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NEUTRAL KAON SYSTEM IN DENSE MATTER AND HEAVY-ION COLLISIONS

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Abstract

Above a critical matter density the propagating modes of the neutral kaon system are essentially eigenstates of strangeness, but below it they are almost complete eigenstates of CP. We estimate the real and imaginary parts of the energies of these modes and their mixing at all densities up to nuclear matter density 2×10^{14} g/cm^3. In a heavy ion collision the strong interactions create eigenstates of strangeness, and these propagate adiabatically until the density has fallen to the critical value, whereupon the system undergoes a sudden transition to (near) eigenstates of CP. We estimate the critical density to be 20 g/cm^3, and that this density will be reached about 2×10^5 fm/c after the end of the collision.

Neutral kaon systems are extremely interesting for a variety of reasons [1]. When they are created in a collision among hadrons or leptons they are essentially in eigenstates of strangeness because the strong and electromagnetic interactions conserve flavor. When they propagate freely in vacuum the weak interactions are operative and both C and P are violated. The eigenstates of the full Hamiltonian are then almost completely flavor mixed as short- and long-lived kaons:

$$
\begin{aligned}
|K_S\rangle &= \left[(1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle\right] / \sqrt{2(1+|\epsilon|^2} \\
|K_L\rangle &= \left[(1+\epsilon)|K^0\rangle + (1-\epsilon)|\bar{K}^0\rangle\right] / \sqrt{2(1+|\epsilon|^2} \,.
\end{aligned}
\tag{1}
$$

Here $|\epsilon| \approx 2\times 10^{-3}$ is the measure of CP violation. When a beam of long-lived kaons is sent through ordinary matter, short-lived kaons are generated due to the different interactions between the components of the former, namely K^0 and $\bar{K}^0$, and atomic nuclei. This is called kaon regeneration. The goal of this paper is to study the collective modes of propagation of the neutral kaons in *dense* matter, and to determine the fate of these modes after they are created in a collision between large nuclei at high energy.

We will use a strong interaction basis with

$$
\begin{aligned}
|K^0\rangle &= |d\bar{s}\rangle = |1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
|\bar{K}^0\rangle &= |\bar{d}s\rangle = |2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \,.
\end{aligned}
\tag{2}
$$

Because it is so small and plays no special role in our analysis we shall set $\epsilon = 0$. Then $|K_S\rangle$ and $|K_L\rangle$ are eigenstates of CP with eigenvalues $+1$ and -1, respectively.

First we do a relativistic analysis. Poles of the propagator determine the time evolution of small amplitude excitations. These are obtained from solutions to the equation

$$
\omega^2 - k^2 - \Pi_{vac}(\omega^2 - k^2) - \Pi_{mat}(\omega, k) = 0 \,, \tag{3}
$$

where the vacuum and matter contributions to the self-energy are indicated. Generally it is an excellent approximation to evaluate these 2×2 matrices on the mass shell. This means that Π_{vac} is a constant and Π_{mat} depends on the momentum k only. The usual analysis gives

$$
\Pi_{vac} = \begin{pmatrix} A & B \\ B & A \end{pmatrix}^2 \tag{4}
$$

where A and B are complex numbers. In terms of measurables they are [2]:

$$
\begin{aligned}
A &= m_K - i(\Gamma_S + \Gamma_L)/4 \\
B &= \Delta m/2 + i\Delta\Gamma \\
m_K &= (m_S + m_L)/2 = 497.67 \text{ MeV}
\end{aligned}
$$

$$\begin{aligned}
\Delta m &= 3.52 \times 10^{-12} \text{ MeV} \\
\Delta\Gamma &= \Gamma_S - \Gamma_L \\
\Gamma_S &= 7.38 \times 10^{-12} \text{ MeV} \\
\Gamma_L &= 1.27 \times 10^{-14} \text{ MeV} \\
\Delta m &= (0.478 \pm 0.003)\Delta\Gamma \,.
\end{aligned} \tag{5}$$

The matter contribution is diagonal in flavor and is expressed as

$$\Pi_{mat} = \begin{pmatrix} F & 0 \\ 0 & \bar{F} \end{pmatrix} . \tag{6}$$

In matter with an excess of baryons over antibaryons the strange and antistrange components, F and $\bar{F}$, are different. Of course, this is the origin of kaon regeneration in matter. One reason they are different is because the valence antiquark $\bar{d}$ in the $\bar{K}^0$ can annihilate on a valence quark d in the proton or neutron, producing a hyperon. This is not possible with a K^0.

