Ref.TH.1925-CERN

FINAL STATE INTERACTIONS IN LARGE TRANSVERSE MOMENTUM LEPTON AND HADRON PRODUCTION

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A B S T R A C T

It is shown that, in standard parton models, initial and final state interactions do not contribute to the leading asymptotic behaviour of the single particle inclusive cross-section for the production of a large p_T lepton or hadron in hadron-hadron collisions. However, they do have an important effect on the distribution of accompanying small p_T hadrons : superimposed on the large p_T event is a "pionization" distribution characteristic of ordinary hadron-hadron collisions. Thus, for example, the cross-section for massive lepton pair production is given simply by the Drell-Yan term, though this does not by itself describe the accompanying final state hadrons.

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Ref.TH.1925-CERN
18 September 1974

1. INTRODUCTION

There have been many calculations [1] of the inclusive cross-section for the production of a large q^2 lepton pair in pp collisions. Nearly all these calculations have been based on the Drell-Yan mechanism, Fig. 1.1, where the virtual photon that decays into the lepton pair is produced by the simple fusion of a parton emitted by one of the incident protons and an antiparton emitted by the other. At asymptotic energies one expects the Drell-Yan mechanism to produce two jets of accompanying hadrons, separated by a gap in longitudinal rapidity [2].

It has been known for some time [3] that the Drell-Yan mechanism does not by itself describe the whole reaction. The inclusion of initial and final state interactions produces effects that scale asymptotically in the same way as the Drell-Yan term, to within possible factors of log s. There has been some disagreement about the contribution of these effects to the inclusive cross-section $d\sigma/dq^2$; Henyey and Savit [4] argue that it is negative, while Cardy and Winbow [5] claim that it is zero.

In this paper we corroborate the latter result : initial and final state interactions have no net effect on the leading asymptotic behaviour of the inclusive cross-section $d\sigma/dq^2$. However, they do affect the distribution of the hadrons in the final state. In addition to the two jets expected from the Drell-Yan term, one should expect also hadrons in the "pionization" region in the centre of the rapidity plot, as is found in most high energy pp collisions.

Events in which there is pionization need not be considered in the calculation of the total inclusive cross-section $d\sigma/dq^2$, because their contribution to this is exactly cancelled by certain interference terms, leaving only the Drell-Yan term. Let $|C>$ be any final state that is accessible through the Drell-Yan mechanism (we ignore, for now, the question of the possible quark quantum numbers of these states). Part of the amplitude for producing $|C>$ is A_C^{DY}, calculated from the Drell-Yan term. There is also a part A_C' where any number of initial or final state interactions has occurred, but with no pionization, that is no particle produced in the central region, Fig. 1.2a. The total amplitude for producing $|C>$ is

$$A_c = A_c^{DY} + A_c' . \tag{1.1}$$

Let $|P>$ be a final state accessible only through pionization, Fig. 1.2b, so that there are particles in the centre of the rapidity plot. It is not possible to give a completely precise experimental prescription for the distinction between the states $|C>$ and the states $|P>$, because of the usual problem of deciding just how far into the central region of the rapidity plot the particles in the states $|C>$ extend ; the precise definition of the set of states $|P>$ is that each is orthogonal to all the states $|C>$ that can be reached through the Drell-Yan mechanism. If the amplitude for producing $|P>$ is A_P, the inclusive cross-section $d\sigma/dq^2$ is calculated from

$$\sum_C |A_C|^2 + \sum_P |A_P|^2 = \sum_C \left[|A_C^{DY}|^2 + |A_C'|^2 + 2\,\mathrm{Re}\, A_C^{DY} A_C'^{*} \right] + \sum_P |A_P|^2 . \qquad (1.2)$$

We show in Section 3 that, to within possible factors of $\log s$, each of the four contributions to this sum scales in the same way, but in Section 2 we show that there is destructive interference and

$$\sum_C \left[|A_C'|^2 + 2\,\mathrm{Re}\, A_C^{DY} A_C'^{*} \right] + \sum_P |A_P|^2 = 0. \qquad (1.3)$$

These results are valid at asymptotic energies in theories of the conventional type, where initial and final state interactions correspond to Pomeron exchange and the coupling of the Pomeron to a parton or particle that is far off-shell is assumed to be small. Further, the Pomeron coupling is assumed not to have a Preparata essential singularity [6]. The result (1.3) holds to each order in the Pomeron coupling, that is for the inclusion of any number of Pomeron in the corresponding Mueller diagram [7]. [The first term in (1.3) is absent for a single Pomeron].

An obvious question is whether there are similar effects in deep inelastic electroproduction or e^+e^- annihilation, but it is known [3] that for the conventional theories the answer is no : in these reactions the contribution from final states reached through pionization decreases more rapidly at asymptotic energies than the Bjorken scaling contributions. Thus the effects do not seem capable of solving the problem of the non-observation of quark quantum numbers in the final state, and we have nothing to say about this problem here.

