Neutrino masses, mixing and oscillations [1]

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1 Introduction

During many years neutrino physics was a very important branch of elementary particle physics. In the last few years the interest to neutrinos particularly increased. This is connected first of all with the success of the Super-Kamiokande experiment in which very convincing evidence in favour of oscillations of atmospheric neutrinos were obtained.

It is plausible that tiny neutrino masses and neutrino mixing are connected with the new large scale in physics. This scale determines the smallness of neutrino masses with respect to the masses of charged leptons and quarks. In such a scenario neutrinos with definite masses are truly neutral Majorana particles (quarks and leptons have charges and are Dirac particles) It is evident, however, that many new experiments are necessary to reveal the real origin of neutrino masses and mixing.

Experimental neutrino physics is a very difficult and exciting field of research. Now it is a time when many new ideas and methods are being proposed. In CERN and other laboratories projects of new neutrino experiments are developing. Possibilities of new neutrino facility, neutrino factory, are investigated in different laboratories. Thus it is a very appropriate time to discuss neutrino physics at the CERN-JINR school.

In these lectures I will consider different possibilities of neutrino mixing. Then, I will discuss in some details neutrino oscillations in vacuum and in matter. In the last part of the lectures I will consider the present experimental situation.

I tried to give in these lectures some important results and details of derivation of some results. I hope that lectures will be useful for those who want to study physics of massive neutrinos. More results and details can be found in the books [1]– [5] and reviews [6]– [17].

Most references to original papers can be found in [17]

2 Neutrino mixing

According to the Standard Model of electroweak interaction the Lagrangian of the interaction of neutrinos with other particles is given by the Charged Current (CC) and the Neutral Current (NC) Lagrangians:

$$\mathcal{L}_I^{\mathrm{CC}} = -\frac{g}{2\sqrt{2}}\, j_\alpha^{\mathrm{CC}}\, W^\alpha + \mathrm{h.c.}\,, \tag{2.1}$$

$$\mathcal{L}_I^{\mathrm{NC}} = -\frac{g}{2\cos\theta_W}\, j_\alpha^{\mathrm{NC}}\, Z^\alpha\,. \tag{2.2}$$

Here g is the electroweak interaction constant, θ_W is the weak (Weinberg) angle and W^α and Z^α are the fields of the W^{+-} and Z^0 vector bosons. If neutrino masses are equal to zero in this case CC and NC interactions conserve electron L_e, muon L_μ and tauon L_τ lepton numbers

$$\sum L_e = const, \sum L_\mu = const, \sum L_\tau = const \tag{2.3}$$

Table 2.1. Lepton numbers of neutrinos and charged leptons.
Lepton numbers of all other particles are equal to zero.

	L_e	L_μ	L_τ
$(\nu_e\,,\,e^-)$	+1	0	0
$(\nu_\mu\,,\,\mu^-)$	0	+1	0
$(\nu_\tau\,,\,\tau^-)$	0	0	+1

The values of the lepton numbers of charged leptons, neutrinos and other particles are given in the Table 2.1.

According to the *neutrino mixing hypothesis* masses of neutrinos are different from zero and *neutrino mass term* does not conserve lepton numbers. For the fields of ν_{lL} that enter into CC and NC Lagrangians (2.1) and (2.2) we have, in this case,

$$\nu_{lL} = \sum_i U_{li}\,\nu_{iL} \tag{2.4}$$

where ν_i is the field of neutrino with mass m_i and U is the unitary mixing matrix.

The relation (2.4) leads to violation of lepton numbers due to small neutrino mass differences and neutrino mixing. To reveal such effects special experiments (neutrino oscillation experiments, neutrinoless double β-decay experiments and others) are necessary. We will discuss such experiments later. Now we will consider *different possibilities of neutrino mixing.*

Let us notice first of all that the relation 2.4 is similar to the analogous relation in the quark case. The standard CC current of quarks have the form

$$j_\alpha^{CC} = 2(\overline{u_L}\gamma_\alpha d'_L + \bar{c}_L\gamma_\alpha s'_L + \bar{t}_L\gamma_\alpha b'_L) \tag{2.5}$$

Here

$$d'_L = \sum_{q=d,s,b} V_{uq}q_L, \qquad s'_L = \sum_{q=d,s,b} V_{cq}q_L, \qquad b'_L = \sum_{q=d,s,b} V_{tq}q_L \tag{2.6}$$

where V is Cabibbo–Kobayashi–Maskawa quark mixing matrix. There can be, however, a fundamental difference between mixing of quarks and neutrino mixing. Quarks are charged four–component Dirac particles: quarks and antiquarks have different charges.

For neutrinos with definite masses there are two possibilities:

1. In case the total lepton number $L = L_e + L_\mu + L_\tau$ is conserved, neutrino with definite masses ν_i are four–component *Dirac particles* (neutrinos and antineutrinos differ by the sign of L);

2. If there are no conserved lepton numbers, neutrinos with definite masses ν_i are two-component *Majorana particles* (there are no quantum numbers in this case that can allow to distinguish neutrino from antineutrino).

The nature of neutrino masses and the character of neutrino mixing is determined by the *neutrino mass term*.

2.1 Dirac Neutrinos

If the neutrino mass term is generated by the same standard Higgs mechanism, that is responsible for the mass generation of quarks and charged leptons, then for the neutrino mass term we have

$$\mathcal{L}^{\mathrm{D}} = -\sum_{l,l'} \overline{\nu_{l'R}}\, M^{\mathrm{D}}_{l'l}\, \nu_{lL} + \mathrm{h.c.} \tag{2.7}$$

where M^D is the complex 3×3 matrix and ν_{lR} is the right–handed singlet. In the case of mass term (2.7) the total Lagrangian is invariant under global gauge invariance

$$\nu_{lL} \to e^{i\alpha}\nu_{lL}\,, \qquad \nu_{lR} \to e^{i\alpha}\nu_{lR}\,, \qquad l \to e^{i\alpha} l\,, \tag{2.8}$$

where α is a constant that does not depend on the flavor index l. The invariance under the transformation (2.8) means that the total lepton number $L = L_e + L_\mu + L_\tau$ is conserved

$$\sum L = const \tag{2.9}$$

Now let us diagonalize the mass term (2.7). The complex matrix M^{D} can be diagonalized by biunitary transformation

$$M^{\mathrm{D}} = V m U^\dagger\,, \tag{2.10}$$

where $V^\dagger V = 1$, $U^\dagger U = 1$ and $m_{ik} = m_i \delta_{ik}, \quad m_i > 0$.

With the help of (2.10), from (2.7) for the neutrino mass term we obtain the standard expression

$$\mathcal{L}^{\mathrm{D}} = -\sum_{l',l,i} \overline{\nu_{l'R}}\, V_{l'i}\, m_i (U^\dagger)_{il}\, \nu_{lL} + \mathrm{h.c.} = -\sum_{i=1}^{3} m_i \overline{\nu_i} \nu_i \tag{2.11}$$

Here

$$\nu_i = \nu_{iL} + \nu_{iR} \qquad (i = 1, 2, 3)$$

and

$$\nu_{iL} = \Sigma_l (U^\dagger)_{il} \nu_{lL}$$
$$\nu_{iR} = \Sigma_l (V^\dagger)_{il} \nu_{lR}$$

For the neutrino mixing we have

$$\nu_{lL} = \sum_i U_{li}\, \nu_{iL} \tag{2.12}$$

Processes in which the total lepton number is conserved, like $\mu \to e + \gamma$ and others, are, in principle, allowed in the case of mixing of Dirac massive neutrinos. It can be shown, however, that the probabilities of such processes are much smaller than the experimental upper bounds.

Neutrinoless double β–decay,

$$(A, Z) \to (A, Z+2) + e^- + e^- \,,$$

due to the conservation of the total lepton number is forbidden in the case of Dirac massive neutrinos.

2.2 Majorana neutrinos

Neutrino mass terms that are generated in the framework of the models beyond the Standard Model, like the Grand Unified SO(10) Model, do not conserve lepton numbers L_e, L_μ and L_τ. Let us build the most general neutrino mass term that does not conserve L_e, L_μ and L_τ.

Neutrino mass term is a linear combination of the products of left–handed and right-handed components of neutrino fields. Notice that $(\nu_L)^C = C(\overline{\nu}_L)^T$ is the right–handed component and $(\nu_R)^C = C(\overline{\nu}_R)^T$ is the left–handed component. Here C is the charge conjugation matrix, that satisfies the relations $C\gamma_\alpha^T C^{-1} = -\gamma_\alpha$, $C^T = -C$, $C^\dagger C = 1$ [2]

The most general Lorentz–invariant neutrino mass term in which flavor neutrino fields ν_{lL} and right–handed singlet fields ν_{lR} enter has the following form

$$\mathcal{L}^{\mathrm{D-M}} = -\frac{1}{2}\overline{(n_L)^C}\, M n_L + \mathrm{h.c.} \tag{2.13}$$

Here

$$n_L = \begin{pmatrix} \nu'_L \\ (\nu'_R)^C \end{pmatrix} \quad \text{with} \quad \nu'_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} \quad \text{and} \quad \nu'_R = \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix}, \tag{2.14}$$

M is complex 6×6 matrix. Taking into account that $\overline{(\nu_L)^C} = -\nu_L^T C^{-1}$ we have

$$\mathcal{L}^{\mathrm{D-M}} = \frac{1}{2} n_L^T\, \mathcal{C}^{-1}\, M\, n_L + \mathrm{h.c.}\,. \tag{2.15}$$

From this expression it is obvious that there is no global gauge invariance in the case of the mass term (2.13), i.e. that the mass term (2.13) does not conserve lepton numbers.

The matrix M is symmetric. In fact, taking into account the commutation properties of fermion fields we have

$$n_L^T\, \mathcal{C}^{-1}\, M\, n_L = -n_L^T\, (\mathcal{C}^T)^{-1}\, M^T\, n_L = n_L^T\, \mathcal{C}^{-1}\, M^T\, n_L\,. \tag{2.16}$$

[2]In fact, L and R components satisfy the relations

$$\frac{1+\gamma_5}{2}\nu_L = 0 \qquad \frac{1-\gamma_5}{2}\nu_R = 0$$

From the first of these relations we have $\overline{\nu}_L(1-\gamma_5)/2 = 0$. Further, from this last relation we obtain $[(1-\gamma_5)/2]^T \overline{\nu}_L^T = 0$. Multiplying this relation by the matrix C from the left and taking into account that $C\gamma_5^T C^{-1} = \gamma^5$ we have $[(1-\gamma_5)/2](\nu_L)^C = 0$. Thus, $(\nu_L)^C$ is right–handed component. Analogously we can show that $(\nu_R)^C$ is left-handed component.

From this relation it follows that

$$M^T = M$$

The symmetric 6×6 matrix can be presented in the form

$$M = \begin{pmatrix} M_L & (M_{\rm D})^T \\ M_{\rm D} & M_R \end{pmatrix} . \tag{2.17}$$

where $M_L = M_L^T$, $M_R = M_R^T$ and M^D are 3×3 matrices. With the help of (2.17) for the mass term (2.15) we have

$$\mathcal{L}^{\rm D-M} = \mathcal{L}_L^{\rm M} + \mathcal{L}^{\rm D} + \mathcal{L}_R^{\rm M} . \tag{2.18}$$

Here $\mathcal{L}^D$ is the Dirac mass term, that we have considered before, and the new terms

$$\mathcal{L}_L^{\rm M} = -\frac{1}{2} \sum_{l',l} \overline{(\nu_{l'L})^c}\, M_{l'l}^L \, \nu_{lL} + \text{h.c.} , \tag{2.19}$$

$$\mathcal{L}_R^{\rm M} = -\frac{1}{2} \sum_{l',l} \overline{(\nu_{l'R})^c}\, M_{l'l}^R \, \nu_{lR} + \text{h.c.} , \tag{2.20}$$

which do not conserve lepton numbers are called left–handed and right–handed Majorana mass terms, respectively. The mass term (2.13) is called Dirac–Majorana mass term.

A symmetrical matrix can be diagonalized with the help of unitary transformation

$$M = (U^\dagger)^T m U^\dagger .$$

Here U is unitary matrix and $m_{ik} = m_i \delta_{ik}$, $m_i > 0$. Using the relation (11) we can write the mass term (2.15) in the standard form

$$\mathcal{L}^{\rm D-M} = -\frac{1}{2} \overline{(U^\dagger n_L)}^C \, m U^+ n_L + \text{h.c.} = -\frac{1}{2} \overline{\nu} m \nu = -\frac{1}{2} \sum_{i=1}^{6} m_i \overline{\nu}_i \nu_i , \tag{2.21}$$

where

$$\nu = U^+ n_L + (U^+ n_L)^C = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \vdots \\ \nu_6 \end{pmatrix} , \tag{2.22}$$

Thus the fields ν_i (i=1,2...6) are the fields of neutrinos with mass m_i. From (2.22) it follows that the fields ν_i satisfy the *Majorana condition*

$$\nu_i^C = \nu_i , \tag{2.23}$$

Let us obtain now the relation that connects the left–handed flavor fields ν_{lL} with the massive fields ν_{iL}. From (2.22) for the left–handed components we have

$$n_L = U \nu_L . \tag{2.24}$$

From this relation for the flavor field ν_{lL} it follows

$$\nu_{lL} = \sum_{i=1}^{6} U_{li}\nu_{iL} \qquad (l = e, \mu, \tau)\,, \tag{2.25}$$

Thus, in the case of Dirac–Majorana mass term, the flavor fields are linear combinations of left–handed components of six massive Majorana fields. From (2.25) it follows that the fields ν_{lR}^{C} are orthogonal linear combinations of the same massive Majorana fields

$$(\nu_{lR})^{C} = \sum_{i=1}^{6} U_{\bar{l}i}\nu_{iL}\,. \tag{2.26}$$

In the case of Majorana field particles and antiparticles, quanta of the field, are identical. In fact, for fermion fields $\nu(x)$ we have in general case

$$\nu(x) = \int \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2p^0}} \left[c_r(p)u^r(p)e^{-ipx} + d_r^{\dagger}(p)C\left(\overline{u}^r(p)\right)^T e^{ipx}\right] d^3p \tag{2.27}$$

where $c_r(p)(d_r^{\dagger}(p))$ is the operator of absorption of particle (creation of antiparticle) with momentum p and helicity r. If the field $\nu(x)$ satisfies the Majorana condition (2.23), then we have

$$c_r(p) = d_r(p) \tag{2.28}$$

Let us stress that it is natural that the neutrinos with definite masses in the case of Dirac–Majorana mass term are Majorana neutrinos: in fact there are no conserved quantum numbers that could allow us to distinguish particles and antiparticles.

