CERN - B DECAY [illegible]

[illegible] JULY 23

THE INCLUSIVE DECAY

$B \to X_u \ell \nu$

TO ORDER α_s

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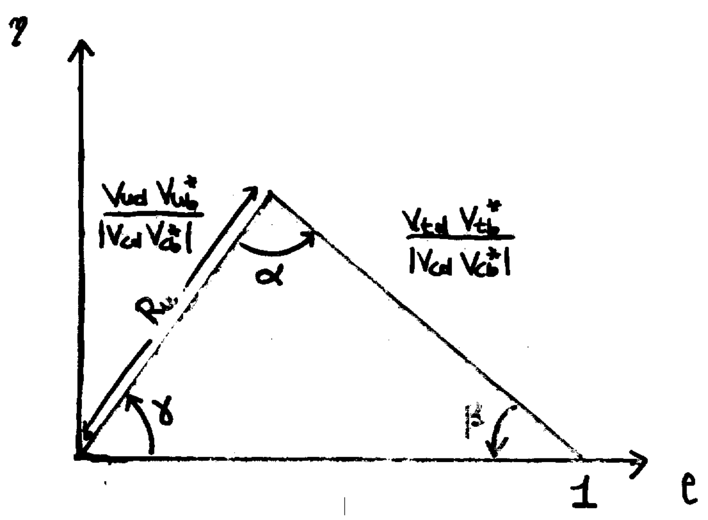
- Methods to determine V_{ub}
- $B \to X_u \ell \nu$ differential decay distributions to order α_s
- Conclusions

B SEMILEPTONIC DECAYS ARE IMPORTANT TO DETERMINE V_{cb} AND V_{ub}

In the Wolfenstein parametrization the CKM matrix is:

$$V = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \Theta(\lambda^4)$$

FROM UNITARITY: $\quad V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$



THIS IS THE WIDEST TRIANGLE OBTAINED FROM THE UNITARITY RELATIONS

$\hookrightarrow$ LARGE CP IN B DECAYS

WHAT DO WE KNOW ?

$$|V| = \begin{pmatrix} 0.9745 - 0.9757 & 0.219 - 0.224 & \boxed{0.002 - 0.005} \\ 0.219 - 0.224 & 0.9736 - 0.9750 & 0.036 - 0.046 \\ \boxed{0.004 - 0.014} & 0.034 - 0.046 & 0.9989 - 0.9993 \end{pmatrix}$$

V_{td} and V_{ub} are those affected by the largest uncertainties

MEASURING DIFFERENT PHYSICAL QUANTITIES PROVIDES CONSTRAINTS ON THE CKM MATRIX ELEMENTS

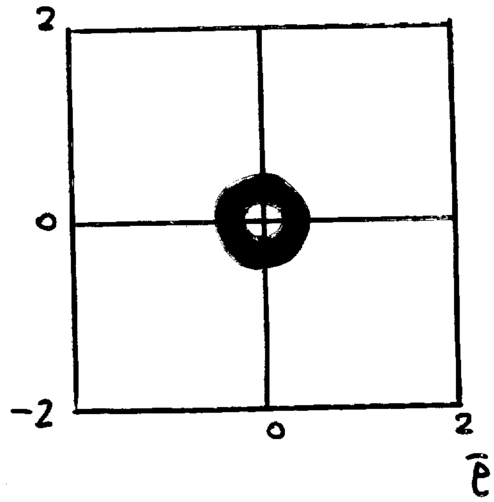
→ THE COMPATIBILITY BETWEEN THE VARIOUS MEASUREMENTS TESTS THE CKM PICTURE (AND HENCE THE S.M.) OF QUARK MIXING

$|V_{ub}/V_{cb}|$ DETERMINES THE LENGHT R_u OF ONE OF THE SIDES OF THE UNITARITY TRIANGLE

CONSTRAINTS FROM THE RATIO $|V_{ub}/V_{cb}|$

$$|V_{ub}/V_{cb}| = 0.08 \pm 0.005 \pm 0.02$$

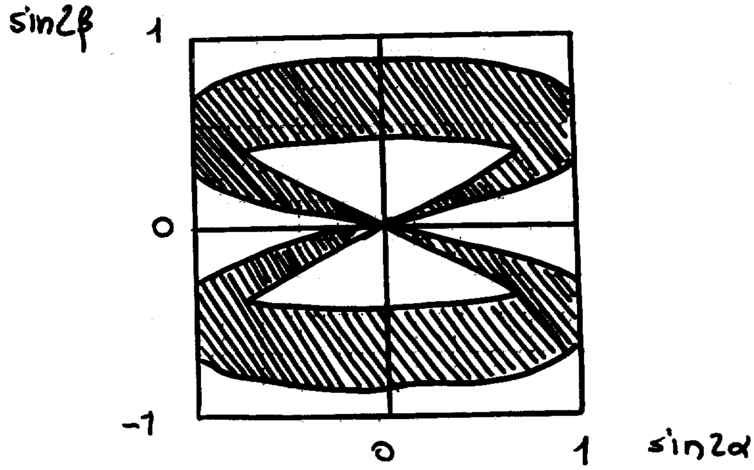
In the plane $(\bar{\rho}, \bar{\eta})$



$$\bar{\rho} = \rho\left(1 - \frac{\lambda^2}{2}\right)$$

$$\bar{\eta} = \eta\left(1 - \frac{\lambda^2}{2}\right)$$

In the plane $(\sin 2\alpha, \sin 2\beta)$



BaBar Physics Book ('98)