Diagonalization of the equation

$$(k^2 + \Pi) \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \omega^2 \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \tag{7}$$

yields the energy eigenvalues

$$\omega_\pm^2 = k^2 + A^2 + B^2 + \frac{1}{2}(F + \bar{F}) \pm \sqrt{4A^2B^2 + \frac{1}{4}(F - \bar{F})^2} \tag{8}$$

and eigenstates

$$\beta_\pm = \left(-\chi \pm \sqrt{1 + \chi^2}\right) \alpha_\pm \tag{9}$$

where $\chi = (F - \bar{F})/4AB$. The states evolve in time as $\exp(-i\omega_\pm t)$. In baryon-free matter, even at finite temperature, there is no difference between F and $\bar{F}$ (from the point of view of the strong and electromagnetic interactions). Then, with the above conventions, $\beta_\pm = \pm\alpha_\pm$, and the upper sign corresponds to K_L and the lower sign to K_S. These are the familiar modes in vacuum except that the square of the mass is shifted by F, which generally has both real and imaginary parts.

At nuclear matter density, 0.155 nucleons/fm^3, the situation is very different. One should expect that $|F - \bar{F}|$ is only somewhat smaller than typical nuclear energies, implying something on the order of (100 MeV)^2. Since

$$2AB = m_K(\Delta m + i\Delta\Gamma/2) = (4.19 \times 10^{-5} \text{ MeV})^2 + i(4.28 \times 10^{-5} \text{ MeV})^2 \tag{10}$$

this means that $|\chi| >> 1$. In this case, the off-diagonal matrix elements in the self-energy, due to the weak interactions, are totally ignorable, and the energy eigenstates are also eigenstates of strangeness.

Let us now estimate $F - \bar{F}$ as a function of density. At low to moderate densities the self-energy may be expressed in terms of the scattering amplitude for kaons scattering on the constituents of the medium, here taken to be an equal mixture of protons and neutrons [3].

$$\begin{aligned} F(\omega, k) &= -4\pi \int \frac{d^3q}{(2\pi)^3} 4n_{FD}(q) \sqrt{\frac{s}{m_N^2 + q^2}} f(s) \\ &= -4\pi\rho_N \left(1 + \frac{m_K}{m_N}\right) \langle f \rangle \, . \end{aligned} \tag{11}$$

In the first line, $n_{FD}(q)$ is the Fermi-Dirac distribution for a nucleon with momentum q, the extra factor of 4 appears because of summation over spin and isospin of the nucleons, f is the forward scattering amplitude in the center-of-momentum frame, and s is the usual Mandelstam variable. The angular brackets in the second line denote the particular momentum averaging, and ρ_N is the spatial density of nucleons. If the nucleons are not distributed according to a Fermi-Dirac distribution then one ought to use the actual momentum distribution; this will affect the numerical value of $\langle f \rangle$ to some extent. Formula (11) is simply a version of the low density virial expansion familiar in statistical physics. Its applicability has been discussed in [3]. Essentially it requires that the average separation of nucleons be larger than both the real part of the forward scattering amplitude and the inverse of the average relative momentum between a nucleon and a kaon. One may think of this formula quantum mechanically as representing indices of refraction (real part) and attentuation (imaginary part). Indeed, it is only a variation of the usual formulae used in kaon regeneration sudies.