2.3 The simplest case of one generation (Majorana neutrinos)

It is instructive to consider in detail the Dirac–Majorana mass term in the simplest case of one generation. We have

$$\begin{aligned}\mathcal{L}^{\mathrm{D-M}} &= -\frac{1}{2}\,m_L\,(\overline{\nu}_L)^c\,\nu_L - m_{\mathrm{D}}\,\overline{\nu}_R\,\nu_L - \frac{1}{2}\,m_R\,\overline{\nu}_R\,(\nu_R)^c + \text{h.c.}\\ &= -\frac{1}{2}\,(\overline{n}_L)^c\,M\,n_L + \text{h.c.}\,,\end{aligned} \tag{2.29}$$

where

$$n_L \equiv \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix}, \qquad M \equiv \begin{pmatrix} m_L & m_{\mathrm{D}} \\ m_{\mathrm{D}} & m_R \end{pmatrix}. \tag{2.30}$$

Let us assume that the parameters m_L, m_R and m_D are real (the case of CP invariance). In order to diagonalize the mass term (2.29) let us write the matrix M in the form

$$M = \frac{1}{2}\,\mathrm{Tr}M + \underline{M}\,, \tag{2.31}$$

where Tr $M = m_L + m_D$ and

$$\underline{M} = \begin{pmatrix} -\frac{1}{2}(m_R - m_L) & m_{\rm D} \\ m_{\rm D} & \frac{1}{2}(m_R - m_L) \end{pmatrix}. \tag{2.32}$$

For the symmetrical real matrix we have

$$\underline{M} = \mathcal{O}\,\underline{m}\mathcal{O}^T. \tag{2.33}$$

Here

$$\mathcal{O} = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix} \tag{2.34}$$

is an orthogonal matrix, and $\underline{m}_{ik} = \underline{m}_i\delta_{ik}$, where

$$\underline{m}_{1,2} = \mp\frac{1}{2}\sqrt{(m_R - m_L)^2 + 4m_D^2} \tag{2.35}$$

are eigenvalues of the matrix $\underline{M}$.

From (2.33), (2.34) and (2.35) for the parameters $\cos\vartheta$ and $\sin\vartheta$ we easily find the following expressions

$$\cos 2\vartheta = \frac{m_R - m_L}{\sqrt{(m_R - m_L)^2 + 4\,m_D^2}}\,, \quad \tan 2\vartheta = \frac{2m_D}{(m_R - m_L)}\,. \tag{2.36}$$

For the matrix M from (2.33) and (2.35) we have

$$M = Om'O^T$$

where

$$m'_{1,2} = \frac{1}{2}\,(m_R + m_L) \mp \sqrt{(m_R - m_L)^2 + 4\,m_D^2}\,. \tag{2.37}$$

The eigenvalues m'_i can be positive or negative. Let us write

$$m'_i = m_i\eta_i\,, \tag{2.38}$$

where $m_i = |m_i|$ and η_i is the sign of the i-eigenvalue. With the help of (2.33) and (2.38) we have

$$M = (U^\dagger)^T m U^\dagger$$

Here

$$U^\dagger = \sqrt{\eta}O^T$$

where $\sqrt{\eta}$ takes the values 1 and i.

Now using the general formulas (2.21) and (2.22) for the mass term we have

$$\mathcal{L}^{D-M} = -\frac{1}{2}\sum_{i=1,2} m_i\overline{\nu}_i\nu_i \tag{2.39}$$

Here $\nu_i = \nu_i^C$ is the field of the Majorana particles with mass m_i. The fields ν_L and $(\nu_R)^C$ are connected with massive fields by the relation

$$\left(\begin{array}{c} \nu_L \\ (\nu_R)^C \end{array} \right) = U \left(\begin{array}{c} \nu_{1L} \\ \nu_{2L} \end{array} \right) , \tag{2.40}$$

where $U = O(\sqrt{\eta})^*$ is a 2×2 mixing matrix.

Let us consider now three special cases.

1. No mixing

   Assume $m_D = 0$. In this case $\theta = 0$, $m_1 = m_L$, $m_2 = m_R$ and $\eta = 1$ (assuming that m_L and m_R are positive). From (2.40) we have

   $$\nu_L = \nu_{1L} \qquad (\nu_R)^C = \nu_{2L} \, . \tag{2.41}$$

   Thus, if $m_{\rm D} = 0$ there is no mixing. For the Majorana fields ν_1 and ν_2 we have

   $$\nu_1 = \nu_L + (\nu_L)^C \tag{2.42}$$

   $$\nu_2 = \nu_R + (\nu_R)^C \, . \tag{2.43}$$

2. Maximal mixing

   Assume $m_R = m_L$, $m_D \neq 0$. From Eq. (2.36), (2.37) and (2.40) we have

   $$\theta = \frac{\pi}{4}, \qquad m_{1,2} = m_L \mp m_D \, . \tag{2.44}$$

   (assuming $|m_D| < m_L$) and

   $$\nu_L = \frac{1}{\sqrt{2}}\nu_{1L} + \frac{1}{\sqrt{2}}\nu_{2L}; \qquad (\nu_R)^C = -\frac{1}{\sqrt{2}}\nu_{1L} + \frac{1}{\sqrt{2}}\nu_{2L} \, . \tag{2.45}$$

   Thus if the diagonal elements of the mass matrix M are equal, then we have maximal mixing.

3. See–saw mechanism of neutrino mass generation

   Assume $m_L = 0$ and

   $$m_D \ll m_R \tag{2.46}$$

   From (2.35) and (2.37) we have in this case

   $$m_1 \simeq \frac{m_D^2}{m_R}, \quad m_2 \simeq m_R, \quad \theta \simeq \frac{m_D}{m_R} \quad (\eta_1 = -1, \eta_2 = 1) \, . \tag{2.47}$$

   Neglecting terms linear in $m_D/m_R \ll 1$, from (2.40) we have

   $$\nu_L \simeq -i\nu_{1L}, \qquad (\nu_R)^C \simeq \nu_{2L} \, . \tag{2.48}$$

For the Majorana fields we have

$$\nu_1 \simeq i\nu_L - i(\nu_L)^C, \qquad \nu_2 = \nu_R + (\nu_R)^C \,. \tag{2.49}$$

Thus if the condition (2.46) is satisfied, in the spectrum of masses of Majorana particles there are one light particle with the mass $m_1 << m_D$ and one heavy particle with the mass $m_1 >> m_D$. The condition $m_L = 0$ means that the lepton number is violated only by the right–handed term $-\frac{1}{2} m_R \overline{\nu}_R (\nu_R)^C$ that is characterized by the large mass m_R. It is natural to assume that the parameter m_D which characterizes the Dirac term $-m_D \overline{\nu}_R \nu_L$ is of the order of lepton or quark masses. The mass of the light Majorana neutrino m_1 will be in this case much smaller than the mass of lepton or quark. This is famous *see-saw mechanism.* This mechanism connects the smallness of the neutrino masses with respect to the masses of other fundamental fermions with violation of the lepton numbers at very large scale (usually $m_D \simeq M_{GUT} \simeq 10^{16}$ GeV.

In the case of the see–saw for three families in the spectrum of masses of Majorana particles there are three light masses m_1, m_2, m_3 (masses of neutrinos) and three very heavy masses M_1, M_2, M_3. Masses of neutrinos are connected with the masses of heavy Majorana particles by the see–saw relation

$$m_i \simeq \frac{(m_{\mathrm{f}}^i)^2}{M_i} \ll m_{\mathrm{f}}^i \qquad (i = 1, 2, 3)\,. \tag{2.50}$$

where m_{f}^i is the mass of lepton or quark in i-family. The see–saw mechanism is a plausible explanation of the experimentally observed smallness of neutrino masses. Let us stress that if neutrino masses are of the see-saw origin then

a. neutrinos with definite masses are Majorana particles;

b. there are three massive neutrinos;

c. there must be a hierarchy of neutrino masses $m_1 \ll m_2 \ll m_3$.

3 Neutrino oscillations

The most important consequences of the neutrino mixing are so called *neutrino oscillations.* Neutrino oscillations were first considered by B. Pontecorvo many years ago in 1957-58. Only one type of neutrino was known at that time and there was general belief that neutrino is a massless two–component particle. B. Pontecorvo draw attention that there is no known principle which requires neutrino to be massless (like gauge invariance for the photon) and that the investigation of neutrino oscillations is a very sensitive method to search for effects of small neutrino masses. We will consider here in detail the phenomenon of neutrino oscillations.

Assume that there is neutrino mixing

$$\nu_{\alpha L} = \sum_i U_{\alpha i}\, \nu_{iL} \,. \tag{3.51}$$

where $U^\dagger U = 1$ and ν_i is the field of neutrino (Dirac or Majorana) with the mass m_i. The field $\nu_{\alpha L}$ in (3.51) are flavor fields ($\alpha = e, \mu, \tau$) and in general also sterile ones ($\alpha = s_1, ...$).

Let us assume that neutrino mass differences are small and different neutrino masses cannot be resolved in neutrino production and detection processes.

For the state of neutrino with momentum $\vec{p}$ we have

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}\,|\nu_i\rangle\,. \tag{3.52}$$

where $|\nu_i\rangle$ is the vector of state of neutrino with momentum $\vec{p}$, energy

$$E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p} \qquad (p \gg m_i)\,. \tag{3.53}$$

and (up to the terms m_i^2/p^2) helicity is equal to -1. If at the initial time $t = 0$ the state of neutrino is $|\nu_\alpha\rangle$ at the time t for the neutrino state we have

$$|\nu_\alpha\rangle_t = \sum_i U^*_{\alpha i}\, e^{-iE_i t}\,|\nu_i\rangle\,. \tag{3.54}$$

The vector $|\nu_\alpha\rangle$ is the superposition of the states of all types of neutrino. In fact, from (3.52), using unitarity of the mixing matrix, we have

$$|\nu_i\rangle = \sum_{\alpha'} |\nu_{\alpha'}\rangle U_{\alpha' i}\,. \tag{3.55}$$

From (3.54) and (3.55) we have

$$|\nu_\alpha\rangle_t = \sum_{\alpha'} |\nu_{\alpha'}\rangle \mathcal{A}_{\nu'_\alpha;\nu_\alpha}(t)\,. \tag{3.56}$$

where

$$\mathcal{A}_{\nu'_\alpha;\nu_\alpha}(t) = \sum_i U_{\alpha' i} e^{-iE_i t} U^*_{\alpha i}\,. \tag{3.57}$$

is the amplitude of the transition $\nu_\alpha \to \nu_{\alpha'}$ at the time t. The transition amplitude $\mathcal{A}_{\alpha';\alpha}(t)$ has a simple meaning: the term $U^*_{\alpha i}$ is the amplitude of the transition from the state $|\nu_\alpha\rangle$ to the state $|\nu_i\rangle$; the term $e^{-iE_i t}$ describes the evolution in the state with energy E_i; the term $U_{\alpha' i}$ is the transition amplitude from the state $|\nu_i\rangle$ to the state $|\nu'_\alpha\rangle$.

The different $|\nu_i\rangle$ gives coherent contribution to the amplitude $\mathcal{A}_{\nu'_\alpha;\nu_\alpha}(t)$. From (3.57) it follows that the transitions between different states can take place only if: i) at least two neutrino masses are different; ii) the mixing matrix is non–diagonal. In fact, if all neutrino masses are equal we have $a(t) = e^{-iEt}\sum U_{\alpha' i} U^*_{\alpha i} = e^{-iEt}\delta_{\alpha'\alpha}$. If the mixing matrix is diagonal (no mixing), we have $\mathcal{A}_{\nu'_\alpha;\nu_\alpha}(t) = e^{-iE_\alpha t}\delta_{\alpha'\alpha}$.