DETERMINATION OF V_{ub}

- FROM EXCLUSIVE CHANNELS $B \to (\pi, \rho) \ell \nu$

  $\to$ It is necessary to know the hadronic matrix elements $B \to (\pi, \rho)$

  $\hookrightarrow$ Theoretical uncertainties

- FROM THE INCLUSIVE DECAY $B \to X_u \ell \nu$

  $\to$ Trying to subtract the background of $B \to X_c \ell \nu$

IN ORDER TO DETERMINE EXPERIMENTALLY V_{ub} FROM THE INCLUSIVE PROCESS IT IS NECESSARY TO OPTIMIZE THE DISCRIMINATION OF $b \to u$ FROM $b \to c$

FROM A THEORETICAL POINT OF VIEW IT IS NECESSARY TO STUDY THE DIFFERENTIAL DECAY DISTRIBUTIONS IN MORE KINEMATICAL VARIABLES:

- CHARGED LEPTON ENERGY
- HADRONIC INVARIANT MASS
- HADRON ENERGY

TRADITIONAL METHOD

ONE USES THE ENDPOINT REGION OF THE CHARGED LEPTON ENERGY SPECTRUM

↳ IT SHOULD BE APPLIED A CUT ON E_ℓ TO ELIMINATE THE CHARM BACKGROUND

PROBLEMS:

- $\Delta E \sim 350$ MeV → only few events
- Breakdown of OPE in the theoretical calculation → not reliable

ALTERNATIVE WAY

MOST OF $B \to X_u \ell \nu$ DECAYS ARE EXPECTED TO HAVE HADRONIC INVARIANT MASS $\sqrt{s_H} < m_D$ WHILE ONLY A SMALL FRACTION OF THESE DECAYS HAVE LEPTON ENERGIES IN THE ENDPOINT REGION

FOR THE LEPTON ENDPOINT REGION THE CONTRIBUTION OF STATES WITH MASS NEAR m_D IS KINEMATICALLY SUPPRESSED

→ THE ENDPOINT REGION IS DOMINATED BY (π, ρ) STATES

CALCULATION OF THE TRIPLE DIFFERENTIAL DISTRIBUTION AT ORDER α_s

↳ IT ALLOWS TO PUT ARBITRARY CUTS ON THE KINEMATICAL VARIABLES (Lepton Energy, Hadron Energy, Hadronic Invariant Mass)

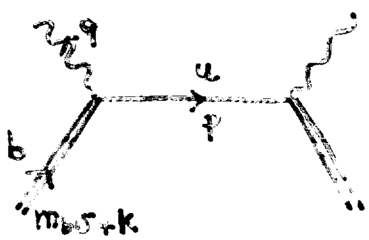
INCLUSIVE DECAYS OF HEAVY HADRONS :

- EXPANSION IN THE INVERSE POWERS OF m_b
- THE FIRST TERM REPRODUCES THE SPECTATOR MODEL RESULT: FREE b-QUARK DECAY
- ABSENCE OF $\Theta\left(\frac{1}{m_b}\right)$ CORRECTIONS

BIGI, URALTSEV, VAINSHTEIN
PL B293 (92) 430
B297 (93) 477 (E)

CHAY, GRINSTEIN, GEORGI
PL B247 (90) 399

AT THE LEADING ORDER ($\alpha_s=0$) I SHOULD COMPUTE THE IMAGINARY PART O



- THE CALCULATION IS PERFORMED IN QCD
- THE b-QUARK MOMENTUM IS $m_b v+k$ WITH $k\sim\Theta(\Lambda_{QCD})$ ⇒
  - TERMS PROPORTIONAL TO K ARE $\Theta\left(\frac{1}{m_b}\right)$
  - THE LEADING TERM IS SELECTED PUTTING K=0
- $\Theta\left(\frac{1}{m_b^2}\right)$ CORRECTIONS HAVE BEEN COMPUTED

MANOHAR, WISE PR D49 (94) 1310
BLOK ET AL, PR D49 (94) 3356
D50 (94) 3572 (E

$$W_{\mu\nu} = -\frac{1}{\pi} \operatorname{Im} T_{\mu\nu}(p, v) \qquad \equiv \text{HADRONIC TENSOR}$$

$$T_{\mu\nu} = -i \int d^4x \, e^{i(p - m_b v)x} \, \frac{\langle B(v) | T\{J_\mu^\dagger(x) J_\nu(0)\} | B(v) \rangle}{2M_B}$$

$$W_{\mu\nu} = W_1 \left(p_\mu v_\nu + p_\nu v_\mu - v.p \, g_{\mu\nu} - i \varepsilon_{\mu\nu\alpha\beta} p^\alpha v^\beta \right) - W_2 \, g_{\mu\nu}$$
$$+ W_3 \, v_\mu v_\nu + W_4 (p_\mu v_\nu + p_\nu v_\mu) + W_5 \, p_\mu p_\nu$$

