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Constraints on $\tan\beta$ in the MSSM from the Upper Bound on the Mass of the Lightest Higgs boson

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Abstract

We investigate the possibilities for constraining $\tan\beta$ within the MSSM by combining the theoretical result for the upper bound on the lightest Higgs-boson mass as a function of $\tan\beta$ with the informations from the direct experimental search for this particle. We discuss the commonly used "benchmark" scenario, in which the parameter values $m_t = 175$ GeV and $M_{\rm SUSY} = 1$ TeV are chosen, and analyze in detail the effects of varying the other SUSY parameters. We furthermore study the impact of the new diagrammatic two-loop result for m_h, which leads to an increase of the upper bound on m_h by several GeV, on present and future constraints on $\tan\beta$. We suggest a slight generalization of the "benchmark" scenario, such that the scenario contains the maximal possible values for $m_h(\tan\beta)$ within the MSSM for fixed m_t and $M_{\rm SUSY}$. The implications of allowing values for m_t, $M_{\rm SUSY}$ beyond the "benchmark" scenario are also discussed.

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1 Theoretical basis

Within the MSSM the masses of the $\mathcal{CP}$-even neutral Higgs bosons are calculable in terms of the other MSSM parameters. The mass of the lightest Higgs boson, m_h, has been of particular interest: one-loop calculations [1,2] have been supplemented in the last years with the leading two-loop corrections, performed in the renormalization group (RG) approach [3–6], in the effective potential approach [7, 8] and most recently in the Feynman-diagrammatic (FD) approach [9,10]. These calculations predict an upper bound on m_h of about $m_h \lesssim 135$ GeV.

For the numerical evaluations in this draft we made use of the Fortran code *subhpole*, corresponding to the RG calculation [5], and of the program *FeynHiggs* [11], corresponding to the recent result of our FD calculation.

In order to fix our notations, we list the conventions for the input from the scalar top sector of the MSSM: the mass matrix in the basis of the current eigenstates $\tilde{t}_L$ and $\tilde{t}_R$ is given by

$$\mathcal{M}^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m_t^2 + \cos 2\beta \, (\frac{1}{2} - \frac{2}{3} s_W^2) M_Z^2 & m_t X_t \\ m_t X_t & M^2_{\tilde{t}_R} + m_t^2 + \frac{2}{3} \cos 2\beta \, s_W^2 M_Z^2 \end{pmatrix}, \tag{1}$$

where

$$m_t X_t = m_t (A_t - \mu \cot \beta \,) \; . \tag{2}$$

For the numerical evaluation, a common choice is

$$M_{\tilde{t}_L} = M_{\tilde{t}_R} =: M_{\mathrm{SUSY}}; \tag{3}$$

this has been shown to yield upper values for m_h which comprise also the case where $M_{\tilde{t}_L} \neq M_{\tilde{t}_R}$, when M_{SUSY} is identified with the heavier one [10]. We furthermore use the short-hand notation

$$M_S^2 := M_{\mathrm{SUSY}}^2 + m_t^2 \; . \tag{4}$$

While the case $X_t = 0$ is labelled as 'no-mixing', it is customary to to assign 'maximal-mixing' to the value of X_t for which the the mass of the lightest Higgs boson is maximal. As can be seen in Fig. 1, where m_h is shown as a function of X_t/M_{SUSY} within the FD and the RG approach, the 'maximal-mixing' case corresponds to $X_t \approx 2\, M_{\mathrm{SUSY}}$ in the FD approach, while it corresponds to $X_t = \sqrt{6}\, M_{\mathrm{SUSY}} \approx 2.4\, M_{\mathrm{SUSY}}$ in the RG approach. It should be noted in this context that, due to the different renormalization schemes utilized in the FD and the RG approach, the (scheme-dependent) parameters X_t and M_{SUSY} have a different meaning in the two approaches, which has to be taken into account when comparing the corresponding results. While the resulting shift in M_{SUSY} turns out to be small, sizable differences occur between the numerical values of X_t in the two schemes, see Refs. [10, 12].