Popular models [1),8)] for the production of large p_T hadrons closely resemble the Drell-Yan mechanism : see Fig. 1.3. Each incident proton emits a parton or a virtual meson (this varies from model to model), and these then scatter at wide angle. The effects of initial and final state interactions between the two outer bunches of hadrons are similar to those discussed above : they cancel in the single particle distribution $E d\sigma/d^3p$ for the production of a hadron with large transverse momentum $\underline{p}$, but affect the distribution in rapidity of the small p_T hadrons. There are experimental indications [1),9)] that this is at least approximately the case, that is large p_T events have superimposed on them characteristics of ordinary events.

For definiteness, the discussion in this paper will deal explicitly with the case of lepton production. In Section 3 we re-examine the arguments of Cardy and Winbow. This is necessary because they base their analysis on a supposed analogy with a discussion of the theory of Regge cuts given by Abramovskii, Gribov and Kancheli [10)]. However, it is now known [11)] that the latter discussion requires modification. Fortunately, it turns out that this does not matter here, because the analogy with the Regge cut problem is not completely accurate.

The Cardy-Winbow discussion deals with only a single Pomeron [12)]. However, their result holds also when any number of Pomerons is involved. We show this in Section 2, where we present an argument that is both simpler and more general than that of Section 3.

2. THE INCLUSIVE CROSS-SECTION

In this Section we argue on rather general grounds that the sum of all initial and final state interactions vanishes to leading order in s, and so the Drell-Yan term by itself gives the dominant contribution to the inclusive cross-section $d\sigma/dq^2$ for massive lepton pair production. Since we are discussing the leading term in s, the initial and final state interactions presumably correspond to Pomeron exchange. Our discussion will apply to the complete interaction, resulting from the exchange of any number of Pomerons. The corresponding Mueller diagram [7)] is drawn in Fig. 2.1. It encompasses all possible time orderings, that is both initial and final state interactions, and cross-interactions between initial and final states ; we illustrate this by explicit examples below (Fig. 2.3). To calculate the inclusive cross-section $d\sigma/dq^2$ we have to calculate the discontinuity in the missing mass variable of the amplitude evaluated on the appropriate sheet [7)], with $p_i = p_i'$ and $q = q'$.

The asymptotic amplitude is first represented as an integral over the internal eight-point function $T(p_1,p_2,p_1',p_2',k_1,k_2,k_1',k_2')$, which we define to include the parton propagators. We then trace how the expected singularities of T are translated into singularities in the variables of integration. Finally, we show that, in each of the two terms whose difference is the required discontinuity in the missing-mass variable, the singularities appear in such a way that it is possible to close at least two of the contours of integration in a half-plane away from all singularities and hence obtain a zero result for the coefficient of the leading term in s.

The fundamental assumption that makes this proof possible is that T has no essential singularities and is sufficiently damped in the variables k_i^2, $k_i'^2$, so that the various integrals are uniformly convergent and integration contours can be closed at infinity without introducing new contributions.

Define variables as follows [3] $[s = (p_1+p_2)^2]$

$$k_1 = x_1 p_1 + y_1 p_2/s + \kappa_1 \, ; \quad k_1' = x_1' p_1 + y_1' p_2/s + \kappa_1'$$
$$k_2 = x_2 p_1/s + y_2 p_2 + \kappa_2 \, ; \quad k_2' = x_2' p_1/s + y_2' p_2 + \kappa_2' , \qquad (2.1)$$

where $\kappa_i \cdot p_1 = \kappa_i \cdot p_2 = \kappa_i' \cdot p_1 = \kappa_i' \cdot p_2 = 0$ and so the κ_i, κ_i' are spacelike and effectively two-dimensional. Because we want to put $q = q'$ (the primed <u>external</u> momenta are distinguished from the unprimed ones only because the corresponding scalar invariants are not necessarily evaluated on the same side of their branch cuts), we have

$$k_1 + k_2 = k_1' + k_2' = q ,$$

and so

$$x_1 \sim x_1'$$
$$y_2 \sim y_2' . \qquad (2.2)$$

The conditions $q^2 > 0$, $0 < q^0 < (p_1+p_2)^0$ give

$$
\begin{aligned}
&0 < x_1 < 1 \\
&0 < y_2 < 1 \quad .
\end{aligned}
\tag{2.3}
$$

Of the variables on which the internal eight-point function T depends, we have

$$
\begin{aligned}
k_1^2 &\sim x_1 y_1 + x_1^2 m^2 - \underset{\sim}{K}_1^2 \\
s_1 \equiv (p_1 - k_1)^2 &\sim (x_1-1) y_1 + (x_1-1)^2 m^2 - \underset{\sim}{K}_1^2 \\
u_1 \equiv (p_1' - k_1' + k_1)^2 &\sim y_1' - y_1 + m^2 - (\underset{\sim}{K}_1' - \underset{\sim}{K}_1)^2 \ ,
\end{aligned}
\tag{2.4a}
$$

where m is the mass of the external nucleons. There are analogous expressions for

$$
\begin{aligned}
&k_2^2, \quad k_1'^2, \quad k_2'^2 \\
&s_2 = (p_2 - k_2)^2, \quad s_1' = (p_1' - k_1')^2, \quad s_2' = (p_2' - k_2')^2 \\
&u_2 = (p_2' - k_2' + k_2)^2, \quad u_1' = (p_1 - k_1 + k_1')^2, \quad u_2' = (p_2 - k_2 + k_2')^2 .
\end{aligned}
\tag{2.4b}
$$

The remaining variables in T are unimportant here.