TREE LEVEL $\rightarrow$ $W_1 = 2\,\delta(p^2)$ $\qquad W_{i \neq 1} = 0$

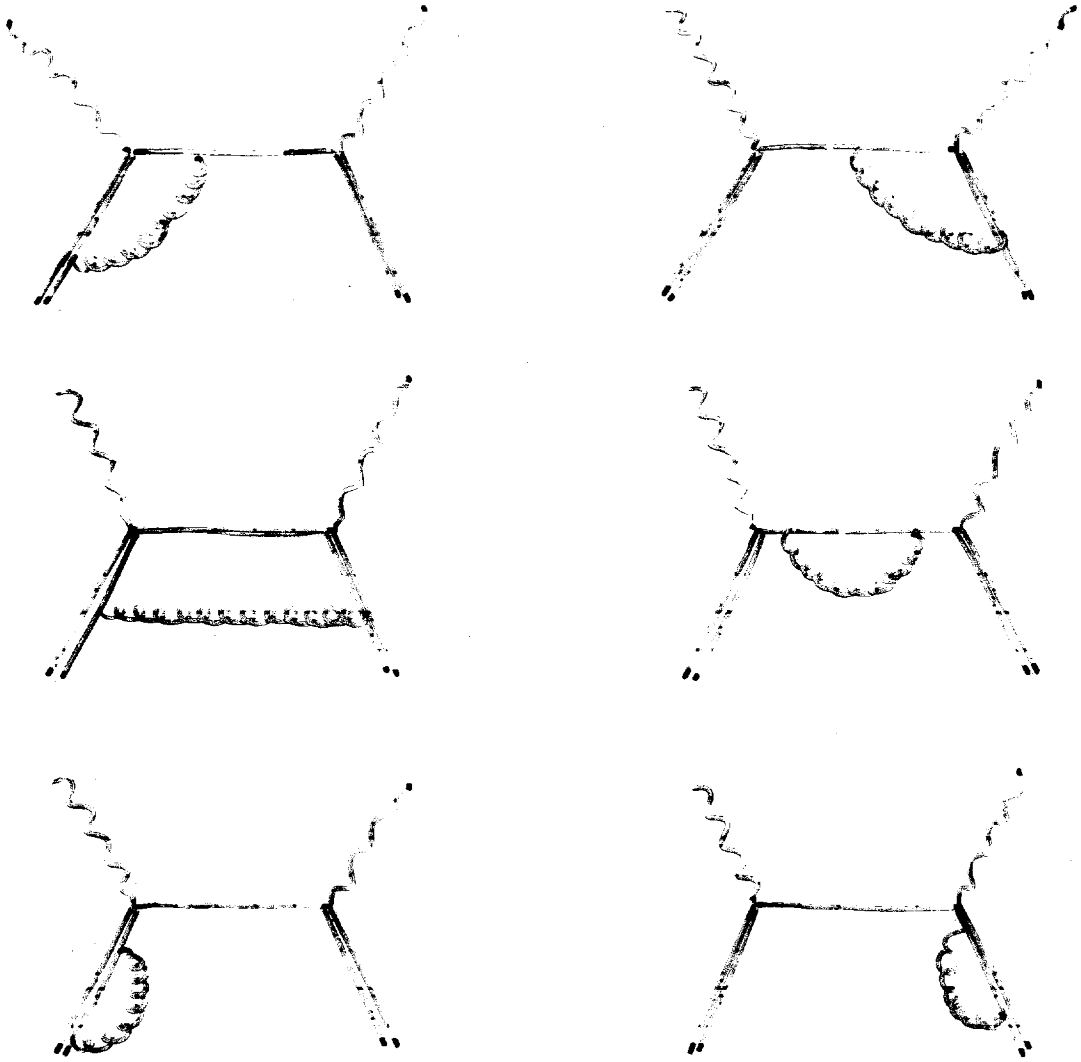
$m_\ell = 0 \qquad x = \frac{2E_\ell}{m_b} \qquad \hat{p}^2 = \frac{p^2}{m_b^2} \qquad z = \frac{2 v.p}{m_b}$

