds for $|\langle m\rangle|$ can be obtained. The upper bounds depend on the values of the nuclear matrix elements, the calculation of which is a complicated problem. From the ^{76}Ge data it follows that

$$|\langle m\rangle| < (0.5-1)\ \mathrm{eV}\,. \qquad (175)$$

Table 3: Lower bounds of the life-time $T_{1/2}$ of the $(\beta\beta)_{0\nu}$–decay.

Experiment	Element	Lower bound of $T_{1/2}$
Heidelberg–Moscow	$^{76}\mathrm{Ge}$	$> 1.6\times10^{25}$ y
Caltech–PSI–Neuchatel	$^{136}\mathrm{Xe}$	$> 4.4\times10^{23}$ y
Milano	$^{130}\mathrm{Te}$	$> 7.7\times10^{22}$ y

In future experiments on the search for $(\beta\beta)_{0\nu}$–decay (Heidelberg–Moscow, NEMO, CUORE and others) the sensitivity $|\langle m\rangle| < 0.1$ eV will be achieved.

7. NEUTRINO OSCILLATION EXPERIMENTS

We will now discuss the existing experimental data on the search for neutrino oscillations. At present there is convincing evidence in favour of neutrino oscillations, which has been obtained in atmospheric neutrino experiments, first of all in the Super-Kamiokande experiment. Strong indications in favour of

[4]We have used the relation $\nu_i^T = -\nu_i C$ that follows from the Majorana condition $\nu_i^C = C\overline{\nu}_i^T = \nu_i$. It is obvious that in the case of Dirac neutrinos the propagator is equal to zero.

[5]The term m_i^2 in the denominator is small with respect to characteristic p in nuclei ($\simeq$ 10 MeV) and can be neglected.

neutrino masses and mixing have been obtained in all solar neutrino experiments. Finally, some indications in favour of $\nu_\mu \to \nu_e$ transitions have been obtained in the LSND accelerator experiment. In many reactor and accelerator short baseline experiments, and in the reactor long baseline experiments CHOOZ, no indication in favour of neutrino oscillations has been found. We will start with a discussion of the results of solar neutrino experiments.

7.1 Solar neutrinos

The energy of the Sun is generated in the reactions of the thermonuclear pp and CNO cycles. The main pp cycle is illustrated in Fig. 1.

The energy of the sun is produced in the transition

$$4\,p + 2\,e^- \to \,^4\mathrm{He} + 2\,\nu_e\,. \tag{176}$$

If we assume that solar ν_e's do not transfer into other neutrino types [$P(\nu_e \to \nu_e) = 1$] we can obtain a relation between the luminosity of the Sun, $L_\odot$ and the flux of solar neutrinos. Let us consider a neutrino with energy E. From (176) it follows that

$$\frac{1}{2}(Q - 2E) \tag{177}$$

is the luminous energy corresponding to the emission of one neutrino. Here

$$Q = 4m_p + 2m_e - m_{^4\mathrm{He}} \simeq 26.7\ \mathrm{MeV} \tag{178}$$

is the energy release in the transition (176). If we multiply (177) by the total flux of solar ν_e's from different reactions and integrate over the neutrino energy E we will obtain the flux of luminous energy from the Sun

$$\frac{1}{2}\int (Q - 2E) \sum_i I_i(E) dE = \frac{L_\odot}{4\pi R^2}\,. \tag{179}$$

Here $L_\odot \simeq 3.86 \times 10^{33}$ erg/s is the luminosity of the Sun, R is the Sun–Earth distance, and $I_i^0(E)$ is the flux of neutrinos from the source i (i = pp, ...). Notice that in the derivation of the relation (179) we have assumed that the Sun is in a stationary state.

The luminosity relation (179) is the solar model independent constraint on the solar neutrino fluxes. The flux $I_i(E)$ can be written in the form

$$I_i(E) = X_i(E)\Phi_i\,, \tag{180}$$

where Φ_i is the total flux, and the function $X_i(E)$ describes the form of the spectrum ($\int X_i(E)dE = 1$). The functions $X_i(E)$ are known functions, determined by the weak interaction. The luminosity relation (179) can be written in the form

$$Q \sum_i \left(1 - 2\frac{\overline{E}_i}{Q}\right)\Phi_i = \frac{L_\odot}{2\pi R^2}\,, \tag{181}$$

where $\overline{E}_i = \int E X_i(E) dE$ is the average energy of neutrinos from the source i. The main sources of solar neutrinos are listed in Table 4.

