STOCHASTIC PRECOOLING OF FRAGMENT BEAMS AT GSI IN DARMSTADT: CONCEPTS AND STATUS

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Abstract

Stochastic Precooling at the ESR storage ring of GSI in Darmstadt will be used to cool 'hot' secondary beams on the injection orbit, before rf stacking and further electron cooling on the stack orbit [1]. The secondary beams arise from nuclear reactions at the GSI fragment separator. A maximum momentum spread of $\delta p/p_0 < \pm 0.35\%$, as well as transverse emittances of $< 20\pi$ mm mrad are to be cooled. The beam energy will be about 500 MeV/u and the number of particles per shot will be $< 10^8$. The system will be used for heavy ions up to uranium. It will operate on all three phase planes.

1. BASIC DESIGN FEATURES

The Palmer method is used for longitudinal cooling. Because of the excellent signal-to-noise ratio of heavy ion beams, notch filtering is unnecessary. There are two pick-up and two kicker stations. The presence of the stacked beam leads to the constraint of placing all electrodes in regions of sufficient dispersion. The optical parameters of the pick-up and kicker stations are given in table 1. We use the signal line P1–K1 for both vertical and longitudinal cooling. The line P2–K2 serves for horizontal cooling. As the P2 signal contains a large contribution of unwanted longitudinal Schottky noise, the P1 and P2 signals are adequately superposed at K2 in order to get a clean betatron signal.

The system works in the frequency band 0.9–1.7 GHz where Schottky bands overlap during the first few seconds of cooling. This is due to the large value $\eta \approx 0.25$ of the frequency dispersion. The time of flight between pick-up and kicker for particles at the high and low momentum ends $\delta p/p_s = \pm 0.35\%$ differs by an amount of up to ± 223 ps. In the chosen frequency band, this would lead to intolerable phase errors. Therefore the signal from the left and right plates of the pick-ups can be delayed by $T_D = \pm 200$ ps in order to restore synchronism. The delays can be varied in steps of 50 ps during each cooling cycle.

The pick-up and kicker modules consist of eight quadruplets of superelectrodes placed around the injected beam. The electrodes are conventional quarter wave plates in 50 Ω geometry. They are mounted on microstrip lines which also serve for the signal combination inside superelectrodes. For the signal transmission to the vacuum feedthroughs, vacuum coaxial lines are used. Near the electrodes, resistive dampers are installed to suppress higher waveguide modes. All modules are completely designed, the first unit (P2) has been built.

The cut-off frequency of the large ESR vacuum chamber is at about 750 MHz. Electromagnetic feedback from the kickers to the pick-ups is suppressed by means of resistive dampers installed in the chambers of ring quadrupoles.

Twelve horizontal and eight vertical steerers are available in order to center the beam at the pick-ups and kickers. With the ordinary electron-cooled beams, precision measurements of the

location	D	β_x	β_z	μ_x	μ_z
P1	3.99 m	1.6 m	17.3 m	0^0	0^0
P2	5.76 m	39.5 m	4.3 m	65^0	66^0
K1	3.99 m	1.6 m	17.3 m	414^0	432^0
K2	5.76 m	39.5 m	4.3 m	479^0	498^0

Table 1. Ion optical parameters at pick-up and kicker locations

sensitivity as a function of position are planned, in order to allow for quantitative comparisons with theory.
Electronic and rf components are ready. A total rf power of 2 kW is available. The power amplifiers are of the CERN band I type [2].

2. ELEMENTS OF THEORY

2.1. Microscopic Description

The clearest way to describe the dynamics of the cooling process is to use Hamiltonian time dependent perturbation theory. The non-perturbed Hamiltonian is expressed by the action and angle variables for a coasting beam [3]. The vector potential seen by the beam at the kickers is treated as a perturbation. The complete Hamiltonian reads

$$
\begin{aligned}
H(\psi_x, J_x, \psi_y, J_y, \theta, -J_s; s) &= -\frac{J_s}{R} + \frac{J_x}{\beta_x(s)} + \frac{J_y}{\beta_y(s)} \\
&- \frac{Qe}{p_s} A_s\left(x(\psi_x, J_x, J_s), y(\psi_y, J_y), s, t(\psi_x, J_x, \theta, J_s)\right). \qquad (1)
\end{aligned}
$$

