

**CP, T AND CPT VERSUS TEMPORAL ASYMMETRIES  
FOR ENTANGLED STATES OF THE  $B_d$ -SYSTEM**

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**ABSTRACT**

The observables used in the  $K$ -system to characterize T and CPT violation are no longer useful for the  $B_d$ -system, since the width difference  $\Delta\Gamma$  between the physical states is vanishingly small. We show that only  $\text{Im}(\varepsilon)$  and  $\text{Re}(\delta)$  can survive if  $\Delta\Gamma = 0$ , and build alternative CP-odd, CPT-odd, T-odd and temporal asymmetries for the  $B_{\pm} \rightarrow B^0, \bar{B}^0$  transitions. These quantities enable us to test T and CPT invariances of the effective Hamiltonian for the  $B$ -system. The method needs the CP eigenstates  $B_{\pm}$ , which can be tagged unambiguously to  $\mathcal{O}(\lambda^3)$  from the entangled states of a  $B$ -factory.

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The time evolution of a neutral-meson system is governed by an effective Hamiltonian. The study of its symmetries corresponds to the problem of mixing, excluding effects from direct decays. The non-invariance of this Hamiltonian is termed as indirect violation. In the case of the neutral  $K$ -system, such a study has been carried out by the CP-LEAR experiment [1] for CP-, T-violating and CP-, CPT-violating observables constructed from the preparation of definite flavour states  $K^0$ - $\bar{K}^0$ . After time evolution of these tagged mesons, the semileptonic decay projects again on a definite flavour state. Although some doubts [2] remain associated with possible CPT violation in the semileptonic decays, the experimental results are interpreted in terms of non-invariance of the mixing Hamiltonian. However, the observables for both T-odd and CPT-odd quantities do not need only a fundamental violation of these symmetries but also non-vanishing absorptive components in the effective Hamiltonian. In the case of the neutral  $K$ -system, such an ingredient is provided by the different lifetimes of the physical states with definite mass,  $K_S$  and  $K_L$ . This study when applied to entangled states of  $K^0$ - $\bar{K}^0$ , as produced by  $\Phi$  decay at DAPHNE, allows [3] additional separation of the different T-odd and CPT-odd asymmetries in mixing and decay.

None of the T-odd and CPT-odd observables based on flavour tag and discussed for the  $K$ -system is useful for the  $B_d$  case. In this system, one expects [4] a negligible value of the width difference  $\Delta\Gamma$  between the physical states. As a consequence, the observables based on flavour tags vanish even if there is a fundamental violation of the symmetries. But the  $B_d$  entangled states obtained in the  $B$  factories from the  $\Upsilon(4S)$  decay can be used for appropriate alternative tags. In this letter we propose the construction of alternative asymmetries in order to test CP, T and CPT invariances. To this end we study the possibilities offered by CP eigenstates of the  $B_d$ -system. These states can be identified [5] unambiguously to  $\mathcal{O}(\lambda^3)$ , which is sufficient to discuss both CP-conserving and CP-violating amplitudes in the effective Hamiltonian for  $B_d$  mesons. Here  $\lambda$  is the flavour-mixing parameter of the CKM matrix [6]. We show that the time-dependent CP-, T-, and CPT-odd asymmetries for the semileptonic decay of the CP-tagged state contain fundamental information on the CP-, T-violating parameter  $\varepsilon$  and the CP-, CPT-violating parameter  $\delta$  in the effective Hamiltonian. More precisely, when  $\Delta\Gamma = 0$ , the parameters signalling the violation of these symmetries are  $\text{Im}(\varepsilon)$  and  $\text{Re}(\delta)$ . We also consider the temporal asymmetry, defined from the inversion in the order of appearance of the final decay products from the entangled state. The main goal of this work is thus the connection of the four CP-odd, CPT-odd, T-odd and temporal asymmetries for the transitions  $B_{\pm} \rightarrow B^0, \bar{B}^0$  with the parameters  $\text{Re}(\delta)$  and  $\text{Im}(\varepsilon)$ , which quantify CP-, CPT-violation and CP-, T-violation, respectively, in the effective Hamiltonian.

