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POLARIZATION SUM RULES

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A B S T R A C T

Sum rules for observable combinations of amplitudes in $0^-\frac{1}{2}^+$ scattering are derived, leading to direct predictions for polarization parameters at intermediate energies. We give predictions for πN, KN and $\bar{K}N$ scattering, and discuss tests of phase-shift solutions.

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Qualitative features of high-energy amplitudes (e.g. zero-structures, spin-dependences) can be predicted from low-energy phase shifts by exploiting analyticity, usually through FESR [see, for example, Elvekjaer and Martin[1] and references therein]. Here we wish to extend such considerations to deal directly with certain observable combinations of amplitudes. We show how to obtain FESR which extrapolate particular experimental quantities from low to intermediate energies, and we apply them to make predictions for polarization measurements in πN, KN, and $\bar{K}N$ scattering in the 2-4 GeV/c region. Existing data confirm the method, and expectations are given for future experiments.

Tests of low-energy phase shifts are also discussed and (for example) evidence is presented against the "Sens β" type of K^+p phase-shift solution[2]. Firstly we give theoretical details, then predictions.

Sum Rules. Consider integrals of the form

$$Q_k = \frac{1}{N_k}\int_{\nu_0}^{\nu_m} \nu^k Q(\nu)\, d\nu$$

where $Q(\nu)$ is some observable quantity at fixed t (suppressed), $\nu = (s - u)/4M$ (M = target mass) and ν_0 is the physical threshold. Normalization is $N_k = \int_{\nu_0}^{\nu_m} \nu^k\, d\nu$. The aim is to establish these low-energy averages Q_k as genuine FESR integrals, capable of interpretation in terms of the higher-energy ($\nu \geq \nu_m$) behaviour of Q. To achieve this we must exhibit a function $H(\nu)$ with usual ν-plane analyticity at fixed t, whose discontinuity across the physical cut is equal to $Q(\nu)$. Moreover $H(\nu)$ must contain, for $|\nu| \geq \nu_m$, no inverse integer-power terms ("right-signature fixed poles") which would give high-energy contributions unrelated to Disc H [see, for example, Sec. 6.2 of Collins[3]]. Our main point is that for at least some observable quantities $Q(\nu)$ a good approximation to the appropriate H-function exists, and the FESR can be constructed.

Firstly take the case Q = P ≡ polarization parameter in $0^-\frac{1}{2}^+$ scattering, [$P\, d\sigma/dt = 2\, \mathrm{Im}\, (NF^*)$], and consider

$$H(\nu) = \frac{1}{2} \ln\left(\frac{N+iF}{N-iF}\right)$$

where the non-flip (N) and flip (F) s-channel helicity amplitudes are normalized to $d\sigma/dt = |N|^2 + |F|^2$. With the large-ν small-t approximations $N = M(A + \nu B)/4s\sqrt{\pi}$ and $F = \sqrt{-t}A/4s\sqrt{\pi}$ [4], we avoid kinematical singularities, and so $H(\nu)$ has, apart from effects of possible zeros of the transversity amplitudes $N \pm iF$, the required analyticity properties, i.e. essentially those of the A, B invariant amplitudes. Note that Born-term poles cancel. Because N, F have no fixed poles, these are absent from H.

On the physical cut Disc H = ½ ln $[(1 + P)/(1 - P)]$ and so since $|P| \leq 1$ only a small ($< \frac{1}{3} |P|^3$) error is committed by the approximation

$$\text{Disc } H \approx P(\nu) .$$

Our calculations show this to be a good approximation.