We can write the amplitude described by Fig. 2.1 as

$$
\begin{aligned}
A(s, q^2, \underset{\sim}{q}) \sim \frac{1}{4s^2} \int_0^1 dx_1\, dy_2\, dx_1'\, dy_2' \int_{-\infty}^{\infty} dy_1\, dx_2\, dy_1'\, dx_2' \int d^2K_1\, d^2K_2\, d^2K_1'\, d^2K_2' \\
\delta(x_1 - x_1')\, \delta(y_2 - y_2')\, \delta^{(2)}(\underset{\sim}{K}_1 + \underset{\sim}{K}_2 - \underset{\sim}{K}_1' - \underset{\sim}{K}_2')\, \delta^{(3)}(x_1 \underset{\sim}{p}_1 + y_2 \underset{\sim}{p}_2 + \underset{\sim}{K}_1 + \underset{\sim}{K}_2 - \underset{\sim}{q}) \\
\delta(x_1 y_2 - q^2/s)\, \theta(x_1 + y_2)\, T \quad .
\end{aligned}
\tag{2.5}
$$

Now we shall show that, on each of the two Riemann sheets on which we need to evaluate **A**, either the y_1 and x_2 integrations, or the y_1' and x_2' integrations, vanish. To do this, we have to identify the locations of the singularities of T in the complex planes of these variables.

Consider, for definiteness, the variable y_1. According to (2.4), the variables

$$k_1^2, \; s_1, \; u_1, \; u_1' \tag{2.6}$$

of T depend on y_1, and the dependence in each case is linear. Thus the singularities of T in each of the variables (2.6) are reflected in a simple fashion in the y_1 plane. In order to determine whether each singularity lies above or below the real y_1 axis, we have to decide the correct $i\epsilon$ prescription [13] for each of the variables (2.6).

In the usual language of inclusive processes [7] the reaction pp → "γ"X is represented by the missing mass discontinuity shown in Fig. 2.2, where the + and - signs [13] signify that q^2 and $s = (p_1+p_2)^2$ are evaluated on the upper sides of their cuts, while q'^2 and $s' = (p_1'+p_2')^2$ are evaluated on the lower sides of their cuts. The signs in the centre of the two right-hand bubbles refer to the singularities in the missing-mass variable

$$M^2 = (p_1+p_2-q)^2 \; .$$

In the amplitude of Fig. 2.1 the singularities in these external variables q^2, s, q'^2, s' and M^2 are generated [13] in the integration by the singularities of the internal amplitude T. Singularities in the internal variables (2.4) generate singularities in the external variables as follows :

$$\begin{aligned} (k_1^2) &+ (k_2^2) \rightarrow (q^2) \\ (u_1) &+ (u_2) \rightarrow (s) \\ (k_1'^2) &+ (k_2'^2) \rightarrow (q'^2) \\ (u_1') &+ (u_2') \rightarrow (s') \end{aligned} \tag{2.7}$$

$$\left.\begin{array}{l}(s_1) + (s_2) \\ (s_1') + (s_2')\end{array}\right\} \rightarrow (M^2) \quad . \tag{2.7}$$

These results can be understood most readily by considering simple examples of Feynman graphs, Fig. 2.3 (in these graphs, the Reggeon can be thought of as an infinite sum over ladder graphs). In Fig. 2.3a, putting the lines k_1^2 and k_2^2 on their mass shells, that is going to the poles of T in these variables, one generates a normal threshold in q^2. Putting the lines a_1 and a_2 on their mass shells, that is going to poles of T in s_1 and s_2, generates a normal threshold in M^2. The amplitude T has no dependence on the u and u' variables, so for this simple graph no singularities occur in s or s'. A more complicated graph, which has singularities in u_1 and u_2 and so also in s, is drawn in Fig. 2.3b. Here the lines (a_1,b_1) generate a normal threshold in the internal variable s_1, and the lines (a_2,b_2) generate a similar singularity in s_2 ; together, these four lines generate a normal threshold singularity in the external variable M^2. By twisting each cross of the graph, we can redraw Fig. 2.3b as in Fig. 2.3c. Now we see readily that the pairs (c_1,d_1), (c_2,d_2), respectively, generate singularities in the internal variables u_1, u_2, and together these generate a singularity in the external variable s.

From (2.7), and the fact that the $i\epsilon$ prescriptions for the external variables are determined in each of the two terms on the right of Fig. 2.2, the $i\epsilon$ prescriptions for the internal variables are fixed. For the second term on the right of Fig. 2.2 these are :

$$\begin{array}{l} k_1^2, k_2^2, u_1, u_2 \quad : \quad +i\epsilon \\ k_1'^2, k_2'^2, u_1', u_2', s_1, s_2, s_1', s_2' \; : \; -i\epsilon \quad . \end{array} \tag{2.8}$$

Returning now to the y_1 integration, we see from (2.3) and (2.4) that y_1 varies linearly as $+k_1^2$ and $+u_1$, but as $-s_1$ and $-u_1'$. Hence from (2.8), in the second term on the right-hand side of Fig. 2.2 all the singularities in the variables (2.6) appear in the lower half of the y_1 plane. By closing the contour of the y_1 integration in the upper half plane, we obtain a zero result for the leading term *). A similar result applies to the x_2 integration.

