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A NEW CALCULATION FOR DIRECT CP VIOLATION : ε'/ε *

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ABSTRACT

The method and results of a new calculation for the hadronic matrix elements of the decays $K \to \pi\pi$ are described. The article concentrates on amplitudes relevant for the CP-violating parameter $(\varepsilon'/\varepsilon)$ and summarizes the predictions for values of this parameter.

1. Introduction and General Framework

An outstanding problem of the electroweak theory is the origin of CP violation and the prediction of CP-asymmetries which occur in several decays. A crucial parameter in K-meson decays is $(\varepsilon'/\varepsilon)$, which in the superweak theory is predicted to be zero. In the standard model, on the other hand, the ratio is predicted to have a well defined range of values. In this talk, I will describe a new calculation for the ratio and try to point out the uncertainties which enter the calculation. Some of the uncertainties come from the experimental accuracy of physical parameters and will improve as the data become more precise. Others are related to basic theoretical issues where improvements are more difficult. In any case the subject is very interesting, because we expect new experimental results[1] with an ultimate accuracy of $\pm 2 \times 10^{-4}$.

Direct CP violation measures relative phases of the decay amplitudes for

$$K^0 \to \pi^0\pi^0 \qquad \text{and} \qquad K^0 \to \pi^+\pi^- .$$

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The two pions in these decays can be in two isospin states, $I = 0$ and 2, and are described by the amplitudes A_0 and A_2, respectively. The amplitudes acquire phases through final-state strong interactions and also through the couplings of weak interactions. We can use Watson's theorem to write them as

$$\langle \pi\pi, I|Hw|K^0\rangle = A_I e^{i\delta_I} \quad (1)$$

$$\langle \pi\pi, I|Hw|\bar{K}^0\rangle = A_I^* e^{i\delta_I} \quad (2)$$

with δ_I being a phase of strong origin, which is extracted from $\pi - \pi$ scattering. The remaining amplitude A_I contains a phase of weak origin to be defined as

$$A_I = |A_I| e^{i\theta_I} \,. \quad (3)$$

The parameter of direct CP violation is defined as

$$\frac{\varepsilon'}{\varepsilon} = \frac{\omega}{\sqrt{2}\,|\varepsilon|} \left[\frac{Im\, A_2}{Re\, A_2} - \frac{Im\, A_0}{Re\, A_0}\right] \quad (4)$$

with $\omega = |\frac{A_2}{A_0}| = \frac{1}{22.2}$. It is now necessary to calculate the two amplitudes including their weak phases.

The theoretical calculation is based on the effective $\Delta S = 1$ Hamiltonian derived from QCD

$$\mathcal{H}_{\text{eff}}^{\Delta S=1}(\mu < m_c) = \frac{G}{\sqrt{2}} \lambda_u \sum_{i=1}^{8} C_i(\mu)\, Q_i(\mu) \quad \text{with} \quad (5)$$

$$C_i(\mu) = z_i(\mu) + \tau\, y_i(\mu)\,, \qquad \tau = -\frac{\lambda_t}{\lambda_u} \qquad \text{and} \qquad \lambda_q = V_{qs}^* V_{qd}\,. \quad (6)$$

The operators $Q_1, \ldots, Q_8$ are well-known and will not be given here. The factors $C_i(\mu)$ are Wilson coefficients, which have been calculated in next-to-leading order in QCD by two groups[2,3]. The terms with the z_i's contribute to the real parts of the amplitudes A_0 and A_2 ($\Delta I = 1/2$ rule). The y_i's, on the other hand, contribute to the imaginary parts.

A second group of parameters are the CKM matrix elements whose values have improved. We have also included a scale dependence in the operator $Q_i(\mu)$, because when we write the four-quark operators in terms of pseudo-scalar mesons, a μ-dependence is introduced. In particular the bosonized expressions for the operators

$$Q_6 = -8 \sum \bar{s}_L\, q_R\, \bar{q}_R\, d_L \quad (7)$$

and $$Q_8 = -8 \sum_q \frac{3}{2} e_q\, \bar{s}_L\, q_R\, \bar{q}_R\, d_L \qquad \text{with} \qquad e_q = (2/3, -1/3, -1/3) \quad (8)$$

are proportional to R^2, with $R = 2m_K^2/(m_s + m_d)$, which cancels to a large extent the scale dependence of the Wilson coefficients $y_6(\mu)$ and $y_8(\mu)$.[4] This means that the remaining scale dependence of the matrix elements for these two operators must be small in order to match with the residual dependence of the products $y_j\langle Q_j\rangle$ with $j = 6$ and 8. This good property holds for the density operators and suggests that the accuracy of the results may depend on the operator under consideration.