$\bar{x} = 1 - x$

$$\frac{d^3\Gamma}{dx\,dz\,d\hat{p}^2} = 12\,\Gamma_0 \Big\{ (1 + \bar{x} - z)(z - \bar{x} - \hat{p}^2) \frac{m_b^2}{2} W_1 +$$
$$+ (1 - z + \hat{p}^2) \frac{m_b}{2} W_2 + \left[\bar{x}(z - \bar{x}) - \hat{p}^2 \right] \frac{m_b}{4} (W_3 + 2 m_b W_4 + m_b^2 W_5) \Big\}$$

where :
$$\Gamma_0 = \frac{G_F^2 |V_{ub}|^2 m_b^5}{192 \pi^3}$$

IN ORDER TO COMPUTE W_c TO ORDER α_s THE FOLLOWING DIAGRAMS SHOULD BE CONSIDERED



$$\frac{m_b^2}{2} W_1^{(1)} = -\delta(\hat{p}^2)\left[4\ln^2 z - 6\ln z + \frac{2\ln z}{1-z} + 4L_2(1-z) + \pi^2 + \frac{15}{2}\right] -$$

$$- f_{\mathrm{IR}}(\hat{\lambda}^2, z, \hat{p}^2) + \frac{4}{\hat{p}^2}\left[\frac{1}{t}\ln\frac{1+t}{1-t} + \ln\frac{\hat{p}^2}{z^2}\right] +$$

$$+1 - \frac{(8-z)(2-z)}{z^2t^2} + \left[\frac{2-z}{2z} + \frac{(8-z)(2-z)}{2z^2t^2}\right]\frac{1}{t}\ln\frac{1+t}{1-t},$$

$$\frac{m_b}{2} W_2^{(1)} = \frac{8-z}{4} + \frac{32-8z+z^2}{4zt^2} - \left[\frac{zt^2}{8} + \frac{4-z}{4} + \frac{32-8z+z^2}{8zt^2}\right]\frac{1}{t}\ln\frac{1+t}{1-t},$$

$$\frac{m_b}{4} W_3^{(1)} = -\frac{8-3z}{8} + \frac{32+22z-3z^2}{4zt^2} - \frac{3(12-z)}{8t^4} +$$

$$+ \left[\frac{zt^2}{16} + \frac{5(4-z)}{16} - \frac{64+56z-7z^2}{16zt^2} + \frac{3(12-z)}{16t^4}\right]\frac{1}{t}\ln\frac{1+t}{1-t},$$

$$\frac{m_b^2}{2} W_4^{(1)} = \frac{2}{1-z}\left[\frac{z\ln z}{1-z} + 1\right]\delta(\hat{p}^2) - 1 - \frac{32-5z}{2zt^2} + \frac{3(12-z)}{2zt^4} -$$

$$- \left[\frac{8-3z}{4z} - \frac{22-3z}{2zt^2} + \frac{3(12-z)}{4zt^4}\right]\frac{1}{t}\ln\frac{1+t}{1-t},$$

$$\frac{m_b^3}{4} W_5^{(1)} = \frac{2}{1-z}\left[\frac{1-2z}{1-z}\ln z - 1\right]\delta(\hat{p}^2) - \frac{8+z}{2z^2t^2} - \frac{3(12-z)}{2z^2t^4} +$$

$$+ \left[\frac{1}{4z} - \frac{2-z}{2z^2t^2} + \frac{3(12-z)}{4z^2t^4}\right]\frac{1}{t}\ln\frac{1+t}{1-t}.$$

$$t = \sqrt{1 - 4\hat{p}^2/z^2}$$

$$f_{\mathrm{IR}}(\hat{\lambda}^2, z, \hat{p}^2) = \delta(\hat{p}^2)\left[\ln^2\hat{\lambda}^2 + (5 - 4\ln z)\ln\hat{\lambda}^2\right] + \theta(\hat{p}^2 - \hat{\lambda}^2)\,\frac{4(\hat{p}^2-\hat{\lambda}^2)}{(\hat{p}^2-\hat{\lambda}^2)^2 + z^2\hat{\lambda}^2} +$$

$$+ \frac{\theta(\hat{p}^2-\hat{\lambda}^2)}{\hat{p}^2}\left[\left(1 - \frac{\hat{\lambda}^2}{\hat{p}^2}\right)\left(3 + \frac{\hat{\lambda}^2}{\hat{p}^2}\right) + 4\ln\left(\frac{\hat{\lambda}^2}{\hat{p}^2} + \frac{\hat{p}^2}{z^2}\right)\right].$$

SOME SINGLE-DIFFERENTIAL DISTRIBUTIONS WERE ALREADY KNOWN AND ARE REPRODUCED

- CHARGED LEPTON ENERGY SPECTRUM

JEŻABEK AND KÜHN
NP B320 (89) 20

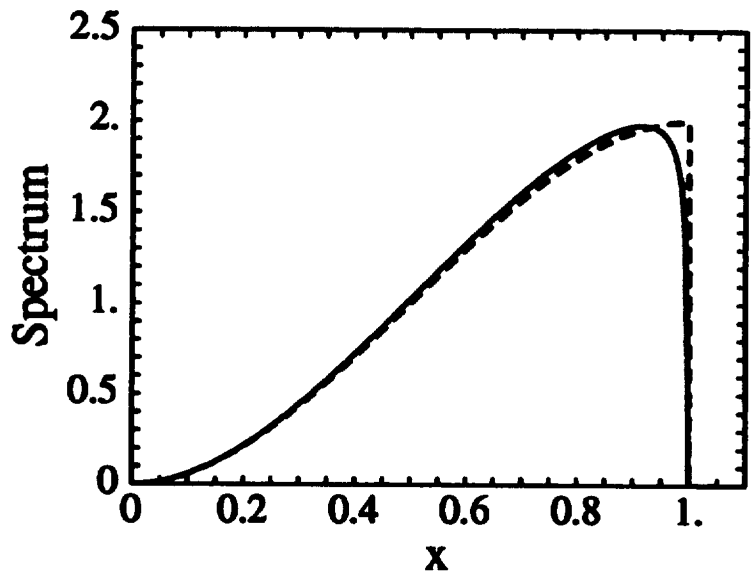
$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dx} = 2x^2(3-2x)\left[1 - \frac{C_F\alpha_s}{2\pi}G(x)\right] \qquad 0 \leq x \leq 1$$