There is an identical expression for antikaons with $\bar{F}$ replacing F and $\bar{f}$ replacing f. The difference $\bar{f} - f$ has been estimated, or may be inferred, from several sources. For example, Eberhard and Uchiyama [4] calculated the real and imaginary parts of the forward scattering amplitudes for neutral kaons incident on proton and nuclear targets. These calculations were based on measured total cross sections for K^+ and K^- on protons and neutrons, on published values of K-nucleon elastic scattering, and charge symmetry. From their figures 2 and 3 we estimate that

$$\langle \bar{f} - f \rangle \approx 0.3(1 + i) \text{ fm} \, .$$

Here the averaging is for kaon momenta ranging from 0.3 to 1 GeV, relevant for the conditions in hot and dense nuclear matter created in a heavy ion collision. Shuryak and Thorsson [5] have calculated the scattering amplitudes for charged kaons incident on nucleons based on partial wave analyses of data. From their figures 1 and 2 we estimate that

$$\text{Re} \, \langle \bar{f} - f \rangle \approx 1.5 \text{ fm} \, .$$

Here the averaging is for $\sqrt{s}$ ranging between 1.5 to 1.8 GeV corresponding to the range of kaon momenta quoted above. (Unfortunately they have only displayed the real parts explicitly.) We do not know the reason for this discrepancy. Fortunately the precise numbers are totally irrelevant for the mixing of the neutral kaons in dense matter, since even the smallest estimate of the strong interaction induced self-energy overwhelms that due to the weak interactions. For example, using the estimate from Eberhard and Uchiyama, we obtain

$$F - \bar{F} \approx (150\,\mathrm{MeV})^2 (1+i) \left(\frac{\rho_N}{0.1/\mathrm{fm}^3} \right) , \tag{12}$$

which ought to be compared to eq. (10). The fact that the real part of $F - \bar{F}$ is positive implies that the nonrelativistic one-body potential, $U = F/2m_K$, is relatively more repulsive for kaons than for antikaons. This is quite natural for the reasons stated earlier. The fact that the imaginary part is positive is also to be expected because the imaginary part of the scattering amplitude and cross section are related by the optical theorem, $\sigma = (4\pi/k_{cm})\mathrm{Im}\, f$, and antikaons have a much bigger inelastic cross section with nucleons than kaons.

An alternative approach to the kaon self-energy at moderate to high density is to use an effective Lagrangian incorporating the relevant degrees of freedom and symmetries [6]. One finds that [7]

$$\begin{aligned} F(\omega, k) &= -\frac{\Sigma_{KN}}{f_\pi^2}\rho_S + \frac{3}{4}\frac{\rho_N}{f_\pi^2}\omega \\ \bar{F}(\omega, k) &= -\frac{\Sigma_{KN}}{f_\pi^2}\rho_S - \frac{3}{4}\frac{\rho_N}{f_\pi^2}\omega\,. \end{aligned} \tag{13}$$

Here Σ_{KN} is the kaon-nucleon sigma term, estimated to be of order 350 MeV. One distinquishes the scalar nucleon density, $\rho_S = \langle \bar{N} N \rangle$, from the conserved vector density, $\rho_N = \langle \bar{N} \gamma^0 N \rangle$, although they only begin to differ significantly above twice nuclear density. The difference in sign in the above equations results from the fact that the scalar density treats particles and antiparticles the same whereas the vector density distinguishes particles from antiparticles. Thus

$$\mathrm{Re}\,(F - \bar{F}) = \frac{3}{2}\frac{\rho_N}{f_\pi^2}\omega = (260\,\mathrm{MeV})^2 \left(\frac{\omega}{m_K} \right) \left(\frac{\rho_N}{0.1/\mathrm{fm}^3} \right) \tag{14}$$

which lies between the estimates from Eberhard-Uchiyama and Shuryak-Thorsson. So far, to our knowledge, no one has calculated the imaginary part in this approach.