The main differences between the RG and the FD calculation have been investigated in Ref. [13]. They arise at the two-loop level. The dominant two-loop contribution of $\mathcal{O}(\alpha\alpha_s)$

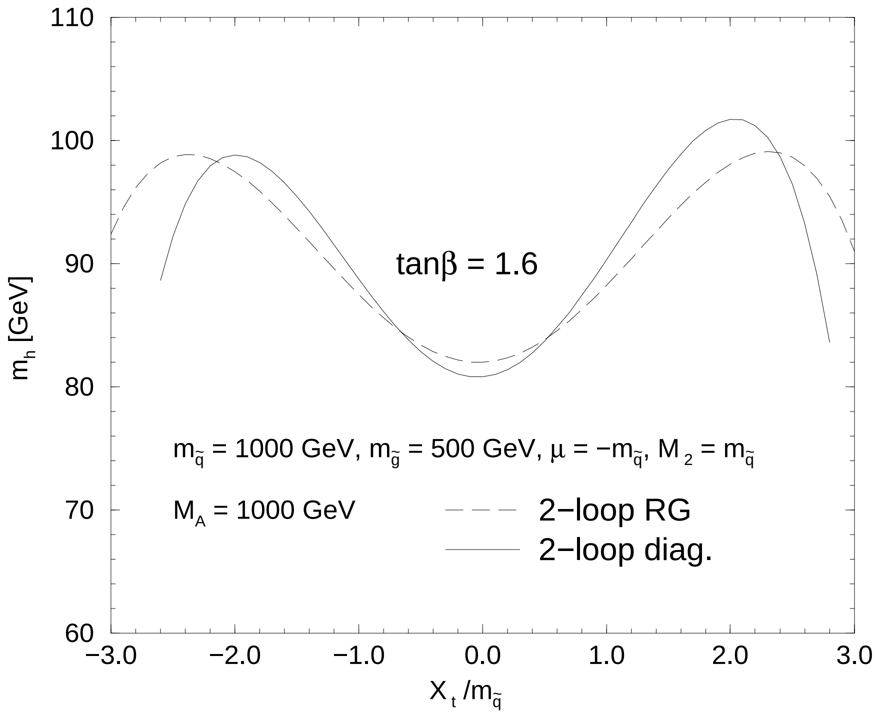

Figure 1: m_h *is shown as a function of* $X_t/m_{\tilde{q}}$ *for* $\tan\beta = 1.6$ *evaluated in the Feynman-diagrammatic (program* FeynHiggs*) and in the renormalization group (program* subhpole*) approach, where* $m_{\tilde{q}} \equiv M_{\rm SUSY}$*. The maximal value of* m_h *is obtained for* $X_t \approx 2\, m_{\tilde{q}}$ *in the FD approach and* $X_t \approx 2.4\, m_{\tilde{q}}$ *in the RG approach.*

to m_h^2 in the FD approach reads:

$$\Delta m_h^{2,\alpha\alpha_s} = \Delta m_{h,\log}^{2,\alpha\alpha_s} + \Delta m_{h,\rm non-log}^{2,\alpha\alpha_s} \ , \tag{5}$$

$$\Delta m_{h,\log}^{2,\alpha\alpha_s} = -\frac{G_F\sqrt{2}}{\pi^2}\frac{\alpha_s}{\pi}\,\overline{m}_t^4\left[3\log^2\left(\frac{\overline{m}_t^2}{M_S^2}\right) + 2\log\left(\frac{\overline{m}_t^2}{M_S^2}\right) - 3\frac{X_t^2}{M_S^2}\log\left(\frac{\overline{m}_t^2}{M_S^2}\right)\right] , \tag{6}$$

$$\Delta m_{h,\rm non-log}^{2,\alpha\alpha_s} = -\frac{G_F\sqrt{2}}{\pi^2}\frac{\alpha_s}{\pi}\,\overline{m}_t^4\left[4 - 6\frac{X_t}{M_S} - 8\frac{X_t^2}{M_S^2} + \frac{17}{12}\frac{X_t^4}{M_S^4}\right] ; \tag{7}$$

therein $\overline{m}_t$ denotes the running top-quark mass

$$\overline{m}_t \equiv \overline{m}_t(m_t) \approx \frac{m_t}{1 + \frac{4}{3\pi}\alpha_s(m_t)} \ . \tag{8}$$