$$G(x) = \ln^2(1-x) + 2L_2(x) + \frac{2}{3}\pi^2 + \frac{82-153x+86x^2}{12x(3-2x)} +$$

$$+ \frac{41-36x+42x^2-16x^3}{6x^2(3-2x)}\ln(1-x)$$

THE SINGULARITIES ARE INTEGRABLE AND:

$$\Gamma = \Gamma_0\left[1 - \frac{C_F\alpha_s}{2\pi}\left(\pi^2 - \frac{25}{4}\right)\right]$$



- - - TREE LEVEL

—— $\mathcal{O}(\alpha_s)$

α_s CORRECTIONS MODIFY ONLY THE ENDPOINT

- HADRON ENERGY SPECTRUM

CZARNECKI JEŻABEK KÜHN

Act. Phys. Pol. B20 (89) 961

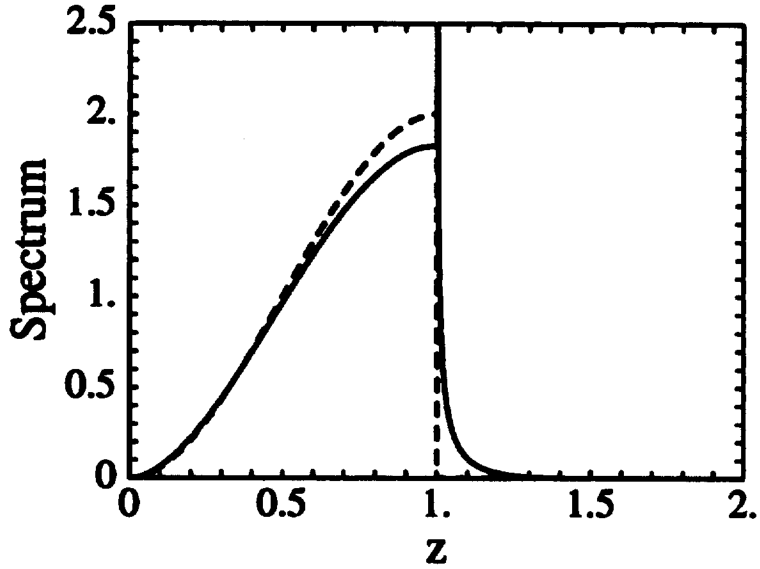
$$\frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}z} = 2z^2(3-2z)\left[1 - \frac{C_F\alpha_s}{2\pi}H_<(z)\right], \qquad 0 \le z \le 1,$$

$$\frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}z} = \frac{C_F\alpha_s}{2\pi}H_>(z), \qquad 1 \le z \le 2,$$

with

$$H_<(z) = 2L_2(1-z) + \pi^2 + \frac{9-4z}{3-2z}\ln z - \frac{4860 - 3720z + 585z^2 - 42z^3 + 4z^4}{360(3-2z)},$$

$$\begin{aligned} H_>(z) = {} & 2z^2(3-2z)\left[\ln^2(z-1) - 2\ln^2 z - 4L_2\left(\frac{1}{z}\right) + \frac{\pi^2}{3}\right] + \\ & + \frac{(2-z)(1248 + 3798z - 2946z^2 + 517z^3 - 34z^4 + 4z^5)}{180} + \\ & + \frac{5 + 12z + 12z^2 - 8z^3}{3}\ln(z-1). \end{aligned}$$



- - - TREE LEVEL

—— $\mathcal{O}(\alpha_s)$

HADRONIC INVARIANT MASS DISTRIBUTION

$M_B = m_b + \bar{\Lambda}$

$s_H = p^2 + 2\bar{\Lambda}\, v \cdot p + \bar{\Lambda}^2$

$\hat{s}_H = \frac{s_H}{m_b^2} \qquad \varepsilon = \frac{\bar{\Lambda}}{m_b}$

$$\frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{s}_H} = T(\hat{s}_H,\varepsilon) - \frac{C_F\alpha_s}{2\pi}\, J_<(\hat{s}_H,\varepsilon)\,, \qquad \varepsilon^2 \le \hat{s}_H \le \varepsilon(1+\varepsilon)\,,$$