What happens to the neutral kaons after production in a heavy ion collision? Based on the above analysis of $F - \bar{F}$ we see that the dimensionless ratio χ is proportional to the nucleon density and has a very small imaginary part relative to the positive real part. Using the results of Eberhard-Uchiyama we get

$$\chi = 6.3 \times 10^{12} \left(\frac{\rho_N}{0.1/\mathrm{fm}^3} \right) . \tag{15}$$

From eq. (9) one finds that $|\alpha_+| \gg |\beta_+|$ and $|\beta_-| \gg |\alpha_-|$. Hence the eigenstates are $|+\rangle = |1\rangle$ and $|-\rangle = |2\rangle$ with corrections of order $1/\chi$. As the matter expands the density decreases until χ becomes very small. Then the eigenstates are the vacuum ones, $|K_L\rangle$ and $|K_S\rangle$. *If* this evolution is adiabatic it would have extremely interesting consequences. A K^0 produced in the collision would evolve into a K_L (rather than into an equal mixture of K_L and K_S), and a $\bar{K}^0$ would evolve into a K_S. Based on valence quark counting [8] one expects that

$$\left(\frac{K_L}{K_S}\right)_{\rm observed} = \left(\frac{K^0}{\bar{K}^0}\right)_{\rm in\ matter} = \left(\frac{K^+}{K^-}\right)\left(\frac{\pi^-}{\pi^+}\right) . \tag{16}$$

That is, the observed ratio of K_L to K_S should be equal to the ratio of K^0 to $\bar{K}^0$ produced in dense matter, which then is equal to the ratio of charged kaons times the ratio of charged pions. The latter product of ratios should not change during the very late dilute expansion phase of a heavy ion collision. It has been measured [9] in central Au-Au collisions at the AGS at Brookhaven National Laboratory ($E_{\rm beam}$ = 11 GeV/nucleon) with the value 6. It has also been measured [10] in central Pb-Pb collisions at the SPS at CERN ($E_{\rm beam}$ = 160 GeV/nucleon) with the value 2. Hence, contrary to all other high energy accelerator experiments, the ratio of long-lived to short-lived kaons would be far from one! But does the neutral kaon system evolve adiabatically?

To answer this question, consider what happens in the kaon's own frame of reference. As time goes on, the surrounding density of matter decreases as $1/t^3$. This naturally follows from dimensional analysis, and it also emerges from a calculation of the free expansion of an ideal relativistic gas [11]. A good estimate of the local nucleon density is $\rho_N(t) = \rho_0(t_0/t)^3$. For numerical estimates we shall use $\rho_0 = 0.1$ nucleons/fm^3 and $t_0 = 10$ fm/c, which are characteristic of central collisions between nuclei of atomic number near 200 at the AGS and the SPS [12]. Furthermore, the average relative speed in an encounter between a kaon and a nucleon will decrease with time. The reason is that the fireball has an initial radius of $R \approx 10$ fm or so. After an elapsed time $\Delta t = t - t_0$ nucleons with a relative speed greater than $R/\Delta t$ are unlikely to ever encounter the kaon. If we are interested in what happens at late times in the expansion, when the strong and weak interactions affecting the neutral kaon system become comparable, the relevant averaged scattering amplitudes are not those discussed above. Rather, one can use only the s-wave scattering lengths. A compilation of data yields [13]

$$\bar{f}_0 - f_0 = 0.1 + 0.6i \text{ fm} , \tag{17}$$

with an uncertainty of about 0.1 fm in both the real and imaginary parts. This all leads to the difference of one-body potentials being

$$U(t) - \bar{U}(t) = (7.5 + 45i)\left(\frac{10 \text{ fm/c}}{t}\right)^3 \text{ MeV} . \tag{18}$$