The imaginary parts of the amplitudes occuring in Eq. (4) are those produced by the weak interaction. Thus we obtain amplitudes

$$Im\, A_I = \frac{G_F}{\sqrt{2}}\, Im\, \lambda_t |\sum_i y_i(\mu)\, \langle Q_i\rangle| \,. \tag{9}$$

Since the phase originating from the strong interactions is already extracted in Eq. (1), absolute values for the $\langle Q_i\rangle_I$ should be taken. Collecting all terms together we arrive at the general expression

$$\frac{\varepsilon'}{\varepsilon} = \frac{G_F}{2}\, \frac{\omega}{|\varepsilon| Re\, A_0}\, Im\, \lambda_t \left[\Pi_0 - \frac{1}{\omega}\,\Pi_2\right] , \tag{10}$$

$$\text{with} \qquad \Pi_0 = |\sum_i y_i(\mu)\, \langle Q_i\rangle_0|\, (1 - \Omega_{\eta+\eta'}) \tag{11}$$

$$\Pi_2 = |\sum_i y_i(\mu)\, \langle Q_i\rangle_2| \,, \tag{12}$$

where $\Omega_{\eta+\eta'} \sim 0.25 \pm 0.05$ takes into account the effect of the isospin breaking in the quark masses ($m_u \neq m_d$).[5] We have written Eq. (10) as a product of factors in order to emphasize the importance and uncertainty associated with each of them. The first factor contains known parameters and takes the numerical value $G_F\omega/(2|\varepsilon|Re\, A_0) =$ 346 GeV^{-3}.

The second factor in Eq. (9) originates from the CKM matrix. In the Wolfenstein parametrization

$$Im\, \lambda_t = Im(V_{ts}^* V_{td}) = V_{us}|V_{cb}|^2 \eta \,. \tag{13}$$

A recent estimate[6] gives the range

$$Im\, \lambda_t = (1.38 \pm 0.33) \cdot 10^{-4} \,. \tag{14}$$

We shall use this value but one must keep in mind it may still change somewhat[7]. The remaining factors are Π_0 and Π_2, which according to Eqs. (11) and (12) depend on the products of Wilson coefficients times the matrix elements. The lowest order values for the coefficients of the two dominant terms, y_6 and y_8, are given in Table (1) as functions of the scale μ and for two values of $\Lambda_{\text{QCD}} = 245$ and 325 MeV. More details on the Wilson coefficients can be found in a forthcoming paper.[10]

μ	0.6 GeV	0.7 GeV	0.8 GeV	0.9 GeV	1.0 GeV	for
y_6	-0.133	-0.116	-0.106	-0.098	-0.092	Λ_{QCD} =
y_8/α	0.217	0.180	0.155	0.138	0.125	245 MeV
y_6	-0.187	-0.154	-0.135	-0.122	-0.113	Λ_{QCD} =
y_8/α	0.324	0.249	0.206	0.178	0.158	325 MeV

Table 1. Wilson coefficients to lowest order for two values of Λ_{QCD}.

Taking all results together, the remaining task is the calculation of the matrix elements $\langle Q_i \rangle_0$ and $\langle Q_i \rangle_2$.

2. Hadronic Matrix Elements

The matrix elements for <u>all</u> the operators are described in two recent papers[8,9] and a forthcoming paper.[10] They are motivated by the $1/N_c$ expansion introduced in ref.[11]. The calculation involves a twofold expansion in powers of external momenta p and in powers of $1/N_c$. The latter expansion corresponds to chiral loops, which are required by unitarity and can be large.

The momentum expansion for the current $\otimes$ current operators are of order p^2, p^4 and higher. For the density $\otimes$ density operators, they are of order p^0, p^2 and higher. Since the latter operators are dominant in ε'/ε and since the space of this report is limited, I will describe results for the density operators; at the same time I emphasize that all operators are included in the numerical results.

The above classification of the expansion in a series is shown schematically in Table 2:

tree–level	p^0	p^2	p^4	...
1–loop	p^0/N_c	p^2/N_c	...	
2–loop	p^0/N_c^2	...	...	

Table 2

The calculation for the terms $\mathcal{O}(p^0)$, $\mathcal{O}(p^2)$ and $\mathcal{O}(p^0/N_c)$ has already been completed and published.[8] The calculation of $\mathcal{O}(p^2/N_c)$ was completed recently.[10]

There are several important properties in the results which I summarize.