For the first term on the right of Fig. 2.2, a completely parallel analysis indicates that the y_1' and x_2' integrations now vanish. Hence the contributions of diagrams with interactions vanish to leading order simply due to the usual analyticity properties and our assumption concerning the convergence of the integrals

3. ANALYSIS OF THE DISCONTINUITIES

We have shown in Section 2 that the total discontinuity of Fig. 2.1 in the missing mass vanishes to leading order in s. The total discontinuity essentially has three separate contributions, corresponding to the three classes of slicing L_1, L_2, L_3 shown in Fig. 3.1. Included in L_1 and L_3 are partial [14] Pomeron slicings, such as illustrated in the explicit model of Fig. 3.2. The slicings L_1 and L_3 result in the interference term $\sum_c 2\mathrm{Re}\, A_C^{DY} A_C'^*$ of (1.3), that is interference between the Drell-Yan amplitude of Fig. 1.1 and contributions of the type of Fig. 1.2a (though with any number of Pomerons exchanged). The slicing L_2 gives the other two terms of (1.3), that is contributions from either type of mechanism of Fig. 1.2 by itself.

In this Section we show that the separate slicings L_1, L_2, L_3 are non-zero, but verify that their sum vanishes.

The slicings L_1, L_2, L_3 are constructed by inserting the appropriate discontinuity of the internal amplitude T into the integral (2.5) in place of T. In terms of the variables defined in (2.4) and the missing mass $M^2 = (p_1+p_2-q)^2$, these discontinuities are

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*) Although we have argued explicitly only that the normal thresholds in each of the variables (2.6) dispose themselves in such a way that the y_1 integration vanishes, it can be shown that the same is true of any other Landau singularities that may appear in these variables.

$$D_1 = \mathrm{disc}_{s_1}\,\mathrm{disc}_{s_2}\,T(s_i, M^{2(-)}, s_i'^{(-)})$$

$$D_2 = \mathrm{disc}_{M^2}\,T(s_i^{(+)}, M^2, s_i'^{(-)}) \qquad (3.1)$$

$$D_3 = -\mathrm{disc}_{s_1'}\,\mathrm{disc}_{s_2'}\,T(s_i^{(+)}, M^{2(+)}, s_i').$$

Here the superscripted signs denote the appropriate $i\epsilon$ prescription for the corresponding variable. For the reasons explained in Section 2, in each case the variables k_i^2 and u_i are assigned a $+i\epsilon$ prescription, and $k_i'^2$ and u_i' are assigned $-i\epsilon$. The negative sign appears in D_3 as a result of the usual Feynman rules combined with the Cutkosky prescription for discontinuities. Consider the y_1 integration in (2.5). Recall that, from (2.3) and (2.4), y_1 depends linearly on k_1^2 and u_1, and on $-s_1$ and $-u_1'$. Hence the above $i\epsilon$ prescriptions have the consequence that, for the integral of either D_2 or D_3, the locations of the possible normal-threshold singularities in the y_1 plane are as in Fig. 3.3. Because of our assumption that T is sufficiently damped at large k_1^2, we can deform the contour of y_1 integration so that it encloses only the s_1 cut, that is the integrals over D_2 and D_3 can be taken over $\mathrm{disc}_{s_1} T$, as is that over D_1. By similarly considering the x_2, y_1' and x_2' integrations, we see that each of the three integrals can be taken over the discontinuities in each of the four variables s_1, s_2, s_1', s_2'. Hence instead of (3.1) we can use

$$D_1 = \Delta T(s_i, M^{2(-)}, s_i')$$

$$D_2 = \Delta\,\mathrm{disc}_{M^2}\,T(s_i, M^2, s_i') \qquad (3.2a)$$

$$D_3 = -\Delta T(s_i, M^{2(+)}, s_i')$$

where

$$\Delta = \mathrm{disc}_{s_1}\,\mathrm{disc}_{s_2}\,\mathrm{disc}_{s_1'}\,\mathrm{disc}_{s_2'} \qquad (3.2b)$$

This is the end of the argument : if we add together the three discontinuities in (3.2), so as to combine the contributions of the three slicings L_1, L_2 and L_3, the result is evidently zero.

The contributions do not vanish separately. This is easiest to see in the case of L_2, which has an integrand that is positive definite. If the asymptotic behaviour of the internal eight-point function is dominated by a $J = 1$ singularity, the asymptotic behaviour of the contribution of the slicing L_2 is the same as that of the Drell-Yan term up to factors of log s.

For completeness, we must now return to two points that we have not treated satisfactorily. The first point pertains to the role played by other channel singularities, corresponding to variables which we have not exhibited explicitly in (3.2a). For example, the invariant $\sigma = (p_1-k_1+p_2)^2$ may also have singularities, as in the graph of Fig. 3.4. In terms of the variables of Section 2,

$$\sigma = s(1-x_1) + (1-x_1)^2 m^2 + m^2 - \underset{\sim}{K}_1^2 - (1-x_1)y_1 , \tag{3.3}$$

so that σ is of order s. In the Regge limit of the eight-point function the singularities associated with σ appear as singularities in the variable σ/s, as [7] just two branch points at 0 and ∞ in this variable, and so at $\pm\infty$ in y_1. Because of the assumed convergence of the y_1 integral the presence of these singularities does not affect the contour distortions that we have made (this property was also used implicitly in the argument of Section 2). Further, when we have made the distortion there is no need to consider which $i\epsilon$ prescription must be applied to the σ singularities. This is because we end up with an integral over the s_2 discontinuity, and so the Steinmann relations [15], which apply to simultaneous discontinuities in overlapping channel variables such as s_2 and σ, mean that the σ singularities have disappeared.

