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Soft-Gluon Resummation in Heavy Quarkonium Physics

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Abstract

Soft-gluon resummation within the framework of heavy quarkonium hadroproduction is considered. A few selected cases are studied in detail. A sizeable increase of the cross sections with respect to the next-to-leading order predictions with central factorization/renormalization scale choice can generally be observed. Improvements in the dependence of the cross sections on the two scales, especially when they are kept equal, are also found.

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1 Introduction

In the twilight zone between perturbative and non-perturbative QCD, there lives heavy quarkonium physics. Indeed, while the large mass ($m \gg \Lambda_{QCD}$) of the charm and bottom quarks allows for a perturbative evaluation of their production cross sections, still the physics of their binding into observable charmonium and bottomonium states unavoidably involves low-scale, and therefore non-perturbative, phenomena.

An understanding of a systematic way to disentangle short- and long-distance effects was reached by Bodwin, Braaten and Lepage in [1]. By making use of the Non-Relativistic QCD (NRQCD) [2] effective field theory, they were able to provide a framework in which heavy quarkonium calculations could be carried out to, in principle, arbitrary high order. Non-perturbative soft effects are factored into NRQCD matrix elements, and perturbative calculations can provide their coefficient functions. Heavy quarkonium (H) production cross sections therefore take the form:

$$d\sigma(H+X) = \sum_n d\hat{\sigma}(Q\overline{Q}[n] + X', \mu_\Lambda)\langle \mathcal{O}^H[n]\rangle^{(\mu_\Lambda)} \, . \tag{1.1}$$

In this expression $Q\overline{Q}[n]$ represents a heavy quark–antiquark pair in a given colour and spin/orbital/total angular momentum state. Notice that the quantum numbers n might differ from the ones of the observable quarkonium H, and at the same time $Q\overline{Q}$ might even be in a colour-octet state: soft gluons "hidden" into the non-perturbative matrix elements $\langle \mathcal{O}^H[n]\rangle$ take care of binding the pair and, at the same time, building up the correct quantum numbers. μ_Λ represents the NRQCD factorization scale, separating short- and long-distance effects. The relative importance of the various contributions in eq. (1.1) can be estimated by using NRQCD velocity scaling rules [3], which allow the truncation of the series at any given order of accuracy.

The coefficient functions $d\hat{\sigma}(Q\overline{Q}[n]+X)$ can be evaluated in perturbative QCD. Next-to-leading order (NLO) calculations for a wide range of processes in hadron–hadron collisions have been presented in [4]. Results for the partonic total cross sections will only depend on the scaling variable $x = 4m^2/s$, s being the partonic centre-of-mass energy.

It is apparent from the fully analytic results listed in [4] that, when $x \to 1$, i.e. when the partonic threshold is approached, large NLO contributions can develop and hence spoil the convergence of the perturbative series. This calls for an all-order resummation of such contributions, which is precisely the goal of this paper. We will work using techniques developed in [5, 6, 7, 8], and more recently applied to heavy-quark production processes in [9, 10].

In the following sections we shall first review the structure of the NLO results and the resummation formalism, and then present some numerical results.

2 Next-to-leading Order Results and Resummation

In ref. [4] results for the total production cross sections $\hat\sigma(ij \to Q\overline{Q}[n] + k)$ are given up to NLO. The colliding partons ij can be gluons, light quark and antiquark, or a quark and a gluon. Analogously, the outgoing parton k will be either a gluon or a light quark, and results have been given for the $Q\overline{Q}$ pair to be either an S-state (both scalar and vector, both colour singlet and octet) or a ^3P_J state.

For ease of reference let us write down here the cross sections for the processes that display the large soft-gluon behaviour.

The total cross section of the process $gg \to \mathcal{Q}g$, with $\mathcal{Q}$ representing the $Q\overline{Q}$ pair in a given state $n = {}^1S_0^{[1,8]}, {}^3P_0^{[1,8]}, {}^3P_2^{[1,8]}$, reads

$$\begin{aligned}
\sigma^H[gg \to \mathcal{Q}\,g] &= \sigma_0^H[gg \to \mathcal{Q}] \left(\delta(1-x) + \frac{\alpha_s(\mu)}{\pi} \Big\{ \mathrm{A_{tot}}[\mathcal{Q}]\,\delta(1-x) \right. \\
&+ \left[x\overline{P}_{gg}(x) \log\frac{4m^2}{x\mu_{\mathrm{F}}^2} + 2x(1-x)P_{gg}(x)\left(\frac{\log(1-x)}{1-x}\right)_+ \right. \\
&+ \left. \left. \left(\frac{1}{1-x}\right)_+ f_{gg}[\mathcal{Q}](x) \right] \Big\} \right) . \qquad (2.1)
\end{aligned}$$

In this equation $P_{gg}(x)$ and $\overline{P}_{gg}(x)$ are related to the gluon–gluon Altarelli–Parisi splitting vertex:

$$P_{gg}(x) = 2C_A \left[\frac{x}{1-x} + \frac{1-x}{x} + x(1-x) \right] \qquad (2.2)$$

$$\overline{P}_{gg}(x) = 2C_A \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] \qquad (2.3)$$

where the plus-distribution is defined by[1]

$$\int_0^1 dx\ [d(x)]_+\, t(x) = \int_0^1 dx\ d(x)\ [t(x) - t(1)]\ ; \qquad (2.4)$$