By transforming the FD result into the $\overline{\rm MS}$ scheme, it has been shown analytically that the RG and the FD approach agree in the logarithmic terms [12]. The non-logarithmic terms $\Delta m_{h,\rm non-log}^{2,\alpha\alpha_s}$, however, are genuine two-loop terms, which are only present in the FD result [12]. In the maximal-mixing scenario, these terms can enhance the lightest Higgs-boson mass by up to 5 GeV.

The new two-loop terms obtained within the FD approach lead to a reduction of the theoretical uncertainty of the Higgs-mass prediction due to unknown higher-order corrections (see Ref. [12] for a discussion). Another source of theoretical uncertainty is related to the experimental errors of the input parameters, such as m_t. In the case of the SUSY parameters, direct experimental information is lacking completely. For this reason it is convenient to discuss specific scenarios, where certain values of the parameters are assumed.

2 The benchmark scenario

In recent years it has become customary to discuss the restrictions on $\tan\beta$ from the search for the lightest Higgs boson within the so-called "benchmark" scenario, which is specified by the parameter values

$$m_t = 175 \text{ GeV}, \qquad M_{\text{SUSY}} = 1 \text{ TeV}, \tag{9}$$

where M_{SUSY} denotes the common soft SUSY breaking scale for all sfermions (see e.g. Refs. [14–18] for recent analyses within this framework). According to Refs. [14, 18–20], the other SUSY parameters within the benchmark scenario are chosen as

$$\begin{aligned} \mu &= -100 \text{ GeV} \\ M_2 &= 1630 \text{ GeV} \\ M_A &\leq 500 \text{ GeV} \\ A_t &= 0 \quad (\text{"no mixing"}) \\ A_t &= \sqrt{6} M_{\text{SUSY}} \quad (\text{"maximal mixing"}), \end{aligned} \tag{10}$$

where μ is the Higgs mixing parameter, M_2 denotes the soft SUSY breaking parameter in the gaugino sector, and M_A is the $\mathcal{CP}$-odd Higgs-boson mass. The maximal possible Higgs-boson mass as a function of $\tan\beta$ within this scenario is obtained for $A_t = \sqrt{6}\, M_{\text{SUSY}}$ and $M_A = 500$ GeV. Exclusion limits on $\tan\beta$ within this scenario follow by combining the information from the theoretical upper bound in the $\tan\beta$–m_h plane with the direct search results for the lightest Higgs boson.

The tree-level value for m_h within the MSSM is determined by M_A, $\tan\beta$ and the Z-boson mass M_Z. Beyond the tree-level, the main correction to m_h stems from the t–$\tilde{t}$-sector. Thus, the most important parameters for the corrections to m_h are m_t, M_{SUSY} and X_t.

Since the benchmark scenario relies on specifying the two parameters $m_t = 175$ GeV and $M_{\text{SUSY}} = 1$ TeV, it is of interest to investigate whether the other inputs in the benchmark scenario are allowed to vary in such a way that the maximal possible value for m_h, once m_t and M_{SUSY} are fixed, is contained in this scenario. This is however not the case:

- Compared to the "benchmark" value of $M_2 = 1630$ GeV, the value of m_h is enhanced by about 2.5 GeV (depending slightly on the value of $\tan\beta$) by choosing a small value for M_2, e.g. $M_2 = 100$ GeV (see Ref. [10], where a scan over the MSSM parameter space has been performed showing that the maximal values for m_h are obtained for small values of M_2 and μ).