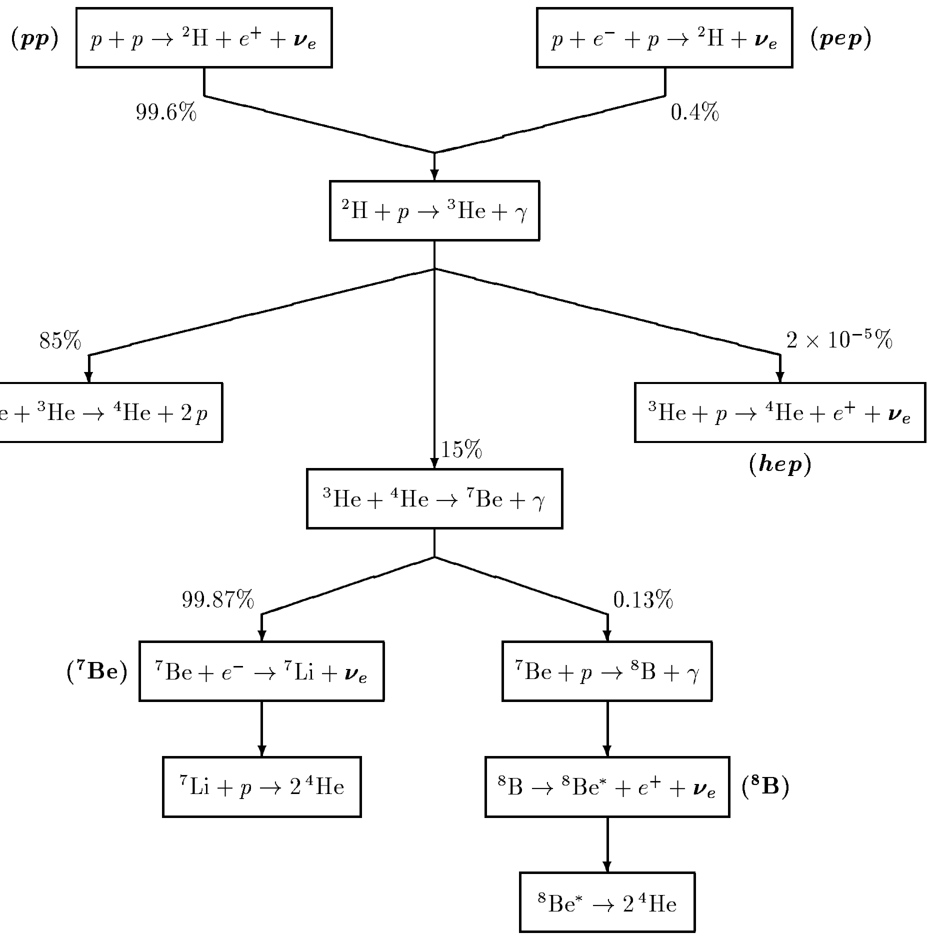

Fig. 1: The pp cycle (the figure is taken from Ref. [17]).

Table 4: Main sources of solar ν_e's.

Reaction	Maximal energy	Standard Solar Model flux $(\text{cm}^{-2}\,\text{s}^{-1})$
$p\,p \to d\,e^+\,\nu_e$	$\leqslant 0.42$ MeV	6.0×10^{10}
$e^-\,{}^7\text{Be} \to \nu_e\,{}^7\text{Li}$	0.86 MeV	4.9×10^{9}
${}^8\text{B} \to {}^8\text{Be}\,e^+\,\nu_e$	$\leqslant 15$ MeV	5.0×10^{6}

As can be seen from the Table, the main source of solar neutrinos is the reaction $p+p \to d + e^+ + \nu_e$. This reaction is the source of low-energy neutrinos. The source of monochromatic medium-energy neutrinos is the process

$$e^- + {}^7\mathrm{Be} \to \nu_e + {}^7\mathrm{Li} \,. \tag{182}$$

The reaction ${}^8\mathrm{B} \to {}^8\mathrm{Be} + e^+ + \nu_e$ is the source of high-energy neutrinos. The results of solar neutrino experiments are presented in Table 5.