The flavour states are connected by  $\text{CP}|B^0\rangle = \text{CP}_{12}^*|\bar{B}^0\rangle$ . Contrary to the corresponding phase of CPT operation, which is rephasing-invariant, the phase  $\text{CP}_{12}$  is parametrization-variant. We keep track of its presence. The corresponding CP eigenstates are thus

$$|B_{\pm}\rangle = \frac{1}{\sqrt{2}}(I \pm \text{CP})|B^0\rangle = \frac{1}{\sqrt{2}}(|B^0\rangle \pm \text{CP}_{12}^*|\bar{B}^0\rangle). \quad (1)$$

These states are well defined iff the CP operator is unique. The identification of the CP operator needs the explicit separation of the Hamiltonian into CP-conserving and CP-violating

components. Its determination is based [5] on the requirement of CP conservation, to  $\mathcal{O}(\lambda^3)$ , in the  $(sd)$  and  $(bs)$  sectors. This fixes, up to a common rephasing of all the quark fields, the CP phases  $(\theta_s - \theta_d)$  and  $(\theta_b - \theta_s)$ , respectively. Thanks to the cyclic relation between CP phases,

$$e^{i(\theta_b - \theta_d)} = e^{i(\theta_b - \theta_s)} e^{i(\theta_s - \theta_d)} \quad (2)$$

is also fixed. Since in the  $(bd)$  sector both CP-conserving and CP-violating amplitudes are present at  $\mathcal{O}(\lambda^3)$ , this fixing allows us to classify CP violation in  $B_d$  decays by referring it to this well defined CP-conserving direction. In this sense, a  $B_d$  decay that is governed by the couplings of the  $(sd)$  or  $(bs)$  unitarity triangles, or by the  $V_{cd}V_{cb}^*$  side of the  $(bd)$  triangle, will not show any CP violation to  $\mathcal{O}(\lambda^3)$ . We may say that such a channel is free from direct CP violation.

The physical states  $B_{1,2}$  of definite mass and lifetime are written in terms of the CP eigenstates  $B_{\pm}$  as

$$\begin{aligned} |B_1\rangle &= \frac{1}{\sqrt{2(1 + |\varepsilon_1|^2)}} [|B_+\rangle + \varepsilon_1 |B_-\rangle] \\ |B_2\rangle &= \frac{1}{\sqrt{2(1 + |\varepsilon_2|^2)}} [|B_-\rangle + \varepsilon_2 |B_+\rangle] . \end{aligned} \quad (3)$$

The complex parameters  $\varepsilon_{1,2}$  describe the CP mixing in the corresponding physical state. Contrary to other definitions of these parameters found in the literature, ours is rephasing-invariant [7] without invoking the introduction of a particular decay channel. Being independent of the parametrization,  $\varepsilon_1$  and  $\varepsilon_2$  are physical quantities iff the CP operator is well defined as discussed above.

If CPT is a good symmetry,  $\varepsilon_2 = \varepsilon_1$ . We thus may alternatively use the parameters  $\varepsilon$  and  $\delta$ , which admit a simpler interpretation in terms of symmetries

$$\varepsilon \equiv \frac{\varepsilon_1 + \varepsilon_2}{2}, \quad \delta \equiv \varepsilon_1 - \varepsilon_2 . \quad (4)$$

The states (3) or  $(\varepsilon, \delta)$  are obtained from the diagonalization of the non-Hermitian effective Hamiltonian  $H = M - \frac{i}{2}\Gamma$ , with  $M^+ = M$ ,  $\Gamma^+ = \Gamma$ . In the limit  $\Delta\Gamma = 0$  [4], the anti-Hermitian part of  $H$  is proportional to the identity matrix, and so  $H$  can be diagonalized by a unitary transformation, and its physical states will be orthogonal. Explicitly, assuming that CPT violation is small and we may neglect terms that are quadratic in  $\Delta \equiv H_{22} - H_{11}$ , we obtain (for  $\Delta\Gamma = 0$ ) the parameters

$$\begin{aligned} \text{Re}(\varepsilon) &= 0, & \frac{\text{Im}(\varepsilon)}{1 + |\varepsilon|^2} &= \frac{\text{Im}(M_{12}\text{CP}_{12}^*)}{\Delta m} \\ \frac{\text{Re}(\delta)}{1 + |\varepsilon|^2} &= \frac{\Delta}{\Delta m}, & \text{Im}(\delta) &= 0 \end{aligned} \quad (5)$$

As can be seen,  $\varepsilon$  becomes purely imaginary and  $\delta$  real in the limit of negligible  $\Gamma_{12}$ , in which  $\Delta\Gamma \ll \Delta m$ . We then find  $\varepsilon_2 = -\varepsilon_1^*$  and the orthogonality of the states (3) is apparent.