Let us numerate neutrino masses in such a way that $m_1 < m_2 < \ldots < m_n$. For the transition probability, from (3.57), we have the following expression:

$$\begin{aligned} P_{\nu_\alpha\to\nu_{\alpha'}} &= \left|\sum_i U_{\alpha' i}\,\left[\left(e^{-i(E_i - E_1)t} - 1\right) + 1\right]\,U^*_{\alpha i}\right|^2 \\ &= \left|\delta_{\alpha\alpha'} + \sum_i U_{\alpha' i}\, U^*_{\alpha i}\left(e^{-i\,\Delta m^2_{i1}\,\frac{L}{2p}} - 1\right)\right|^2\,, \end{aligned} \tag{3.58}$$

where $\Delta m^2_{i1} = m^2_i - m^2_1$ and $L \simeq t$ is the distance between neutrino source and neutrino detector. Thus the neutrino transition probability depends on the ratio $\frac{L}{E}$, the range of values of which is determined by the conditions of an experiment.

It follows from Eq. (3.58) that the transition probability depends in the general case on $(n-1)$ neutrino mass squared differences and parameters that characterize the mixing matrix U. The $n \times n$ matrix U is characterized by $n_\theta = n(n-1)/2$ angles. The number of phases for Dirac and Majorana cases is different. If neutrino with definite masses ν_i are Dirac particles the number of phases is equal to $n^D_\phi = (n-1)(n-2)/2$. If ν_i are Majorana particles the number of phases is equal to $n^{M_j}_\phi = n(n-1)/2$.

Notice that from (3.58) it follows that transition probability is invariant under the transformation

$$U_{\alpha i} \to e^{-i\beta_\alpha}\, U_{\alpha i}\, e^{i\alpha_i} \tag{3.59}$$

where β_α and α_i are arbitrary real phases. From (3.59) it follows that the number of phases that enter into the transition probability is equal to $n_\phi = (n-1)(n-2)/2$ in both Dirac and Majorana cases. We come to the conclusion that additional Majorana phases do not enter into the transition probability . Thus, by investigation of neutrino oscillations it is impossible to distinguish the case of Dirac neutrinos from the case of Majorana neutrinos.

Let us consider now oscillations of antineutrinos. For the vector of state of antineutrino with momentum $\vec{p}$ from (3.51) we have

$$|\overline{\nu}_\alpha\rangle = \sum_i U_{\alpha i} |\overline{\nu}_i\rangle \qquad \text{(Dirac case)} \tag{3.60}$$

$$|\overline{\nu}_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle \qquad \text{(Majorana case)} \tag{3.61}$$

where $|\overline{\nu}_i\rangle$ ($|\nu_i\rangle$) is the state of antineutrino (neutrino) with momentum $\vec{p}$, energy $E_i = \sqrt{p^2 + m^2_i} \simeq p + m^2_i/2p$ and helicity equal to $+1$ (up to m^2_i/p^2 terms).

In analogy with (3.57) for the amplitude of the transition $\overline{\nu}_\alpha \to \overline{\nu}_{\alpha'}$ in both Dirac and Majorana cases we have

$$\mathcal{A}_{\overline{\nu}_{\alpha'};\overline{\nu}_\alpha}(t) = \sum_i U^*_{\alpha' i} e^{-iE_i t} U_{\alpha i}\,. \tag{3.62}$$

If we compare (3.57) and (3.62) we come to the conclusion that

$$\mathcal{A}_{\overline{\nu}_{\alpha'};\overline{\nu}_\alpha}(t) = \mathcal{A}_{\nu_\alpha;\nu_{\alpha'}}(t)\,. \tag{3.63}$$

Thus for the transition probabilities we have the following relation

$$\mathrm{P}(\nu_\alpha \to \nu_{\alpha'}) = \mathrm{P}(\overline{\nu}_{\alpha'} \to \overline{\nu}_\alpha)\,. \tag{3.64}$$

This relation is the consequence of CPT invariance. If CP invariance in the lepton sector takes place then for Dirac neutrinos we have

$$U^*_{\alpha i} = U_{\alpha i} \tag{3.65}$$

while for Majorana neutrinos, from CP invariance, we have

$$U_{\alpha i}\eta_i = U^*_{\alpha i}\,; \tag{3.66}$$

where $\eta_i = \pm i$ is the CP parity of the Majorana neutrino with mass m_i. From (3.57), (3.63), (3.65) and (3.66) it follows that in case of CP invariance we have

$$\mathrm{P}(\nu_\alpha \to \nu'_\alpha) = \mathrm{P}(\overline{\nu}_\alpha \to \overline{\nu}'_\alpha)\,. \tag{3.67}$$

Let us go back to the Eq. (3.58). It is obvious from (3.58) that if the conditions of an experiment are such that $\Delta m^2_{i1}\frac{L}{p} \ll 1$ for all i then neutrino oscillations cannot be observed. To observe neutrino oscillations it is necessary that for at least one neutrino mass squared difference the condition $\Delta m^2\frac{L}{p} \gtrsim 1$ is satisfied. We will discuss this condition later.

3.1 Two neutrino oscillations

Let us consider in details the simplest case of the oscillations between two neutrinos $\nu_\alpha \leftrightarrows \nu_{\alpha'}$ ($\alpha' \neq \alpha; \alpha, \alpha'$ are equal to μ, e or τ, μ,...). The index i in Eq. (3.58) takes values 1 and 2 and for the transition probability we have

$$\mathrm{P}(\nu_\alpha \to \nu'_\alpha) = |\delta_{\alpha'\alpha} + U_{\alpha'_2}U^*_{\alpha 2}(e^{-i\Delta m^2_{21}\frac{L}{2p}} - 1)|^2 \tag{3.68}$$

For $\alpha' \neq \alpha$ we have from (3.68)

$$\mathrm{P}(\nu_\alpha \to \nu'_\alpha) = \mathrm{P}(\nu_{\alpha'} \to \nu_\alpha) = \frac{1}{2}\mathrm{A}_{\alpha'\alpha}(1 - \cos\Delta m^2\frac{L}{2p}) \tag{3.69}$$

Here the amplitude of oscillations is equal to

$$\mathrm{A}_{\alpha';\alpha} = 4|U_{\alpha'2}|^2|U_{\alpha 2}|^2 \tag{3.70}$$

and $\Delta m^2 = m_2^2 - m_1^2$. Due to unitarity of the mixing matrix

$$|U_{\alpha 2}|^2 + |U_{\alpha' 2}|^2 = 1 \qquad (\alpha' \neq \alpha) \tag{3.71}$$

Let us introduce the mixing angle θ

$$|U_{\alpha 2}|^2 = \sin^2\theta \qquad |U_{\alpha' 2}|^2 = \cos^2\theta \tag{3.72}$$

Thus the oscillation amplitude $\mathrm{A}_{\alpha';\alpha}$ is equal to

$$\mathrm{A}_{\alpha';\alpha} = \sin^2 2\theta \tag{3.73}$$

The survival probabilities $\mathrm{P}(\nu_\alpha \to \nu_\alpha)$ and $\mathrm{P}(\nu_{\alpha'} \to \nu_{\alpha'})$ can be obtained from (3.68) or from the condition of the conservation of the total probability $\mathrm{P}(\nu_\alpha \to \nu_\alpha) + \mathrm{P}(\nu_\alpha \to \nu_{\alpha'}) = 1$. We have

$$\mathrm{P}(\nu_\alpha \to \nu_\alpha) = \mathrm{P}(\nu_{\alpha'} \to \nu_{\alpha'}) = 1 - \frac{1}{2}\sin^2 2\theta(1 - \cos\frac{\Delta m^2 L}{2p}) \tag{3.74}$$

Thus in the case of two neutrinos the transition probabilities are characterized by two parameters $\sin^2 2\theta$ and Δm^2.

Let us notice that in the case of transitions between two neutrinos only moduli of the elements of the mixing matrix enter into expressions for the transition probabilities. This means that in this case the CP relation (3.64) is satisfied automatically. Thus, in order to observe effects of CP violation in the lepton sector the transitions between three neutrinos must take place (this is similar to the quark case: for two families of quarks CP is conserved due to unitarity of the mixing matrix).

We also notice that the expression (3.69) for the transition probability can be written in the form

$$\mathrm{P}(\nu_\alpha \to \nu_{\alpha'}) = \frac{1}{2}\sin^2 2\theta \left(1 - \cos 2\pi \frac{L}{L_0}\right) \tag{3.75}$$

where

$$L_0 = 4\pi \frac{E}{\Delta m} \tag{3.76}$$

is the oscillation length. The expression (3.69) is written in the units $\hbar = c = 1$. We can write it in the form

$$\mathrm{P}(\nu_\alpha \to \nu_{\alpha'}) = \frac{1}{2}\sin^2 2\theta \left(1 - \cos\, 2.54\Delta m^2 \frac{E}{L}\right) \tag{3.77}$$

where Δm^2 is neutrino mass squared difference in eV^2, L is the distance in m (km) and E is the neutrino energy in MeV (GeV). For the oscillation length we have

$$L_0 = 2.47 \frac{E(\mathrm{MeV})}{\Delta m^2(\mathrm{eV}^2)}\,\mathrm{m} \tag{3.78}$$

The Eq. (3.69) and (3.74) describe periodical transitions (oscillations) between different types of neutrinos due to difference of neutrino masses and to neutrino mixing. The transition probability depends periodically on L/E. At the values of L/E at which the condition $2.54\,\Delta m^2(L/E) = \pi(2n+1) \quad (n = 0, 1, ...)$ is satisfied, the transition probability is equal to the maximal value $\sin^2 2\theta$. If the condition $2.54\,\Delta m^2(L/E) = 2\pi n$ is satisfied, the transition probability is equal to zero.

In order to see neutrino oscillations it is necessary that the parameter Δm^2 is large enough so that the condition $\Delta m^2(L/E) \geq 1$ is satisfied. This condition allows us to estimate the minimal value of the parameter Δm^2 that can be revealed in an experiment on the search for neutrino oscillations. For short and long baseline experiments with accelerator (reactor) neutrinos for Δm^2_{min} we have, respectively 10 – 1 eV^2, 10^{-2} – 10^{-3} eV^2 (10^{-1} – 10^{-2} eV^2, 10^{-2} – 10^{-3} eV^2). For atmospheric and solar neutrinos for Δm^2_{min} we have 10^{-2} – 10^{-3} eV^2 and 10^{-10} – 10^{-11} eV^2 , respectively. Let us notice that in the case of $\Delta m^2(L/E) \ll 1$, due to averaging over neutrino spectrum and over distances between neutrino production and detection points, the term $\cos \Delta m^2(L/2p)$ in the transition probability disappears and the averaged transition probabilities are given by $\overline{\mathrm{P}}(\nu_\alpha \to \nu_{\alpha'}) = \frac{1}{2}\sin^2 2\theta$ and $\overline{\mathrm{P}}(\nu_\alpha \to \nu_\alpha) = 1 - \frac{1}{2}\sin^2 2\theta$.

3.2 Three neutrino oscillations in the case of neutrino mass hierarchy

The two neutrino transition probabilities (3.69) and (3.74) are usually used for the analysis of experimental data. Let us consider now the case of the transitions between three flavor neutrinos.

General expressions for transition probabilities between three neutrino types are characterized by 6 parameters and have a rather complicated form. We will consider the case of hierarchy of neutrino masses

$$m_1 \ll m_2 \ll m_3$$

which corresponds to the oscillations of solar and atmospheric neutrinos (we have in mind that Δm^2_{21} can be relevant for oscillations of solar neutrinos and Δm^2_{31} can be relevant for oscillations of atmospheric neutrinos; from the analysis of the experimental data it follows that $\Delta m^2_{sol} \simeq 10^{-5}$ eV^2 (or 10^{-10} eV^2) and $\Delta m^2_{atm} \simeq 10^{-3}$ eV^2; see later). We will see that transition probabilities have in this case the rather simple two–neutrino form.

Let us consider neutrino oscillations in experiments for which the largest neutrino mass squared difference Δm^2_{31} is relevant. For such experiments

$$\Delta m^2_{12} \frac{L}{2p} \ll 1 \tag{3.79}$$

and for the probability of the transition $\nu_\alpha \to \nu_{\alpha'}$, from (3.58) we obtain the following expression

$$\mathrm{P}(\nu_\alpha \to \nu_{\alpha'}) = \left|\delta_{\alpha'\alpha} + U_{\alpha'3} U^*_{\alpha 3} \left(e^{-i\Delta m^2_{31} \frac{L}{2p}} - 1\right)\right|^2 \tag{3.80}$$

For the transition probability $\nu_\alpha \to \nu_{\alpha'}$ ($\alpha' \neq \alpha$) from (3.80) we have

$$\mathrm{P}(\nu_\alpha \to \nu'_\alpha) = \frac{1}{2} \mathrm{A}_{\alpha';\alpha} \left(1 - \cos \Delta m^2_{31} \frac{L}{2p}\right) \tag{3.81}$$

where the amplititude of oscillations is given by

$$\mathrm{A}_{\alpha';\alpha} = 4|U_{\alpha'3}|^2 |U_{\alpha 3}|^2 \tag{3.82}$$

Using unitarity of the mixing matrix, for the survival probability we obtain, from (3.81) and (3.82),

$$\mathrm{P}(\nu_\alpha \to \nu_\alpha) = 1 - \sum_{\alpha' \neq \alpha} \mathrm{P}(\nu_\alpha \to \nu_{\alpha'}) = 1 - \frac{1}{2} \mathrm{B}_{\alpha;\alpha} \left(1 - \cos \Delta m^2_{31} \frac{L}{2p}\right) \tag{3.83}$$

where

$$\mathrm{B}_{\alpha;\alpha} = 4|U_{\alpha 3}|^2 (1 - |U_{\alpha 3}|^2) \tag{3.84}$$

It is natural that Eq. (3.81) and (3.82) have the same dependence on the parameter L/E as the standard two–neutrino formulas (3.68) and (3.74): only the largest Δm^2 is relevant for the oscillations. The oscillation amplitudes $\mathrm{A}_{\alpha;\alpha}$ and $\mathrm{B}_{\alpha;\alpha}$ depend on the moduli squared of the mixing matrix elements that connect neutrino flavors with the heaviest neutrino ν_3. Further, from the unitarity of the mixing matrix it follows that

$$|U_{e3}|^2 + |U_{\mu 3}|^2 + |U_{\tau 3}|^2 = 1 \tag{3.85}$$

Thus, in three–neutrino case with hierarchy of neutrino masses, the transition probabilities in experiments for which Δm^2_{31} is relevant are described by three parameters: Δm^2_{31}, $|U_{e3}|^2$ and $|U_{\mu 3}|^2$ (remember that in the two neutrino case there are two parameters, Δm^2 and $\sin^2 2\theta$).