[1]Notice that in ref. [4] the results are instead written in terms of ρ-distributions defined by

$$\int_\rho^1 dx\ [d(x)]_\rho\, t(x) = \int_\rho^1 dx\ d(x)\ [t(x) - t(1)]\ ,$$

$\sigma_0^H[gg \to \mathcal{Q}]$ represents the Born cross section for the production of the quarkonium H via the intermediate state $\mathcal{Q}$, i.e. according to eq. (1.1),

$$\sigma_0^H[gg \to \mathcal{Q}] = \hat{\sigma}_0[gg \to \mathcal{Q}]\langle \mathcal{O}^H(\mathcal{Q})\rangle \ . \tag{2.5}$$

The constants $\mathrm{A_{tot}}[\mathcal{Q}]$ and the functions $f_{gg}[\mathcal{Q}](x)$ depend, as indicated, on the particular $Q\overline{Q}[n]$ state produced. They can be taken, together with $\hat{\sigma}_0[gg \to \mathcal{Q}]$, from the results published in [4]. Finally, μ and μ_F are respectively the renormalization and factorization scales. It is worth noticing that the NRQCD factorization scale μ_Λ does not explicitly appear, at this order, in the cross section for the production of the $Q\overline{Q}$ pair. It is therefore not indicated in the NRQCD matrix element either.

The presence of very large corrections near $x = 1$ is clearly visible in eq. (2.1), in the form of $(\log(1-x)/(1-x))_+$ and $(1/(1-x))_+$ distributions. Their form can be even more clearly appreciated by going to Mellin moments space. The Mellin transform is defined by

$$f_N \equiv \int_0^1 dx\, x^{N-1} f(x) \tag{2.6}$$

and, in the large-N limit (corresponding to the $x \to 1$ one), eq. (2.1) becomes

$$\begin{aligned}\sigma_N^H[gg \to \mathcal{Q}^{[c]}g] &= \sigma_{0,N}^H[gg \to \mathcal{Q}^{[c]}]\left(1 + \frac{\alpha_s(\mu)}{\pi}\left\{ \mathrm{A_{tot}}[\mathcal{Q}^{[c]}]\right.\right.\\ &+ 2C_A \log^2 N + 2C_A \log N \left(2\gamma_E - \log\frac{4m^2}{\mu_\mathrm{F}{}^2}\right)\\ &+ 2C_A\left(\gamma_E^2 + \frac{\pi^2}{6} - \gamma_E \log\frac{4m^2}{\mu_\mathrm{F}{}^2}\right)\\ &\left.\left.+ C_A\left(\gamma_E + \log N\right)\delta_{c8} + \mathcal{O}(1/N)\right\}\right) ,\end{aligned} \tag{2.7}$$

where $\gamma_E = 0.5772...$ is the Euler constant and the superscript $^{[c]}$ refers to the colour state (singlet or octet) of the $Q\overline{Q}$ pair $\mathcal{Q}$. In this form one can easily see the leading double log, due to soft *and* collinear radiation from the initial light partons, and the subleading single logs. The δ_{c8} in the last line indicates that this term is only present when $\mathcal{Q}$ is in a colour-octet state, since it is due to soft radiation from a coloured final state. It is worth noting that, aside for the constant terms in $\mathrm{A_{tot}}[\mathcal{Q}]$ and the colour of the final state, this structure does not depend on what exactly $\mathcal{Q}$ is.

with $\rho = 4m^2/S$ and S being the hadronic centre-of-mass energy. As a consequence, the constants $\mathrm{A_{tot}}[\mathcal{Q}]$ to be inserted in the NLO cross sections given here differ from the ones published in [4], missing now all the β-dependent terms ($\beta \equiv \sqrt{1-\rho}$).

Another process displaying large threshold corrections is $q\bar{q} \to {}^3S_1^{[8]} g$. The total cross section reads

$$\begin{aligned}
\sigma^H[q\bar{q} \to {}^3S_1^{[8]}\, g] &= \sigma_0^H[q\bar{q} \to {}^3S_1^{[8]}] \left(\delta(1-x) + \frac{\alpha_s}{\pi} \left\{ \mathrm{A}_{\rm tot}[{}^3S_1^{[8]}]\delta(1-x) \right.\right. \\
&+ \left[x\overline{P}_{qq}(x) \log\frac{4m^2}{x\mu_{\rm F}^2} + C_F x(1-x) + 2x(1-x)P_{qq}(x) \left(\frac{\log(1-x)}{1-x}\right)_+ \right] \\
&+ \left.\left. \left(\frac{1}{1-x}\right)_+ f_{q\bar{q}}[{}^3S_1^{[8]}](x) \right\}\right) , \qquad (2.8)
\end{aligned}$$

where

$$P_{qq}(x) = C_F \left(\frac{1+x^2}{1-x}\right) \qquad \text{and} \qquad \overline{P}_{qq}(x) = C_F \frac{1+x^2}{(1-x)_+} . \qquad (2.9)$$

