ARCHIVES

Ref.TH.1811-CERN

RESONANCE AND BACKGROUND ADDITION WITH APPLICATION TO A POLE MODEL OF THE Δ (1220) RESONANCE *)

M.G. Olsson **)

CERN -- Geneva

ABSTRACT

A simple method of adding an elastic resonance and background consistent with unitarity and dispersion relation results is described. When applied to the (3,3) resonance case it is found that a pole model with σ term of about 40 MeV provides an excellent representation of the phase shift. The usual unitarization method is discussed and is shown to be inadequate.

---

*) Supported in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation and in part by the U.S. Atomic Energy Commission under contract AT (11-1)-881.

**) Permanent address Department of Physics University of Wisconsin, Madison, Wisconsin 53706.

Ref.TH.1811-CERN

1 February 1974

If an elastic resonance and a background of the same partial wave are present, care must be taken so that the total amplitude is unitary. The resonance alone can be written as [1],

$$\tan \delta_r = \frac{1}{\varepsilon} \qquad\qquad \varepsilon = \frac{M_r^2 - s}{M_r \Gamma(s)} \tag{1}$$

and the background alone is taken to be,

$$\tan \delta_B = B(s) \tag{2}$$

The customary [2] method for ensuring that the total amplitude is unitary is to add the phase shifts of the resonance and the background

$$\delta = \delta_r + \delta_B$$

$$\tan \delta = \frac{1 + \varepsilon B}{\varepsilon - B} \tag{3}$$

We see immediately that if the phase shift passes through 90° at $W=M_o$ and if the background B is positive, the resonance energy M_r will lie above M_o.

In a dispersion relation analysis Höhler, Jakob, and Strauss [3] found the Δ mass and $\Delta N\pi$ coupling constant to be

$$M_r \simeq 1219 \text{ MeV} \quad , \quad \lambda^* \equiv \frac{g^{*2}}{4\pi} \simeq .264\, m_{\pi^+}^{-2} \tag{4}$$

Since the (3,3) phase shift passes through 90° at $M_o \simeq 1230.5$ MeV [4] the resonance energy M_r falls below M_o. This analysis was carried out by comparing the pole model amplitude to a dispersion relation result using the experimental (3,3) phase shift but dropping the N exchange Born term. Since the N exchange contribution is positive, the 90° point M_o will be larger than M_r. The value of the coupling constant λ^*, although smaller than previous values [3] is the correct one to account for the crossing symmetric power expansion parameters [3] : Since M_r is less than M_o the phase shift addition method is inadequate. The procedure described below will give the resonance mass and coupling constant in substantial agreement with Eq. (4).

The total amplitude in the (3,3) partial wave will be considered to be a sum of a resonance and background. From Eqs. (1) and (2) we take this amplitude to be [5]

$$qf = \frac{1}{\varepsilon - i\gamma} + \frac{B}{1 - iB} \tag{5}$$

Following De Baenst et al. [6] the factor $\gamma(s)$ has been introduced to allow the amplitude (5) to satisfy the unitarity equation

$$(qf)^{-1} = \cot\delta - i \tag{6}$$

Equating the expressions (5) and (6) results (after some algebra) in

$$\gamma = \frac{1 + B^2 + 2\varepsilon B}{1 - B^2}$$

$$\tan\delta = \frac{1 + \varepsilon B}{\varepsilon + B} \tag{7}$$

This expression for $\tan\delta$ should be compared with the corresponding formula (3) resulting from adding phase shifts. The resonance energy will now lie below the $\delta = 90^\circ$ point $(M_r < M_o)$ for positive B as desired.

A pole model for elastic πN scattering consistent with current algebra and PCAC constraints has been previously presented [7]. The (3,3) partial wave projection of this model amplitude is

$$qf = \frac{1}{\varepsilon} + B(s) \tag{8}$$

where

$$\varepsilon = \frac{M_r^2 - s}{M_r \Gamma_\Delta(s)} \quad , \quad \Gamma_\Delta(s) = \lambda^* \frac{(E+M)(W+M_r)}{6s} q^3$$

The background term $B(s)$ comes primarily from nucleon exchange B_N with a possible σ term correction B_σ to be discussed later. The N exchange part is

$$B_N(s) = \left(\frac{g^2}{4\pi}\right)\frac{1}{2Wq}\left[(E+M)(W-M)Q_1(z) - (E-M)(W+M)Q_2(z)\right]$$
$$z = \frac{2\omega E - m_\pi^2}{2q^2}$$

In the static limit B_N reduces to the usual [8]

$$B_N^S = \frac{4}{3}\,\frac{f^2 q^3}{\omega_L m_\pi^2} \quad , \quad f^2 = \left(\frac{g^2}{4\pi}\right)\left(\frac{m_{\pi^+}}{2M}\right)^2 \qquad (9)$$

which differs from B_N by only 10 % in the resonance region.