Betatron motion is characterized by the action variables $J_{x,y}$ and the angle variables $\psi_{x,y}$. Longitudinal motion is described by the action variable $J_s = Rp/p_s$ where R is the effective radius of the design orbit and p_s denotes the corresponding momentum. θ can be interpreted as an azimuthal angle. A_s is the longitudinal component of the vector potential at the kickers. The transformation equations for the coordinates x, y, and t are

$$
\begin{aligned}
x &= D(s)\frac{\delta p}{p_s}(J_s) + x_+(s)\exp(i\psi_x) + x_-(s)\exp(-i\psi_x), \qquad (2)\\
y &= y_+(s)\exp(i\psi_y) + y_-(s)\exp(-i\psi_y), \qquad (3)\\
t &= \frac{1}{v(J_s)}\left[\theta R + \delta C(J_s)\right] + t_+(s)\exp(i\psi_x) + t_-(s)\exp(-i\psi_x). \qquad (4)
\end{aligned}
$$

The familiar betatron amplitudes are described by the coefficients

$$
x_\pm(s) = \frac{\pm 1}{2i}\sqrt{2J_x\beta_x(s)}. \qquad (5)
$$

An analogous relation holds for the y components. The equation for the particle arrival time t at s contains the well-known momentum dependent term $\delta C = \delta p/p_s \int D/\rho \, ds$ as well as a less familiar expression which is caused by oscillations of the arrival time due to betatron motion:

$$
t_\pm(s) = -\frac{\sqrt{2J_x}}{2v(J_s)}\left[\frac{D(s)}{\sqrt{\beta_x(s)}}\left(1 \pm \frac{\beta_x'(s)}{2i}\right) \pm iD'(s)\sqrt{\beta_x(s)}\right]. \qquad (6)
$$

This expression is only non-zero at locations with finite dispersion. It has a considerable impact on Hamilton's equation of motion for the horizontal action variable which reads

$$\frac{dJ_x}{ds} = -\frac{\partial H}{\partial \psi_x} = \frac{Qe}{p_s}\left(\frac{\partial A_s}{\partial x}\frac{\partial x}{\partial \psi_x} + \frac{\partial A_s}{\partial t}\frac{\partial t}{\partial \psi_x}\right). \tag{7}$$

The first term in the brackets is equivalent to the Panofsky-Wenzel theorem [4],[5].
The second term describes the change of horizontal emittance by longitudinal kicks at locations of finite dispersion [6]. In the Hamiltonian formalism, this effect is mediated by the oscillatory behaviour of the arrival times due to betatron motion.
Writing (7) as an integral equation with the unperturbed solution in the integrand yields the first order solution of (7):

$$J_x(s) \approx J_x - \frac{Qe}{p_s}\int_{s_0}^{s} ds' \left(\frac{\partial A_s}{\partial x}\frac{\partial x}{\partial \psi_x} + \frac{\partial A_s}{\partial t}\frac{\partial t}{\partial \psi_x}\right)\Bigg|_{\text{unperturbed solution}}. \tag{8}$$

The integral over vector potentials is replaced by discrete sums over kicks during subsequent revolutions. A single interaction at one kicker can be represented as the *convolution* of the voltage V_k at the input port of the kicker k with a sensitivity function S_k:

$$\int ds\, A_s(x, s, t(s)) = S_k(x, t(s_k)) * V_k(t(s_k)). \tag{9}$$

The calculation of S at a given geometry is a considerable task [7].
As the coordinate x at the nth revolution at the kicker k depends on the betatron phase angle, it is necessary to expand the sensitivity function [8]:

$$S_k(x(n,k), t) = \sum_{l=-\infty}^{\infty} S_k^l(J_s, J_x, t)\exp(il\psi_x(n,k)). \tag{10}$$

For small betatron amplitudes, the coefficients S_k^l are approximately

$$S_k^l(J_s, J_x, t) \approx \frac{(2J_x\beta_x(s_k))^{l/2}}{(2i)^l l!}\,\frac{\partial^l S_k(x,t)}{\partial x^l}\bigg|_{x\,=\,D\delta p/p_s}. \tag{11}$$