Homestake, GALLEX and SAGE are radiochemical experiments. In the Kamiokande and the Super-Kamiokande experiments, recoil electrons (angle and energy) in elastic neutrino–electron scattering are detected. In these experiments the direction of the neutrinos is determined, and it is confirmed that the detected events are from solar neutrinos.

In the Homestake experiment, because of the high threshold ($E_{\mathrm{th}} = 0.81$ MeV), mainly ^{8}B neutrinos are detected: $\simeq 77\%$ of the events are due to ^{8}B neutrinos and $\simeq 15\%$ of the events are due to ^{7}Be neutrinos. In the GALLEX and SAGE experiments ($E_{\mathrm{th}} = 0.23$ MeV) neutrinos are detected from all the reactions: $\simeq 54\%$ of the events are due to pp neutrinos, $\simeq 27\%$ of the events are due to ^{7}Be neutrinos, and $\simeq 10\%$ of the events are due to ^{8}B neutrinos. In the Kamiokande and Super-Kamiokande experiments, due to the high threshold ($E_{\mathrm{th}} = 7$ MeV for Kamiokande and $E_{\mathrm{th}} = 5.5$ MeV for the Super-Kamiokande), only high-energy ^{8}B neutrinos are detected.

The results of the solar neutrino experiments are presented in Table 5. As can be seen from the Table, the detected event rates in all the solar neutrino experiments are significantly smaller than the predicted ones.[6] The most natural explanation of the data of the solar neutrino experiments can be obtained in the framework of neutrino mixing. In fact, if neutrinos are massive and mixed, solar ν_e's on the way to the earth can transfer into neutrinos of other types that are not detected in the radiochemical Homestake, GALLEX, and SAGE experiments. In the Kamiokande and Super-Kamiokande experiments, all flavour neutrinos ν_e, ν_μ, and ν_τ are detected. However, the cross section of ν_μ (ν_τ) $- e$ scattering is about six times smaller than the cross section of $\nu_e - e$ scattering.

All existing solar neutrino data can be explained if we assume that solar neutrino fluxes are given by the Standard Solar Model (SSM) and that there are transitions between two neutrino types determined by the two parameters: mass squared difference Δm^2 and mixing parameter $\sin^2 2\theta$. We will present the results of the analysis of the data later on.

Now we shall make some remarks about a model-independent analysis of the data. First of all, from the luminosity relation (179) for the total flux of solar neutrinos, we have the following lower bound

$$\Phi = \sum_i \Phi_i \geq \frac{L_\odot}{2\pi R^2 Q} \,. \tag{183}$$

Furthermore, for the counting rate in the gallium experiments we have

$$Q_{Ga} = \int_{E_{\mathrm{th}}} \sigma(E) \sum I_i(E) dE = \sum_i \overline{\sigma}_i \Phi_i \geq \overline{\sigma}_{pp} \Phi = (76 \pm 2)\,\mathrm{SNU} \,. \tag{184}$$

By comparing this lower bound with the results of the GALLEX and SAGE experiments (see Table 5), we come to the conclusion that there is no contradiction between the experimental data and luminosity constraint if we assume that there are no transitions of solar neutrinos into other states [$P(\nu_e \to \nu_e) = 1$].

[6] Note that in the framework of neutrino oscillations, the possibility of a deficit of the solar ν_e's was discussed by B. Pontecorvo in 1968, before the results of the Homestake experiment were obtained.

Table 5: Results of solar neutrino experiments.
[1 SNU = 10^{-36} events/(atoms · sec)]