- While in the benchmark scenario only M_A values up to 500 GeV are considered, higher M_A values lead to an increase of m_h. For $M_A = 1000$ GeV, m_h is enhanced by up to 1.5 GeV.

- While within the benchmark scenario "maximal mixing" is defined as

$$A_t = X_t + \mu \cot\beta \; = \sqrt{6}\, M_{\rm SUSY} \;, \tag{11}$$

  the maximal Higgs-boson masses are in fact obtained (in the RG approach) for

$$X_t = \sqrt{6}\, M_{\rm SUSY} \quad ({\rm RG}) \;. \tag{12}$$

  This changes m_h by $\mathcal{O}(300$ MeV$)$ for $\tan\beta = \mathcal{O}(1.6)$ and $\mu = -100$ GeV. As mentioned above, in the FD calculation one has to take

$$X_t = 2\, M_{\rm SUSY} \quad ({\rm FD}) \tag{13}$$

  for maximal mixing.

- In the benchmark scenario, according to the implementation in the HZHA event generator [19], the running top-quark mass has been defined by including corrections up to $\mathcal{O}(\alpha_s^2)$. Compared to the definition (8), which includes only corrections up to $\mathcal{O}(\alpha_s)$, this leads to a reduction of the running top-quark mass by about 2 GeV. From the point of view of a perturbative calculation up to $\mathcal{O}(\alpha\alpha_s)$ it is however not clear whether corrections of $\mathcal{O}(\alpha_s^2)$ in the running top-quark mass, which is inserted into an expression of $\mathcal{O}(\alpha)$, will in fact lead to an improved result. On the contrary, as a matter of consistency of the perturbative evaluation it appears to be even favorable to restrict the running top-quark mass to its $\mathcal{O}(\alpha_s)$ expression (8). Adopting this more conservative choice leads to an increase of m_h by up to 1.5 GeV.

All four effects shift the Higgs-boson mass to higher values. For the analyses below we will use the current experimental value for the top-quark mass, $m_t = 174.3$ GeV [21], i.e. we consider the benchmark scenario with $m_t = 174.3$ GeV and $M_{\rm SUSY} = 1$ TeV. Two of the effects discussed above are displayed in Fig. 2, where also the maximal values for m_h according to the discussion above ($m_h^{\rm max}$-scenario: $M_2 = 100$ GeV, $M_A = 1000$ GeV, $X_t = \sqrt{6}\, M_{\rm SUSY}$ (RG), $X_t = 2\, M_{\rm SUSY}$ (FD), $\overline{m}_t$ as defined in eq. (8)) obtained in the RG approach with $m_t = 174.3$ GeV and $M_{\rm SUSY} = 1$ TeV are displayed. Comparing the $m_h^{\rm max}$-scenario with the benchmark scenario, the values for m_h are higher by about 5 GeV.

So far we have only discussed the increase in the maximal value of the Higgs-boson mass which is obtained using the slight generalization of the benchmark scenario discussed above. Now we also take into account the impact of the new FD two-loop result for m_h, which contains previously unknown non-logarithmic two-loop terms. The corresponding result in the $\tan\beta$–m_h plane (program *FeynHiggs*) is shown in Fig. 3 in comparison with the benchmark scenario and the $m_h^{\rm max}$-RG scenario (program *subhpole*). The maximal value for