In ordinary geometries, the dominating terms are those with $l = 0$.
Because of (7) we introduce functions $K_{x;k}$ describing the coupling of the applied rf field to horizontal betatron phase space at the kicker k,

$$K_{x;k}^l(J_s, J_x, t) = \sum_{\pm}\left[\pm ix_{\pm}\frac{\partial S_k^{l\mp1}(J_s, J_x, t)}{\partial x} \pm it_{\pm}\frac{\partial S_k^{l\mp1}(J_s, J_x, t)}{\partial t}\right], \tag{12}$$

the sum being over positive and negative signs. The $K_{x;k}^{\pm1}$ terms are the dominant ones. Similarly, there is a coupling function to longitudinal phase space:

$$K_{s;k}^l(J_s, J_x, t) = -\frac{R}{v}\frac{\partial S_k^l(J_s, J_x, t)}{\partial t}. \tag{13}$$

Using these coefficients it is possible to derive simple, general expressions describing the cooling process.

2.2. Cooling drift

After 'sampling' over many revolutions it turns out that the change of the action variables is due to kicks only at the harmonics m and betatron sidebands l of the particle revolution frequency:

$$\omega_{m,l} = (m - lQ_x)\,\omega \tag{14}$$

With the usual assumptions about reciprocity ([8], [4]), the pick-up response can be described by the coupling of the electric field with the beam current. We therefore introduce the pick-up sensitivity $E_p = -\partial S_p/\partial t$. Now the drift of the horizontal action can be written

$$F_x = \left\langle \frac{\Delta J_x}{T} \right\rangle = \frac{(Qe\omega)^2 Z}{8\pi^2 p_s} \sum_{p,k} \sum_{m,l} \tilde{E}_p^l(J_s, J_x, \omega_{m,l}) G_{p,k}(\omega_{m,l}) \tilde{K}_{x;k}^l(J_s, J_x, \omega_{m,l}) \exp(i\phi). \quad (15)$$

The first sum extends over all pick-ups and kickers, the second sum over all integers m and l. Z is the pick-up impedance, and the tilde ˜ denotes the Fourier transform. $G_{p,k}$ is the voltage amplification between pick-up and kicker, without consideration of electrical delays. If we assume the electrical length of the amplification chain to be adjusted to the time of flight of the particle with momentum p_s plus some variable delay $T_{\rm D}^{p,k}$, the phase ϕ in (15) becomes

$$\phi = l\,(\mu_k - \mu_p) + \omega_{m,l} \left(-t_{p,k} \eta_{p,k} \frac{\delta p}{p_s} \mp T_{\rm D}^{p,k} \right). \quad (16)$$

Here, $\mu_k - \mu_p$ is the betatron phase advance between pick-up and kicker, $t_{p,k}$ is the time of flight of the design particle, and η is the local time of flight dispersion between p and k.
The advantage of the formalism presented above is due to the fact that for getting the average change of the longitudinal action J_s, one simply has to change the $\tilde{K}_{x;k}$ terms in (15) into $\tilde{K}_{s;k}$ terms, everything else remaining unchanged.

2.3. Schottky power and diffusion

The Schottky power densities and the voltage correlation at two pick-ups are delta correlated [9], i.e. $< \tilde{V}_p(\Omega)\tilde{V}_{p'}(\Omega') >= 2\pi C_{p,p'}(\Omega)\delta(\Omega + \Omega')$, with

$$C_{p,p'}(\Omega) = \frac{(QeZ)^2 \omega_s R}{8\pi|\eta|} \sum_{\rm bands} \frac{\int dJ_x \Psi(J_s, J_x) \tilde{E}_p^l(J_s, J_x, \Omega) \left(\tilde{E}_{p'}^l(J_s, J_x, \Omega)\right)^*}{|m - lQ_x|} \exp(i\phi), \quad (17)$$

where $\Psi = \partial^2 N/\partial J_s \partial J_x$ is the particle distribution function, normalized to the number of particles. J_s is chosen such that the Schottky frequency $\omega_{m,l}$ is equal to Ω. In the case of Schottky overlap, there is more than one choice of m and l. ϕ is a phase similar to (16). Using the appropriate gain factors, the expression $C_{k,k'}$ for the power at the kickers is easily calculated. The components of the diffusion tensor are proportional to $C_{k,k'}$ and the appropriate coupling functions K. For example, the off-diagonal term is