Since only moduli of the elements of the mixing matrix enter into transition probabilities, the relation

$$\mathrm{P}(\nu_\alpha \to \nu_{\alpha'}) = \mathrm{P}(\overline{\nu}_\alpha \to \overline{\nu}_{\alpha'}) \tag{3.86}$$

holds (as in the two–neutrino case). Thus the violation of the CP–invariance in the lepton sector cannot be revealed in the case of three neutrinos with mass hierarchy. Notice that the relation

$$\mathrm{P}(\nu_\alpha \to \nu_\alpha) = \mathrm{P}(\nu_{\alpha'} \to \nu_{\alpha'})\,, \tag{3.87}$$

which takes place in the case of two neutrino oscillations, is not valid in the three–neutrino case.

Let us consider now neutrino oscillations in the case of experiments for which Δm^2_{21} is relevant ($\Delta m^2_{21}\,\frac{L}{2p} \gtrsim 1$). From (3.57) for the survival probability we obtain in this case the following expression

$$\mathrm{P}(\nu_\alpha \to \nu_\alpha) = \left| \sum_{i=1,2} |U_{\alpha i}|^2 e^{-i\Delta m^2_{i1}\frac{L}{2p}} + |U_{\alpha 3}|^2 e^{-i\Delta m^2_{31}\frac{L}{2p}} \right|^2 \tag{3.88}$$

Due to averaging over neutrino spectra and source–detector distances, the interference term $\cos \Delta m^2_{31}\,(L/2p)$ in Eq. (3.88) disappears and for the probability we have

$$\mathrm{P}(\nu_\alpha \to \nu_\alpha) = |\sum_{i=1,2} |U_{\alpha i}|^2 e^{-i\Delta m^2_{i1}\frac{L}{2p}}|^2 + |U_{\alpha 3}|^4 \tag{3.89}$$

Further, from the unitarity relation $\sum_{i=1}^{3} |U_{\alpha i}|^2 = 1$ we have

$$\sum_{i=1,2} |U_{\alpha i}|^4 = (1 - |U_{\alpha 3}|^2)^2 - 2|U_{\alpha 1}|^2 |U_{\alpha 2}|^2 \tag{3.90}$$

Using (3.90) we can present the survival probability in the form

$$\mathrm{P}(\nu_\alpha \to \nu_\alpha) = (1 - |U_{\alpha 3}|^2)^2 \mathrm{P}^{(1,2)}(\nu_\alpha \to \nu_\alpha) + |U_{\alpha 3}|^4 \tag{3.91}$$

Here

$$\mathrm{P}^{(1,2)}(\nu_\alpha \to \nu_\alpha) = 1 - \frac{1}{2}\sin^2 2\overline{\theta}_{12}(1 - \cos^2 \Delta m^2_{21}\,\frac{L}{2p}) \tag{3.92}$$

and the angle $\overline{\theta}_{12}$ is determined by the relations

$$\cos^2\overline{\theta}_{12} = \frac{|U_{\alpha 1}|^2}{\sum_{i=1,2}|U_{\alpha i}|^2}, \qquad \sin^2\overline{\theta}_{12} = \frac{|U_{\alpha 2}|^2}{\sum_{i=1,2}|U_{\alpha i}|^2}, \tag{3.93}$$

The probability $\mathrm{P}^{(1,2)}(\nu_e \to \nu_e)$ has the two–neutrino form and it is characterized by two parameters: Δm^2_{31} and $\sin^2 2\overline{\theta}_{12}$. We have derived the expression (3.92) for the case of the oscillations in vacuum. Let us notice that similar expression is valid for the case of the neutrino transitions in matter.

The expressions (3.81), (3.83) and (3.92) can be used to describe neutrino oscillations in atmospheric and long baseline neutrino experiments (LBL) as well as in solar neutrino experiments. In the framework of neutrino mass hierarchy, in the probabilities of transition of atmospheric (LBL) and solar neutrinos enter different Δm^2 (Δm^2_{31} and $\Delta m^2_{2,1}$, respectively) and the only element that connects oscillations of atmospheric (LBL) and solar neutrinos is $|U_{e3}|^2$. From LBL reactor experiment CHOOZ and Super–Kamiokande experiment it follows that this element is small (see later). This means that oscillations of atmospheric (LBL) and solar neutrinos are described by different elements of the neutrino mixing matrix.

4 Neutrino in matter

Up to now we have considered oscillations of neutrinos in vacuum. If there is neutrino mixing the effects of the matter can significantly enhance the probability of the transitions between different types of neutrinos (MSW effect). We will consider here this effect in some details.

Let consider neutrinos with momentum $\vec{p}$. The equation of the motion for a free neutrino has the form

$$i\frac{\partial|\psi(t)\rangle}{\partial t} = H_0|\psi(t)\rangle \tag{4.94}$$

Let us develop the state $|\psi(t)\rangle$ over states of neutrinos with definite flavor $|\nu_\alpha\rangle$ ($\alpha = e, \mu, \tau$). We have

$$|\psi(t)\rangle = \sum_\alpha |\nu_\alpha\rangle a_\alpha(t) \tag{4.95}$$

where $a_\alpha(t)$ is the wave function of neutrino in the flavor representation. From (4.94) for $a_\alpha(t)$ we obtain the equation

$$i\frac{\partial a_\alpha(t)}{\partial t} = \sum_{\alpha'}\langle\nu_\alpha|H_0|\nu_{\alpha'}\rangle a_{\alpha'}(t) \tag{4.96}$$

Now we will develop the state $|\nu_\alpha\rangle$ over the eigenstates $|\nu_i\rangle$ of the free Hamiltonian H_0:

$$H_0|\nu_i\rangle = E_i|\nu_i\rangle\,, \tag{4.97}$$

$$E_i = \sqrt{p^2+m_i^2} \simeq p + \frac{m_i^2}{2p}\,. \tag{4.98}$$

We have:

$$|\nu_\alpha\rangle = \sum_i |\nu_i\rangle\langle\nu_i|\nu_\alpha\rangle \tag{4.99}$$

If we compare (4.99) and (3.52) we find

$$\langle\nu_i|\nu_\alpha\rangle = U^*_{\alpha i} \qquad \langle\nu_\alpha|\nu_i\rangle = U_{\alpha i} \tag{4.100}$$

Further we have

$$\langle\nu_\alpha|H_0|\nu_{\alpha'}\rangle = \sum_i \langle\nu_\alpha|\nu_i\rangle\langle\nu_i|H_0|\nu_i\rangle\langle\nu_i|\nu_{\alpha'}\rangle = \sum_i U_{\alpha i}\frac{m_i^2}{2p}U^\dagger_{i\alpha'} + p\delta_{\alpha\alpha'} \tag{4.101}$$

The last term of 4.101, which is proportional to unit matrix, cannot change the flavor state of neutrino. This term can be excluded from the equation of motion by redefining the phase of the function $a(t)$. We have:

$$i\frac{\partial a(t)}{\partial t} = U\frac{m^2}{2p}U^\dagger a(t) \tag{4.102}$$

This equation can be easily solved. Let us multiply (4.102) by the matrix $U^\dagger$ from the left. Taking into account unitarity of the mixing matrix we have:

$$i\frac{\partial a'(t)}{\partial t} = \frac{m^2}{2p}a'(t) \tag{4.103}$$

where $a'(t) = U^\dagger a(t)$. The solution of equation (4.103) has the form

$$a'(t) = e^{-i\frac{\Delta m^2}{2p}t}a'(0)\,. \tag{4.104}$$

For the function $a(t)$ in flavor representation, from (4.103) and (4.104), we find

$$a(t) = Ue^{-i\frac{\Delta m^2}{2p}t}U^\dagger a(0) \tag{4.105}$$

and for the amplitude of the $\nu_\alpha \to \nu_{\alpha'}$ transition in vacuum from (4.105) we obtain the expression

$$\mathcal{A}_{\nu_{\alpha'};\nu_\alpha}(t) = \sum_i U_{\alpha' i}e^{-i\frac{\Delta m_i^2}{2p}}U^*_{\alpha i} \tag{4.106}$$

which (up to the irrelevant factor e^{-ipt}) coincides with (3.57).

Let us now introduce the effective Hamiltonian of interaction of flavor neutrino with matter. Due to coherent scattering of neutrino in matter, the refraction index of neutrino is given by the following classical expression:

$$n(x) = 1 + \frac{2\pi}{p^2}\mathrm{f}(0)\rho(x) \tag{4.107}$$

Here f(0) is the amplitude of elastic neutrino scattering in forward direction, and $\rho(x)$ is the number density of matter (the axis x is the direction of $\vec{p}$). The effective interaction of neutrinos with matter is determined by the second term of Eq. (4.107) :

$$H_I(x) = p[n(x) - 1] = \frac{2\pi}{p}\mathrm{f}(0)\rho(x) \tag{4.108}$$

NC scattering of neutrinos on electrons and nucleons (due to the Z-exchange) cannot change the flavor state of neutrinos. This is connected with $\nu_e - \nu_\mu - \nu_\tau$ universality of NC: the corresponding effective Hamiltonian is proportional to the unit matrix[3].

CC interaction (due to the W-exchange) gives contribution only to the amplitude of the elastic ν_e -e scattering

$$\nu_e + e \rightarrow \nu_e + e \tag{4.109}$$

For the corresponding effective Hamiltonian we have

$$\mathcal{H}_I(x) = \frac{G_F}{\sqrt{2}} 2\overline{\nu}_{eL}\gamma^\alpha \nu_{eL}\overline{e}\gamma_\alpha(1-\gamma_5)e + h.c. \tag{4.110}$$

The amplitude of process (4.109) is given by

$$\mathrm{f}_{\nu_e e} = \frac{1}{\sqrt{2}\pi} G_F p \tag{4.111}$$

and, from (4.108) and (4.111), for the effective Hamiltonian in flavor representation we have

$$H_I(x) = \sqrt{2} G_F \rho_e(x)\beta \tag{4.112}$$

where $(\beta)_{\nu_e;\nu_e} = 1$, while all other elements of the matrix β are equal to zero and $\rho_e(x)$ is the electron number density at the point x.

The effective Hamiltonian of the neutrino interaction with matter can be also obtained by calculating of the average value of the Hamiltonian (4.110) in the state which describes matter and neutrino with momentum $\vec{p}$ and negative helicity . Taking into account that for non–polarized media

$$\langle \mathrm{mat}\, |\overline{e}(\vec{x})\gamma^\alpha e(\vec{x})|\, \mathrm{mat}\rangle = \rho_e(\vec{x})\delta_{\alpha 0}\,, \tag{4.113}$$

$$\langle \mathrm{mat}|\overline{e}(\vec{x})\gamma^\alpha \gamma_5 e(\vec{x})|\mathrm{mat}\rangle = 0\,, \tag{4.114}$$

from (4.110) we obtain (4.112).

The evolution equation of neutrino in matter can be written, from (4.102) and (4.112), in the following form ($t = x$):

$$i\frac{\partial a(x)}{\partial x} = (U\frac{m^2}{2p}U^\dagger + \sqrt{2}G_F\rho_e(x)\beta)a(x) \tag{4.115}$$

[3]Let us notice that if there are flavour and sterile neutrinos NC interactions with matter must be taken into account.

Let consider in detail the simplest case of two flavor neutrinos (say, ν_e and ν_μ). In this case we have

$$U = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix} \tag{4.116}$$

where θ is the mixing angle. Further it is convenient to write the Hamiltonian in the form

$$H = \frac{1}{2}\mathrm{Tr}H + H^m \tag{4.117}$$

where Tr $H = \frac{1}{2p}(m_1^2 + m_2^2) + \sqrt{2}G_F\rho_e$. The first term of (4.117), which is proportional to the unit matrix, can be omitted. For the Hamiltonian we have then

$$H^m(x) = \frac{1}{4p}\begin{pmatrix} -\Delta m^2\cos 2\vartheta + A(x) & \Delta m^2 \sin 2\vartheta \\ \Delta m^2 \sin 2\vartheta & \Delta m^2 \cos 2\vartheta - A(x) \end{pmatrix} \tag{4.118}$$

where $\Delta m^2 = m_2^2 - m_1^2$ and $A(x) = 2\sqrt{2}G_F\rho_e(x)p$. The effect of matter is described by the quantity $A(x)$. Notice that this quantity enters only into the diagonal elements of the Hamiltonian and has the dimensions of M^2.