If we unitarize the two terms of Eq. (8) separately we obtain the results of Eqs. (1) and (2). We now have two alternatives for the unitarization of the sum. The actual (3,3) phase shift can be fit well [4] by the Breit - Wigner form,

$$\tan\delta = \frac{M_0\Gamma(s)}{M_0^2 - s} \quad , \quad \Gamma(s) = \Gamma_0\left(\frac{q}{q_0}\right)^3\left(\frac{1+(Rq_0)^2}{1+(Rq)^2}\right) \qquad (10)$$

where $M_0 = 1230.6\ \mathrm{MeV}$, $\Gamma_0 = 111\ \mathrm{MeV}$

The parameters M_r and λ^* appearing in the pole model amplitude (8) can be determined by the conditions,

$$\delta = 90^\circ \quad \text{at} \quad W = M_0$$
$$\lim_{W\to M_0}(M_0^2 - s)\tan\delta = M_0\Gamma_0 \qquad (11)$$

For the two unitarization methods we find the pole parameters to be

1. Phase shift addition $M_r = 1245$ MeV, $\lambda^* = 0.442\ M_{\pi^+}^{-2}$
2. Present method $M_r = 1220.8$ MeV, $\lambda^* = 0.302\ M_{\pi^+}^{-2}$

The first method is inconsistent with the dispersion relation determined parameters (4). Now that the pole parameters M_r and λ^* have been fixed by the resonance position and phase shift slope at resonance we can use Eqs. (3) and (7) to test the momentum dependence of the phase shift for the two unitarization methods. It is seen from Fig. 1 that the predictions are similar despite the differing parameters. One difference is, however, worth noting. Phase shift addition gives a low energy phase shift in substantial agreement with the experimental data [4] while the proposed method predicts low energy phase shifts one or two degrees smaller. As we shall see the σ term contribution to $B(s)$ is positive and will improve the agreement of the proposed method while decreasing agreement with the addition method.

The remaining contributions to $B(s)$ are Δ exchange, ρ exchange, and σ exchange. Of these only σ exchange can amount to more than a few per cent of the N exchange part. In Schnitzer's model [9] the $\sigma\pi\pi$ vertex at low momentum transfer follows from unitarization of π-π scattering. The σ term contribution to the total πN elastic amplitude for the simplest chiral summetry breaking is [7),8)]

$$A^{(+)} = \frac{1}{2F_\pi^2}\left(\frac{2t}{m_\pi^2} - 1\right) g_\sigma(t) \quad , \quad A^{(-)} = B^{(+)} = B^{(-)} = 0$$

For small momentum transfers we take $g_\sigma(t) \simeq g_\sigma$ and the partial wave projection is

$$B_\sigma(s) \simeq .55 \left(\frac{q^3}{W}\right)\left(\frac{g_\sigma}{m_\pi}\right)$$

where g_σ is the σ term believed to be less than about 100 MeV. Using the proposed unitarization scheme and $g_\sigma = 40$ MeV we find

$$M_r = 1217.5 \text{ MeV} \quad , \quad \lambda^* = .30\, m_{\pi^+}^{-2}$$

The corresponding phase shifts at the two lowest energies are ($W = 1157$ MeV, $\delta = 19.9^\circ$, $\delta_{exp} = 19.8 \pm 0.1^\circ$) and ($W = 1173$ MeV, $\delta = 30.0^\circ$, $\delta_{exp} = 30.1 \pm 1^\circ$). We have seen that, using the unitarization method described a pole model can account for the (3,3) phase shift in the low energy region. This result is achieved with parameters similar to those derived from dispersive analyses [3] or fits to the scattering lengths or crossing symmetric expansion parameters [3),7)].

ACKNOWLEDGEMENTS

I would like to thank Dr. C. Michael for helpful discussions and for reading the manuscript.

REFERENCES

1) The kinematic quantities have their usual meaning; M_r resonance mass; M nucleon mass; m_π pion mass; q is the C.M. pion momentum; ω and ω_L are the C.M. and the laboratory incident pion energy respectively; $W = \sqrt{s}$ is the total C.M. energy; $E = W-\omega$. The $NN\pi$ coupling constant $g^2/4\pi = 14.6$.

2) R.H. Dalitz, Ann.Rev.Nucl.Sci. 13, 339 (1963);
C. Michael, Phys.Letters 21, 93 (1966);
C.J. Goebel and K.W. Mc Voy, Phys.Rev. 164, 1932 (1967).

3) G. Höhler, H.P. Jakob and R. Strauss, Nucl.Phys. B39, 237 (1972);
The definitions of M_r and λ^* are the same as used in this paper and in reference 7).

4) J.R. Carter, D.V. Bugg and A.A. Carter, Nucl.Phys. B58, 378 (1973);
Particle Data Group (1972), Phys.Letters 39B, 1 (1972).

5) The phase shift addition method can be established by taking $\gamma = 1$ and modifying the resonance part by a multiplicative phase factor.