and, as before, $\sigma_0^H[q\bar{q} \to {}^3S_1^{[8]}]$, $\mathrm{A}_{\rm tot}[{}^3S_1^{[8]}]$ and $f_{q\bar{q}}[{}^3S_1^{[8]}](x)$ can be found in [4]. This becomes, in Mellin moments space,

$$\begin{aligned}
\sigma_N^H[q\bar{q} \to {}^3S_1^{[8]} g] &= \sigma_{0,N}^H[q\bar{q} \to {}^3S_1^{[8]}] \left(1 + \frac{\alpha_s(\mu)}{\pi} \left\{ \mathrm{A}_{\rm tot}[{}^3S_1^{[8]}] \right.\right. \\
&+ 2C_F \log^2 N + 2C_F \log N \left(2\gamma_E - \log\frac{4m^2}{{\mu_{\rm F}}^2}\right) \\
&+ 2C_F \left(\gamma_E^2 + \frac{\pi^2}{6} - \gamma_E \log\frac{4m^2}{{\mu_{\rm F}}^2}\right) \\
&+ \left.\left. C_A\left(\gamma_E + \log N\right) + \mathcal{O}(1/N) \right\}\right) . \qquad (2.10)
\end{aligned}$$

Comparing this equation to eq. (2.7) we can see that the structure is identical, with the replacement $C_A \to C_F$ for radiation coming from initial-state quarks rather than gluons. This cross section can also be seen to be free of an explicit μ_Λ dependence.

Resummation techniques for these large threshold logarithms have been developed in [5, 6, 7, 8] and more recently applied to heavy quark production [9, 10]. Soft gluon resummation for processes also involving non-perturbative matrix elements, namely radiative and semileptonic B decays within Heavy Quark Effective Theory, have also been considered in [11, 12].

Upon inspection, one can see that the soft limits of the amplitudes projected onto some specific $Q\overline{Q}[n]$ state are identical, up to this order, to the ones for open heavy quark production. Such soft limits, derived in [4], read, for the gluon–gluon channel and for colour-singlet and -octet production respectively,

$$\sum_{\rm col,pol} \left|\overline{A^{[1]}_{\rm soft}}\right|^2 = 4\pi\alpha_s C_A \frac{2ab}{(ak)(bk)} \sum_{\rm col,pol} \left|\overline{A^{[1]}_{\rm Born}}\right|^2 , \qquad (2.11)$$

$$\sum_{\text{col,pol}} \left|\overline{A^{[8]}_{\text{soft}}}\right|^2 = 4\pi\alpha_s C_A \left[\frac{2ab}{(ak)(bk)} - \frac{2ab}{(Pk)^2}\right] \sum_{\text{col,pol}} \left|\overline{A^{[8]}_{\text{Born}}}\right|^2 \,, \tag{2.12}$$

where a, b, P, k are, respectively, the momenta of the incoming gluons, the outgoing heavy $Q\overline{Q}$ pair, and the emitted soft gluon. A similar expression holds for ${}^3S_1^{[8]}$ production in $q\overline{q}$ collisions, where we find

$$\sum_{\text{col,pol}} \left|\overline{A^{[8]}_{\text{soft}}}\right|^2 = 4\pi\alpha_s \left[C_F \frac{2q\bar{q}}{(qk)(\bar{q}k)} - C_A \frac{2q\bar{q}}{(Pk)^2}\right] \sum_{\text{col,pol}} \left|\overline{A^{[8]}_{\text{Born}}}\right|^2 \,, \tag{2.13}$$

q and $\bar{q}$ being the momenta of the two incoming light quarks.

Making use of these soft matrix elements one can easily reproduce the large-N limits given above. Moreover, one can then achieve next-to-leading log (NLL) resummation for heavy quarkonium production processes by means of the following formula:

$$\hat{\sigma}^{res}_N[ij \to \mathcal{Q}^{[c]} g] = \hat{\sigma}_{0,N}[ij \to \mathcal{Q}^{[c]}] \left(1 + \frac{\alpha_s(\mu)}{\pi} C_{ij}[\mathcal{Q}^{[c]}]\right) \Delta_{ij,c,\,N+1}\left(\alpha_s(\mu), \mu, \mu_{\text{F}}\right) \tag{2.14}$$

the $C_{ij}[\mathcal{Q}^{[c]}]$ being the constant, N-independent terms readable from eqs. (2.7) and (2.10):

$$C_{ij}[\mathcal{Q}^{[c]}] = \text{A}_{\text{tot}}[\mathcal{Q}^{[c]}] + 2 \begin{pmatrix} C_A \\ C_F \end{pmatrix} \left(\gamma_E^2 + \frac{\pi^2}{6} - \gamma_E \log \frac{4m^2}{{\mu_{\text{F}}}^2}\right) + C_A\,\gamma_E\;\delta_{c8} \,, \tag{2.15}$$

where the coefficient of the second term is either C_A or C_F for $ij = gg$ or $q\overline{q}$ respectively, and the last term is only present when a colour octet state is produced.