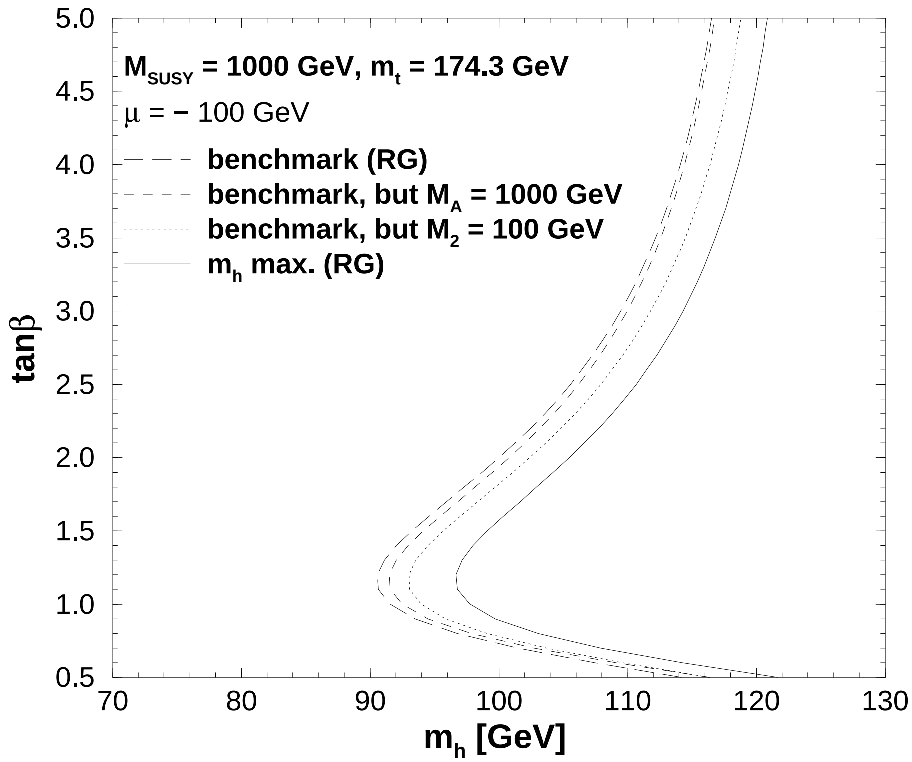

Figure 2: *m_h is shown as a function of* $\tan\beta$*, evaluated in the RG approach. The left (long-dashed) curve displays the benchmark scenario. For the dotted (dashed) curves one deviation from the benchmark scenario,* $M_2 = 100$ GeV *(*$M_A = 1000$ GeV*), is taken into account. The solid curve displays the maximal possible* m_h *value for* $m_t = 174.3$ GeV *and* $M_{\rm SUSY} = 1$ TeV.

m_h within the FD result is higher by up to 4 GeV compared to the $m_h^{\rm max}$-RG scenario and by up to 9 GeV compared to the benchmark scenario.

The increase in the maximal value for m_h by about 4 GeV from the new FD result and by further 5 GeV if the benchmark scenario is slightly generalized has a significant effect on exclusion limits for $\tan\beta$ derived from the Higgs-boson search. Employing the benchmark scenario and the RG result, an excluded $\tan\beta$ range already appears for an experimental bound on m_h of slightly above 90 GeV, see Fig. 2. However, taking into account the above sources for an increase in the maximal value for m_h the current data from the Higgs-boson search hardly allow any $\tan\beta$ exclusion yet, see Fig. 3. Concerning the assumed m_h limit obtained at the end of LEP2, the accessible $\tan\beta$ region is largely reduced from the $m_h^{\rm max}$-RG to the $m_h^{\rm max}$-FD calculation.