$$D_{xs} = \left\langle \frac{\Delta J_x \Delta J_s}{T} \right\rangle = \left(\frac{Qe\omega}{2\pi p_s} \right)^2 \sum_{m,l} \sum_{k,k'} \tilde{K}_{x;k}^l(J_s, J_x, \omega_{m,l}) C_{k,k'}(\omega_{m,l}) \left(\tilde{K}_{s;k'}^l(J_s, J_x, \omega_{m,l}) \right)^*. \quad (18)$$

The other diffusion matrix elements are gained simply by exchanging K_x and K_s coupling functions.

2.4. Scaling Laws for Heavy Ions

In the following we shall assume that the thermal noise can be neglected in comparison with the Schottky noise. This is always true for heavy ion beams with intensities that are appreciated as useful by the experimental physicists. Furthermore we suppose to be power-limited, i.e. cooling is above the diffusion limit or the Ψ function is far from equilibrium. Then it makes sense to look at the scaling of the drift and diffusion coefficients under constant power conditions, which means that $G \propto \sqrt{P}/Q$. Then the drift coefficients (and the initial cooling rates) scale according to $F_i \propto Q/A$ and the diffusion due to Schottky noise scales according to $D_{ij} \propto Q^2/A^2$. Hence the cooling rates and equilibrium beam temperatures are roughly independent of the heavy ion species provided they are fully stripped.

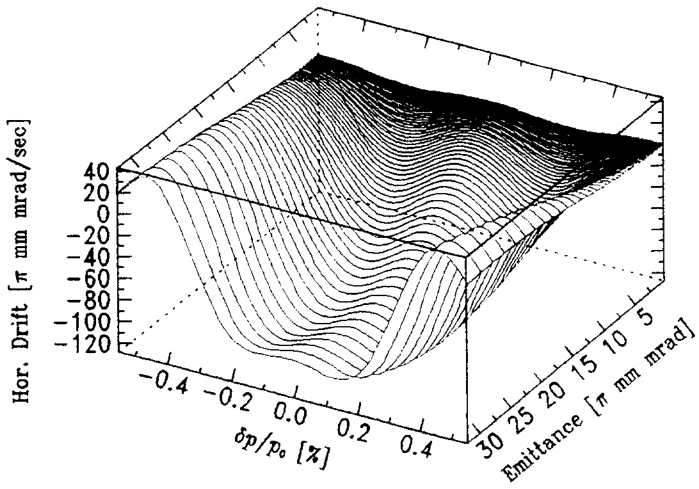

Figure 1. Horizontal drift in model calculation

3. PRELIMINARY MODEL CALCULATION

The model is calculated for a $^{238}U^{92+}$ beam with 10^7 particles. The initial distribution extends up to the limits at which the system is intended to cool, decreasing smoothly and ending closely beyond these limits. The mean power at the K1 and K2 kickers is set to about 100 W. The power due to thermal noise is of the order of a few (1–2) per cent of the total power. At the horizontal kicker K2, the power due to both betatron sidebands is about 40 W compared to 40 W from the longitudinal bands. If the signal from P1 was not used at K2 for compensating for the large longitudinal component of the P2 signal, G_{22} would have to be lowered by 7 dB, leading to a reduction of the horizontal cooling rate by about a factor of 2.2. This indicates that a proper bias between the G_{12} and G_{22} amplifications may be crucial for getting optimum cooling rates. The component of the horizontal diffusion D_{xx} which is due to horizontal kicks is larger than the one due to longitudinal kicks by about a factor of ten. Correlations between both lead to negligible corrections.

The enhancement of the cooling rate due to the variable delays T_D is verified by the calculations. As an example, fig. 1 shows the horizontal drift F_x as a function of $\delta p/p_s$ and 'emittance' $2J_x$. The valleys along the off-momentum lines at about ±0.2% are due to delays of ±150 ps in the P2–K2 line. Cooling times are in the order of seconds.

Detailed numerical solutions of the 2D Fokker-Planck equation are in preparation.

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