The resummation function $\Delta_{ij,c,\,N}$ up to NLL accuracy reads

$$\Delta_{ij,c,\,N}\left(\alpha_s(\mu), \mu, \mu_{\text{F}}\right) \;=\; \exp\left\{\ln N \; g^{(1)}_{ij}(b_0\,\alpha_s(\mu)\ln N) + g^{(2)}_{ij,c}(b_0\,\alpha_s(\mu)\ln N, \mu, \mu_{\text{F}})\right\} \tag{2.16}$$

The functions $g^{(1)}$ and $g^{(2)}$ resum the LL and NLL terms, respectively. Their explicit form is [9, 10]

$$g^{(1)}_{q\bar{q}}(\lambda) = \frac{C_F}{\pi b_0 \lambda}\left[2\lambda + (1-2\lambda)\ln(1-2\lambda)\right] \,, \quad g^{(1)}_{gg}(\lambda) = \frac{C_A}{C_F}\, g^{(1)}_{q\bar{q}}(\lambda) \,, \tag{2.17}$$

and

$$\begin{aligned} g^{(2)}_{q\bar{q},1}(\lambda, \mu, \mu_{\text{F}}) \;&=\; -\gamma_E \frac{2C_F}{\pi b_0}\ln(1-2\lambda) + \frac{C_F b_1}{\pi b_0^3}\left[2\lambda + \ln(1-2\lambda) + \frac{1}{2}\ln^2(1-2\lambda)\right] \\ &-\; \frac{C_F K}{2\pi^2 b_0^2}\left[2\lambda + \ln(1-2\lambda)\right] - \frac{C_F}{\pi b_0}\ln(1-2\lambda)\ln\frac{{\mu_{\text{F}}}^2}{4m^2} \end{aligned}$$

$$
\begin{aligned}
& - \; \frac{C_F}{\pi b_0}\,[\ln(1-2\lambda)+2\lambda]\ln\frac{\mu^2}{{\mu_{\rm F}}^2}\;, \\
g^{(2)}_{gg,1}(\lambda,\mu,\mu_{\rm F}) \;&=\; \frac{C_A}{C_F}\, g^{(2)}_{q\bar{q},1}(\lambda,\mu,\mu_{\rm F})\;, \\
g^{(2)}_{ij,8}(\lambda,\mu,\mu_{\rm F}) \;&=\; g^{(2)}_{ij,1}(\lambda,\mu,\mu_{\rm F}) - \frac{C_A}{2\pi b_0}\ln(1-2\lambda)\;,
\end{aligned}
\tag{2.18}
$$

where b_0, b_1 are the first two coefficients of the QCD β-function[2]:

$$
b_0 = \frac{11C_A - 4T_F n_f}{12\pi}\;, \quad b_1 = \frac{17C_A^2 - 10C_A T_F n_f - 6C_F T_F n_f}{24\pi^2}\;, \tag{2.19}
$$

and K is given by

$$
K = \left(\frac{67}{18} - \frac{\pi^2}{6}\right) C_A - \frac{10}{9} T_F n_f\;. \tag{2.20}
$$

It is easy to see that expanding these formulas up to order α_s, and using eq. (2.14), one recovers the fixed-order results given in eqs. (2.7) and (2.10).

Finally, observable hadron-level cross sections will be obtained by convoluting eq. (2.14) with hadronic parton distribution functions and multiplying by the proper NRQCD non-perturbative matrix element. We shall therefore have

$$
\sigma_N^{H,res}[\mathcal{Q}] = F_{i,N+1}(\mu_{\rm F})\, F_{j,N+1}(\mu_{\rm F})\, \hat{\sigma}_N^{res}[ij \to \mathcal{Q}\, g]\, \langle \mathcal{O}^H(\mathcal{Q})\rangle \tag{2.21}
$$

and an improved hadronic cross section, including the full NLO result plus NLL resummation, will be obtained as

$$
\sigma^{H,\,{\rm NLO+NLL}}[\mathcal{Q}] = \sigma^{H,res}[\mathcal{Q}] - (\sigma^{H,res}[\mathcal{Q}])_{\alpha_s^3} + \sigma^{H,\,{\rm NLO}}[\mathcal{Q}]\;, \tag{2.22}
$$

where $\sigma^{H,\,{\rm NLO}}[\mathcal{Q}]$ is the full NLO result as calculated in [4] and $(\sigma^{res}[ij \to \mathcal{Q}\, g])_{\alpha_s^3}$ is the order-α_s^3 truncation of the resummed result, here subtracted to avoid double-counting.

3 Numerical Results

Phenomenological cross sections are obtained via inverse Mellin transform of eq. (2.14), to be performed numerically. Many problems are to be found at this stage, both of conceptual and technical nature.

[2]It is worth mentioning the factor of 2π difference between this definition of b_0 and the one employed in ref. [4].