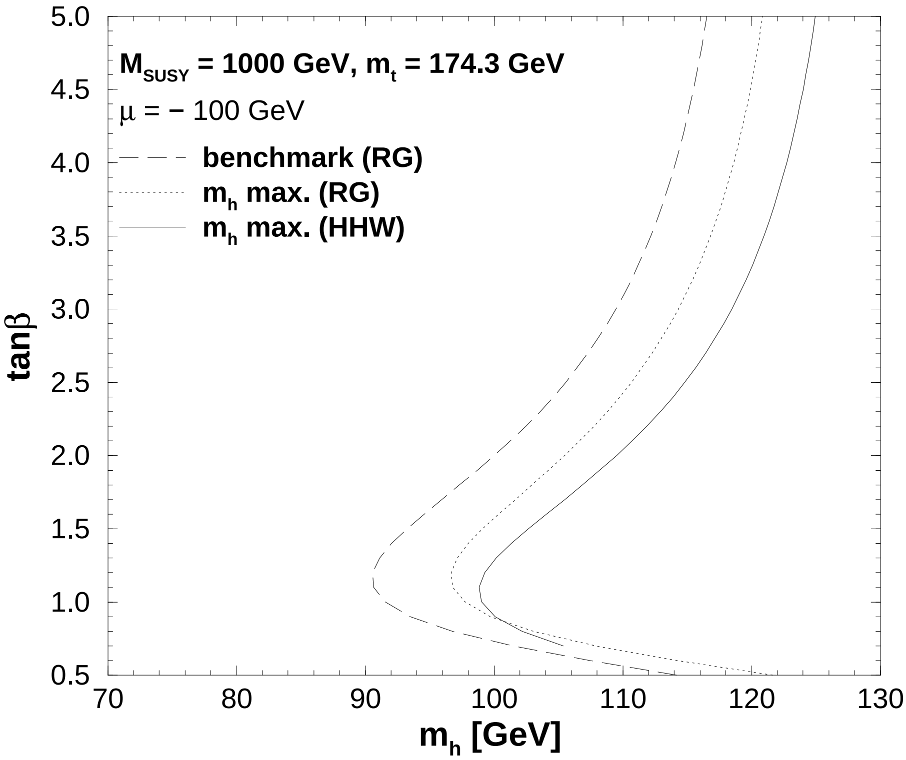

Figure 3: *m_h is shown as a function of* $\tan\beta$. *The dashed curve displays the benchmark scenario. The dotted curve shows the $m_h^{\rm max}$-RG scenario (program* subhpole*), while the solid curve represents the $m_h^{\rm max}$-FD scenario (HHW, program* FeynHiggs*).*

3 Constraints on $\tan\beta$ "beyond the benchmark"

Since the dominant radiative corrections to the lightest Higgs-boson mass are proportional to m_t^4, the theoretical prediction for m_h depends sensitively on the precise value of the top-quark mass. The experimental uncertainty in the top-quark mass of currently $\Delta m_t = 5.1$ GeV [21] thus has a strong effect on the prediction for the upper bound on m_h, where larger values of m_t give rise to larger values of m_h. An increase in m_t by $\Delta m_t = 5.1$ GeV leads to an increase in m_h of up to 6 GeV, as shown in Fig. 4, where also the effect of increasing m_t by two standard deviations is displayed.

Besides the top-quark mass, the other main entry of the benchmark scenario is the choice $M_{\rm SUSY} = 1$ TeV. Similarly to the case of m_t, allowing for higher values of $M_{\rm SUSY}$ leads to higher values of m_h. Since $M_{\rm SUSY}$ enters only logarithmically in the prediction for m_h, the dependence on it is more moderate. An increase of $M_{\rm SUSY}$ from 1 TeV to 2 TeV enhances m_h by up to 4 GeV.

Allowing values of m_t one or even two standard deviations above the current experimental central value and increasing also the input value of $M_{\rm SUSY}$ clearly has a large effect on

