An Investigation of the Soft Pion Relation in Quenched Lattice QCD

UKQCD Collaboration
K.C. Bowler[a], L. Del Debbio[b], J.M. Flynn[b], L. Lellouch[c], V.I. Lesk[b*], J. Nieves[d], C.M. Maynard[a] and D.G. Richards[a†].

[a]Department of Physics and Astronomy, University of Edinburgh EH9 3JZ, UK

[b]Department of Physics and Astronomy, University of Southampton, SO17 1BJ, UK

[c]Theory Division, CERN, CH 1211, Geneva 23, Switzerland

[d]Departamento de Fisica Moderna, Universidad de Granada, Granada 18071, Spain

A lattice determination of the form factor and decay constants for the semileptonic decay of heavy pseudoscalar (PS) mesons at zero recoil is presented from which the soft pion relation is satisfied. Chiral extrapolation of the form factor is performed at constant q^2. Pole dominance is used to extrapolate the form factor in heavy quark mass. At the B mass, the form factor at zero recoil lies somewhat below the ratio of decay constants; the relation remains satisfied within error.

1. Introduction

Heavy-quark and chiral symmetries combine to predict the following relation for a heavy-light meson H the mass of whose heavy quark is larger than some arbitrary hadronic scale, and for a soft pion:

$$f_0(q^2_{\rm max})_{H\to\pi} = f_H/f_\pi \qquad (1)$$

where $f_0(q^2)$ is the form factor in the 0^+ channel for the vector current hadronic matrix element $\langle H(p)|V^\mu|\pi(p-q)\rangle$. The two quantities on the right-hand side are the corresponding PS meson decay constants. Equation 1 is the soft pion relation for heavy-light PS mesons [?].

Results are given for $f_0(q^2_{\rm max})_{H\to\pi}$ and f_H/f_π for heavy-light PS mesons and are extrapolated to the B mass. Errors are quoted as

$$f_B = 190(5)_{\rm stat.}(10)_{\rm sys.}{\rm MeV}. \qquad (2)$$

2. Extracting the form factors

The form factors for the hadronic matrix element of semileptonic heavy pseudoscalar decay to a pion are extracted from mass extrapolations of the appropriate matrix elements of quenched lattice QCD with SW action [?] with a^{-1}=2.6(1) GeV, β=6.2 on a $24^3\times 48$ lattice using 216 gauge configurations generated with a combination of the Cabbibo-Marinari algorithm [?] and an over-relaxed algorithm [?]. The matrix elements are calculated using the method of extended quark propagators with all combinations of the following hopping parameters:

$$\begin{aligned}\kappa_{\rm heavy} &\in \{0.1200, 0.1233, 0.1266, 0.1299\}\\ \kappa_{\rm light} &\in \{0.1346, 0.1351, 0.1353\}\\ \kappa_{\rm spectator} &\in \{0.1346, 0.1351\}.\end{aligned}$$

From the light PS meson masses $\kappa_{\rm crit}$=0.13582(1)(2) is found.

The action and operators are improved to O(a) and renormalized to a continuum scheme, using coefficients determined non-perturbatively where possible [?, ?]. The following values are used:

Z_V	=	0.792	Z_A	=	0.807
b_V	=	1.41	b_A	=	1.11
c_V	=	-1.58×10^{-2}	c_A	=	-3.71×10^{-2}
c_{SW}	=	1.61	b_M	=	0.583

Of these, only c_V, b_A and b_M are determined from a perturbative scheme [?].

*Presenter
†Jefferson Laboratory, Newport News, VA 23606, USA

In the continuum, the form factors f_+ and f_0 are defined from hadronic matrix elements as follows:
$\langle H(p)|V^\mu|\pi(p-q)\rangle =$
$f_+(q^2)\left(2\,p^\mu - \left[\left(M_H^2 - M_\pi^2\right)/q^2 + 1\right] q^\mu\right) +$
$f_0(q^2)\left(\left[M_H^2 - M_\pi^2\right]/q^2\right)\ q^\mu$

3. Pion physics on the lattice

Lattice correlators in this study are directly relevant to the physics of heavy-light mesons with mass 1300 − 2200 MeV, and of light-light mesons with mass 350 − 850 MeV. Form factors at the same value of q^2 for decays to a pion can be estimated by assuming the following dependence of a form factor $f \in \{f_+, f_0\}$ on the meson masses:

$$f(q^2, M, m) = f(q^2, M_{\rm chiral}, 0) + a \times \Delta M + b \times m^2 \quad (3)$$

and fitting the data to determine $f(0)$ and a. Similarly light-light meson and heavy-light meson decay contants f_L and f_H are extrapolated in the meson mass according to the following:

$$f_L(m) = f_L(0) + a' \times m^2 \quad (4)$$

$$f_H(M) = f_H(M_{\rm chiral}) + a'' \times \Delta M \quad (5)$$

where $\Delta M \equiv M - M_{\rm chiral}$.

The q^2 dependence of the form factors can be modelled using the following parameterization [?].

$$f_+(q^2) = \frac{c_B(1-\alpha)}{(1-q^2/M_*^2)(1-\alpha q^2/M_*^2)} \quad (6)$$

$$f_0(q^2) = \frac{c_B(1-\alpha)}{(1-q^2/\beta M_*^2)} \quad (7)$$

where M_* is the heavy-light vector meson mass. $f_0(q^2_{\rm max})$ is extracted by extrapolating f_0 upwards in q^2 using the best fit curve from a fit of 7 (fig. 1). One finds the soft pion relation to be well satisfied at simulated values of heavy quark mass.