Experiment	Observed rate	Expected rate
Homestake $\nu_e\,{}^{37}\mathrm{Cl} \to e^-\,{}^{37}\mathrm{Ar}$ $E_{\mathrm{th}} = 0.81$ MeV	$2.56 \pm 0.16 \pm 0.16$ SNU	7.7 ± 1.2 SNU
GALLEX $\nu_e\,{}^{71}\mathrm{Ga} \to e^-\,{}^{71}\mathrm{Ge}$ $E_{\mathrm{th}} = 0.23$ MeV	$77.5 \pm 6.2^{+4.3}_{-4.7}$ SNU	129 ± 8 SNU
SAGE $\nu_e\,{}^{71}\mathrm{Ga} \to e^-\,{}^{71}\mathrm{Ge}$ $E_{\mathrm{th}} = 0.23$ MeV	$66.6 \pm {}^{+6.8}_{-7.1}\,{}^{+3.8}_{-4.0}$ SNU	— · —
Kamiokande $\nu e \to \nu e$ $E_{\mathrm{th}} = 7.0$ MeV	$(2.80 \pm 0.19 \pm 0.33)\ 10^6\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$	$(5.15^{+1.00}_{-0.72})\ 10^6\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$
Super − Kamiokande $\nu e \to \nu e$ $E_{\mathrm{th}} = 5.5$ MeV	$(2.44 \pm 0.05^{+0.09}_{-0.07})\ 10^6\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$	— · —

It is possible, however, to show in a model-independent way that the results of *different solar neutrino experiments* are not compatible if we assume $P(\nu_e \to \nu_e) = 1$. In fact, let us compare the results of the Homestake and Super-Kamiokande experiments. We will consider the total neutrino fluxes Φ_i to be free parameters. From the results of the Super-Kamiokande experiment we can determine the flux of ^{8}B neutrinos, $\Phi_{^8B}$ (see Table 5). If we now calculate the contribution of ^{8}B neutrinos into the counting rate of the Homestake experiment we obtain

$$Q_{Cl}^{^8B} = (2.78 \pm 0.27)\,\mathrm{SNU}\ . \tag{185}$$

The difference between the measured event rate and $Q_{Cl}^{^8B}$ gives the contribution to the Chlorine event rate of ^{7}Be and other neutrinos. We have

$$Q_{Cl}^{^7Be+\ldots} = Q_{Cl}^{ex} - Q_{Cl}^{^8B} = (-0.22 \pm 0.35)\,\mathrm{SNU}\ . \tag{186}$$

All existing solar models predict a much larger contribution of ^{7}Be neutrinos to the Chlorine event rate:

$$Q_{Cl}^{^7Be}(SSM) = (1.15 \pm 0.1)\,\mathrm{SNU}\ . \tag{187}$$

The large suppression of the flux of ^{7}Be neutrinos (together with the observation of ^{8}B neutrinos) is a problem for all solar models. The ^{8}B nuclei are produced in the reaction $p\,+\,{}^7\mathrm{Be} \to {}^8\mathrm{B}\,+\,\gamma$, and in order to observe neutrinos from ^{8}B decay, enough ^{7}Be nuclei must exist in the Sun's interior. We can come to the same conclusion about the suppression of the flux of ^{7}Be neutrinos if we compare the results of the Gallium and Super-Kamiokande experiments.

All existing solar neutrino data can be described if there are oscillations between two neutrino flavours, the neutrino fluxes being given by the SSM values. If we assume that the oscillation parameters Δm^2 and $\sin^2 2\theta$ are in the region in which matter MSW effects are important, then from the fit of the data two allowed regions of the oscillation parameters can be obtained. For the best fit values it was found

$$\Delta m^2 = 5 \times 10^{-6}\ \mathrm{eV}^2 \quad \sin^2 2\theta = 5 \times 10^{-3}\ (\mathrm{SMA}) \tag{188}$$

$$\Delta m^2 = 2 \times 10^{-5}\ \mathrm{eV}^2 \quad \sin^2 2\theta = 0.76\ (\mathrm{LMA})\ . \tag{189}$$

The data can also be described if we assume that the oscillation parameters are in the region in which matter effects can be neglected (the case of vacuum oscillations). For the best fit values it was found in this case

$$\Delta m^2 = 4.3 \times 10^{-10}\ \mathrm{eV}^2 \qquad \sin^2 2\theta = 0.79\ (\mathrm{VO})\,. \tag{190}$$

In the Super-Kamiokande experiment during 825 days, 11240 solar neutrino events were observed. Such large statistics allow the Super-Kamiokande collaboration to measure the energy spectrum of the recoil electrons and day/night asymmetry. No significant deviation from the expected spectrum was observed (perhaps with the exception of the high-energy part of the spectrum). For the day/night asymmetry the following value was obtained

$$\frac{1}{2}\left(\frac{N-D}{N+D}\right) = 0.065 \pm 0.031 \pm 0.013\ . \tag{191}$$