Thus, for example, for $\pi^{\pm}p \to \pi^{\pm}p$, from functions $H_{\pm}$, we can write FESR for the polarizations $P_{\pm}$ by integrating round the usual closed contour [see p. 187 of Collins[3]] the symmetrized quantities

$$\nu^k H^{(\pm)} \equiv \nu^k [H_+(\nu) \pm H_-(\nu)] , \qquad k \text{ odd or even,}$$

to obtain a form

$$P_k^{(\pm)} = \frac{k+1}{\alpha+k+1} \left(\langle P_+ \rangle \pm \langle P_- \rangle \right) + \varepsilon_k^{(\pm)} .$$

The l.h.s. of this equation is the low-energy average

$$P_k^{(\pm)} \equiv \frac{1}{N_k} \int_{\nu_0}^{\nu_m} \nu^k (P_+ \pm P_-)\, d\nu \qquad k = 1, 3 \ldots \text{ or } 0, 2 \ldots$$

and, because of the properties of the $H_{\pm}$, on the r.h.s. appear the desired higher-energy ($\nu \geq \nu_m$) averages $\langle P_{\pm} \rangle$ of the polarization.

The quantity α parametrizes effective high-energy behaviour, $H \propto \nu^{\alpha}$ ($|\nu| \geq \nu_m$), and the error term $\varepsilon_k^{(\pm)}$ absorbs approximations. For α, typical Regge models suggest a small value: $0 \lesssim \alpha \lesssim \frac{1}{2}$. Note that for the individual amplitudes themselves a stronger energy-variation is usual, and models are needed to extract quantitative predictions from FESR. The main contribution to $\varepsilon_k^{(\pm)}$ is expected to arise from complex zeros of $N \pm iF$, and here we find*) an exponential decrease with k. Thus, for k "large" we expect $P_k^{(\pm)}$ to be independent of k -- a crucial consistency check.

In practice, any $k > 0$ usually suffices, and so we have simply

$$P_k^{(\pm)} \approx \langle P_+ \rangle \pm \langle P_- \rangle .$$

Combining even and odd moments gives $\langle P_+ \rangle$ and $\langle P_- \rangle$ separately. For a process like $\pi^- p \to \pi^0 n$, where P is crossing-even, moments k = 1, 3 ... can be used directly. Note that $k \geq 1$ emphasizes more the region near $\nu = \nu_m$ and so the averages $\langle P \rangle$ (and generally $\langle Q \rangle$) are to be interpreted as intermediate-energy predictions.

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*) At high energies $\nu > \nu_m$ on the real axis, if one of the amplitudes N, F dominates (as is the usual case -- although see later discussion), then since the amplitudes are smooth (Regge behaved) the Phragmén-Lindelöff Theorem[5] excludes complex zeros. For $|\nu| < \nu_m$ possible zeros occur in pairs (at say $\pm \nu_B$) in the crossing-symmetric functions $H^{(\pm)}$ and can be joined by branch cuts. $H^{(\pm)}$ thus has terms of the form ln $[(\nu + \nu_B)/(\nu - \nu_B)]$, each leading to an error contribution $\varepsilon_k^{(\pm)}$ equal to $(-\pi/N_k)$ Re (ν_B^{k+1}). This is bounded by $\pi(k + 1) \exp [-(k + 1) \ln (\nu_m/|\nu_B|)]$, decreasing with k since $|\nu_B| < \nu_m$.

What other observables Q can be treated this way? Another case we have considered is $Q = \tilde{R} \equiv$ spin-correlation parameter in $0^{-}\frac{1}{2}^{+}$ scattering defined by $\tilde{R}\, d\sigma/dt = 2\,\mathrm{Re}\,(NF^{*})$. Here we have functions

$$H(\nu) = \frac{1}{2}(\nu_0^2-\nu^2)^{1/2} \ln\left(\frac{N+F}{N-F}\right),$$

where

$$\mathrm{Disc}\, H = -i\sqrt{\nu^2-\nu_0^2}\,\{\tilde{R} + O(\tfrac{1}{3}\tilde{R}^3)\}.$$