The restrictions imposed by each symmetry are the following:

- CP conservation imposes  $\text{Im}(M_{12}\text{CP}_{12}^*) = 0$  and  $H_{11} = H_{22}$ ;
- CPT invariance requires  $H_{11} = H_{22}$ ;
- T invariance imposes  $\text{Im}(M_{12}\text{CP}_{12}^*) = 0$ .

As a consequence, CPT invariance leads to  $\Delta = 0$  and thus  $\delta = 0$ , irrespective of the value of  $\varepsilon$ . Similarly, T invariance leads to  $\varepsilon = 0$ , independently of the value of  $\delta$ . CP conservation requires both  $\varepsilon = \delta = 0$ . Therefore,

- $\text{Im}(\varepsilon) \neq 0$  indicates the presence of both CP and T violation;
- $\text{Re}(\delta) \neq 0$  means that CP and CPT violation exist.

In a  $B$  factory, charge conjugation and Bose statistics require that the  $B^0\text{-}\bar{B}^0$  state produced in the  $\Upsilon(4S)$  decay be given by

$$|\Psi_{in}\rangle = \frac{1}{\sqrt{2}} \left( |B^0(\vec{k}), \bar{B}^0(-\vec{k})\rangle - |\bar{B}^0(\vec{k}), B^0(-\vec{k})\rangle \right). \quad (6)$$

The correlation implied by Eq. (6) remains at any moment in the evolution of the system and therefore, the decay product observed first in one side of the detector allows the tagging of the meson in the other side at the moment the decay took place. This correlation holds not only for the flavour basis ( $B^0, \bar{B}^0$ ), but also for CP eigenstates ( $B_+, B_-$ ) and for physical states ( $B_1, B_2$ ), independently of the values of  $\varepsilon$  and  $\delta$ .

We will use the notation  $(X, Y)$  for the final state. Here  $X$  is the decay product observed with momentum  $\vec{k}$  at a time  $t_0$ , whereas  $Y$  is detected with momentum  $-\vec{k}$  at a later time  $t$ . We will describe the process in terms of the time variables  $\Delta t = t - t_0$  and  $t' = t_0 + t$ . The probability of finding the arbitrary final state  $(X, Y)$  from the initial state (6) may be written as:

$$|(X, Y)|^2 = \frac{1}{16} \left| \frac{1 + \varepsilon_2}{1 + \varepsilon_1} \right|^2 |\langle X|B_1\rangle|^2 |\langle Y|B_1\rangle|^2 e^{-\Gamma t'} \times \\ \times \{(\eta_+ + \eta_-) - (\eta_+ - \eta_-) \cos(\Delta m \Delta t) - 2\eta_{\text{im}} \sin(\Delta m \Delta t)\}, \quad (7)$$

$\Gamma$  being the common width of  $B_{1,2}$ . Since  $\Delta\Gamma = 0$ , in this and the following expressions one should take  $\varepsilon_2 = -\varepsilon_1^*$ . The  $\eta$  coefficients are defined according to

$$\eta_+ = |\eta_X + \eta_Y|^2, \quad \eta_- = |\eta_X - \eta_Y|^2, \quad \eta_{\text{im}} = \text{Im}[(\eta_X + \eta_Y)(\eta_X^* - \eta_Y^*)], \quad (8)$$

with

$$\eta_X \equiv \frac{\langle X|B^0\rangle - \frac{1-\varepsilon_2}{1+\varepsilon_2}\text{CP}_{12}^*\langle X|\bar{B}^0\rangle}{\langle X|B^0\rangle + \frac{1-\varepsilon_1}{1+\varepsilon_1}\text{CP}_{12}^*\langle X|\bar{B}^0\rangle} = \frac{1 + \varepsilon_1}{1 + \varepsilon_2} \cdot \frac{\varepsilon_2 \langle X|B_+\rangle + \langle X|B_-\rangle}{\langle X|B_+\rangle + \varepsilon_1 \langle X|B_-\rangle}, \quad (9)$$

and an analogous expression for  $\eta_Y$ . We have written Eq. (9) in terms of both flavour and CP eigenstates, in view of the tags we propose below. One can check that, for  $Y = X$ , only  $\eta_+$  remains and the probability  $|(X, X)|^2$  vanishes for  $\Delta t = 0$ . This is a kind of EPR correlation, imposed here by Bose statistics [8].