From the analysis of the latest Super-Kamiokande data, the following best-fit values of the oscillation parameters were found:

$$\Delta m^2 = 5 \times 10^{-6}\ \mathrm{eV}^2 \qquad \sin^2 2\theta = 5 \times 10^{-3}\ (\mathrm{SMA}) \tag{192}$$

$$\Delta m^2 = 3.2 \times 10^{-5}\ \mathrm{eV}^2 \qquad \sin^2 2\theta = 0.8\ (\mathrm{LMA}) \tag{193}$$

$$\Delta m^2 = 4.3 \times 10^{-10}\ \mathrm{eV}^2 \qquad \sin^2 2\theta = 0.79\ (\mathrm{VO})\ . \tag{194}$$

These values are compatible with the ones in Eqs. (188), (189), and (190), which were found from the analysis of the event rates measured in all solar neutrino experiments.

A new solar neutrino experiment SNO started recently in Canada. The target in this experiment is heavy water (1 kton of D_2O) and Cherenkov light is detected by $\simeq 10^4$ photomultipliers. Neutrinos will be detected through the observation of the CC reaction

$$\nu_e + d \to e^- + p + p\ , \tag{195}$$

as well as the NC reaction

$$\nu + d \to \nu + n + p\ , \tag{196}$$

and $\nu - e$ elastic scattering

$$\nu + e \to \nu + e\ . \tag{197}$$

The detection of neutrinos via the CC process (195) will allow the spectrum of ν_e on the Earth to be measured. The detection of neutrinos via the NC process (196) (neutrons will be detected) will allow the total flux of flavour neutrinos ν_e, ν_μ, ν_τ to be determined. From the comparison of the NC and CC event rates, model-independent conclusions on the transition of solar ν_e's into other flavour states can be made.

The next solar neutrino experiment will be BOREXINO. In this experiment, 300 t of liquid scintillator with very high purity will be used. Solar neutrinos will be detected through the observation of the recoil electrons in the process

$$\nu + e \to \nu + e\,. \tag{198}$$

The energy threshold in the BOREXINO experiment will be very low, about 250 keV. This will allow monoenergetic ^{7}Be neutrinos to be detected. If vacuum oscillations are the origin of the solar neutrino problem, a seasonal variation of the ^{7}Be neutrino signal (due to the excentricity of the Earth's orbit) will be observed.

7.2 Atmospheric neutrinos

Atmospheric neutrinos are produced mainly in the decays of pions and muons

$$\pi \to \mu + \nu_\mu\ , \qquad \mu \to e + \nu_e + \nu_\mu\ , \tag{199}$$

pions being produced in the interaction of cosmic rays in the Earth's atmosphere. Notice that in the existing detectors neutrino and antineutrino events cannot be distinguished. At small energies, ≤ 1 GeV, the ratio of fluxes of ν_μ's and ν_e's from the chain (199) is equal to two. At higher energies this ratio is larger than two (not all muons decay in the atmosphere) but it can be predicted with an accuracy better than 5% (the absolute fluxes of muon and electron neutrinos are presently predicted with an accuracy not better than 20–25%). This is the reason why the results of the measurements of total fluxes of atmospheric neutrinos are presented in the form of a double ratio

$$R = \frac{(N_\mu/N_e)_{\rm data}}{(N_\mu/N_e)_{\rm MC}}\ , \tag{200}$$

where $(N_\mu/N_e)_{\rm data}$ is the ratio of the total number of observed muon and electron events, and $(N_\mu/N_e)_{\rm MC}$ is the ratio predicted from Monte Carlo simulations.