The second point pertains to whether the normal threshold singularities indicated in Fig. 3.3 are actually present for each of the various slicings L_1, L_2, L_3. It may happen that because of the different $i\epsilon$ prescriptions in the discontinuities, some of the branch points are absent in the various slicings. If this were true, then again, the cancellation would not necessarily take place. Indeed, applying De Grand's argument [11] for Mandelstam double Regge graph to our case, we would expect that the second term which involves a discontinuity in M^2 would not itself contain discontinuities in the overlapping variables u_1, u_1', u_2 and u_2'. Thus these cuts in the lower half plane in Fig. 3.3 would disappear in the integral over the second term in Eq. (3.1), whereas they would be present in the first and third. However, the singularities in the parton propagator remain in all cases and so prevent the contour from being closed in the lower half plane. If we consider the usual additive separation of cuts in s_1 and u_1 which precedes a decomposition of the amplitude into signatured amplitudes :

$$T \sim f_s(s_1) + f_u(u_1)$$

then we may make use of the added damping provided by the off-shell parton propagator to show that the integral over the second term is in any case zero since it has singularities only in the lower half plane ; therefore it makes no difference to the asymptotic limit of the integral whether these cuts are present or not. In the Mandelstam diagram this was not the case because the additional damping effect of the parton propagator was not there. Thus in the one case in which we suspect the singularity structure is apt to differ from term to term, we find that the expected cancellation is unaffected. However, to complete the discussion, we would need to consider other possible differences, including Landau singularities of any order. This would carry the argument to a level of detail which we shall not enter here. It is certainly true that no normal thresholds cause difficulties other than those in u_1, u_1', u_2, and u_2' discussed above. We have not found any higher order singularities which present problems and do not believe they do.

4. CONCLUSION

Let us briefly review the major points. We have studied the contributions of diagrams with explicit initial and final state interactions (Fig. 1.2) to the process of lepton production at large $p_\perp$ via parton-antiparton annihilation. Of course our results also apply to parton models for large $p_\perp$ hadron production processes which have a similar diagrammatic structure. We conclude that on quite general grounds these terms with interactions do not contribute to the leading asymptotic behaviour of the single particle inclusive cross-section in these large $P_\perp$ processes. However, as we explicitly indicated in Section 3, this zero results from the cancellation of several non-zero terms which correspond to quite different structure for the unobserved multiparticle final state. Hence the contributions with interactions will, in fact, be very important when one asks more detailed questions about the structure of the final state. In particular we expect that a component of these large $p_\perp$ events will have superimposed on them a multiparticle structure similar to that of usual low $p_\perp$, "pionization" events.

These conclusions follow from two fundamental assumptions. We assume that the amplitudes which describe the emission of partons are sufficiently damped in the parton invariant masses to insure the convergence of the relevant integrals and that the complete amplitude displays conventional analytic structure.

ACKNOWLEDGMENTS

We wish to thank our colleagues for many helpful discussions and especially Professor D. Amati for the hospitality of the CERN Theory Division which made this collaboration possible. We also thank Drs. J.L. Cardy and G.A. Winbow for sending an advance draft of their paper.

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FIGURE CAPTIONS

Fig. 1.1 The Drell-Yan mechanism for massive lepton pair production via a large q^2 virtual photon.

Fig. 1.2 Final state interactions (a) without pionization, (b) with pionization. There are similar contributions from initial state interactions, and from cross interactions between initial and final states.

Fig. 1.3 Models for the production of large p_T hadrons.

Fig. 2.1 Mueller diagram for the production of a highly virtual photon, including the exchange of any number of Pomerons (represented by the doubled zig-zag line). The other internal lines are partons.

Fig. 2.2 Discontinuity in the missing mass variable needed to describe the inclusive reaction $pp \rightarrow "\gamma"X$.

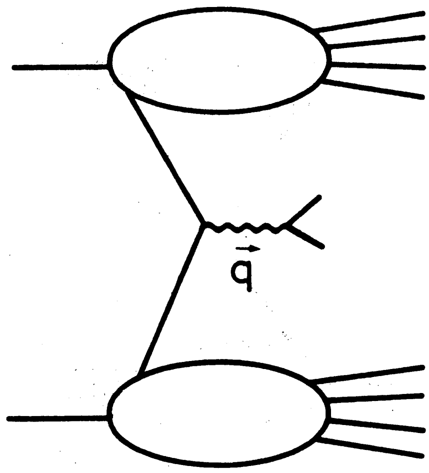
Fig. 2.3 Feynman graphs that are particular examples of Fig. 2.1. Here (b) and (c) are the same graph drawn in two different ways.