Let us first consider the case of constant density. In order to solve equation of motion we will diagonalize the Hamiltonian. We have:

$$H^m = U^m E^m U^{m\dagger} \tag{4.119}$$

where E_i^m is the eigenvalue of the matrix H^m and

$$U^m = \begin{pmatrix} \cos\vartheta^m & \sin\vartheta^m \\ -\sin\vartheta^m & \cos\vartheta^m \end{pmatrix} \tag{4.120}$$

It is easy to see that

$$E_{1,2}^m = \mp\frac{1}{4p}\sqrt{(\Delta m^2\cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}. \tag{4.121}$$

Now, with the help of Eq. (4.119) – (4.121), for the angle θ^m we have

$$\tan 2\theta^m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2\cos 2\theta - A}; \qquad \cos 2\theta^m = \frac{\Delta m^2 \cos 2\theta - A}{\sqrt{(\Delta m^2\cos 2\theta - A)^2 + (\Delta m^2\sin 2\theta)^2}} \tag{4.122}$$

The states of flavor neutrinos are given by

$$|\nu_e\rangle = \cos\theta^m|\nu_{1m}\rangle + \sin\theta^m|\nu_{2m}\rangle; \qquad |\nu_\mu\rangle = -\sin\theta^m|\nu_{1m}\rangle + \cos\theta^m|\nu_{2m}\rangle \tag{4.123}$$

where $|\nu_{im}\rangle$ $(i = 1, 2)$ are eigenvectors of the Hamiltonian of neutrino in matter and θ^m is the mixing angle of neutrino in matter.

The solution of the evolution equation

$$i\frac{\partial a(x)}{\partial x} = H_m a(x) \tag{4.124}$$

can be now easily found. With the help of (4.119) we have

$$i\frac{\partial a'(x)}{\partial x} = E^m a'(x) \tag{4.125}$$

where

$$a'(x) = (U^m)^\dagger a(x). \tag{4.126}$$

The equation (4.125) has the following solution:

$$a'(x) = e^{-iE^m(x-x_0)}a'(x_0) \tag{4.127}$$

where x_0 is the point where the neutrino was produced. Finally, from (4.126) and (4.127), we have

$$a(x) = U^m e^{-iE^m(x-x_0)}(U^m)^\dagger a(x_0) \tag{4.128}$$

The amplitude of the $\nu_\alpha \to \nu_{\alpha'}$ transition in matter turns out to be

$$\mathcal{A}_{\nu_{\alpha'};\nu_\alpha} = \sum_{i=1,2} U^m_{\alpha' i} e^{-iE^m_i(x-x_0)} U^*_{\alpha i} \tag{4.129}$$

and, from (4.129) and (4.120), we obtain the following transition probabilities, in full analogy with the two–neutrino vacuum case:

$$P^m(\nu_e \to \nu_\mu) = P^m(\nu_\mu \to \nu_e) = \frac{1}{2}\sin^2 2\theta^m(1 - \cos\Delta E^m L)\,, \tag{4.130}$$

$$P^m(\nu_e \to \nu_e) = P^m(\nu_\mu \to \nu_\mu) = (1 - P^m(\nu_e \to \nu_\mu)\,. \tag{4.131}$$

Here $\Delta E^m = E^m_2 - E^m_1 = \frac{1}{2p}\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}$ and $L = x - x_0$ is the distance that neutrino passes in matter.

For the oscillation length of neutrino in matter with constant density we have

$$L^m_0 = 4\pi \frac{p}{\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}} \tag{4.132}$$

The mixing angle and oscillation length in matter can differ significantly from the vacuum values. It follows from (4.122) that if the condition[4]

$$\Delta m^2 \cos 2\theta = A = 2\sqrt{2}G_F \rho_e p \tag{4.133}$$

is satisfied, the mixing in matter is maximal ($\theta^m = \pi/4$) independently on the value of the vacuum mixing angle θ. Notice also that if the condition (4.133) is satisfied, the distance between the energy levels of neutrinos in matter is minimal and the oscillation length in matter is maximal. We have

$$L^m_0 = \frac{L_0}{\sin 2\theta} \tag{4.134}$$

[4]Eq. (4.131) is the condition at which the diagonal elements of the Hamiltonian of neutrino in matter vanish. It is evident that in such a case the mixing is maximal.

where $L_0 = 4\pi p/(\Delta m)$ is the oscillation length in vacuum. If the distance L in the transition probabilities (4.131) is large (as in the Sun case) the effect of $\nu_e \to \nu_\mu$ transitions is large even in case of a small vacuum mixing angle θ. The relation (4.133) is called resonance condition.

The density of electrons in the Sun is not constant. It is maximal in the center of the Sun and decreases practically exponentially to its periphery. The consideration of the dependence of ρ_e on x allowed to discover possibilities for the large effects of the transitions of solar ν_e's into other states in matter (MSW effect).

Let us consider the evolution equation when the Hamiltonian depends on the distance x that neutrino passes in matter

$$i\frac{\partial a(x)}{\partial x} = H^m(x)a(x) \tag{4.135}$$

The Hermitian Hamiltonian $H^m(x)$ can be diagonalized by a unitary transformation

$$H^m(x) = U^m(x)E^m(x)U^{m\dagger}(x) \tag{4.136}$$

where $U^m(x)U^{m\dagger}(x) = 1$ and E_i^m(x) are eigenvalues of $H^m(x)$. From (4.135) and (4.136) we have

$$U^{m\dagger}(x)i\frac{\partial a(x)}{\partial t} = E^m(x)a'(x) \tag{4.137}$$

where

$$a'(x) = U^{m\dagger}(x)a(x) \tag{4.138}$$

Further, by taking into account that

$$U^{m\dagger}(x)i\frac{\partial a(x)}{\partial x} = i\frac{\partial a'(x)}{\partial x} + iU^{m\dagger}(x)\frac{\partial U^m(x)}{\partial x}a'(x)\,, \tag{4.139}$$

we have the following equation for $a'(x)$:

$$i\frac{\partial a'(x)}{\partial x} = \left(E^m(x) - iU^{m\dagger}(x)\frac{\partial U^m(x)}{\partial x}\right)a'(x)\,. \tag{4.140}$$

In the case $\rho_e =$ const the equation (4.140) coincides with (4.125).

Let us now assume that the function $\rho_e(x)$ depends weakly on x and the second term in Eq. (4.138) can be dropped (adiabatic approximation). It is evident that the solution of the equation

$$i\frac{\partial a_i'(x)}{\partial x} = E_i^m(x)a_i'(x) \tag{4.141}$$

has the form

$$a_i'(x) = e^{-i\int_{x_0}^{x} E_i^m(x)\,dx}\,a_i'(x_0) \tag{4.142}$$

(x_0 being the initial point).

It follows from (4.141) and (4.142) that, in the adiabatic approximation, a neutrino on the way from the point x_0 to the point x remains in the same energy level. From

(4.138) and (4.142) we obtain the following solution of the evolution equation in flavor representation:

$$a(x) = U^m(x)e^{-i\int_{x_0}^{x} E^m(x)\,dx}U^{m\dagger}(x_0)A(X_0)\,. \tag{4.143}$$

Moreover the amplitude of $\nu_\alpha \to \nu_{\alpha'}$ transition in adiabatic approximation is given by

$$\mathcal{A}_{\nu_{\alpha'};\nu_\alpha} = \sum U^m_{\alpha' i}(x)e^{-i\int_{x_0}^{x} E^m_i(x)\,dx}U^{m*}_{\alpha i}(x_0)\,. \tag{4.144}$$

The latter is similar to the expressions (4.106) and (4.129) for the amplitudes of transition in vacuum and in matter with $\rho_e = \text{const}$.

For the case of the two flavor neutrinos

$$U^m(x) = \begin{pmatrix} \cos\vartheta^m(x) & \sin\vartheta^m(x) \\ -\sin\vartheta^m(x) & \cos\vartheta^m(x) \end{pmatrix} \tag{4.145}$$

and $\tan 2\theta^m(x)$ and $cos 2\theta^m(x)$ are given by Eq. (4.122) in which

$$\mathrm{A}(x) = 2\sqrt{2}G_F\rho_e(x)p \tag{4.146}$$

The eigenvalues of the Hamiltonian $H^m(x)$ are given by Eq. (4.121). From (4.145) we have

$$U^{m\dagger}(x)\,\frac{\partial U^m(x)}{\partial x} = \begin{pmatrix} 0 & \dfrac{\partial\theta^m(x)}{\partial x} \\ -\dfrac{\partial\theta^m(x)}{\partial x} & 0 \end{pmatrix} \tag{4.147}$$

and the exact equation (4.140) takes the form

$$i\frac{\partial}{\partial x}\begin{pmatrix} a'_1 \\ a'_2 \end{pmatrix} = \begin{pmatrix} E^m_1 & -i\dfrac{\partial\theta^m}{\partial x} \\ i\dfrac{\partial\theta^m}{\partial x} & E^m_2 \end{pmatrix}\begin{pmatrix} a'_1 \\ a'_2 \end{pmatrix} \tag{4.148}$$

The Hamiltonian H^m in the right–hand side of this equation can be written in the form

$$H_m = \frac{1}{2}(E^m_1 + E^m_2) + \begin{pmatrix} -\frac{1}{2}\Delta E^m & -i\dfrac{\partial\theta^m}{\partial x} \\ i\dfrac{\partial\theta^m}{\partial x} & \frac{1}{2}\Delta E^m \end{pmatrix} \tag{4.149}$$

where $\Delta E^m = E^m_2 - E^m_1$. As we stressed several times, the term of the Hamiltonian which is proportional to the unit matrix is not important for flavor evolution.

From Eq. (4.149) it follows that adiabatic approximation is valid if the condition

$$\left|\frac{\partial\theta^m}{\partial x}\right| \ll \frac{1}{2}\Delta E^m \tag{4.150}$$

is satisfied. With the help of (4.122) it is easy to show that (4.150) can be written in the form

$$4\sqrt{2}G_F p^2\Delta m^2\sin 2\theta\left|\frac{\partial\rho_e}{\partial x}\right| \ll \left[(\Delta m^2\cos 2\theta - A)^2 + (\Delta m^2\sin 2\theta)^2\right]^{3/2}. \tag{4.151}$$

If the resonance condition

$$\Delta m^2 \cos 2\theta = A(x_R) \tag{4.152}$$

is satisfied at the point $x = x_R$, the condition of validity of the adiabatic approximation can be written in the form

$$\frac{2p \cos 2\theta \left| \frac{\partial}{\partial x} \ln \rho_e(x_R) \right|}{\Delta m^2 \sin^2 2\theta} \ll 1 \,. \tag{4.153}$$

From Eq. (4.144) we obtain the following probability for the $\nu_\alpha \to \nu_{\alpha'}$ transition in the adiabatic approximation:

$$\begin{aligned} P(\nu_\alpha \to \nu_{\alpha'}) &= \sum_i |U^m_{\alpha' i}(x)|^2 |U^m_{\alpha i}(x_0)|^2 + \\ &+ 2Re \sum_{i<k} U^m_{\alpha' i}(x) {U^m_{\alpha' k}}^* e^{-i\int_{x_0}^{x} (E^m_i - E^m_k)\, dx} U^{m*}_{\alpha i}(x_0) U^m_{\alpha k}(x_0) \,. \end{aligned} \tag{4.154}$$

For solar neutrinos the second term in the r.h.s. of this expression disappears due to averaging over the energy and the region in which neutrinos are produced. Hence for the averaged transition probability we have

$$\overline{P}(\nu_\alpha \to \nu_{\alpha'}) = \sum_i |U^m_{\alpha' i}(x)|^2 |U^m_{\alpha i}(x_0)|^2 \tag{4.155}$$

Thus, in the adiabatic approximation, the averaged transition probability is determined by the elements of the mixing matrix in matter at the initial and final points. For the case of two neutrino flavors we have the following simple expression for the ν_e survival probability

$$\begin{aligned} \overline{P}(\nu_e \to \nu_e) &= \cos^2 \theta^m(x) \cos^2 \theta^m(x_0) + \sin^2 \theta^m(x) \sin^2 \theta^m(x_0) \\ &= \frac{1}{2} \left(1 + \cos 2\theta^m(x) \cos 2\theta^m(x_0)\right) \end{aligned} \tag{4.156}$$

From Eq. (4.156) it is easy to see that if the neutrino passes the point $x = x_R$ where the resonance condition is satisfied, a large effect of disappearance of ν_e will be observed. In fact, the condition (4.152) is fulfilled if $\cos 2\theta > 0$ (neutrino masses are labelled in such a way that $\Delta m^2 > 0$). At the production point x_0 the density is larger than at point x_R and $A(x_0) > \Delta m^2 \cos 2\theta$. From (4.122) it follows than $\cos 2\theta(x_0) < 0$. Thus, if the resonance condition is fulfilled, we see from Eq. (4.156) that $P(\nu_e \to \nu_e) < \frac{1}{2}$. If the condition

$$A(x_0) \gg \Delta m^2 \tag{4.157}$$

is satisfied for neutrinos produced in the center of the Sun,then $\cos 2\theta^m(x_0) \simeq -1$ and, for neutrinos passing through the Sun, the survival probability is equal to:

$$\overline{P}(\nu_e \to \nu_e) \simeq \frac{1}{2}(1 - \cos 2\theta) \tag{4.158}$$

It is obvious from this expression that the ν_e survival probability at small θ is close to zero: all ν_e's are transformed into ν_μ's.