Normalizing by $N_k = \int_{\nu_0}^{\nu_m} \nu^k \sqrt{\nu^2 - \nu_0^2}\, d\nu$ we have then in, for example, $\pi^{\pm}p \to \pi^{\pm}p$ entirely analogous forms of FESR:

$$R_k^{(\pm)} \approx \langle \tilde{R}_+ \rangle \pm \langle \tilde{R}_- \rangle .$$

For $\tilde{A} \equiv (|N|^2 - |F|^2)/(d\sigma/dt)$ we do not know a suitable H-function. Simply making low-energy averages A_k of $\tilde{A}$ gives very moment-dependent results, grossly violating the relation $P^2 + \tilde{R}^2 + \tilde{A}^2 = 1$. However, fixing a phase from empirical flip/non-flip dominance we can use $\langle P \rangle$, $\langle \tilde{R} \rangle$ and the above quadratic relation to estimate $\langle \tilde{A} \rangle$. Measured Wolfenstein parameters R, A are then obtained by a rotation[6].

Here we leave aside higher-spin processes and concentrate on πN, KN and $\bar{K}N$ scattering, where phase-shift reconstructions of low-energy amplitudes are readily possible.

Predictions. Figure 1 (a-c) shows predictions for πN scattering based on Saclay phase shifts[7] with a cut-off ν_m at p_L = 2.7 GeV/c. [Comparison between results using Almehed-Lovelace phase shifts[8] and Saclay phase shifts at the same cut-off (p_L = 2.1 GeV/c) showed mutual consistency. The predictions are of somewhat different magnitude but have similar qualitative features to those with the larger value of ν_m]. The range of t extends to $-2.5\ (\mathrm{GeV/c})^2$. Extrapolation to unphysical t-values was made with conventional Legendre-series expansions. Overlapping s- and u-channel cuts were dealt with carefully, whereas we neglected the third double spectral function ρ_{su}. The bands in Fig. 1 cover moments k = 1, 2, 3. The general agreement between moments confirms the validity of the method, indicating negligible complex singularities and small, indeterminate, values for the energy-dependence parameters α. As expected, results for moment k = 0 were rather different. Integrating the full (rather than approximate) expression for Disc H made no change in the results, within the bands.

In Fig. 1 $P(\pi^{\pm}p)$ is compared with the extensive data at p_L = 2.7 GeV/c [9]. Except for the magnitude at smaller angles the agreement is good up to very large $|t|$. [The less extensive data of Johnson and Hansroul[10] in fact agree better with our predictions at smaller $|t|$]. The data[11] for $P(\pi^-p \to \pi^0 n)$ for $2.5 \lesssim p_L \lesssim 3.5$ GeV/c are not very self-consistent but show roughly 20-30%

polarization for $t > -0.4$ (GeV/c)^2 in agreement with Fig. 1a. For $|t| \gtrsim 0.5$ (GeV/c)^2 there is considerable moment-dependence of this quantity. This is not unexpected and is presumably caused by significant complex singularities in $H(\pi^- p \to \pi^\circ n)$ because neither N nor F dominates here [e.g. Barger and Phillips[12]] see previous footnote.

In calculating R and A the sign of $\tilde{A}$ is fixed by non-flip dominance of elastic, and flip dominance of CEX processes. We remark that the 6 and 16 GeV/c $\pi^{\pm}p$ data on R and A[6] agree well with our predictions. An interesting feature of the $\tilde{R}$ and $\tilde{A}$ results of Fig. 1 (c, d) is that $\tilde{R}(\pi^+ p)$ is substantially positive, while $\tilde{R}(\pi^- p)$ is essentially zero. From the usual arguments[13] -- i.e. interference between dominantly $N^{(I_t=0)}$ and $F^{(I_t=1)}$ -- one would expect mirror symmetry like that of $P(\pi^{\pm}p)$. This comment is, of course, valid at p_L = 6, 16 GeV/c where the data agree with out predictions. Assuming only that $N^{(I_t=1)}$ is small we infer $\mathrm{Im}\, F^{(I_t=0)} \approx -\mathrm{Im}\, F^{(I_t=1)}$ as a result. This relation is well satisfied by the Barger-Phillips 5-pole model[12].