$$\frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{s}_H} = \frac{C_F\alpha_s}{2\pi}\, J_>(\hat{s}_H,\varepsilon)\,, \qquad \varepsilon(1+\varepsilon) \le \hat{s}_H \le (1+\varepsilon)^2\,.$$

$$\varepsilon\, T(\hat{s}_H,\varepsilon) = 2x^2(3-2x)\,,$$

$$\begin{aligned}
\varepsilon\, J_<(\hat{s}_H,\varepsilon) = {} & 2x^2(3-2x)\left[\pi^2 + 2L_2(1-x) - 2L_2\Big(-\frac{x}{\varepsilon}\Big)\right] + \\
& + \frac{1}{3}(\varepsilon d_1 - x d_2 - 2x^2 d_3)(x+\varepsilon)\ln\Big(1+\frac{x}{\varepsilon}\Big) + \\
& + 2x^2(9-4x)\ln x - \frac{x}{3}\Big(\varepsilon d_1 + \frac{x}{2}\, d_4 - \frac{x^2}{3}\, d_5\Big)\,,
\end{aligned}$$

$$\begin{aligned}
\varepsilon\, J_>(\hat{s}_H,\varepsilon) = {} & 2x^2(3-2x)\,\times \\
& \times\left[\frac{\pi^2}{3} + \ln^2 y - 2\ln^2 x + 4L_2\Big(-\frac{1}{\varepsilon}\Big) - 4L_2\Big(\frac{1}{x}\Big) - 2L_2\Big(-\frac{x}{\varepsilon}\Big)\right] + \\
& + \frac{1}{3}(\varepsilon d_1 - x d_2 - 2x^2 d_3)(x+\varepsilon)\ln\Big(\frac{y+\varepsilon}{1+\varepsilon}\Big) + \\
& + \Big(7 + 4y - \frac{y^2}{\varepsilon^2}\, d_6 + \frac{2y^3}{3\varepsilon^2}\, d_7\Big)\ln y + \\
& + \frac{1-y}{18(1+\varepsilon)}(d_8 + x d_9 - 2x^2 d_{10})\,,
\end{aligned}$$

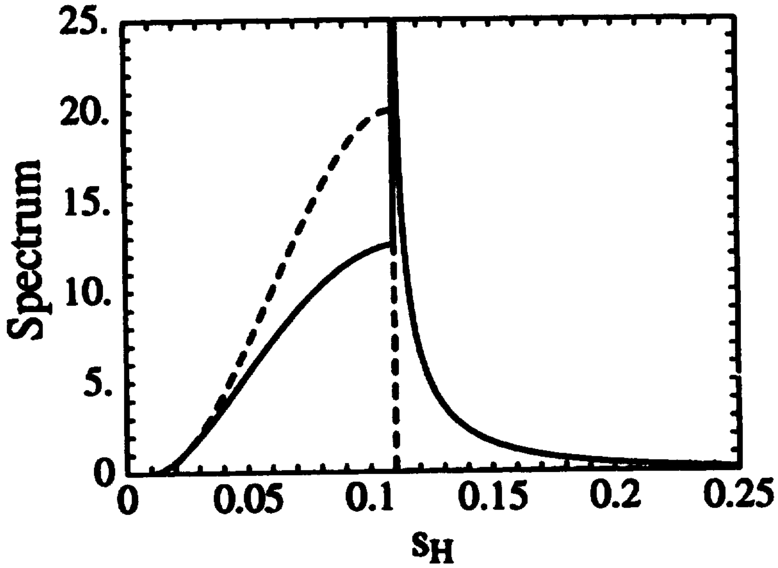
where

$$x = \frac{\hat{s}_H - \varepsilon^2}{\varepsilon}\,, \qquad y = \frac{\hat{s}_H - \varepsilon(1+\varepsilon)}{1+\varepsilon}\,,$$

$d_i \equiv$ polinomials in ε

M. NEUBERT, F.D.

JHEP 9906 (99) 017



--- TREE LEVEL

—— $\mathcal{O}(\alpha_s)$

RADIATIVE CORRECTIONS HAVE AN IMPORTANT ROLE

IN ... V_{ub} ... THE

$$\Gamma(\hat{m}^2,\varepsilon) \equiv \int_{\varepsilon^2}^{\hat{m}^2} \mathrm{d}\hat{s}_H \, \frac{\mathrm{d}\Gamma}{\mathrm{d}\hat{s}_H}, \qquad \varepsilon^2 \le \hat{m}^2 \le (1+\varepsilon)^2 , \qquad \varepsilon = \frac{\Lambda}{m_b}$$

$$\Gamma(\hat{m}^2,\varepsilon) = \Gamma_0 \left[t(\hat{m}^2,\varepsilon) - \frac{C_F \alpha_s}{2\pi}\, j_<(\hat{m}^2,\varepsilon) \right], \qquad \varepsilon^2 \le \hat{m}^2 \le \varepsilon(1+\varepsilon),$$

$$\Gamma(\hat{m}^2,\varepsilon) = \Gamma_0 \left[1 - \frac{C_F \alpha_s}{2\pi}\, j_>(\hat{m}^2,\varepsilon) \right], \qquad \varepsilon(1+\varepsilon) \le \hat{m}^2 \le (1+\varepsilon)^2 .$$