Let us consider evolution of neutrino states in such a case. From Eq. (4.122) it follows that, at the production point, $\theta^m(x_0) \simeq \pi/2$. From (4.123) we have then

$$|\nu_e\rangle \simeq |\nu_{2m}\rangle\,; \qquad |\nu_\mu\rangle = -|\nu_{1m}\rangle \qquad (x = x_0) \tag{4.159}$$

Thus at the production point flavor states are states with definite energy. In the adiabatic approximation there are no transitions between energy levels. In the final point $\rho_e = 0$ and at small θ we have

$$|\nu_2\rangle \simeq |\nu_\mu\rangle, \qquad |\nu_1\rangle \simeq |\nu_e\rangle \qquad (x = x_0) \tag{4.160}$$

Thus, all ν_e's transfer to ν_μ's. The resonance condition (4.152) was written in units $\hbar = c = 1$. We can rewrite it in the following form

$$\Delta m^2 \cos 2\theta \simeq 0.7 \cdot 10^{-7} E\rho \mathrm{eV}^2$$

where ρ is the density of matter in g$\cdot$ cm^{-3} and E is the neutrino energy in MeV. In the central region of the Sun $\rho \simeq 10^2 \mathrm{g} \cdot \mathrm{cm}^{-3}$ and the energy of the solar neutrinos is $\simeq 1MeV$. Thus the resonance condition is satisfied at $\Delta m^2 \simeq 10^{-5}$ eV^2.

The expression (4.155) gives the averaged survival probability in the adiabatic approximation. In the general case we have

$$\overline{P}(\nu_\alpha \to \nu_{\alpha'}) = \sum |U^m_{\alpha' i}(x)|^2 P_{ik} |U^m_{\alpha k}(x_0)|^2 \tag{4.161}$$

where P_{ik} is the probabilty of transition from the state with energy E^m_k to the state with energy E^m_i. Let us consider the simplest case of the transition between two types of neutrinos. From the conservation of the total probability we have

$$P_{11} = 1 - P_{21}\,, \qquad P_{22} = 1 - P_{12}\,, \qquad P_{12} = P_{21} \tag{4.162}$$

Thus in the case of two neutrinos all transition probabilities P_{ik} are expressed through P_{12}. With the help of (4.145), (4.161) and (4.162), for the ν_e survival probability we have:

$$\overline{P}(\nu_e \to \nu_e) = \frac{1}{2} + \left(\frac{1}{2} - P_{12}\right) \cos 2\theta^m(x) \cos 2\theta^m(x_0) \tag{4.163}$$

In the literature there exist different approximate expressions for the transition probability P_{12}. In the Landau–Zenner approximation, based on the assumption that the transition occurs mainly in the resonance region,

$$P_{12} = e^{-\frac{\pi}{2}\gamma_R F} \tag{4.164}$$

where

$$\gamma_R = \frac{\frac{1}{2}\Delta E^m}{|\partial\theta^m/\partial x|} = \frac{\Delta m^2 \sin^2 2\theta}{2p \cos 2\theta |\frac{\partial}{\partial x} \ln \rho_e(x_R)|}\,. \tag{4.165}$$

In the above equation $F = 1$ for linear density and $F = 1 - \tan^2\theta$ for exponential density. The adiabatic approximation is valid if $\gamma_R \gg 1$ [see (4.150)]. In this case $P_{12} \simeq 0$.

This concludes the considerations on the phenomenological theory of neutrino mixing and on the theory of neutrino oscillations in vacuum and in matter. We will start now the discussion of experimental data. There are three methods to search for the effects of neutrino masses and mixing:

I. The precise measurement of the high energy part of β–spectrum;

II. The search for neutrinoless double β–decay;

III. The investigation of neutrino oscillations .

We shall discuss now the results which have been obtained in some of the most recent experiments.

5 Search for effects of neutrino mass in experiments on the measurement of the β-spectrum of ^{3}H

We will discuss here briefly the results of searching for effects of neutrino masses in experiments on the measurement of the high-energy part of the β–spectrum in the decay

$$^3\mathrm{H} \to {}^3\mathrm{He} + e^- + \overline{\nu_e} \tag{5.166}$$

The process (5.166) is a superallowed β–decay: the nuclear matrix element is constant and the β–spectrum is determined by the phase–space factor and the Coulomb interaction of the final e^- and ^{3}He. For the β–spectrum we have

$$\frac{dN}{dT} = C\, pE(Q-T)\sqrt{(Q-T)^2 - m_\nu^2}\,F(E) \tag{5.167}$$

Here p is electron momentum, $E = m_e + T$ is the total electron energy, $Q = m_{^3\mathrm{H}} - m_{^3\mathrm{He}} - m_e \simeq 18.6$ keV is the energy release, $C =$ const and $F(E)$ is the Fermi function, which describes the Coulomb interaction of the final particles. In the Eq. (5.167) the term $(Q-T)$ is the neutrino energy (the recoil energy of ^{3}He can be neglected) and the neutrino mass enters through the neutrino momentum $p_\nu = \sqrt{(Q-T)^2 - m_\nu^2}$. Notice that in the derivation of Eq. (5.167) the simplest assumption was done that ν_e is the particle with mass m_ν.

The Kurie function is then determined as follows

$$K(T) = \sqrt{\frac{dN}{dt}\frac{1}{pEF(E)}} = \sqrt{C}\sqrt{(Q-T)\sqrt{(Q-T)^2 - m_\nu^2}} \tag{5.168}$$

If $m_\nu = 0$, the Kurie function is the stright line $K(T) = \sqrt{C}(Q-T)$ and $T_{max} = 0$. If $m_\nu \neq 0$ then $T_{max} = Q - m_\nu$ and at small m_ν the Kurie function deviates from the stright

Table 5.2. Neutrino mass from ^{3}H experiments.

Experiment	m_ν^2	m_ν
Troitsk	$-1.0 \pm 3.0 \pm 2.0$ eV^2	$<$ 2.5 eV
Mainz	$-0.1 \pm 3.8 \pm 1.8$ eV^2	$<$ 2.8 eV

line in the region close to the maximum allowed energy. Thus, if $m_\nu \neq 0$ in the end point part of the spectrum a deficit of observed events must be measured (with respect to the number of events expected at $m_\nu = 0$).

In experiments on the search for effects of neutrino mass by ^{3}H–method no positive indications in favour of $m_\nu \neq 0$ were found. In these experiments some anomalies were observed. First, practically in all experiments the best–fit values of m_ν^2 are negative. This means that instead of a deficit of events, some excess is observed. Second, in the Troitsk experiment a peak in electron spectrum is observed at the distance of a few eV from the end. The position of the peak is changed periodically with time. There are no doubts that new, more precise experiments are necessary. The results of two running experiments are presented in Table 5.2.

6 Neutrinoless double β-decay

The decay

$$(A, Z) \to (A, Z + 2) + e^- + e^- \tag{6.169}$$

is possible only if the total lepton number L is not conserved, i.e. if neutrinos with definite masses are Majorana particles. There are many experiments in which neutrinoless double β-decay ($(\beta\beta)_{0\nu}$–decay) of ^{76}Ge, ^{136}Xe, ^{130}Te, ^{82}Se, ^{100}Mo and other even–even nuclei is searched for.

Let consider the process (6.169) in the framework of neutrino mixing. The standard CC Hamiltonian of the weak interaction has the form

$$H_I = \frac{G_F}{\sqrt{2}} 2\, \overline{e}_L \gamma^\alpha \nu_{eL} j_\alpha + h.c. \tag{6.170}$$

Here j_α is the weak hadronic current and

$$\nu_{eL} = \sum U_{ei} \nu_{iL} \tag{6.171}$$

where ν_i is the Majorana neutrino field with mass m_i.

The $(\beta\beta)_{0\nu}$ decay is a process of second order in G_F with an intermediate virtual neutrino. Neutrino masses and mixing enter into the neutrino propagator[5]

$$\nu_{eL}{}^{\bullet}(x_1)\nu_{eL}^{T}{}^{\bullet}(x_2) = \sum_i U_{ei}^2 \nu_{iL}{}^{\bullet}(x_1)\nu_{iL}^{T}{}^{\bullet}(x_2) = -\sum U_{ei}^2 \frac{(1-\gamma_5)}{2}\nu_i{}^{\bullet}(x_1)\overline{\nu}_i{}^{\bullet}(x_2)\frac{(1-\gamma_5)}{2}C$$

$$= -\sum U_{ei}^2 \frac{(1-\gamma_5)}{2}\frac{i}{(2\pi)^4}\int \frac{e^{-ip(x_1-x_2)}(\not p + m_i)}{p^2-m_i^2}\, d^4p \frac{(1-\gamma_5)}{2}C \quad (6.172)$$

Taking into account that

$$\frac{(1-\gamma_5)}{2}(\not p + m_i)\frac{(1-\gamma_5)}{2} = m_i\frac{(1-\gamma_5)}{2}\,, \quad (6.173)$$

we come to the conclusion that the matrix element of $(\beta\beta)_{0\nu}$–decay is proportional to[6]

$$< m >= \sum U_{ei}^2 m_i \quad (6.174)$$

From (6.173) it is evident that the proportionality of the matrix element of $(\beta\beta)_{0\nu}$–decay to $< m >$ is due to the fact that the standard CC interaction is the left–handed one. If neutrino masses are equal to zero $(\beta\beta)_{0\nu}$–decay is forbidden (conservation of helicity). Notice that, if there is some small admixture of right–handed currents in the interaction Hamiltonian, the $L-R$ interference gives a contribution proportional to the $\not p$ term in the neutrino propagator. Other mechanisms of $(\beta\beta)_{0\nu}$–decay are also possible (SUSY with violation of R-parity ect.).

In the experiments on the search for $(\beta\beta)_{0\nu}$-decay very strong bounds on the life–time of this process were obtained. The results of some of the latest experiments are presented in Table 6.3. From these data upper bounds for $| < m > |$ can be obtained. The upper bounds depend on the values of the nuclear matrix elements, the calculation of which is a complicated problem. From ^{76}Ge data it follows

$$| < m > | < (0.5-1) \text{ eV} \quad (6.175)$$

In the future experiments on the search for $(\beta\beta)_{0\nu}$–decay (Heidelberg-Moscow, NEMO, CUORE and others) the sensitivity $| < m > | < 0.1$ eV is planned to be achieved.

7 Neutrino oscillation experiments

We will discuss now the existing experimental data on the search for neutrino oscillations. There exist at present convincing evidences in favour of neutrino oscillations, which were obtained in atmospheric neutrino experiments and first of all in the Super–Kamiokande experiment. Strong indications in favour of neutrino masses and mixing were obtained in

[5]We have used the relation $\nu_i^T = -\nu_i C$ that follows from the Majorana condition $\nu_i^C = C\overline{\nu}_i^T = \nu_i$. It is obvious that in the case of Dirac neutrino the propagator is equal to zero.

[6]The term m_i^2 in denominator is small with respect to characteristic p in nuclei ($\simeq$ 10 MeV) and can be neglected.

Table 6.3. Lower bounds of the life–time $T_{1/2}$ of $(\beta\beta)_{0\nu}$–decay

Experiment	Element	Lower bound of $T_{1/2}$
Heidelberg-Moscow	^{76}Ge	$> 1.6 \times 10^{25}$ y
Caltech-PSI-Neuchatel	^{136}Xe	$> 4.4 \times 10^{23}$ y
Milano	^{130}Te	$> 7.7 \times 10^{22}$ y

all solar neutrino experiments. Finally, some indications in favour of $\nu_\mu \to \nu_e$ transitions were obtained in the LSND accelerator experiment. In many reactor and accelerator short baseline experiments and in the reactor long baseline experiments CHOOZ no indication in favour of neutrino oscillations was found. We will start with the discussion of the results of solar neutrino experiments.

7.1 Solar neutrinos

The energy of the Sun is generated in the reactions of the thermonuclear pp and CNO cycles. The main pp–cycle is illustrated in Fig.7.1. The energy of the sun is produced in the transition

$$4\,p + 2\,e^- \to\ ^4\mathrm{He} + 2\,\nu_e\,, \tag{7.176}$$

If we assume that solar ν_e's do not transfer into other neutrino types ($P(\nu_e \to \nu_e) = 1$) we can obtain a relation between the luminosity of the Sun, $L_\odot$ and the flux of solar neutrinos. Let us consider neutrino with energy E. From (7.176) it follows that

$$\frac{1}{2}(Q - 2E) \tag{7.177}$$

is the luminous energy corresponding to the emission of one neutrino. Here

$$Q = 4m_p + 2m_e - m_{^4\mathrm{He}} \simeq 26.7 MeV \tag{7.178}$$

is the energy release in the transition (7.176). If we multiply (7.177) by the total flux of solar ν_e's from different reactions and integrate over the neutrino energy E we will obtain the flux of luminous energy from the Sun

$$\frac{1}{2}\int (Q - 2E) \sum_i I_i(E) dE = \frac{L_\odot}{4\pi R^2}. \tag{7.179}$$

Here $L_\odot \simeq 3.86 \cdot 10^{33}$ erg/s is the luminosity of the Sun, R is the Sun–Earth distance and $I_i^0(E)$ is the flux of neutrinos from the source i ($i =$ pp, ...). Notice that in the derivation of the relation (7.179) we have assumed that the Sun is in a stationary state.