Figure 2 presents predictions for KN and $\overline{K}N$ scattering. Here the cut-off ν_m is at p_L = 1.5 GeV/c, and the bands cover moments k = 1, 2, 3. Again, results involving k = 0 are rather different.

The phase-shift solutions used are as follows: for $K^+p \to K^+p$ the Sens γ solution[2]; for $K^-p \to K^-p$ and $p_L \leq 0.43$ GeV/c we used Kim's solution[14] plus a $\Lambda(1520)$ of standard Breit-Wigner form with parameters from the Particle Data Group[15]; for $0.43 \leq p_L \leq 1.2$ GeV/c the solution of Lea et al. [LOMM][16] was used, and above 1.2 GeV/c we took the phase shifts of Litchfield et al.[17]. For KN I = 0 we chose the so-called BGRT-D (Sens γ) solution[18]. The KNY coupling constant $G^2 = (g^2_{KN\Lambda} + g^2_{KN\Sigma})/4\pi$ is taken from Pietarinen and Knudsen[19], which by SU(3) yields $g^2_{KN\Lambda}/4\pi = 11.0$ and $g^2_{KN\Sigma}/4\pi = 1.4$. A detailed description and discussion of all these choices is given in Ref. 1.

Figure 3 compares predictions with available intermediate-energy ($\nu \approx \nu_m$) data where possible. Data at 1.64 GeV/c [2] agree very well with $P(K^+p)$, confirming that observables for this exotic channel tend to be smooth already from rather low energy. The prediction for $P(K^-p \to K^-p)$ has shape similar to the data at p_L = 1.72 GeV/c [20]. It is clear, however, also from the data of Ref. 20, that for $\nu \approx \nu_m$ this quantity still varies with energy. $P(K^+n \to K^\circ p)$ is compared with the 4 GeV/c SU(3) predictions of Ref. 21. The agreement is good, and gives further evidence that amplitudes in this exotic channel cannot be purely real. A similar comparison for $P(K^-p \to \overline{K}^\circ n)$ is made, and also data at 8 GeV/c [22] are included. As in $K^-p \to K^-p$ there still seems to be energy-variation for $p_L \gtrsim 1.5$ GeV/c.

The main alternatives for the chosen phase-shift solutions are the K^+p Sens β solution[2], and the K^-p Langbein-Wagner (L-W) solution[23] in $0.43 \leq p_L \leq 1.2$ GeV/c. Generally we find that these give results more dependent on moment, k. This is illustrated in Fig. 4, where we have predictions for $R(K^\pm p \to K^\pm p)$. Also shown are reconstructed values at $\nu = \nu_m$ for the different possibilities. Especially for $K^+p \to K^+p$ one expects the observables to be smooth by $p_L = 1.5$ GeV/c, and therefore the sum-rule predictions for R should coincide with the phase-shift reconstructions. Therefore, quite apart from possible experimental tests[24], requirements of analyticity plus smoothness of exotic amplitudes weigh heavily against the β solution, while the γ possiblitiy is perfectly consistent. A similar comparison for $K^-p \to K^-p$ is seen to favour LOMM over L-W, but not decisively. Here R at $\nu = \nu_m$ is calculated with the phase shifts of Litchfield et al.[17]. In both $K^\pm p$ cases, for other observables the distinctions are found to be less clear-cut.

Conclusions

To summarize:

i) FESR for some polarization observables are justifiable;

ii) they are useful in practice -- the failure of simple averages is illustrated in the remarks concerning $\tilde{A}$;

iii) quantitative predictions from amplitude FESR are expected to be more model-dependent;

iv) predictions for yet unmeasured, but soon accessible, experimental quantities are given;

v) the "Sens β" type K^+p phase shifts show inconsistencies, but there is no clear test for K^-p;

vi) extensions to other processes and observables seem possible.