1. The diagrams for the corrections are naturally classified into two categories: factorizable and non–factorizable.[11,8]

    a) It has been shown explicitly[8] that all divergent corrections from factorizable diagrams are absorbed into renormalization of masses, coupling constants and wave functions. Thus for the factorizable sector the definition of the low–energy parameters accounts for the contribution of states beyond the cut–off Λ_c.

b) The non–factorizable diagrams to $\mathcal{O}(p^0/N_c)$ are logarithmically divergent in Λ_c. Thus we expect a good matching with the cut–off Λ_c of the chiral theory equated (identified) with the scale μ of QCD.

2. To $\mathcal{O}(p^2/N_c)$ there are both Λ_c^2 and $ln\,\Lambda_c$ divergences which are needed to smoothen out the μ–dependence.

3. Finally, the $Im\,A_0$ and $Im\,A_2$ involve Wilson coefficients with charm– and top–quark contributing in the intermediate states, so that the short distance estimates should be reliable.

I will now present numerical results for the matrix elements of density operators. Some typical results for the operators $\langle Q_8\rangle_2$ and $\langle Q_6\rangle_0$ are shown in Table (3). The tree level contribution for $\langle Q_8\rangle_2$ is dominated by the $\mathcal{O}(p^0)$ term, which means that the series in powers of momenta converges. The one–loop contribution to $\mathcal{O}(p^0)$ is large. The imaginary part from the loop is negative and sizable. The real part, to this order, is even bigger and has the opposite sign relative to the tree term. This brings a substantial decrease in the absolute value of the matrix element. The $\mathcal{O}(p^2/N_c)$ term is again moderate. The results for $\langle Q_8\rangle_2$ show that the loop corrections are important.

	p^0	p^2	p^0	p^2
tree	56.4	-0.50	0.0	-35.2
one–loop	$-28.8 - 11.5i$	$8.9 + 1.3i$	$5.8 + 0.0i$	$-15 - 16.3i$
	$\langle Q_8\rangle_2$	$\langle Q_8\rangle_2$	$\langle Q_6\rangle_0$	$\langle Q_6\rangle_0$

Table 3. Numerical values for the matrix elements $\langle Q_8\rangle_2$ and $\langle Q_6\rangle_0$ in units of R_K^2 MeV and for $\Lambda_c = 0.8$ GeV.

The gluon penguin term, $\langle Q_6\rangle_0$, is particular because the tree level term of $\mathcal{O}(p^0)$ vanishes. There was suspicion that the loop–term of $\mathcal{O}(p^0/N_c)$ may be sizable, but explicit calculation showed that it is relatively small.[8] We can understand this result, because by unitarity arguments the imaginary part of this term must be zero, as included in Table 3. The next order tree term $\mathcal{O}(p^2)$ gives the dominant contribution.

The one–loop contribution to this term has sizeable imaginary and real parts. The real part is a sensitive function of the cut-off and is preliminary. The largest corrections in $\langle Q_6 \rangle_0$ and $\langle Q_8 \rangle_2$ are unitarity corrections from $\pi - \pi$ rescattering.[12,13]

3. The Parameter $(\varepsilon'/\varepsilon)$

The interested reader can use formulas and numerical values of the previous sections to calculate the ratio ε'/ε. I have already mentioned that the operators Q_6 and Q_8 deliver the dominant contribution to this ratio. In fig. 1 we show the contributions of the operators Q_1 to Q_8 to the parameter $(\varepsilon'/\varepsilon)$ separated in each case into isospin amplitudes $I = 0$ and 2. The dominance of $\langle Q_8 \rangle_2$ and $\langle Q_6 \rangle_0$ is apparent, in spite of the fact that $\Omega_{\eta+\eta'} = 0.25$ has already been subtracted, according to Eq. (11), from the $\langle Q_6 \rangle_0$ matrix element. A second property is that the sum of all the operators is positive. This is a general property as will be discussed below. As mentioned in the