The integration of the probability (7) over  $t'$  between  $\Delta t$  (always positive with our definition) and  $\infty$  gives the intensity for the chosen final state, depending only on  $\Delta t$

$$I(X, Y; \Delta t) = \int_{\Delta t}^{\infty} dt' |(X, Y)|^2. \quad (10)$$

To perform a CP tag, let  $X$  be a CP eigenstate produced along the CP-conserving direction. Since, as discussed above, such a decay is free from direct CP violation, this assures that at  $t = t_0$  the living meson of the other side had the opposite CP eigenvalue. Among others, one example of a decay channel with the properties we are looking for is given by  $X \equiv J/\Psi K_S$ , with CP= -, or by  $X \equiv J/\Psi K_L$ , with CP= +, governed by the ‘‘CP-allowed’’ side  $V_{cb}V_{cs}^*$  of the ( $bs$ ) triangle. Then, the detection of such a final state leads to the preparation of the remaining  $B_d$  meson in the complementary CP eigenstate to  $\mathcal{O}(\lambda^3)$ .

With these considerations, let us assume that, at  $t = t_0$ , the  $B_d$  meson is prepared as a  $B_+$ . After this CP tag, the time evolution acts during  $\Delta t$  and we ask for the probability of its transition to  $B^0$ . In our result (7) for the probability from the entangled state, this  $B_+ \xrightarrow{\Delta t} B^0$  transition corresponds to a final state  $X = J/\Psi K_S$ ,  $Y = \ell^+$ . In Table 1 we present the transitions of  $B_d$  states, connected to the original one by CP-, CPT- and T- symmetry transformations. The last column, showing  $\Delta t$ , is obtained by inverting the order of appearance of the decay products. The second row in the table shows the  $(X, Y)$  characterization for each one of these processes, i.e., the configuration of the final decay products, which will signal such a transition at the meson level.

$B_+ \rightarrow B^0$ ( $J/\Psi K_S, \ell^+$ )	CP	CPT	T	$\Delta t$
Transition	$B_+ \rightarrow \bar{B}^0$	$\bar{B}^0 \rightarrow B_+$	$B^0 \rightarrow B_+$	$\bar{B}^0 \rightarrow B_-$
$(X, Y)$	( $J/\Psi K_S, \ell^-$ )	( $\ell^+, J/\Psi K_L$ )	( $\ell^-, J/\Psi K_L$ )	( $\ell^+, J/\Psi K_S$ )

Table 1: Transitions and  $(X, Y)$  final decay products connected to  $B_+ \rightarrow B^0$  and  $(J/\Psi K_S, \ell^+)$  by the four transformations.

From the five different processes indicated in Table 1, we construct four intensity asymmetries

$$A(X, Y) = \frac{I(X, Y) - I(J/\Psi K_S, \ell^+)}{I(X, Y) + I(J/\Psi K_S, \ell^+)} \quad (11)$$

as a function of  $\Delta t$ . They are odd under the four transformations considered.

To first order in the CPT parameter  $\delta$ , and in the limit  $\Delta\Gamma = 0$ , we obtain the following results:

- The CP-odd asymmetry gives

$$A_{\text{CP}} \equiv A(J/\Psi K_S, \ell^-) = -2 \frac{\text{Im}(\varepsilon)}{1 + |\varepsilon|^2} \sin(\Delta m \Delta t) + \frac{1 - |\varepsilon|^2}{1 + |\varepsilon|^2} \frac{2\text{Re}(\delta)}{1 + |\varepsilon|^2} \sin^2\left(\frac{\Delta m \Delta t}{2}\right) \quad (12)$$

with contributions from both T-violating ( $\Delta t$  odd) and CPT-violating ( $\Delta t$  even) terms.

- The CPT-odd asymmetry yields

$$A_{\text{CPT}} \equiv A(\ell^+, J/\Psi K_L) = \frac{1 - |\varepsilon|^2}{1 + |\varepsilon|^2} \frac{2\text{Re}(\delta)}{1 + |\varepsilon|^2} \frac{1}{1 - 2 \frac{\text{Im}(\varepsilon)}{1 + |\varepsilon|^2} \sin(\Delta m \Delta t)} \sin^2\left(\frac{\Delta m \Delta t}{2}\right) \quad (13)$$

with a needed contribution from  $\delta \neq 0$ , including T-even and T-odd terms.

- The T-odd asymmetry gives

$$A_{\text{T}} \equiv A(\ell^-, J/\Psi K_L) = -2 \frac{\text{Im}(\varepsilon)}{1 + |\varepsilon|^2} \sin(\Delta m \Delta t) \left[ 1 - \frac{1 - |\varepsilon|^2}{1 + |\varepsilon|^2} \frac{2\text{Re}(\delta)}{1 + |\varepsilon|^2} \sin^2\left(\frac{\Delta m \Delta t}{2}\right) \right] \quad (14)$$

which needs  $\varepsilon \neq 0$ , including CPT-even and CPT-odd terms.