We will discuss the results of the Super-Kamiokande experiment. A large water Cherenkov detector is used in this experiment. The detector consists of two parts: the inner one of 50 kton (22.5 kton fiducial volume) is covered with 11146 photomultipliers, and the outer part, 2.75 m thick, is covered with 1885 photomultipliers. The electrons and muons are detected by observing the Cherenkov radiation. The efficiency of particle identification is larger than 98%. The observed events are divided into fully contained events (FC) for which Cherenkov light is deposited in the inner detector, and partially contained events (PC) in which the muon track deposits part of its Cherenkov radiation in the outer detector. FC events are further divided into sub-GeV events ($E_{vis} \leq$ 1.33 GeV), and multi-GeV events $E_{vis} \geq 1.33$ GeV). In the Super-Kamiokande experiment for sub-GeV events and multi-GeV events (FC and PC), the following values of the double ratio R were obtained, respectively (848.3 days):

$$\begin{aligned} R &= 0.680^{+0.023}_{-0.022} \pm 0.053 \\ R &= 0.678^{+0.042}_{-0.039} \pm 0.080\ . \end{aligned} \tag{201}$$

These values are in agreement with the values of R obtained in other water Cherenkov experiments (Kamiokande and IMB), and in the Soudan2 experiment in which the detector is an iron calorimeter.

$$R = 0.65 \pm 0.05 \pm 0.08 \qquad ({\rm Kamiokande}) \tag{202}$$

$$R = 0.54 \pm 0.05 \pm 0.11 \qquad ({\rm IMB}) \tag{203}$$

$$R = 0.61 \pm 0.15 \pm 0.05 \qquad ({\rm Soudan2})\ . \tag{204}$$

The fact that the double ratio R is significantly less than one is an indication in favour of neutrino oscillations.

Important evidence in favour of neutrino oscillations has been obtained by the Super-Kamiokande Collaboration. These data were first reported at the NEUTRINO98 Conference in Japan, in June 1998. A significant up–down asymmetry of multi-GeV muon events was discovered in the Super-Kamiokande experiment.

For atmospheric neutrinos, the distance between the production region and the detector changes from about 20 km for down-going neutrinos ($\theta = 0$, θ being the zenith angle), up to about 13,000 km for up-going neutrinos ($\theta = \pi$). In the Super-Kamiokande experiment for the multi-GeV events the zenith angle θ can be determined. In fact, charged leptons follow the direction of neutrinos (the averaged angle between the charged lepton and the neutrino is $15°$–$20°$). The possible source of the zenith angle dependence of the neutrino events is the magnetic field of the Earth. However, for neutrinos with energies larger than 2–3 GeV, within a few % no θ-dependence of neutrino events is expected.

The Super-Kamiokande Collaboration found a significant zenith angle dependence of the multi-GeV muon neutrinos. For the integral up–down asymmetry of multi-GeV muon neutrinos (FC and PC) the following value was obtained

$$A_\mu = 0.311 \pm 0.043 \pm 0.010\ . \tag{205}$$

Here

$$A = \frac{U - D}{U + D}\ , \tag{206}$$

where U is the number of up-going neutrinos ($\cos\theta \leq -0.2$), and D is the number of down-going neutrinos ($\cos\theta \geq 0.2$). No asymmetry of the electron neutrinos was found:

$$A_e = 0.036 \pm 0.067 \pm 0.02\ . \tag{207}$$

The Super-Kamiokande data can be described if we assume that there are $\nu_\mu \to \nu_\tau$ oscillations. The following best-fit values of the oscillation parameters were found from the analysis of the FC events

$$\Delta m^2 = 3.05 \times 10^{-3}\ \mathrm{eV}^2\ , \qquad \sin^2 2\theta = 0.995 \tag{208}$$

($\chi^2_{\min} = 55.4$ at 67 d.o.f.). It should be noted that if we assume that there are no oscillations, then in this case $\chi^2 = 177$ at 69 d.o.f. From the combined analysis of all the data it was found that

$$\Delta m^2 \simeq (2 - 6) \times 10^{-3}\ \mathrm{eV}^2\ , \qquad \sin^2 2\theta > 0.84\ . \tag{209}$$

If $\nu_\mu \to \nu_s$ oscillations are assumed, at large energies matter effects must be important. From the investigation of the high-energy events (PC and upward-going muon events, muons being produced by neutrinos in the rock under the detector) the Super-Kamiokande Collaboration came to the conclusion that $\nu_\mu \to \nu_s$ oscillations are disfavoured at 95% C.L.