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Figure captions

Fig. 1 : Predictions for πN scattering. Data on π^+p (●) and π^-p (■) from Ref. 9.

Fig. 2 : Predictions for KN and $\overline{K}N$ scattering.

Fig. 3 : Comparison with data from Refs. 2, 20, 22. The broken curves are limits of 4 GeV/c SU(3) predictions from Ref. 21.

Fig. 4 : Predictions with alternative phase-shift solutions, compared with K^+p Sens β, γ (Ref. 2) reconstructions (○, ●), and with the K^-p Litchfield et al. (Ref. 17) reconstruction (▲).

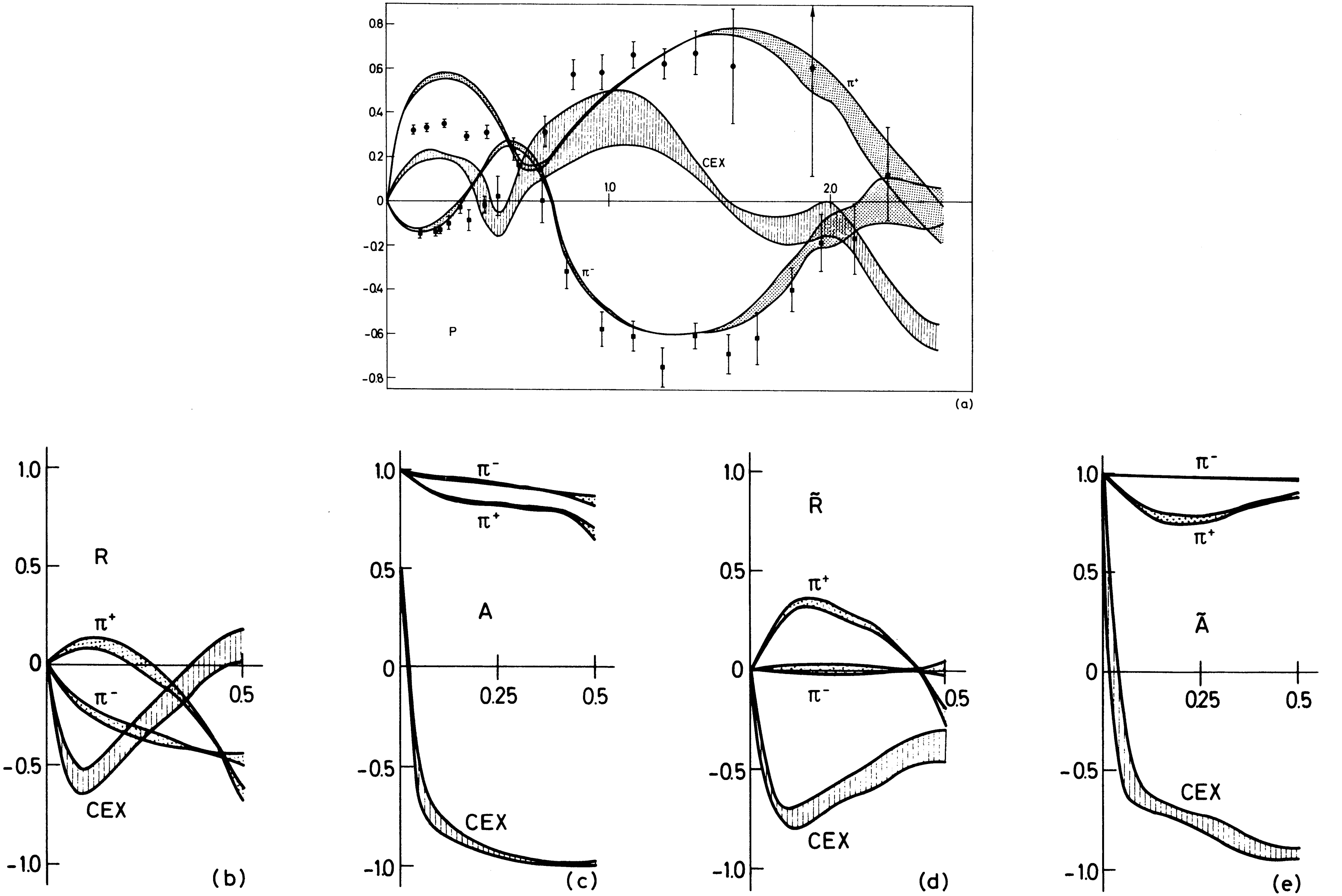
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
1.0
2.0
π⁺
CEX
π⁻
P
(a)
1.0
0.5
0
-0.5
-1.0
R
π⁺
π⁻
CEX
0.5
(b)
A
π⁻
π⁺
CEX
0.25
0.5
(c)
R̃
π⁺
π⁻
CEX
0.5
(d)
Ã
π⁻
π⁺
CEX
0.25
0.5
(e)