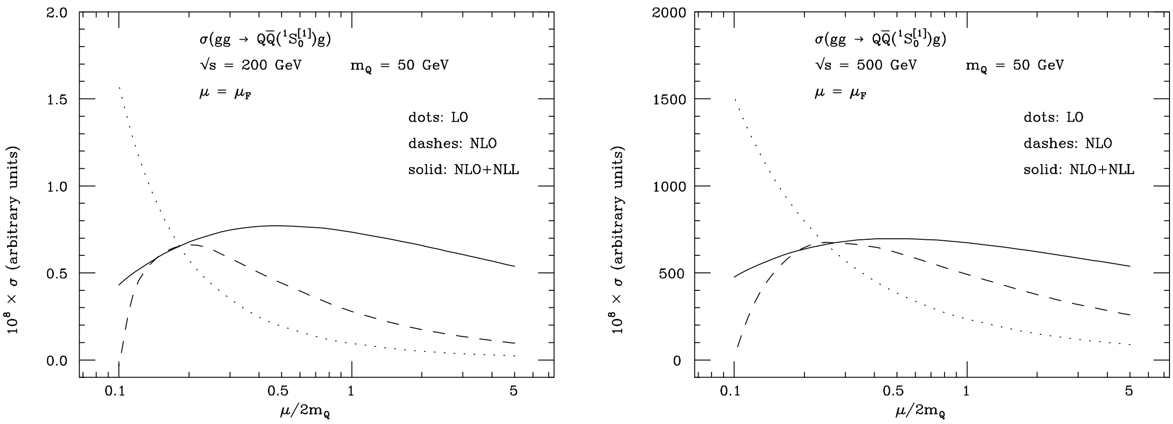

Figure 1: The cross section for production, in pp collisions via the gluon-gluon channel, of a fictitious $^1S_0^{[1]}$ quarkonium state made up of 50 GeV quarks, at 200 and 500 GeV centre-of-mass energy.

For instance, to take care of the presence of the Landau pole we shall adopt the so-called Minimal Prescription introduced in [9]: the integration contour for the inverse Mellin transform passes to the right of all singularities in the complex N-plane, but to the left of the Landau pole at $N = N_L \equiv \exp(1/2b_0\alpha_s)$.

Technical problems are instead related to the difficulty of performing the required integrals, due to the resummation function $\Delta(x)$ being strongly oscillating in x-space close to $x = 1$. The are taken care of by a subtraction procedure similar to the one described in Appendix B of ref. [9].

In the following we shall present some plots showing the effect of the soft-gluon resummation described in the previous section. Hadron level results are obtained with the CTEQ3M parton distribution function (PDF) set, unless otherwise stated. NRQCD matrix elements for $Q\overline{Q}$ level final states (i.e. not real physical quarkonium states) are fixed, for S-states, at the value number of polarization states $\times$ number of colours of the $Q\overline{Q}$ pair. If replaced with the real values, in units of GeV^3 for S-states and GeV^5 for P-states, the cross sections would be in nanobarns.

We shall first study the renormalization/factorization scale dependence of a fictitious heavy quarkonium made up of 50 GeV quarks. Charmonium and bottomonium are very close to the non-perturbative region, and the large value of the strong coupling at such low scales makes the results less readily readable.

In fig. 1 resummation effects are shown at 200 and 500 GeV centre-of-mass energies, as a

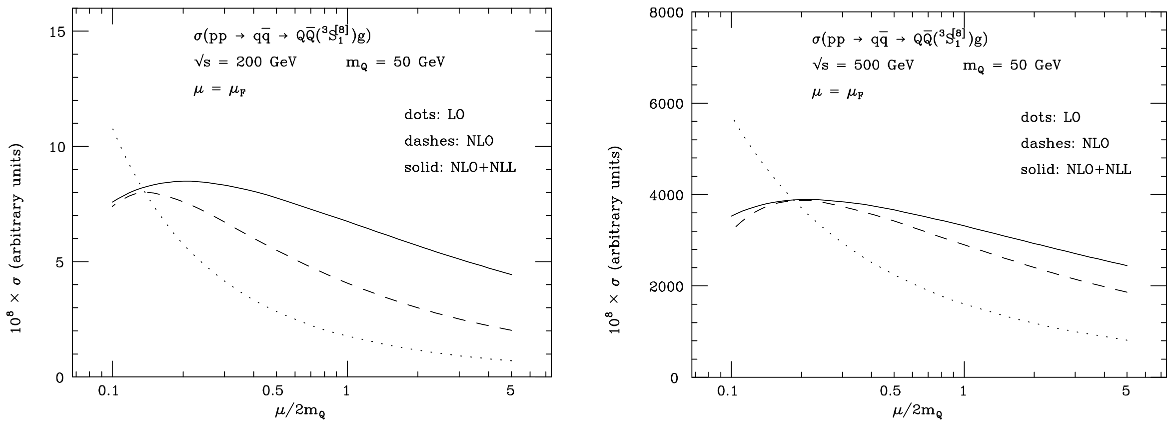

Figure 2: The cross section for production, in pp collisions via the $q\bar{q}$ channel, of a ${}^3S_1^{[8]}$ state with 50 GeV quarks.

function of the renormalization/factorization scale μ. Production of a ${}^1S_0^{[1]}$ state in proton–proton collisions via the gluon–gluon channel is considered here. While the leading order result displays the usual monotonical dependence, and the NLO one presents a maximum at pretty small values of μ, the resummed result can be seen to be markedly less dependent on the arbitrary scale. One can also see resummation effects to be larger closer to the threshold, at $\sqrt{S} = 200$ GeV, than at 500 GeV, as expected. In both cases a sizeable increase with respect to the NLO prediction with $\mu = 2m_Q$ can be observed.