The range of oscillation parameters obtained from the analysis of the atmospheric neutrino data will be investigated in detail in long baseline experiments. The results of the first LBL reactor experiment, CHOOZ, were recently published (in this experiment the distance between the reactors and the detector is $\simeq 1$ km). No indication in favour of the transition of $\overline{\nu_e}$ into other states was found in this experiment. For the ratio R of the number of measured and expected events it was found that

$$R = 1.01 \pm 2.8\%\,(\mathrm{stat}) \pm 2.7\%\,(\mathrm{syst})\ . \tag{210}$$

These data allow $\Delta m^2 > 7 \times 10^{-4}\ \mathrm{eV}^2$ at $\sin^2 2\theta = 1$ (90% C.L.) to be excluded.

In the LBL Kam-Land experiment $\overline{\nu_e}$'s from reactors at a distance of 150–200 km from the detector will be detected. Neutrino oscillations $\overline{\nu}_e \leftrightarrow \overline{\nu}_x$ with $\Delta m^2 \gtrsim 10^{-5}\ \mathrm{eV}^2$ and large values of $\sin^2 2\theta$ will be explored. The BOREXINO Collaboration plans to detect $\overline{\nu}_e$ from reactors at a distance of about 800 km from the detector.

The first LBL accelerator experiment K2K is now running. In this experiment ν_μ's with an average energy of 1.4 GeV, produced at the KEK accelerator, will be detected in the Super-Kamiokande detector (at a distance of about 250 km). The disappearance channel $\nu_\mu \to \nu_\mu$ and the appearance channel $\nu_\mu \to \nu_e$ will be investigated in detail. This experiment will be sensitive to $\Delta m^2 \geq 2 \times 10^{-3}\ \mathrm{eV}^2$ at large $\sin^2 2\theta$.

The LBL MINOS experiment between Fermilab and Soudan (the distance is about 730 km), is under construction. In this experiment all the possible channels of ν_μ transitions will be investigated in the atmospheric neutrino range of Δm^2.

The LBL CERN–Gran Sasso experiments (the distance is about 730 km), ICARUS, NOE, and others, are under construction at CERN and Gran Sasso. The direct detection of τ's from the $\nu_\mu \to \nu_\tau$ transition will be one of the major goals of these experiments.

7.3 LSND experiment

Some indications in favour of $\nu_\mu \leftrightarrow \nu_e$ oscillations were found in the short baseline LSND accelerator experiment. This experiment was carried out at the Los Alamos linear accelerator (with protons of 800 MeV energy). This is a beam-stop experiment: most of the π^+'s in the beam, produced by protons, come to rest in the target and decay (mainly by $\pi^+ \to \mu^+ \nu_\mu$); μ^+'s also come to rest in the target and decay by $\mu_+ \to e^+ \nu_e \overline{\nu_\mu}$. Thus, the beam-stop target is the source of ν_μ, ν_e, and $\overline{\nu}_\mu$ (no $\overline{\nu}_e$ are produced in the decays).

The large scintillator neutrino detector LSND was located at a distance of about 30 m from the neutrino source. In the detector ν_e's were searched for through the observation of the process

$$\overline{\nu}_e + p \to e^+ + n \,. \tag{211}$$

Both e^+ and delayed 2.2 MeV γ's from the capture $n\,p \to d\,\gamma$ were detected.

In the LSND experiment 33.9 ± 8.0 events were observed in the interval of e^+ energies $30 < E < 60$ MeV. Assuming that these events are due to $\overline{\nu}_\mu \to \overline{\nu}_e$ transitions, for the transition probability it was found that

$$P(\overline{\nu}_\mu \to \overline{\nu}_e) = (0.31 \pm 0.09 \pm 0.06) \times 10^{-3} \,. \tag{212}$$

From the analysis of the LSND data the allowed region in the $\sin^2 2\theta - \Delta m^2$ plot was obtained. If the results of the SBL reactor experiments and the SBL accelerator experiments on the search for $\nu_\mu \to \nu_e$ transitions are taken into account, for the allowed values of the oscillation parameters it was found

$$0.2 \lesssim \Delta m^2 \lesssim 2 \text{ eV}^2 \qquad 2 \times 10^{-3} \lesssim \sin^2 2\theta \lesssim 4 \times 10^{-2} \,. \tag{213}$$

The indications in favour of $\nu_\mu \to \nu_e$ oscillations obtained in the LSND experiment will be checked by the BOONE experiment at Fermilab, scheduled for 2001–2002.