Fig. 3.1 Three classes of slicing in Fig. 2.1 that contribute to the total discontinuity in the missing mass.

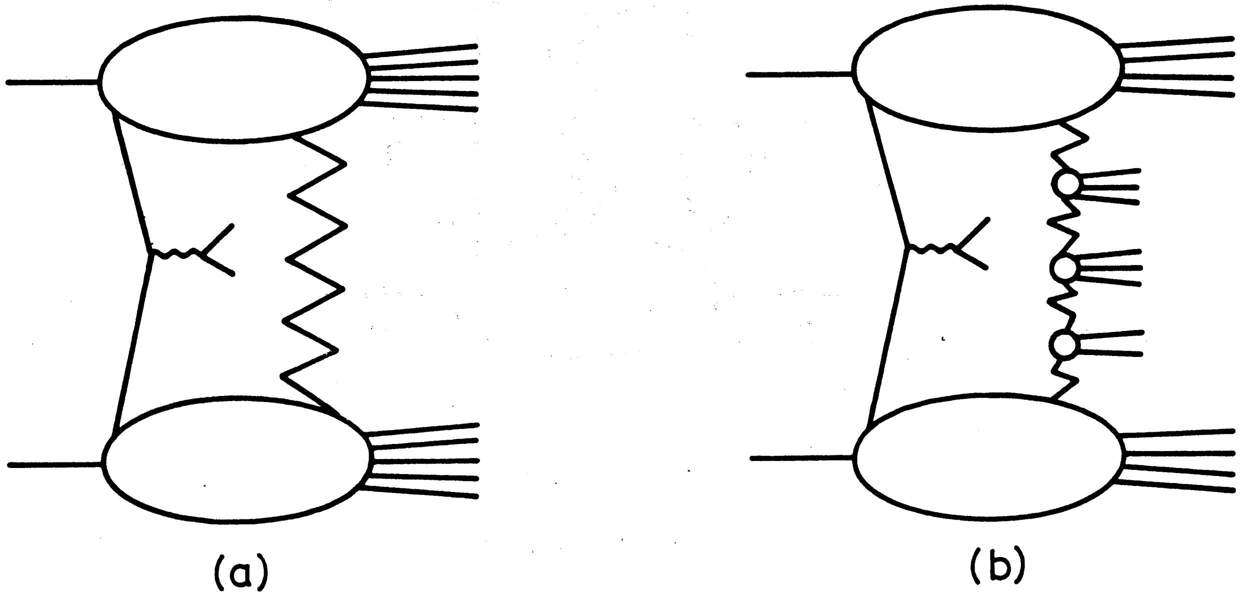
Fig. 3.2 A partial Pomeron slicing that is included in L_1 in Fig. 3.1.

Fig. 3.3 Singularities in the complex y_1 plane for the integrals over D_2 and D_3.

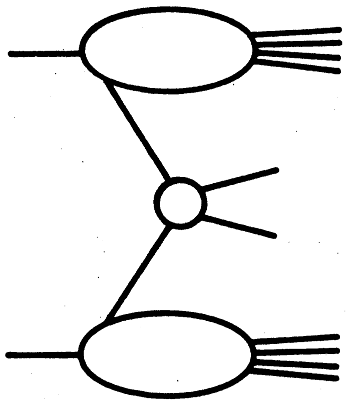
Fig. 3.4 Example of a graph producing singularities in overlapping channels in the eight-point function.