Results for other states produced via the gluon–gluon channel, i.e. ${}^3P_0$ and ${}^3P_2$, both in the colour singlet and octet states, and ${}^1S_0^{[8]}$, appear qualitatively similar. These plots suggest that a small value of the renormalization/factorization scale should be chosen in the NLO calculation in order to obtain a more realistic prediction for the cross section. We can also speculate that the value $\mu = m_Q$, rather than $2m_Q$, should be chosen as the central one for the renormalization and factorization scale, despite the $\log(4m^2/\mu^2)$ terms appearing in the NLO calculation. A variation in the range $[m_Q/2, 2m_Q]$ would then still span a large fraction of the NLO band and give a pretty small uncertainty with the NLO+NLL result.

Qualitatively similar results are found in the $q\bar{q}$ channel for production of a ${}^3S_1^{[8]}$ state. Figure 2 shows the dependence of the cross section on the renormalization/factorization scale at $\sqrt{S} = 200$ and 500 GeV. Again, the NLO+NLL result can be seen to be more stable than the fixed-order NLO one.

One can, of course, also try to vary the renormalization scale μ and the factorization scale $\mu_{\rm F}$ independently. According to the observation made above on the "best" scales

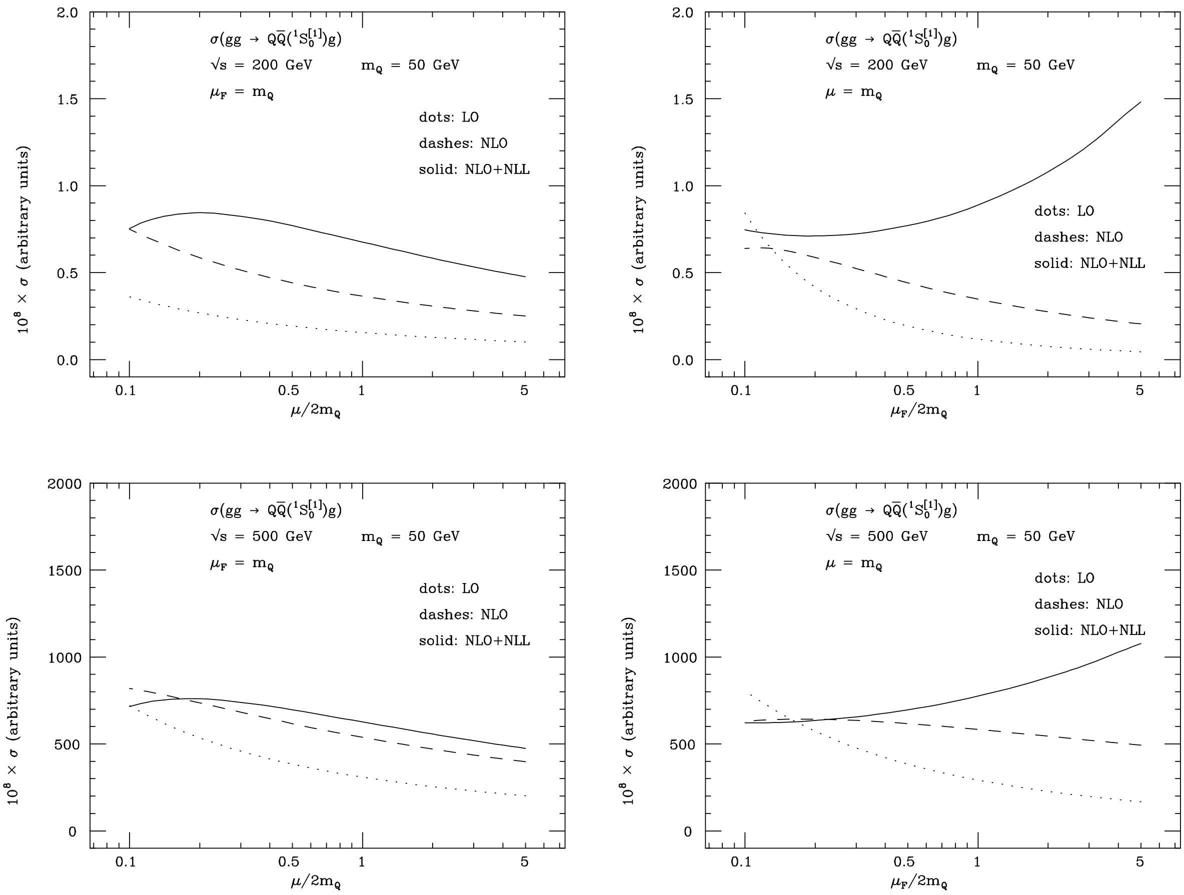

Figure 3: Cross section as in fig. 1, but with independent variation of the renormalization (left plot) and factorization (right plot) scales. Top plots are for $\sqrt{S} = 200$ GeV, bottom ones for $\sqrt{S} = 500$ GeV.

central value, we now fix one of the scales at m_Q (rather than $2m_Q$) and we vary the other in the range $[2m_Q/10, 5 \times 2m_Q]$. We can see in fig. 3 that the remarkable independence seen in fig. 1 is unfortunately at least partially lost, especially when going to large factorization scales. Independent scale variations in the $q\bar{q}$ channel give a similar outcome. These results are qualitatively similar to those reported in [13], where soft-gluon resummation in prompt-photon hadroproduction is studied, indicating that they are not specific to bound states production. They are moreover identical to what can be obtained by making m_Q even as large as the top quark mass, i.e. 175 GeV. This suggests that, when studying the overall dependence on the renormalization and factorization scales, one should always take care to vary them independently.