The problem can be reduced to a two-level Schrödinger equation with a time-dependent complex potential. Taking out the kaon mass leads to

$$i\frac{\partial}{\partial t}\psi(t) = H(t)\psi(t) = \begin{pmatrix} -\frac{i}{4}(\Gamma_S+\Gamma_L)+U(t) & \frac{1}{2}\Delta m+\frac{i}{4}\Delta\Gamma \\ \frac{1}{2}\Delta m+\frac{i}{4}\Delta\Gamma & -\frac{i}{4}(\Gamma_S+\Gamma_L)+\bar{U}(t) \end{pmatrix}\psi(t)\,. \qquad (19)$$

The instantaneous energy eigenvalues are

$$E_\pm = \frac{1}{2}(U+\bar{U}) - \frac{i}{4}(\Gamma_S+\Gamma_L) \pm \sqrt{\left(\frac{1}{2}\Delta m + \frac{i}{4}\Delta\Gamma\right)^2 + \frac{1}{4}(U-\bar{U})^2}\,. \qquad (20)$$

The instantaneous eigenstates are given by eq. (9). The exact solutions can be expanded in terms of these, with time-dependent coefficients $a_\pm(t)$, as

$$\psi(t) = a_+(t)|+,t\rangle + a_-(t)|-,t\rangle \qquad (21)$$

where

$$|\pm,t\rangle = \begin{pmatrix} \alpha_\pm(t) \\ \beta_\pm(t) \end{pmatrix} \exp\left\{-i\int_0^t E_\pm(t')dt'\right\}\,. \qquad (22)$$

There exists a critical density ρ_c defined by the condition that $|\chi| = 1$. With the above input its numerical value is $\rho_c = 1.1\times10^{-14}$ nucleons/fm^3 or about 19 g/cm^3. For densities much greater than ρ_c the strong interactions dominate and the eigenstates of the system are eigenstates of strangeness. For densities much less than ρ_c the weak interactions dominate and the eigenstates of the system are eigenstates of CP. The critical region may be conservatively defined as $8 > |\chi| > 1/8$ or $8 > \rho_N/\rho_c > 1/8$.

The picture that emerges is as follows. For $t < 10^5$ fm/c the matter expands freely, with frequent (on the time scale of $1/\Delta m$ and $1/\Delta\Gamma$) interactions of the kaons with the nucleons maintaining the kaons in eigenstates of strangeness, namely, K^0 and $\bar{K}^0$. For $10^5 < t < 4\times10^5$ fm/c the system is in a transition region where $|\chi| \approx 1$, meaning that interactions with the nucleons are comparable in strength with the internal weak interactions of the kaons. In this region there is nothing to prevent transitions between the states. For $t > 4\times10^5$ fm/c the matter is so dilute that the internal weak interactions of the kaons dominate and they propagate essentially as in vacuum, namely, as K_L and K_S. Thus the states of the neutral kaon system evolve adiabatically at both high and low density. The time to pass through the transition region, about 3×10^5 fm/c, is so short compared to the natural oscillation time of the neutral kaons, $1/\Delta m = 5.6\times10^{13}$ fm/c, that this transition may be treated with the sudden approximation. It is at the time $t \approx 2\times10^5$ fm/c and the density $\rho_c \approx 20$ g/cm^3 that the strangeness eigenstates K^0 and $\bar{K}^0$ decompose into the eigenstates K_L and K_S of CP. This may be shown mathematically from the equations of motion of $a_\pm$.[1] As a consequence of this sudden transition the

[1]However, the analysis is more complicated than that given in textbook discussions of time-dependent perturbation theory and the adiabatic and sudden approximations because the Hamiltonian is not Hermitian, and the instantaneous eigenstates are not orthogonal in the transition region.

observed ratio of long-lived to short-lived kaons should be 1. Finally we should remark that the probability for a kaon to decay before the transition region is reached is negligible because the lifetime of even K_S is much greater than the time to reach the critical density.

The phenomenon described here does not happen in elementary particle collisions such as e^+e^-, $p\bar{p}$, and pp because the net baryon number is either zero or negligibly small. It should be noted that the critical density of 20 g/cm^3 is characteristic of heavy metals[2], perhaps opening the window on new types of experiments with neutral kaons.

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