6) A. De Baenst-Vandenbrouke and P. De Baenst, (1973 preprint) Mathematical Physics Department, University College Dublin, Belfield, Ireland. These authors followed a similar procedure to the present work except that the background amplitude was not initially unitarized. The result of this is to considerably complicate the formalism but does not change the result significantly.

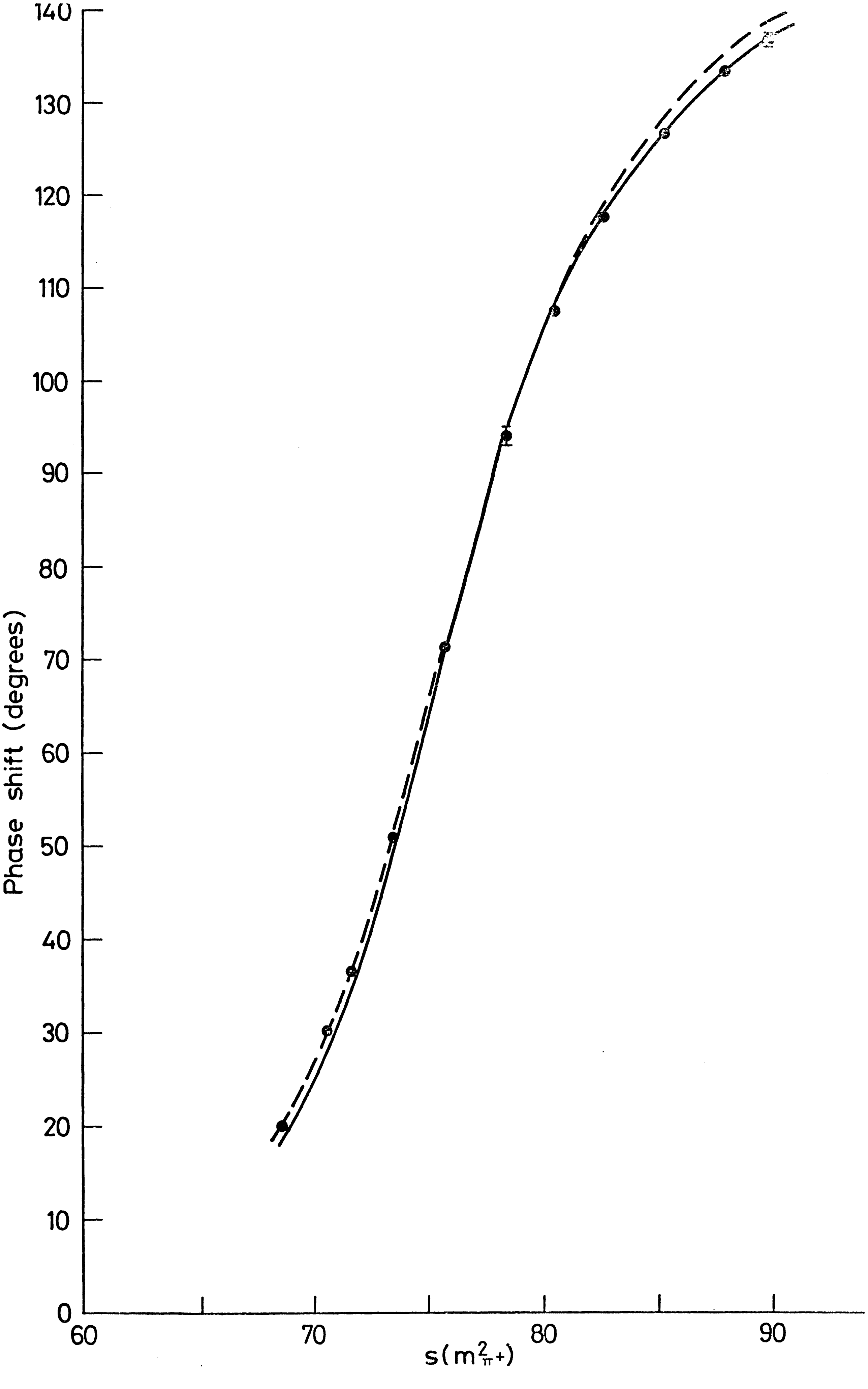
7) M.G. Olsson, Leaf Turner, and E.T. Osypowski, Phys.Rev. D1, 3444 (1973).

8) G.F. Chew, M.L. Goldberger, F.E. Low, and Y. Nambu, Phys.Rev. 106, 1337 (1957); J. Hamilton, High Energy Physics Vol 1, p.193, edited by E.H.S. Burhop, Academic Press Inc. (1967).

9) H.J. Schnitzer, Phys.Rev D5, 1482 (1972); Phys.Rev. D6, 1801 (1972).

Figure Caption

Phase shift for the (3,3) amplitude with experimental data from Ref. 4). The curves represent the momentum dependence predicted by the two unitarization methods given the resonance energy and slope at resonance. The background in each case is nucleon exchange only. The solid line is the proposed method and the dashed curve is the phase shift addition method.


Phase shift (degrees)
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
60
70
80
90
s(m$^2_{\pi^+}$)