We now move to studying real quarkonium states, i.e. made up of charm or bottom

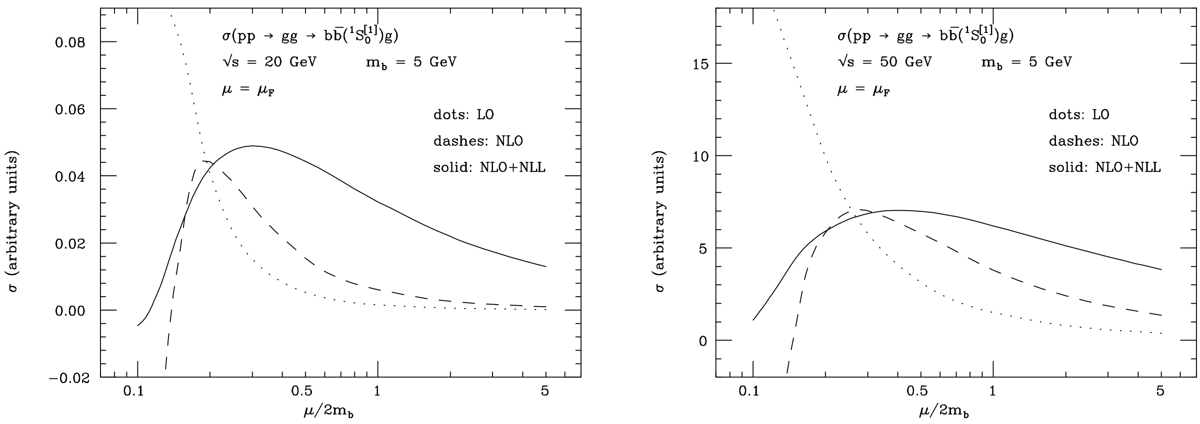

Figure 4: The cross sections for production of the bottomonium state ${}^1S_0^{[1]}$ via the gluon–gluon channel, at 20 and 50 GeV centre-of-mass energy, as functions of the renormalization/factorization scales.

quarks. In these cases, and especially for charmonium, the not-so-large mass of the heavy quark, and hence the relatively large value of the running coupling at these scales, will produce less clear-cut results than the ones previously displayed.

Figure 4 is a scaled-down version of fig. 1, meaning that both the mass of the heavy quark and the centre-of-mass energy are scaled down by a factor of 10, to yield $m_b = 5$ GeV and $\sqrt{S} = 20$ and 50 GeV. We can see that, while the behaviour with respect to scale variations is the same, even after resumming the soft-gluon logs the cross section does however retain a larger dependence than in the $m_Q = 50$ GeV case, as expected. As before, the NLO+NLL result predicts larger cross sections than the NLO one with a central scale choice.

Once again, qualitatively similar results are found for ${}^3P_0$ and ${}^3P_2$, both in the colour singlet and octet states, and for ${}^1S_0^{[8]}$.

Production of a ${}^3S_1^{[8]}$ bottomonium state in $q\bar{q}$ interactions can also be considered. Figure 5 shows the dependence of the cross section on the renormalization/factorization scale at $\sqrt{S} = 20$ and 50 GeV. At 50 GeV centre-of-mass energy the NLO+NLL result can be seen to be slightly more stable than the fixed-order NLO one, confirming the trend observed in gg collisions. Less is instead to be gained closer to the threshold, at $\sqrt{S} = 20$ GeV.

One can finally also try studying charmonium hadroproduction. Figure 6 shows the renormalization/factorization scale dependence for production of ${}^1S_0^{[1]}$ and ${}^3S_1^{[8]}$ charm bound states in pp collisions at 20 GeV centre-of-mass energy. The parton distribution function set CTEQ4lQ (i.e. low Q^2, valid down to $Q = 0.7$ GeV) is used, since very small scales are

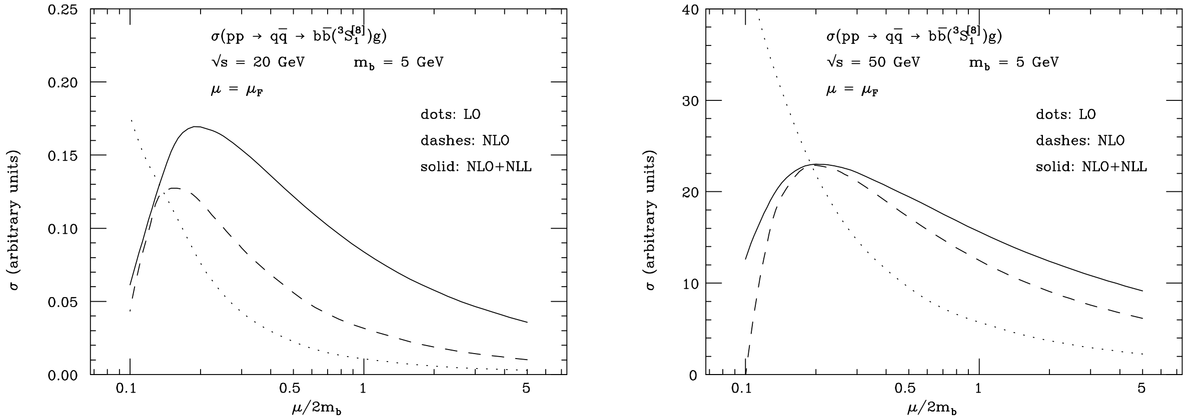

Figure 5: The cross sections for production, in pp collisions via the $q\bar{q}$ channel, of a ${}^3S_1^{[8]}$ bottomonium state.