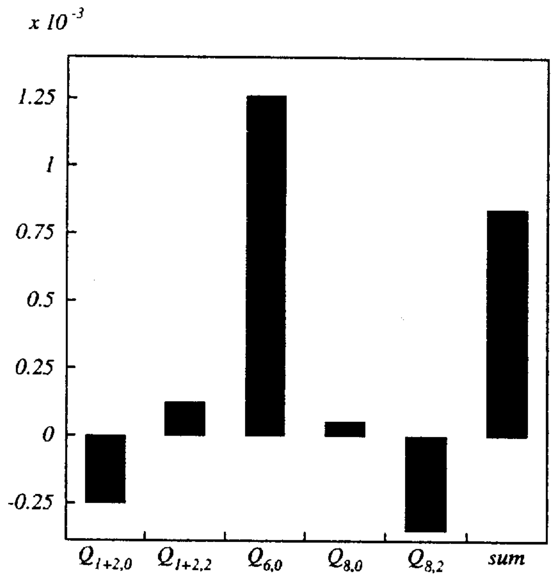

Figure 1: The isospin $I = 0$ and $I = 2$ contributions to ε'/ε of the operators for $\Lambda_{QCD} = 325$ MeV, $m_s(1\text{ GeV}) = 175$ MeV, $\Omega_{\eta+\eta'} = 0.25$, $Im\,\lambda_t = 1.38 \cdot 10^{-4}$ and $\Lambda_c = 700$ MeV.

caption of figure 1, there are five quantities to vary. In order to study the sensitivity of the results, we selected $Im\,\lambda_t = 1.38 \times 10^{-4}$ and varied the other parameters within the ranges indicated in figure 2. We use three values for Λ_{QCD} and the range $600 \leq \Lambda_c \leq 1000$ MeV. We consider two values for $\Omega_{\eta+\eta'}$ and two values for $m_s = 150$ and 175 MeV. The predictions of the standard model for the parameter $(\varepsilon'/\varepsilon)$ range between

$$5 \times 10^{-4} \leq (\varepsilon'/\varepsilon) \leq 22 \times 10^{-4}\,. \tag{15}$$

It is evident that the ratio is positive and stable as a function of $\Lambda_c = \mu$. To obtain smaller values we must increase $\Omega_{\eta+\eta'}$, which I personally consider unreasonable. Higher values of ε'/ε require smaller values of m_s (1 GeV), as the ratio scales like $(1/m_s)^2$.

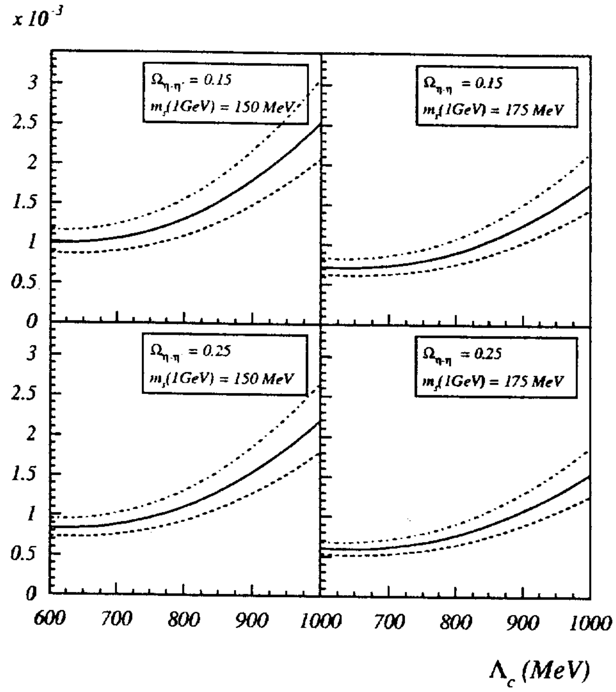

Figure 2: ε'/ε in units of 10^{-3} for various values of the parameters for $\Lambda_{\rm QCD} = 405$ (— · —), 325 (——) and 245 (- - -) MeV.

A detailed comparison with other articles will be done elsewhere.[10] I only mention here that the comparisons made in previous articles[8,?] are still valid.

In conclusion, the calculation of direct CP–violation in the standard model involves many operators and requires a careful study for all of them. The calculation I described is the most sophisticated to this date and leads to a definite range which will become more accurate, as the input parameters are better understood and determined.

Note added in proof: At the conference Drs. M. Pang (KTeV) and P. Cenci (NA48) presented the status of their experiments and reported that they will reach an accuracy of $\pm 2 \times 10^{-4}$. On February 24, 1999, the KTeV collaboration announced a new result: $(\varepsilon'/\varepsilon) = (28 \pm 4) \times 10^{-4}$. All experimental results are now in agreement with the range of values predicted by our new calculation and agree with the earlier predictions of ref. [12].

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