4. The soft pion relation for $B \to \pi$

Heavy quark effective theory predicts the following form for the dependence of f_0 on the heavy quark mass, at constant recoil variable $v \cdot (p-q)$:

$$f_0(M,\omega)\Theta(M)\sqrt{M} = a + b/M + c/M^2 + O(\frac{1}{M^3}) \quad (8)$$

and for a heavy-light pseudoscalar decay constant:

$$f_H(M)\Theta(M)\sqrt{M} = a' + b'/M + c'/M^2 + O(\frac{1}{M^3}) \quad (9)$$

where $\Theta(M)$ is a perturbative matching coefficient function whose value is very close to 1. These prescriptions are used as fit ansatz to determine a best fit quadratic in 1/M to the data in table 1. The curve is shown in (fig. 2). Values for $f_0(q^2_{\rm max})$ and f_H extrapolated to the b mass are presented in table 2. It is instructive to repeat

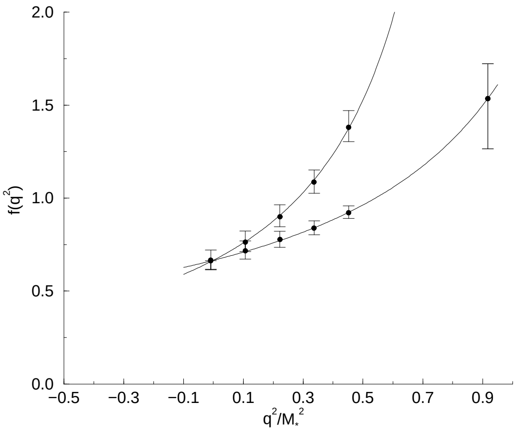

Figure 1. Example fit to $f_+, f_0(q^2)$ for decay to a pion, $\kappa_{\rm heavy} = 0.1200$.

Table 1
The soft pion relation for a massless pion.

$\kappa_{\rm heavy}$	$f_0(q^2_{\rm max})$	f_{PS}/f_π
0.1200	1.5(2)(3)	1.57(6)(9)
0.1233	1.5(2)(2)	1.53(6)(9)
0.1266	1.5(2)(2)	1.48(6)(9)
0.1299	1.4(1)(1)	1.41(5)(9)

Table 2
The soft pion relation for B decay to a massless pion.

	$f_0(q^2_{\rm max})$	f_B/f_π
M_B	1.1(2)(2)	1.45(6)(9)

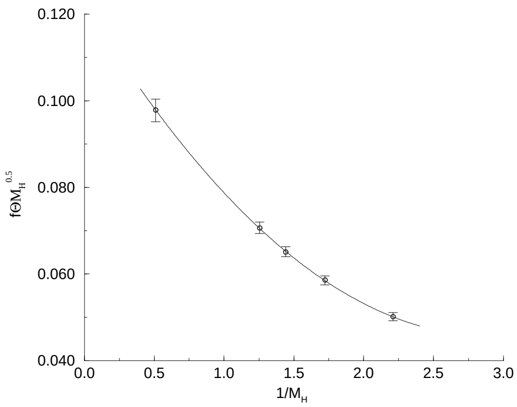

Figure 2. Best fit f_B vs. heavy mass according to eqn. 9.

the whole procedure, extrapolating form factors for a pion which is not massless but has its physical mass. Results are presented in table 3 and are practically unchanged from the massless case.

Table 3
The soft pion relation for B decay to a 140 MeV pion.

	$f_0(q^2_{\rm max})$	f_B/f_π
M_B	1.1(2)(2)	1.44(6)(9)

5. Systematic error

Systematic error in this work arises from:

1. discretization errors of O(a^2)
2. quenched approximation
3. estimation of correlation matrix for fits
4. interpolation of lattice form factors in q^2
5. corrections to heavy quark effective theory
6. choice in setting the scale.

The quoted systematic error is an estimate of the variation arising from the last sources $3-6$, achieved by repeating the analysis with different numbers of bootstrap sets, varying the model function used to interpolate the form factor in q^2, changing the degree of the polynomial in 1/M used as an ansatz for the extrapolation in heavy mass, and setting the scale alternatively against M_ρ and the gluonic scale r_0. Quenching error is not quantified here. Residual discretization errors also may be significant, particularly in the heavy extrapolation where the fitted curve used may be diverted substantially by a correction to the lattice form factor at the largest simulated heavy quark mass, for which $(Ma)^2 \simeq 0.65$.

6. Conclusions

For a simulated heavy-light decay whose mass is within 20% of M_D, the soft pion relation is reproduced in this study. On extrapolating the form factor and the heavy-light decay constant to the B mass, systematic errors become O(30%), to within which precision the soft pion relation holds for B meson decays. Other approaches to date [**?**, **?**] have generally violated this relation, with the form factor lying significantly below the ratio of decay constants. Work is ongoing with the prospect of controlling further the systematic error, and a simulation at $\beta = 6.0$ including a static heavy quark will address a large source of systematic uncertainty as well as providing scaling information [**?**].