8. CONCLUSIONS

The problem of neutrino masses and mixing is the central problem of today's neutrino physics. More than 40 different experiments all over the world are dedicated to the investigation of this problem, and many new experiments are in preparation. The investigation of the properties of neutrinos is one of the most important directions in the search for a new scale in physics. These investigations will be very important for understanding the origin of tiny neutrino masses and neutrino mixing which, according to the existing data, is very different from CKM quark mixing.

If all the existing data are confirmed by the future experiments it will mean that at least four massive neutrinos exist in nature (in order to to provide three independent neutrino mass squared differences: $\Delta m^2_{\text{solar}} \simeq 10^{-5}$ eV^2 (or 10^{-10} eV^2), $\Delta m^2_{\text{atm}} \simeq 10^{-3}$ eV^2, and $\Delta m^2_{\text{LSND}} \simeq 1$ eV^2). From the phenomenological analysis of all the existing data it follows that in the spectrum of the masses of four massive neutrinos there are two close masses separated by the 'large' one, by about 1 eV LSND gap. Taking into account the big-bang nucleosynthesis constraint on the number of neutrinos, it can be shown that the dominant transition of the solar neutrinos is the $\nu_e \to \nu_{\text{sterile}}$ one, and the dominant transition of the atmospheric neutrinos is $\nu_\mu \to \nu_\tau$.

If the LSND indication in favour of $\nu_\mu \to \nu_e$ oscillations is not confirmed by the future experiments, the mixing of three massive neutrinos with mass hierarchy is a plausible scenario.

The nature of massive neutrinos (Dirac or Majorana?) can be determined from the experiments on the search for neutrinoless double β- decay. It can be shown that from the existing neutrino oscillation data it follows that effective Majorana mass $\langle m \rangle$ in the case of three massive Majorana neutrinos with mass hierarchy is not larger than 10^{-2} eV (the present bound is $|\langle m \rangle| \simeq 0.5$ eV and the sensitivity of the next generation of experiments will be $|\langle m \rangle| \simeq 0.1$ eV).

The sensitivity $|\langle m \rangle| \simeq 10^{-2}$ eV is a very important problem of experiments related to the search for neutrinoless double β-decay.

I would like to express my deepest gratitude to R. Bernabei, W.M. Alberico and S. Bilenkaia for their great help in the preparation of these lecture notes.

References

[1] R.N. Mohapatra and P.B. Pal, *Massive Neutrinos in Physics and Astrophysics*, World Scientific Lecture Notes in Physics, Vol. 41, World Scientific, Singapore, 1991.

[2] F. Boehm and P. Vogel, *Physics of Massive Neutrinos*, Cambridge University Press, Cambridge, 1987.

[3] B. Kayser, F. Gibrat-Debu and F. Perrier, *The Physics of Massive Neutrinos*, World Scientific Lecture Notes in Physics, Vol.25, World Scientific, Singapore, 1989.

[4] J.N. Bahcall, *Neutrino Physics and Astrophysics*, Cambridge University Press, 1989.

[5] C.W. Kim and A. Pevsner, *Neutrinos in Physics and Astrophysics*, Contemporary Concepts in Physics, Vol. 8, Harwood Academic Press, Chur, Switzerland, 1993.

[6] S.M. Bilenky and B. Pontecorvo, Phys. Rep. **41** (1978) 225.

[7] P.H. Frampton and P. Vogel, Phys. Rep. **82** (1982) 339.

[8] J.D. Vergados, Phys. Rep. **133** (1986) 1.

[9] S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. **59** (1987) 671.

[10] J.W.F. Valle, Prog. Part. Nucl. Phys. **26** (1991) 91.

[11] L. Oberauer and F. von Feilitzsch, Rept. Prog. Phys. **55** (1992) 1093.

[12] G. Gelmini and E. Roulet, Rept. Prog. Phys. **58** (1995) 1207.

[13] J. Brunner, Fortsch. Phys. **45** (1997) 343.

[14] K. Zuber, Phys. Rep. **305** (1998) 295.

[15] J.W.F. Valle, hep-ph/9809234 (1998).

[16] G. Raffelt, hep-ph/9902271 (1999).

[17] S.M. Bilenky, C. Giunti and W. Grimus, Progr. in Part. and Nucl. Phys. **43** (1999) 1.