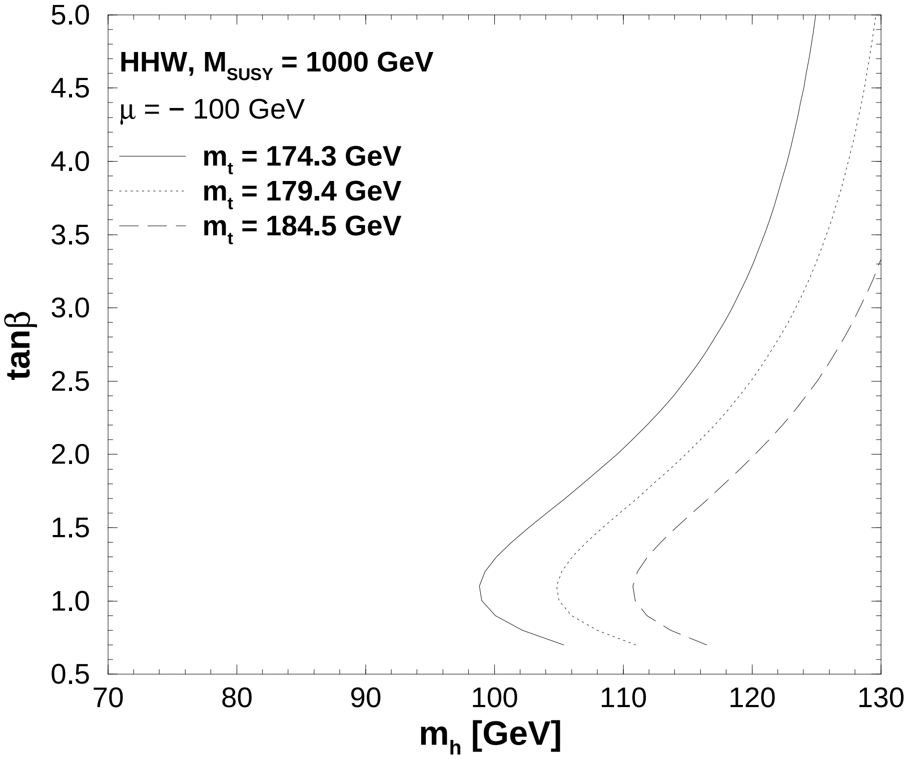

Figure 4: *m_h is shown as a function of* $\tan\beta$*, evaluated in the FD approach. We give the results for three different values of the top-quark mass,* $m_t = 174.3, 179.4, 184.5$ GeV*.*

possible $\tan\beta$ constraints. In Fig. 5 we show a "worst case" scenario, where $m_t = 184.5$ GeV has been chosen, i.e. two standard deviations above the current experimental value, and $M_{\rm SUSY} = 2000$ GeV is taken. It is compared with the benchmark scenario in the RG calculation and with the $m_h^{\rm max}$-FD scenario. In the "worst case" scenario exclusion of a $\tan\beta$ range would become possible only with a limit on m_h of more than about 115 GeV.

In this context one should keep in mind that the benchmark scenario contains not only an assumption about the SUSY parameters but also about the actual model which is tested, namely a SUSY model with a minimal Higgs sector that does not contain $\mathcal{CP}$-violating phases. Extensions of the Higgs sector by additional particle representations can shift the upper bound on the mass of the lightest Higgs boson up to values of about 200 GeV [22].