Fig. 1

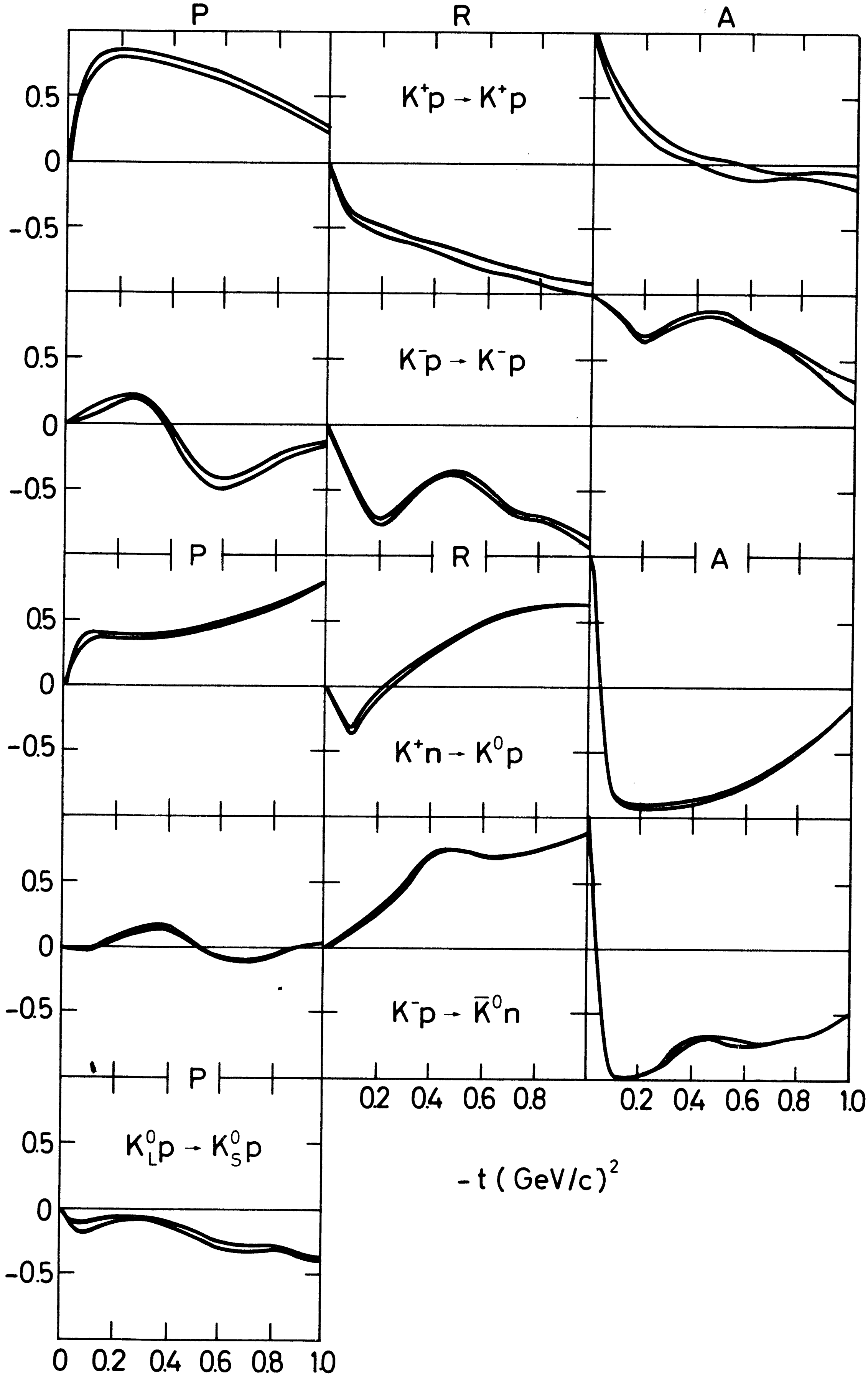
P
R
A
K⁺p → K⁺p
K⁻p → K⁻p
K⁺n → K⁰p
K⁻p → K̄⁰n
K⁰_L p → K⁰_S p
0.5
0
-0.5
0 0.2 0.4 0.6 0.8 1.0
-t (GeV/c)²