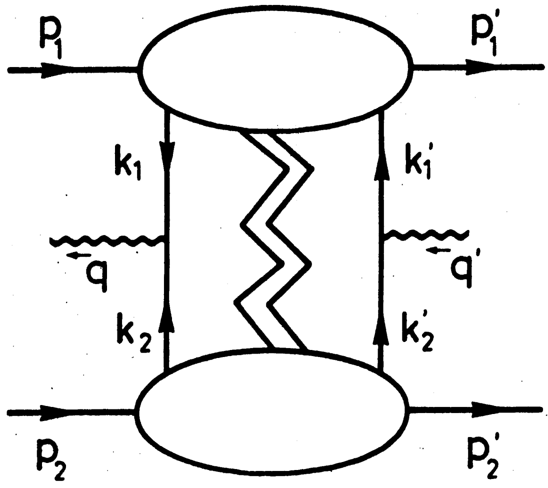
- Figure 1.1 -



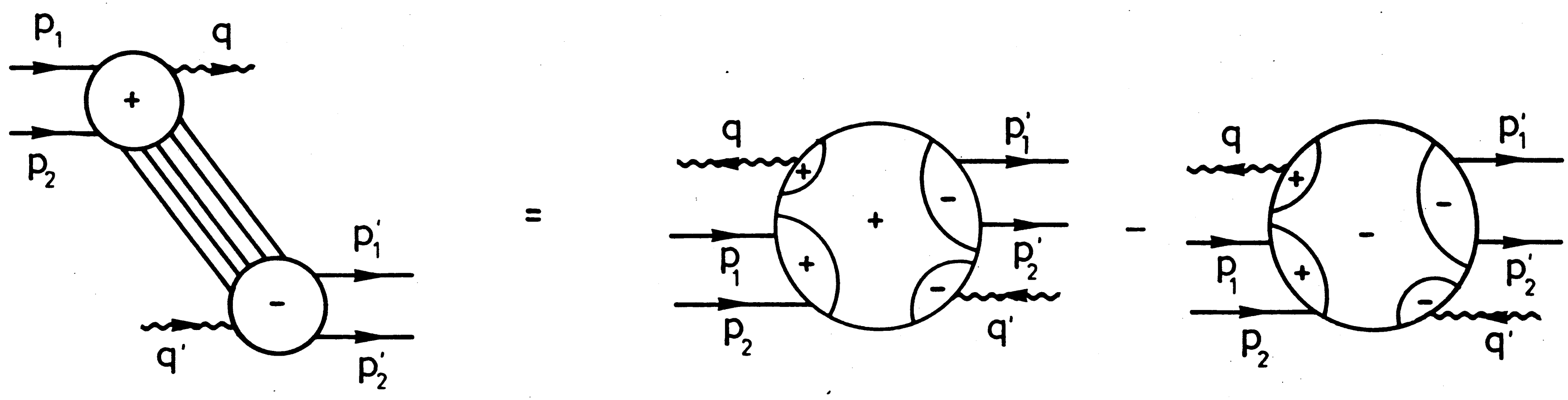
- Figure 1.2 -



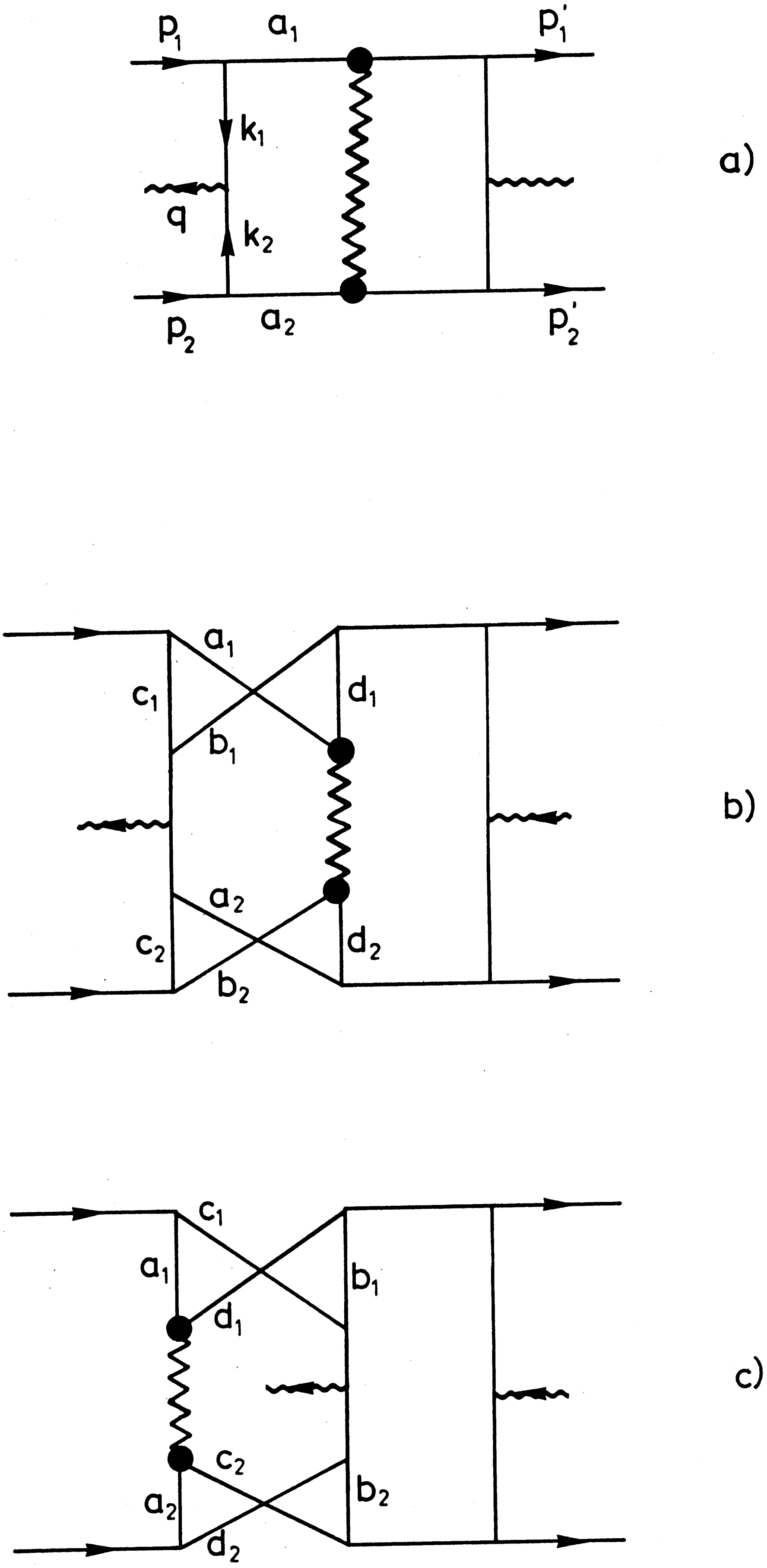
- Figure 1.3 -



- Figure 2.1 -

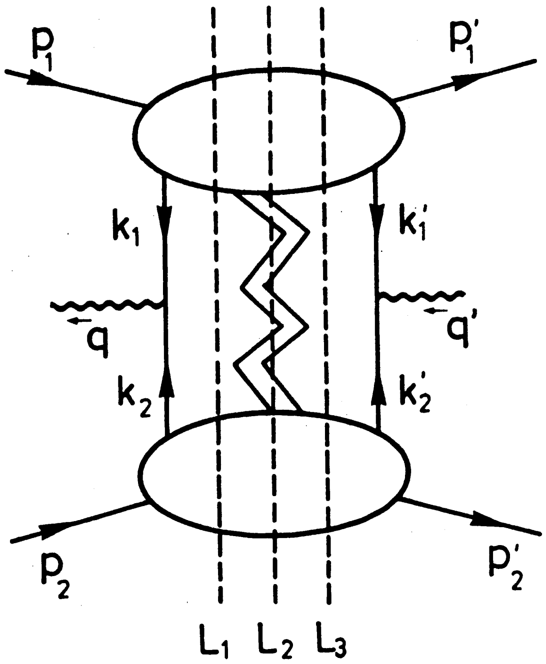


- Figure 2.2 -

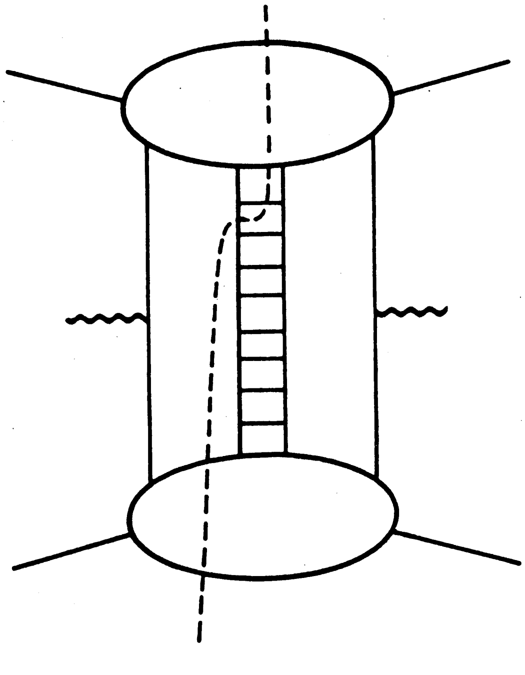


