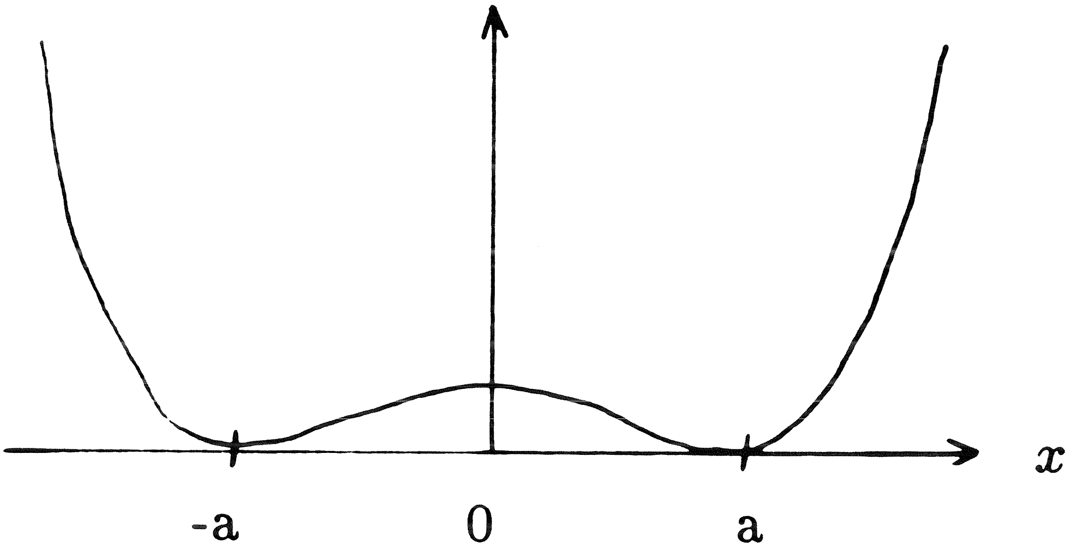
Potential Energy $V(x)$



Quantum Field Theory Anomalies

Massless, single-helicity fermions moving in an external electromagnetic field (4-potential A^μ).

First quantized theory:

$$\left\{ \boldsymbol{\alpha} \cdot \left(\frac{\hbar}{i} \nabla - \frac{e}{c} \mathbf{A} \right) + eA^0 \right\} \psi = E\psi$$

wave-functions: ψ

gauge covariance: $A_\mu \to A_\mu + \partial_\mu \theta$, $\psi \to \psi \times \text{phase}$

probability conservation: $N \equiv \int \psi^\dagger \psi$, $\frac{d}{dt} N = 0$

Second quantized field theory:

$$\psi \longrightarrow \Psi \longleftarrow \text{operator}$$

$$N \longrightarrow \text{charge } Q = \int \Psi^\dagger \Psi \text{ not conserved!}$$

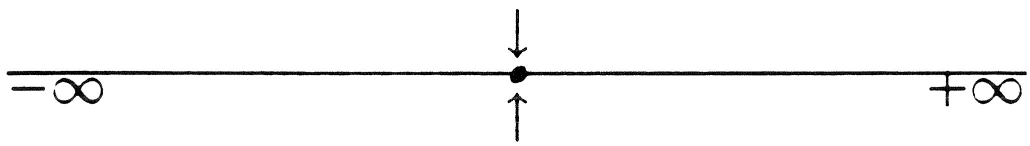
Anomaly equation:

$$0 \neq \frac{d}{dt}Q = \frac{i}{\hbar}[H, Q] \propto \int \mathbf{E} \cdot \mathbf{B}$$

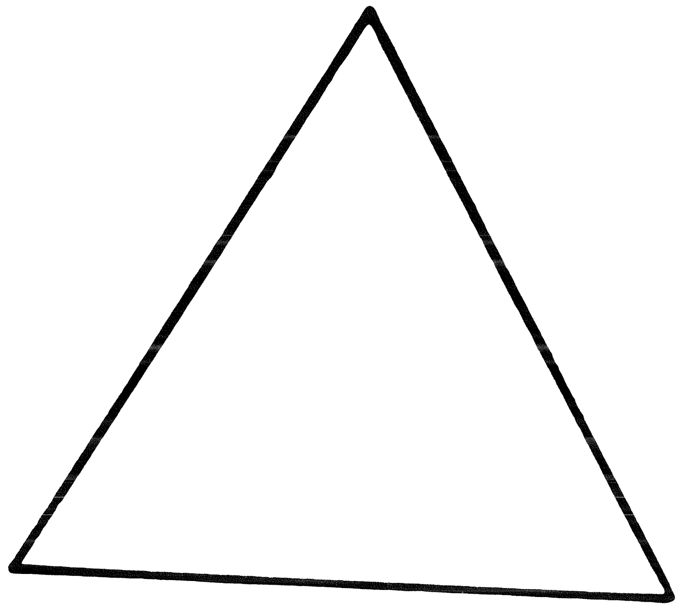
electric ↑ ↑ magnetic

fields

Dirac equation spectrum:



hole↔ particle separation not gauge invariant



Quantum Mechanical Anomaly

$$H = -\frac{1}{2}\nabla^2 + V(\mathbf{r}) \qquad (\hbar = 1 = m)$$

$\uparrow$ scales as $\mathbf{r}^{-2}$

if $V(\mathbf{r})$ scales as $\mathbf{r}^{-2}$

$\Longleftrightarrow$ scale symmetry (no dimensional constants)

$\Longrightarrow$ phase shift E is independent

e.g.

(a) $V(\mathbf{r}) = \lambda/r^2$, λ dimensionless, δ_ℓ is E-independent,

scale symmetry holds!

(b) $V(\mathbf{r}) = \lambda\delta^2(\mathbf{r})$ on the plane, λ dimensionless,

formally scale symmetry holds.

But

$$\operatorname{ctn}\delta_0 = \frac{1}{\pi}\ln\frac{E}{m} + \frac{1}{g}$$

scale symmetry anomalously broken!