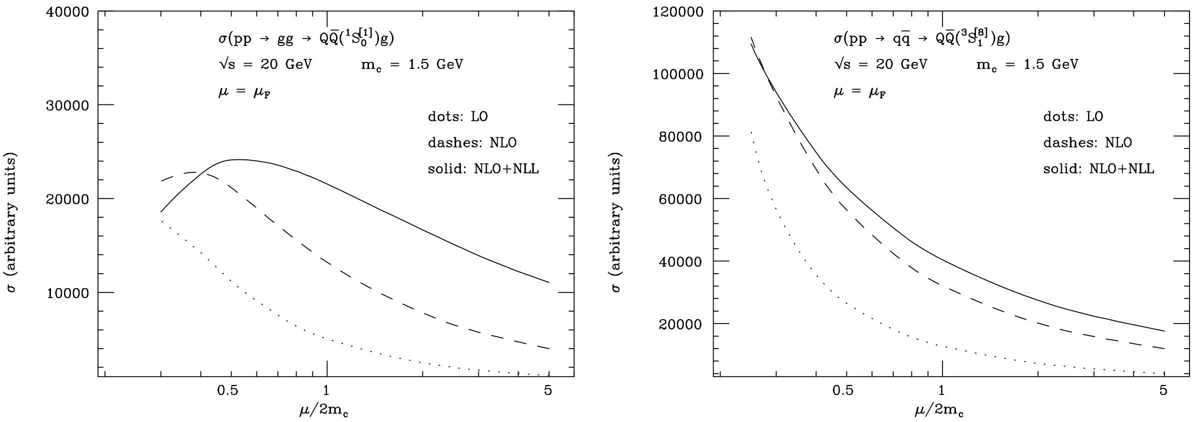

Figure 6: The cross section for production, in pp collisions, of ${}^1S_0^{[1]}$ and ${}^3S_1^{[8]}$ charm bound states. The PDF set is CTEQ4lQ.

probed. Moreover, the scales are not varied below $\mu \sim 0.25 \times 2m_c \sim 0.75$ GeV, already at the borderline of both perturbative QCD and the validity range of the PDF set.

One can see from the plots that the dependence on the renormalization and factorization scales is once more lessened when soft gluon resummation is included. However, the improvement is less than in the bottomonium case, as expected, the scale involved here being extremely close to the non-perturbative region

4 Open Questions

This paper only provides a first step towards a full understanding of soft-gluon effects in heavy quarkonium physics, as a number of questions remain to be addressed.

The NLO cross sections we have studied here do not present the need for an explicit subtraction of infrared singularities to be absorbed into the non-perturbative NRQCD matrix elements. Such singularities are however expected to appear in higher orders, as they do for instance in the two-loop evaluation of J/ψ decay into leptons [14]. The resummation function Δ will then have to be modified accordingly, so as to reflect the presence of this new factorization scale μ_Λ, in the same way it contains the PDF's factorization scale μ_F. This will avoid double counting of contributions already included somewhere else. It is also conceivable that the interplay of this subtraction with the one of Coulomb terms, which are also absorbed, already at the one-loop level, into the NRQCD matrix elements, will have to be carefully considered.

In ref. [4], where the fixed order NLO cross sections were evaluated, the need for an explicit subtraction of infrared singularities, and hence the appearance of the NRQCD factorization scale μ_Λ, is however in one case already met at order α_s^3. This happens when studying the production of a P-wave state via the $q\overline{q}$ channel. When the emitted gluon becomes soft, the ensuing singularity can only be cancelled by adding the corresponding $q\overline{q} \to S$-state process, in full accordance with the general NRQCD factorization formula (1.1) (see also section 6 of ref. [4] for more details). In this case we find indeed that the soft limit of the $q\overline{q} \to P$-state $+g$ process amplitude does not factorize onto $q\overline{q} \to P$-state and no gluons, this cross section being actually zero, but rather onto the matrix element squared for $q\overline{q} \to S$-state. This implies a different framework for achieving soft-gluon resummation, which will conceivably be closely interlaced with the structure of the NRQCD matrix elements.

5 Conclusions

In this paper we have studied the effect of resumming soft gluons in some selected heavy quarkonium hadroproduction processes. The inclusion of resummation effects leads in many cases to sizeably larger cross sections than those given by the fixed order NLO result with central renormalization/factorization scales. Moreover, improvements in the dependence of the cross sections on the renormalization and factorization scales can be observed, especially when they are kept equal. We have also pointed out that less marked improvements appear when we vary them independently, leading to an uncertainty band a little larger than what would be otherwise obtained.

A full systematic analysis of soft-gluon resummation in heavy quarkonium processes, with phenomenologically relevant results, is in progress, as is the one of the open questions briefly described in the previous section.

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