- Lastly, the temporal asymmetry satisfies the equality

$$A_{\Delta t} \equiv A(\ell^+, J/\Psi K_S) = A(\ell^-, J/\Psi K_L) \equiv A_{\text{T}}. \quad (15)$$

It is worth noting that, contrary to the other asymmetries,  $A_{\Delta t}$  is not associated to the transformation of the original transition under a fundamental symmetry. Nevertheless, it may provide information on the symmetry properties of the system. The experimental  $\Delta t$  asymmetry is different from the T-odd asymmetry. In our case, the equality (15) is a consequence of  $\Delta\Gamma = 0$  and it can be used as a consistency test of this assumption. In general, the equality of the probabilities for the T-inverted and  $\Delta t$ -inverted processes is only valid for Hermitian Hamiltonians, up to a global (proportional to unity) absorptive part. It is not satisfied, for instance, for the  $K$ -system. On the contrary, for  $B_d$ -mesons the T-odd asymmetry (14) becomes an odd function of time, once  $\Delta\Gamma = 0$  is assumed.

The results (12)-(15) can be used to test consistencies and extract the parameters

$$\frac{2\text{Im}(\varepsilon)}{1 + |\varepsilon|^2}, \quad \frac{1 - |\varepsilon|^2}{1 + |\varepsilon|^2} \frac{2\text{Re}(\delta)}{1 + |\varepsilon|^2}, \quad (16)$$

fundamental for CP, T violation and CP, CPT violation, respectively. Their connection with the matrix elements of the effective Hamiltonian is given in Eq. (5).

In the Standard Model,  $\delta = 0$ , and the four asymmetries collapse to

$$\begin{aligned} A_{\text{CP}} &= A_{\text{T}} = A_{\Delta t} = -2 \frac{\text{Im}(\varepsilon)}{1 + |\varepsilon|^2} \sin(\Delta m \Delta t) \\ A_{\text{CPT}} &= 0 \end{aligned} \quad (17)$$

In terms of the angles of the  $(bd)$  unitarity triangle, the CP-mixing parameter is then given by

$$\begin{aligned}\frac{2\text{Im}(\varepsilon)}{1+|\varepsilon|^2} &= \sin(2\beta) = -\frac{2\eta(1-\rho)}{(1-\rho)^2+\eta^2} \\ \frac{1-|\varepsilon|^2}{1+|\varepsilon|^2} &= \cos(2\beta) = \frac{(1-\rho)^2-\eta^2}{(1-\rho)^2+\eta^2}\end{aligned}\tag{18}$$

where  $(\rho, \eta)$  are the couplings appearing in the Wolfenstein parametrization [9].  $A_{\text{CP}}$  has been measured [10] by the CDF Collaboration at Fermilab, using a flavour tag, and the result interpreted in the Standard Model with  $\sin(2\beta) = 0.79_{-0.44}^{+0.41}$ . However, the other three asymmetries do not need only a flavour tag but also a CP tag, as shown in Table 1.

To summarize, we have been able to bypass the vanishing of T-odd and CPT-odd asymmetries in the limit  $\Delta\Gamma = 0$ . The crucial point consists in going beyond the flavour tags used for the  $K$ -system. The production of entangled states of  $B_d$  mesons in a  $B$ -factory allows an unambiguous tag of CP eigenstates. This CP tag makes use of decay channels along the CP-conserving direction, such as that illustrated by  $J/\Psi K_S$ . Starting with the  $B_+ \xrightarrow{\Delta t} B^0$  transition, we have considered four asymmetries, which are odd under the CP, CPT and T symmetry transformations and under the temporal inversion of the decay products. We prove that, in the limit  $\Delta\Gamma = 0$ , the T-odd asymmetry equals the temporal asymmetry. CP-odd and CPT-odd asymmetries contain, on the contrary, both  $\Delta t$ -odd and  $\Delta t$ -even terms. Whereas the CPT asymmetry needs  $\delta \neq 0$ , in the presence or absence of  $\varepsilon$ , the CP asymmetry can be generated by a non-vanishing value of either  $\varepsilon$  or  $\delta$ , or of both. All these four asymmetries, experimentally different, provide a consistent set of observables able to disentangle the fundamental parameters  $\varepsilon$  and  $\delta$  of T-violation and CPT-violation, respectively.

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