p1
a1
p1'
k1
q
k2
p2
a2
p2'
a)
a1
c1
d1
b1
a2
c2
d2
b2
b)
c1
a1
b1
d1
c2
a2
b2
d2
c)

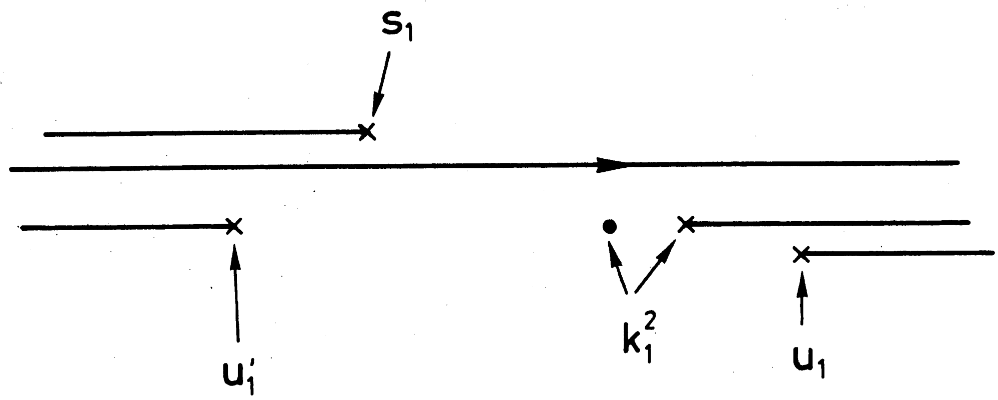
- Figure 2.3 -



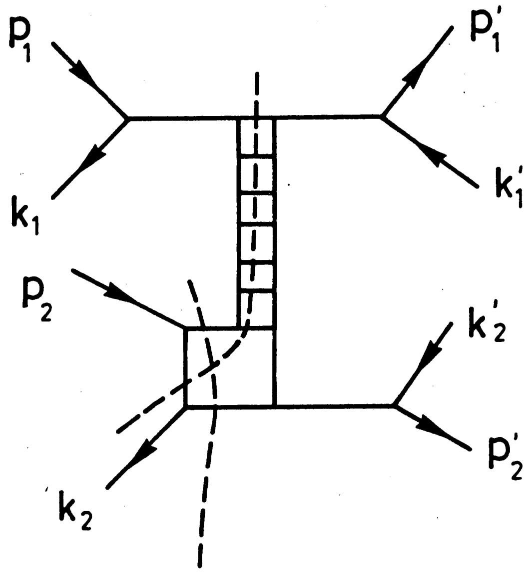
- Figure 3.1 -



- Figure 3.2 -



- Figure 3.3 -



- Figure 3.4 -