$$t(\hat{m}^2,\varepsilon) = \mu^3(2-\mu),$$

$$\begin{aligned} j_<(\hat{m}^2,\varepsilon) = {} & \mu^3(2-\mu)\left[\pi^2 + 2L_2(1-\mu) - 2L_2\Big(-\frac{\mu}{\varepsilon}\Big)\right] + \\ & + \frac{\pi^2}{3} - 2L_2(1-\mu) + \Big(2\mu + \mu^2 + \frac{20}{3}\mu^3 - \frac{5}{2}\mu^4\Big)\ln\mu + \\ & + \frac{1}{6}\Big(\varepsilon^2 e_1 + \mu\varepsilon e_2 - \mu^2 e_3 + \mu^3 e_4\Big)(\mu+\varepsilon)\ln\Big(1+\frac{\mu}{\varepsilon}\Big) - \\ & - \frac{\mu}{6} e_5 - \frac{\mu^2}{12} e_6 - \frac{\mu^3}{18} e_7 + \frac{\mu^4}{72} e_8 , \end{aligned}$$

$$\begin{aligned} j_>(\hat{m}^2,\varepsilon) = {} & -\mu^3(2-\mu)\left[\frac{\pi^2}{3} + \ln^2\varrho - 2\ln^2\mu + 4L_2\Big(-\frac{1}{\varepsilon}\Big) - 4L_2\Big(\frac{1}{\mu}\Big) - 2L_2\Big(-\frac{\mu}{\varepsilon}\Big)\right] + \\ & + \pi^2 + \ln^2\varrho - \frac{1}{6\varepsilon(1+\varepsilon)}\Big(e_9 + \mu e_{10} + \mu^2\varepsilon e_{11} - \mu^3 e_{12}\Big)\varrho\ln\varrho - \\ & - \frac{1}{6}\Big(\varepsilon^2 e_1 + \mu\varepsilon e_2 - \mu^2 e_3 + \mu^3 e_4\Big)(\mu+\varepsilon)\ln\frac{\varrho+\varepsilon}{1+\varepsilon} + \\ & + \frac{1}{(1+\varepsilon)^2}\left(\frac{1}{72}e_{13} - \frac{\mu}{18}e_{14} - \frac{\mu^2}{12}e_{15} + \frac{\mu^3}{18}e_{16} - \frac{\mu^4\varepsilon}{72}e_{17}\right), \end{aligned}$$

$$\mu = \frac{\hat{m}^2 - \varepsilon^2}{\varepsilon} \qquad \varrho = \frac{\hat{m}^2 - \varepsilon(1+\varepsilon)}{1+\varepsilon}$$

e_i = polinomials in ε

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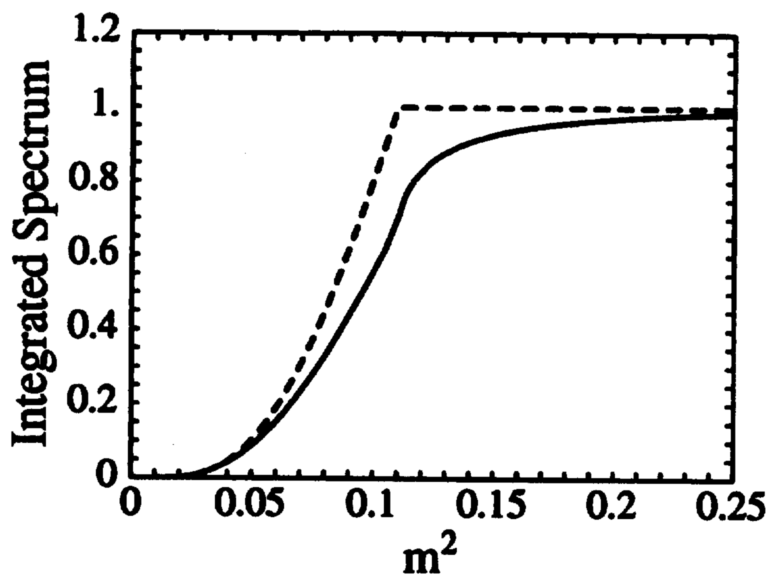


--- TREE LEVEL

— $O(\alpha_s)$

R[illegible] : There are kinematical regions where the non-perturbative effects are important

("Fermi motion" because they are linked to the b quark motion inside the hadron)
ACCMM, NP B208 (82) 365

WELL KNOWN EXAMPLE: ENDPOINT REGION IN THE CHARGED LEPTON ENERGY SPECTRUM

→ Kinematics is relative to the decay of a free quark ([illegible])

while it should be determined by the hadron decay ([illegible])

Non-perturbative effects can be resummed in a shape function $F(k_+)$ which is a universal feature of the B meson in all decays with massless particles in the final state ($B \to X_u l \nu$, $B \to X_s \gamma$)