The luminosity relation (7.179) is solar model independent constraint on the solar neutrino fluxes. The flux $I_i(E)$ can be written in the form

$$I_i(E) = X_i(E)\Phi_i \tag{7.180}$$

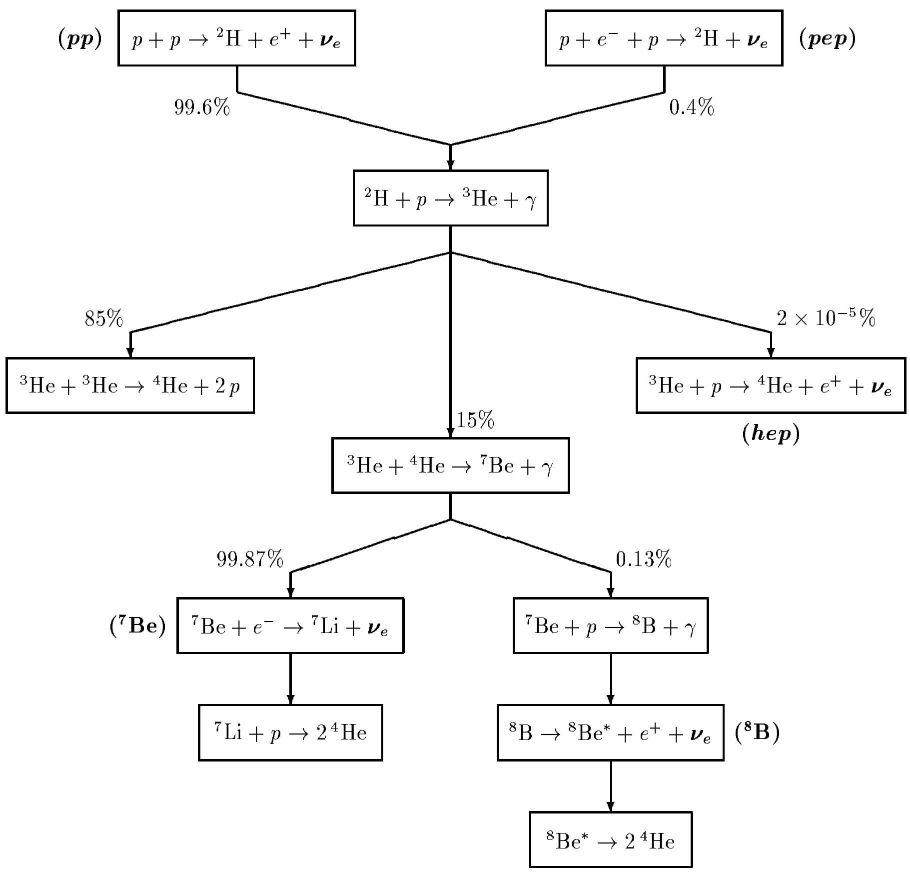

Figure 7.1. The *pp* cycle (the figure is taken from ref. [17]).

Table 7.4. Main sources of solar ν_e's.

Reaction	Maximal energy	Standard Solar Model flux ($\mathrm{cm}^{-2}\mathrm{s}^{-1}$)
$p\,p \to d\,e^+\,\nu_e$	$\leqslant 0.42$ MeV	6.0×10^{10}
$e^-\,{}^7\mathrm{Be} \to \nu_e\,{}^7\mathrm{Li}$	0.86 MeV	4.9×10^9
${}^8\mathrm{B} \to {}^8\mathrm{Be}\,e^+\,\nu_e$	$\leqslant 15$ MeV	5.0×10^6

where Φ_i is the total flux and the function $X_i(E)$ describes the form of the spectrum ($\int X_i(E)dE = 1$). The functions $X_i(E)$ are known functions, determined by the weak interaction. The luminosity relation (7.179) can be written in the form

$$Q\sum_i \left(1 - 2\frac{\overline{E}_i}{Q}\right)\Phi_i = \frac{L_\odot}{2\pi R^2} \tag{7.181}$$

where $\overline{E}_i = \int EX_i(E)dE$ is the average energy of neutrinos from the source i. The main sources of solar neutrinos are listed in Table 7.4

As it is seen from the Table, the main source of solar neutrinos is the reaction $p + p \to d + e^+ + \nu_e$. This reaction is the source of low energy neutrinos. The source of monochromatic medium energy neutrinos is the process

$$e^- + {}^7\mathrm{Be} \to \nu_e + {}^7\mathrm{Li}. \tag{7.182}$$

The reaction ${}^8\mathrm{B} \to {}^8\mathrm{Be} + e^+ + \nu_e$ is the source of the rare high energy neutrinos. The results of solar neutrino experiments are presented in Table 7.5.

Homestake, GALLEX and SAGE are radiochemical experiments. In the Kamiokande and the Super–Kamiokande experiments recoil electrons (angle and energy) in the elastic neutrino–electron scattering are detected. In these experiments the direction of neutrinos is determined and it is confirmed that the detected events are from solar neutrinos.

In the Homestake experiment, because of high threshold ($E_{th} = 0.81$ MeV) mainly ^{8}B neutrinos are detected: $\simeq 77\%$ of events are due to ^{8}B neutrinos and $\simeq 15\%$ of events are due to ^{7}Be neutrinos. In GALLEX and SAGE experiments ($E_{th} = 0.23$ MeV) neutrinos from all reactions are detected: $\simeq 54\%$ of events are due to pp neutrinos, $\simeq 27\%$ of events are due to ^{7}Be neutrinos and $\simeq 10\%$ of events are due to ^{8}B neutrinos. In the Kamiokande and Super–Kamiokande experiments due to the high threshold ($E_{th} = 7$ MeV for Kamiokande and $E_{th} = 5.5$ MeV for the Super–Kamiokande) only high energy ^{8}B neutrinos are detected.

The results of the solar neutrino experiments are presented in Table7.5. As it is seen from the Table, the detected event rates in all solar neutrino experiments are significantly smaller than the predicted one.[7] The most natural explanation of the data of solar

[7]Notice that in the framework of neutrino oscillations the possibility of deficit of solar ν_e's was discussed by B. Pontecorvo in 1968 before the results of the Homestake experiment were obtained.

Table 7.5. Results of solar neutrino experiments.
[1 SNU = 10^{-36} events/(atoms $\cdot$ sec)]

Experiment	Observed rate	Expected rate
Homestake $\nu_e\,{}^{37}\mathrm{Cl} \to e^-\,{}^{37}\mathrm{Ar}$ $E_{th} = 0.81$ MeV	$2.56 \pm 0.16 \pm 0.16$ SNU	7.7 ± 1.2 SNU
GALLEX $\nu_e\,{}^{71}\mathrm{Ga} \to e^-\,{}^{71}\mathrm{Ge}$ $E_{th} = 0.23$ MeV	$77.5 \pm 6.2^{+4.3}_{-4.7}$ SNU	129 ± 8 SNU
SAGE $\nu_e\,{}^{71}\mathrm{Ga} \to e^-\,{}^{71}\mathrm{Ge}$ $E_{th} = 0.23$ MeV	$66.6 \pm {}^{+6.8}_{-7.1}\ {}^{+3.8}_{-4.0}$ SNU	— · —
Kamiokande $\nu e \to \nu e$ $E_{th} = 7.0$ MeV	$(2.80 \pm 0.19 \pm 0.33)\ 10^6\mathrm{cm}^{-2}\mathrm{s}^{-1}$	$(5.15^{+1.00}_{-0.72})\ 10^6\mathrm{cm}^{-2}\mathrm{s}^{-1}$
Super – Kamiokande $\nu e \to \nu e$ $E_{th} = 5.5$ MeV	$(2.44 \pm 0.05^{+0.09}_{-0.07})\ 10^6\mathrm{cm}^{-2}\mathrm{s}^{-1}$	— · —

neutrino experiments can be obtained in the framework of neutrino mixing. In fact, if neutrinos are massive and mixed, solar ν_e's on the way to the earth can transfer into neutrinos of the other types that are not detected in the radiochemical Homestake, GALLEX and SAGE experiments. In Kamiokande and Super–Kamiokande experiments all flavor neutrinos ν_e, ν_μ and ν_τ are detected. However, the cross section of ν_μ (ν_τ) $-e$ scattering is about six times smaller than the cross section of $\nu_e - e$ scattering.

All existing solar neutrino data can be explained if we assume that solar neutrino fluxes are given by the Standard Solar Model (SSM) and that there are transitions between two neutrino types determined by the two parameters: mass squared difference Δm^2 and mixing parameter $\sin^2 2\theta$. We will present the results of such analysis of the data later on.

Now we will make some remarks about a model independent analysis of the data. First of all from the luminosity relation (7.179) for the total flux of solar neutrinos we have the following lower bound

$$\Phi = \sum_i \Phi_i \geq \frac{L_\odot}{2\pi R^2 Q} \tag{7.183}$$

Furthermore, for the counting rate in the gallium experiments we have

$$Q_{Ga} = \int_{E_{th}} \sigma(E) \sum I_i(E) dE = \sum_i \overline{\sigma}_i \Phi_i \geq \overline{\sigma}_{pp} \Phi = (76 \pm 2)\,\mathrm{SNU} \tag{7.184}$$

By comparing this lower bound with the results of the GALLEX and SAGE experiments (see Table 7.5) we come to the conclusion that there is no contradiction between experimental data and luminosity constraint if we assume that there are no transitions of solar neutrinos into other states ($P(\nu_e \to \nu_e) = 1$).

It is possible, however, to show in a model independent way that the results of *different solar neutrino experiments* are not compatible if we assume $P(\nu_e \to \nu_e) = 1$. In fact, let us compare the results of the Homestake and the Super–Kamiokande experiments. We will consider the total neutrino fluxes Φ_i as free parameters. From the results of Super-Kamiokande experiment we can determine the flux of ^{8}B neutrinos, $\Phi_{^8B}$ (see Table7.5). If we calculate now the contribution of ^{8}B neutrinos into the counting rate of the Homestake experiment we get

$$Q_{Cl}^{^8B} = (2.78 \pm 0.27)\,\mathrm{SNU} \tag{7.185}$$

The difference between measured event rate and $Q_{Cl}^{^8B}$ gives the contribution to the Chlorine event rate of ^{7}Be and other neutrinos. We have

$$Q_{Cl}^{^7Be+\dots} = Q_{Cl}^{ex} - Q_{Cl}^{^8B} = (-0.22 \pm 0.35)\,\mathrm{SNU} \tag{7.186}$$

All existing solar models predict much larger contribution of ^{7}Be neutrinos to the Chlorine event rate:

$$Q_{Cl}^{^7Be}(SSM) = (1.15 \pm 0.1) SNU \tag{7.187}$$

The large suppression of the flux of ^{7}Be neutrinos (together with the observation of ^{8}B neutrinos) is the problem for any solar model. The ^{8}B nuclei are produced in the reaction $p\,+\,^7\mathrm{Be} \to\, ^8\mathrm{B}\, +\, \gamma$ and in order to observe neutrinos from ^{8}B decay enough ^{7}Be nuclei must exist in the Sun interior. We can come to the same conclusion about the suppression of the flux of ^{7}Be neutrinos if we compare the results of Gallium and Super–Kamiokande experiments.

All existing solar neutrino data can be described if there are oscillation between two neutrino flavors, the neutrino fluxes being given by the SSM values. If we assume that the oscillation parameters Δm^2 and $\sin^2 2\theta$ are in the region in which matter MSW effect can be important, then from the fit of the data two allowed regions of the oscillation parameters can be obtained. For the best fit values it was found

$$\Delta m^2 = 5 \cdot 10^{-6} \mathrm{eV}^2 \qquad \sin^2 2\theta = 5 \cdot 10^{-3} \qquad (\mathrm{SMA}) \tag{7.188}$$

$$\Delta m^2 = 2 \cdot 10^{-5} \mathrm{eV}^2 \qquad \sin^2 2\theta = 0.76 \qquad (\mathrm{LMA}) \tag{7.189}$$

The data can be also described if we assume that the oscillation parameters are in the region in which matter effects can be neglected (the case of vacuum oscillations). For the best fit values it was found in this case

$$\Delta m^2 = 4.3 \cdot 10^{-10} \mathrm{eV}^2 \qquad \sin^2 2\theta = 0.79 \qquad (\mathrm{VO})\,. \tag{7.190}$$

In the Super–Kamiokande experiment during 825 days 11240 solar neutrino events were observed. Such a large statistics allows the Super–Kamiokande collaboration to measure the energy spectrum of the recoil electrons and day/night asymmetry. No significant deviation from the expected spectrum was observed (may be with the exception of the high energy part of the spectrum). For the day/night asymmetry the following value was obtained

$$\frac{1}{2}\left(\frac{N-D}{N+D}\right) = 0.065 \pm 0.031 \pm 0.013 \tag{7.191}$$

From the analysis of the latest Super–Kamiokande data the following best–fit values of the oscillation parameters were found:

$$\Delta m^2 = 5 \cdot 10^{-6} \mathrm{eV}^2 \qquad \sin^2 2\theta = 5 \cdot 10^{-3} \qquad (\mathrm{SMA}) \tag{7.192}$$

$$\Delta m^2 = 3.2 \cdot 10^{-5} \mathrm{eV}^2 \qquad \sin^2 2\theta = 0.8 \qquad (\mathrm{LMA}) \tag{7.193}$$

$$\Delta m^2 = 4.3 \cdot 10^{-10} \mathrm{eV}^2 \qquad \sin^2 2\theta = 0.79 \qquad (\mathrm{VO}) \tag{7.194}$$

These values are compatible with the ones in Eq. (7.188), (7.189) and (7.190), which were found from the analysis of the event rates measured in all solar neutrino experiments.