Fig. 2

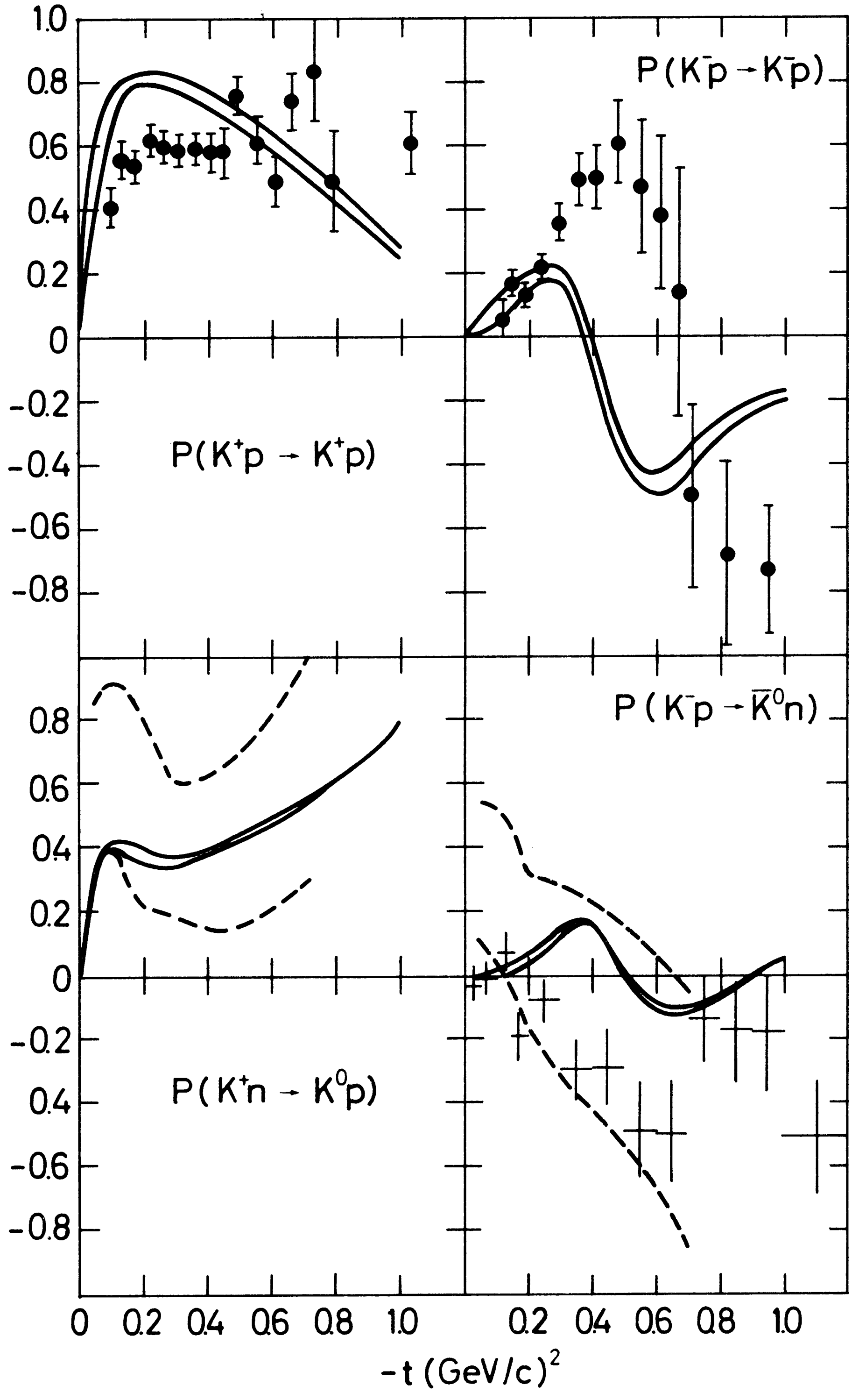

Fig. 3

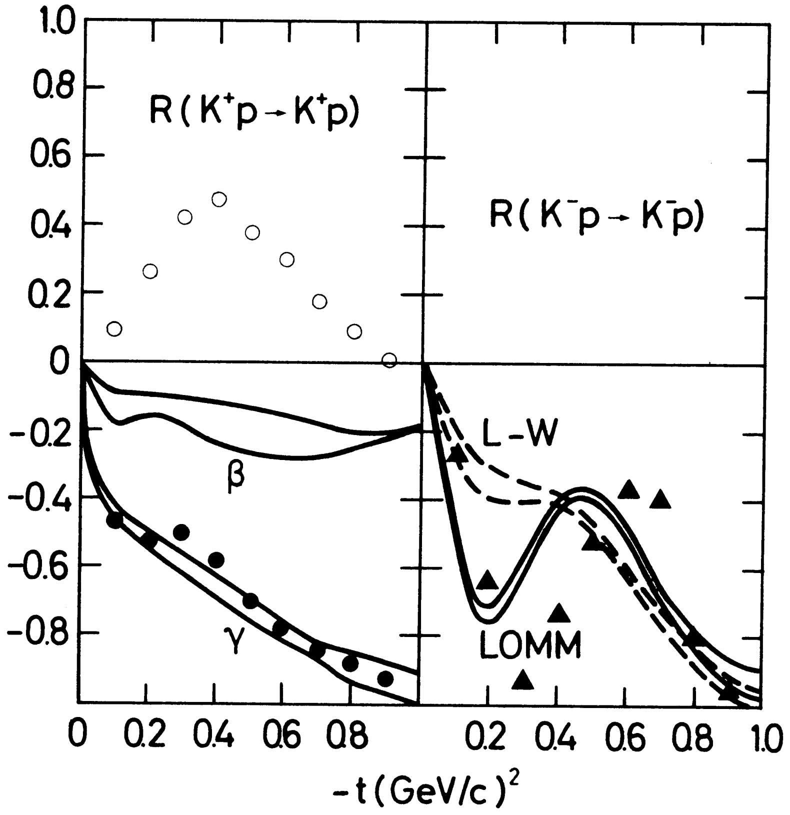
R(K⁺p → K⁺p)
R(K⁻p → K⁻p)
β
γ
L-W
LOMM
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0 0.2 0.4 0.6 0.8 0.2 0.4 0.6 0.8 1.0
-t (GeV/c)²

Fig. 4