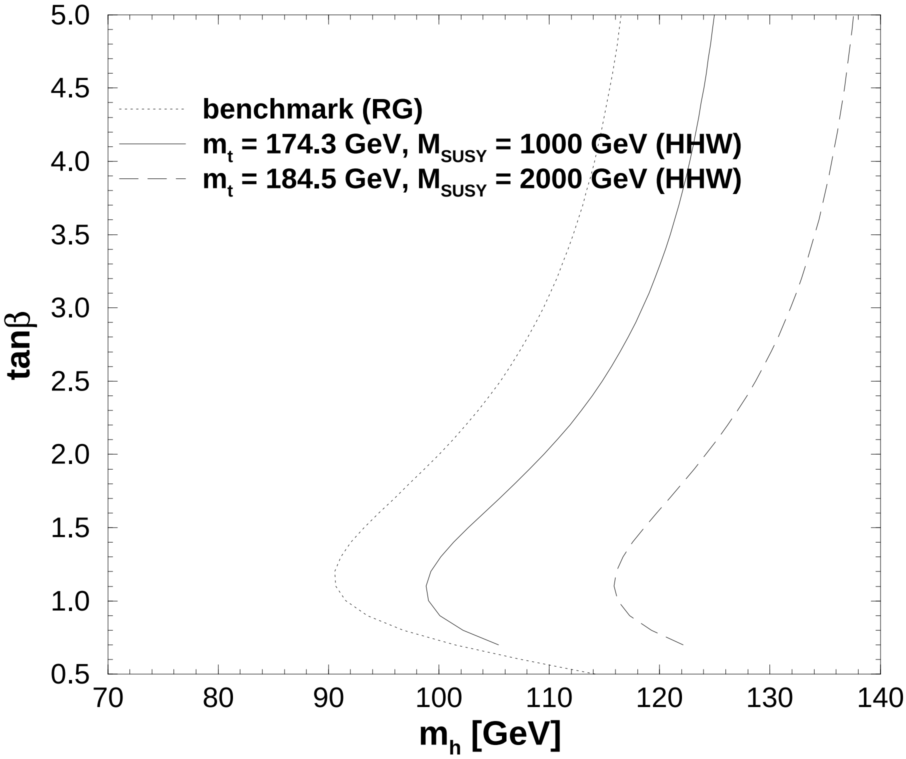

Figure 5: *m_h is shown as a function of* $\tan\beta$. *The dotted curve displays the benchmark scenario in the RG approach, which has been used for phenomenological analyses up to now. The solid curve displays the $m_h^{\rm max}$-FD scenario, while the dashed curve corresponds to the "worst case" scenario with* $m_t = 184.5$ GeV *and* $M_{\rm SUSY} = 2000$ GeV.

4 Conclusions and suggestions

We have investigated the upper bound on the mass of the lightest $\mathcal{CP}$-even Higgs boson in the MSSM, depending on $\tan\beta$. In order to discuss possible exclusion limits on $\tan\beta$ from the direct Higgs-boson search, it is useful to consider definite scenarios with specific assumptions on the relevant input parameters and on the structure of the considered model. Constraints on $\tan\beta$ derived within such a framework are of course to be understood under the assumptions defining the investigated scenario.

In this spirit in particular the "benchmark" scenario has been widely used, in which $m_t = 175$ GeV and $M_{\rm SUSY} = 1$ TeV are chosen. In this note we have analyzed the influence of variations in the other parameters entering the prediction for m_h and we have shown that the settings used for those parameters within the benchmark scenario do not cover the maximal possible value of m_h for $m_t = 175$ GeV and $M_{\rm SUSY} = 1$ TeV. We thus suggest a slight generalization of the definition of the benchmark scenario, where more general values of M_2 and M_A are allowed, a more conservative expression for the running top-quark mass is taken, and the case of maximal mixing in the scalar top sector is defined such that it corresponds to the maximal m_h value. Compared to the definition of the benchmark scenario used so far, the generalization suggested here leads to a shift in the upper bound of m_h of about 5 GeV.

Concerning the definition of the running top-quark mass, since the theoretical value of m_h depends very sensitively on the precise value of the top-quark mass, it might be helpful for comparisons to specify in the benchmark plots not only the value of the pole mass, $m_t = 175$ GeV, but also the corresponding value of the running mass used in the analysis. For the same reason it should also be made clear that the input value $M_{\rm SUSY} = 1$ TeV refers to a low-energy parameter, which can directly be used as input in *FeynHiggs* and *subhpole*, and not to a parameter at a high (i.e. unification) scale, for which one would first need to perform the appropriate running to the low scale.

Independently of the precise definition of the benchmark scenario, we have furthermore analyzed the impact of taking into account the new diagrammatic two-loop result (program *FeynHiggs*) for the mass of the lightest Higgs boson, which contains in particular genuine non-logarithmic two-loop contributions that are not present in the previous result obtained by renormalization group methods (program *subhpole*). The maximal value for m_h obtained with *FeynHiggs* is higher by about 4 GeV than the maximal value calculated with *subhpole*. This leads to a significant reduction of the $\tan\beta$ region accessible at LEP2.

Going beyond the benchmark scenario, we have also discussed a "worst case" scenario, where m_t is chosen to be two standard deviations above the current experimental central value and $M_{\rm SUSY} = 2$ TeV. In this scenario no values of $\tan\beta$ can be excluded as long as the limit on m_h is lower than about 115 GeV.

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