THE PHYSICAL DECAY DISTRIBUTIONS ARE OBTAINED FROM A CONVOLUTION OF THE PREVIOUSLY COMPUTED SPECTRA WITH THE SHAPE FUNCTION

FEW NON PERTURBATIVE CORRECTIONS ARE KNOWN :

→ We know the moments of the shape function

(The reconstruction is not unique)

$$F(k_+) = N\,(1-x)^a\, e^{(1+a)x} \qquad x = \frac{k_+}{\bar\Lambda} \leq 1$$

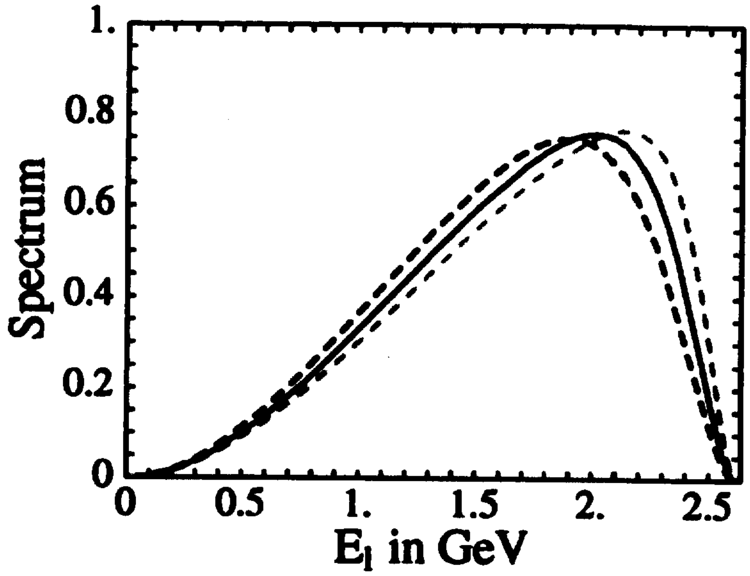
KAGAN, NEUBERT
EPJ C7 (99) 5

The first three moments are known: $A_{[illegible]}$ [illegible] → fixes N

$A_1 = 0$ → by construction

$A_2 =$ [illegible] → fixes a

[illegible]

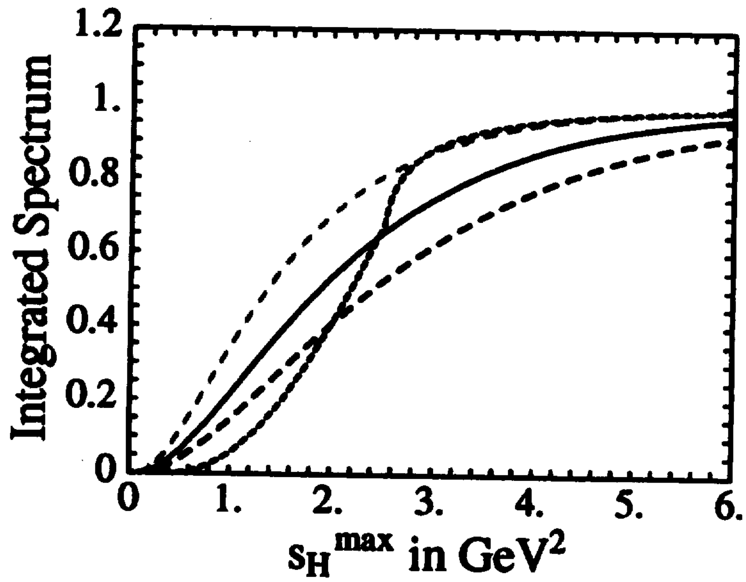


- - - $m_b = 4.65$ GeV

—— $m_b = 4.8$ GeV

- - - $m_b = 4.95$ GeV

FRACTION OF $B \to X_u \ell \nu$ EVENTS WITH HADRONIC INVARIANT MASS BELOW s_H^{max}



THE FRACTION OF EVENTS BELOW THE c THRESHOLD $s_H \leq 3.49\ GeV^2$ IS $(80 \pm 10)\%$

↳ THE CUT $s_H < M_D^2$ SHOULD BE THE BEST WAY TO DISCRIMINATE $b \to u$ FROM $b \to c$

CONCLUSIONS

The calculation to order α_s of $B \to X_u \ell \nu$ triple differential distribution allows to obtain WHATEVER differential distribution in the relevant kinematic variables (E_e, E_H, s_H)

$\hookrightarrow$ POSSIBILITY TO PUT ARBITRARY CUTS ON THE KINEMATIC VARIABLES

(particularly useful for the experimental analysis should be $\frac{d^2\Gamma}{dE_e\, d\hat{s}_H}$)

The physical spectrum is obtained by the convolution with a "shape function" which takes into account non perturbative effects.

RESULTS

$(30 \pm 5)\%$ of $B \to X_u \ell \nu$ events have hadronic energy below the charm threshold

$(80 \pm 10)\%$ have hadronic invariant mass below M_D^2

$\hookrightarrow$ DETERMINATION OF V_{ub}

WITH REDUCED THEORETICAL UNCERTAINTIES