The new solar neutrino experiment SNO started recently in Canada. The target in this experiment is heavy water (1 kton of D_2O) and Cerenkov light is detected by $\simeq 10^4$ photomultipliers. Neutrinos will be detected through the observation of the CC reaction

$$\nu_e + d \to e^- + p + p \tag{7.195}$$

as well as of the NC reaction

$$\nu + d \to \nu + n + p \tag{7.196}$$

and $\nu - e$ elastic scattering

$$\nu + e \to \nu + e \tag{7.197}$$

The detection of neutrinos via the CC process (7.195) will allow to measure the spectrum of ν_e on the Earth. The detection of neutrinos via the NC process (7.196) (neutrons will be detected) will allow to determine the total flux of flavor neutrinos ν_e, ν_μ, ν_τ. From the comparison of NC and CC event rates model independent conclusions on the transition of solar ν_e's into other flavor states can be made.

Next solar neutrino experiment will be BOREXINO. In this experiment 300 tons of liquid scintillator of very high purity will be used. Solar neutrinos will be detected through the observation of the recoil electrons in the process

$$\nu + e \to \nu + e\,. \tag{7.198}$$

The energy threshold in the BOREXINO experiment will be very low, about 250 keV. That will allow to detect the monoenergetic ^{7}Be neutrinos. If vacuum oscillations are the origin of the solar neutrino problem, a seasonal variation of the ^{7}Be neutrino signal (due to excentricity of the Earth orbit) will be observed.

7.2 Atmospheric neutrinos

Atmospheric neutrinos are produced mainly in the decays of pions and muons

$$\pi \to \mu + \nu_\mu, \qquad \mu \to e + \nu_e + \nu_\mu \tag{7.199}$$

pions being produced in the interaction of cosmic rays in the Earth atmosphere. Notice that in the existing detectors neutrino and antineutrino events cannot be distinguished. At small energies, ≤ 1 GeV, the ratio of fluxes of ν_μ's and ν_e's from the chain (7.199) is equal to two. At the higher energies this ratio is larger than two (not all muons decay in the atmosphere) but it can be predicted with accuracy better than 5% (the absolute fluxes of muon and electron neutrinos are predicted presently with accuracy not better than 20 – 25%). This is the reason why the results of the measurements of total fluxes of atmospheric neutrinos are presented in the form of a double ratio

$$R = \frac{(N_\mu/N_e)_{\rm data}}{(N_\mu/N_e)_{\rm MC}} \tag{7.200}$$

where $(N_\mu/N_e)_{\rm data}$ is the ratio of the total number of observed muon and electron events and $(N_\mu/N_e)_{\rm MC}$ is the ratio predicted from Monte Carlo simulations.

We will discuss the results of the Super–Kamiokande experiment. A large water Cerenkov detector is used in this experiment. The detector consists of two parts: the inner one of 50 kton (22.5 kton fiducial volume) is covered with 11146 photomultipliers and the outer part, 2.75 m thick, is covered with 1885 photomultipliers. The electrons and muons are detected through the observation of the Cerenkov radiation. The efficiency of particle identification is larger than 98%. The observed events are divided in fully contained events (FC) for which Cerenkov light is deposited in the inner detector and partially contained events (PC) in which the muon track deposits part of its Cerenkov radiation in the outer detector. FC events are further divided into sub-GeV events ($E_{vis} \leq$ 1.33 GeV) and multi-GeV events $E_{vis} \geq 1.33$ GeV). In the Super-Kamiokande experiment for sub-GeV events and multi-GeV events (FC and PC) the following values of the double ratio R were obtained, respectively (848.3 days):

$$\begin{aligned} R &= 0.680^{+0.023}_{-0.022} \pm 0.053 \\ R &= 0.678^{+0.042}_{-0.039} \pm 0.080 \end{aligned} \tag{7.201}$$

These values are in agreement with the values of R obtained in other water Cerenkov experiments (Kamiokande and IMB) and in the Soudan2 experiment in which the detector is iron calorimeter.

$$R = 0.65 \pm 0.05 \pm 0.08 \qquad \text{(Kamiokande)} \tag{7.202}$$

$$R = 0.54 \pm 0.05 \pm 0.11 \qquad \text{(IMB)} \tag{7.203}$$

$$R = 0.61 \pm 0.15 \pm 0.05 \qquad \text{(Soudan2)} \tag{7.204}$$

The fact that the double ratio R is significantly less than one is an indication in favor of neutrino oscillations.

The important evidence in favour of neutrino oscillations was obtained by the Super–Kamiokande collaboration. These data were first reported at NEUTRINO98 conference in Japan, in June 1998. A significant up–down asymmetry of multi–GeV muon events was discovered in the Super–Kamiokande experiment.

For atmospheric neutrinos the distance between production region and detector changes from about 20 km for down–going neutrinos ($\theta = 0$, θ being the zenith angle) up to about 13,000 km for up–going neutrinos ($\theta = \pi$). In the Super–Kamiokande experiment for the multi–GeV events the zenith angle θ can be determined. In fact, charged leptons follow the direction of neutrinos (the averaged angle between the charged lepton and the neutrino is $15^o - 20^o$). The possible source of the zenith angle dependence of neutrino events is the magnetic field of the Earth. However, for neutrinos with energies larger than 2 – 3 GeV, within a few % no θ-dependence of neutrino events is expected.

The Super-Kamiokande collaboration found a significant zenith angle dependence of the multi–GeV muon neutrinos. For the integral up–down asymmetry of multi–GeV muon neutrinos (FC and PC) the following value was obtained

$$A_\mu = 0.311 \pm 0.043 \pm 0.010 \tag{7.205}$$

Here

$$A = \frac{U - D}{U + D} \tag{7.206}$$

where U is the number of up–going neutrinos ($\cos\theta \leq -0.2$) and D is the number of down–going neutrinos (($\cos\theta \geq 0.2$). No asymmetry of the electron neutrinos was found:

$$A_e = 0.036 \pm 0.067 \pm 0.02 \tag{7.207}$$

The Super–Kamiokande data can be described if we assume that there are $\nu_\mu \to \nu_\tau$ oscillations. The following best–fit values of the oscillation parameters were found from the analysis of FC events

$$\Delta m^2 = 3.05 \cdot 10^{-3} eV^2, \qquad \sin^2 2\theta = 0.995 \tag{7.208}$$

($\chi^2_{min} = 55.4$ at 67 d.o.f.). Let us notice that if we assume that there are no oscillations, then in this case $\chi^2 = 177$ at 69 d.o.f. From the combined analysis of all data it was found

$$\Delta m^2 \simeq (2 - 6) \cdot 10^{-3} eV^2, \qquad \sin^2 2\theta > 0.84 \tag{7.209}$$

If $\nu_\mu \to \nu_s$ oscillations are assumed, at large energies matter effects must be important. From the investigation of the high energy events (PC and upward–going muon events, muons being produced by neutrinos in the rock under the detector) the Super–Kamiokande collaboration came to the conclusion that $\nu_\mu \to \nu_s$ oscillations are disfavoured at 95% C.L.

The range of oscillation parameters which was obtained from the analysis of the atmospheric neutrino data will be investigated in details in long–baseline experiments. The results of the first LBL reactor experiment, CHOOZ, were recently published (in this experiment the distance between reactors and detector is $\simeq 1$ km). No indication in favour ofthe transitions of $\overline{\nu_e}$ into other states was found in this experiment. For the ratio R of the number of measured and expected events it was found

$$R = 1.01 \pm 2.8\% \,(\mathrm{stat}) \pm 2.7\% \,(\mathrm{syst}) \tag{7.210}$$

These data allow to exclude $\Delta m^2 > 7 \cdot 10^{-4}\mathrm{eV}^2$ at $\sin^2 2\theta = 1$ (90% C.L.).

In LBL Kam-Land experiment $\overline{\nu_e}$'s from reactors at the distance of $150 - 200$ km from the detector will be detected. Neutrino oscillations $\overline{\nu}_e \leftrightarrow \overline{\nu}_x$ with $\Delta m^2 \gtrsim 10^{-5}\mathrm{eV}^2$ and large values of $\sin^2 2\theta$ will be explored. The BOREXINO collaboration plans to detect $\overline{\nu}_e$ from reactors at the distance of about 800 km from the detector.

The first LBL accelerator experiment K2K is running now. In this experiment ν_μ's with average energy of 1.4 GeV, produced at KEK accelerator, will be detected in the Super–Kamiokande detector (at a the distance of about 250 km). The disappearance channel $\nu_\mu \to \nu_\mu$ and the appearance channel $\nu_\mu \to \nu_e$ will be investigated in detail. This experiment will be sensitive to $\Delta m^2 \geq 2 \cdot 10^{-3}\mathrm{eV}^2$ at large $\sin^2 2\theta$.

The LBL MINOS experiment between Fermilab and Soudan (the distance is of about 730 km) is under the construction. In this experiment all the possible channels of ν_μ transitions will be investigated in the atmospheric neutrino range of Δm^2.

The LBL CERN-Gran Sasso experiments (the distance is of about 730 km) ICARUS, NOE and others, are under constraction at CERN and Gran Sasso. The direct detection of τ's from $\nu_\mu \to \nu_\tau$ transition will be one of the major goal of these experiments.

7.3 LSND experiment

Some indications in favour of $\nu_\mu \leftrightarrow \nu_e$ oscillations were found in short–baseline LSND accelerator experiment. This experiment was done at the Los Alamos linear accelerator (with protons of 800 MeV energy). This is a beam–stop experiment: most of π^+'s in the beam, produced by protons, come to a rest in the target and decay (mainly by $\pi^+ \to \mu^+ \,\nu_\mu$); μ^+'s also come to a rest in the target and decay by $\mu_+ \to e^+ \,\nu_e \,\overline{\nu_\mu}$. Thus, the beam–stop target is the source of ν_μ, ν_e and $\overline{\nu}_\mu$ (no $\overline{\nu}_e$ are produced in the decays).

The large scintillator neutrino detector LSND was located at a distance of about 30 m from the neutrino source. In the detector ν_e's were searched for through the observation of the process

$$\overline{\nu}_e + p \to e^+ + n \tag{7.211}$$

Both e^+ and delayed 2.2 MeV γ's from the capture $n\,p \to d\,\gamma$ were detected.

In the LSND experiment 33.9 ± 8.0 events were observed in the interval of e^+ energies $30 < E < 60$ MeV. Assuming that these events are due to $\overline{\nu}_\mu \to \overline{\nu}_e$ transitions, for the transition probability it was found

$$P(\overline{\nu}_\mu \to \overline{\nu}_e) = (0.31 \pm 0.09 \pm 0.06) \cdot 10^{-3} \tag{7.212}$$

From the analysis of LSND data the allowed region in $\sin^2 2\theta - \Delta m^2$ plot was obtained. If the results of SBL reactor experiments and SBL accelerator experiments on the search for $\nu_\mu \to \nu_e$ transitions are taken into account for the allowed values of the oscillation parameters it was found

$$0.2 \lesssim \Delta m^2 \lesssim 2\mathrm{eV}^2 \qquad 2 \cdot 10^{-3} \lesssim \sin^2 2\theta \lesssim 4 \cdot 10^{-2} \tag{7.213}$$

The indications in favour of $\nu_\mu \to \nu_e$ oscillations obtained in the LSND experiment will be checked by BOONE experiment at Fermilab, scheduled for 2001-2002.

8 Conclusions

The problem of neutrino masses and mixing is the central problem of today's neutrino physics. More than 40 different experiments all over the world are dedicated to the investigation of this problem and many new experiments are in preparation. The investigation of the properties of neutrinos is one of the most important direction in the search for a new scale in physics. These investigations will be very important for the understanding of the origin of tiny neutrino masses and neutrino mixing which, according to the existing data, is very different from CKM quark mixing.

If all existing data will be confirmed by the future experiments it would mean that at least four massive neutrinos exist in nature (in order to to provide three independent neutrino mass squared differences: $\Delta m^2_{\mathrm{solar}} \simeq 10^{-5}\mathrm{eV}^2$ (or 10^{-10} eV^2), $\Delta m^2_{\mathrm{atm}} \simeq 10^{-3}\mathrm{eV}^2$ and $\Delta m^2_{\mathrm{LSND}} \simeq 1\mathrm{eV}^2$). From the phenomenological analysis of all existing data it follows that in the spectrum of masses of four massive neutrinos there are two close masses separated by the "large" one, by the about 1 eV LSND gap. Taking into account big–bang nucleosynthesis constraint on the number of neutrinos it can be shown that the dominant transition of the solar neutrinos is $\nu_e \to \nu_{\mathrm{sterile}}$ one and the dominant transition of the atmospheric neutrinos is $\nu_\mu \to \nu_\tau$.

If the LSND indication in favour of $\nu_\mu \to \nu_e$ oscillations will be not confirmed by the future experiments, the mixing of three massive neutrinos with mass hierarchy is plausible scenario.

The nature of massive neutrinos (Dirac or Majorana?) can be determined from the experiments on the search for neutrinoless double β- decay. It can be shown that from the existing neutrino oscillation data it follows that effective Majorana mass $< m >$ in the case of three massive Majorana neutrinos with mass hierarchy is not larger than 10^{-2} eV (the present bound is $| < m > | \simeq 0.5$ eV and the sensitivity of the next generation of experiments will be $| < m > | \simeq 0.1$ eV).

The sensitivity $| < m > | \simeq 10^{-2}$ eV is very important problem of experiments on the search for neutrinoless double β-decay.

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