

$B^0 - \bar{B}^0$ oscillations and measurements of $|V_{ub}|/|V_{cb}|$ at LEP

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February 4, 1999

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Abstract

In this paper a review of the LEP analyses on $B^0 - \bar{B}^0$ oscillations and on the measurement of $|V_{ub}|/|V_{cb}|$ is presented. These measurements are of fundamental importance in constraining the ρ and η parameters of the CKM matrix. A review of the current status of the V_{CKM} matrix determination is also given.

Introduction

The data registration at the Z^0 pole has stopped at the end of 1995. The four LEP experiments (ALEPH, DELPHI, L3 and OPAL) have collected about 4M hadronic Z^0 decays per experiment.

In the past three years, the quality of the data analysis has continuously improved, thanks to a better understanding of the behaviour of all components of the detector. At the same time, new ideas, and then, new analyses have been tried. A more performant statistical treatment of the information has been also developed. As a result, the precision on the Δm_d parameter has been improved and above all, the sensitivity for the Δm_s parameter has been tremendously increased. The new and precise LEP analyses on $|V_{ub}|/|V_{cb}|$ are also a consequence of these improvements. Many analyses described in this paper have been presented at the last '98 Summer Conferences and are still preliminary. This paper is organized as follows. The first sections are dedicated to the oscillations and $|V_{ub}|/|V_{cb}|$ analyses. In the last section the present status of the V_{CKM} matrix is given with a special emphasis placed on the impact of the measurements presented in this paper.

The oscillation analyses

The probability that a B^0 meson oscillates into a \bar{B}^0 or stays as a B^0 is given by:

$$P_{B_q^0 \rightarrow B_q^0(\bar{B}_q^0)} = \frac{1}{2} e^{-t/\tau_q} (1 \pm \cos \Delta m_q t) \quad (1)$$

where the effect of CP violation has been neglected. τ_q is the lifetime of the B_q^0 meson, $\Delta m_q = m_{B_1^0} - m_{B_2^0}$ is the mass difference between the two mass eigenstates¹ and gives the period of the time oscillations (the effect of a lifetime difference between the two states has been also neglected).

The Standard Model predicts:

$$\Delta m_d \propto A^2 \lambda^6 [(1 - \rho)^2 + \eta^2] f_{B_d}^2 B_d; \quad \Delta m_s \propto A^2 \lambda^4 f_{B_s}^2 B_s \quad (2)$$

The difference in the λ dependence of these expressions ($\lambda \sim 0.22$) implies that $\Delta m_s \sim 20 \Delta m_d$. It is then clear that a very good proper time resolution is needed to measure the Δm_s parameter. A time dependent study of $B^0 - \bar{B}^0$ oscillations requires:

- the measurement of the decay proper time,
- to know if a B^0 or a \bar{B}^0 decays at $t = t_o$ (decay tag)
- to know if a b or a \bar{b} quark has been produced at $t = 0$ (production tag).

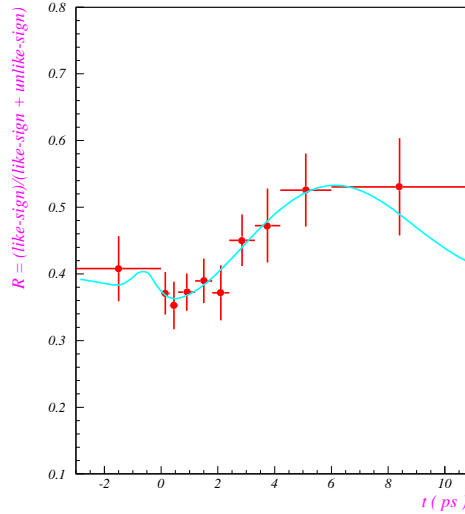


Figure 1: The plot shows the time dependence behaviour of the $B_d^0 - \bar{B}_d^0$ oscillation. The points with error bars are the data. The curve shows the result of the fit using $\Delta m_d = 0.47 \text{ ps}^{-1}$.

The precision on the Δm measurement is given by the following relation:

$$\text{error} = \left(\sqrt{N} f_{B_{d(s)}^0} (2\varepsilon_1 - 1)(2\varepsilon_2 - 1) e^{-\left(\frac{\Delta m_{d(s)} \sigma_t}{2}\right)^2} \right)^{-1} \quad (3)$$

¹ Δm_q is usually given in ps^{-1} . 1 ps^{-1} corresponds to $6.58 \cdot 10^{-4} \text{ eV}$.

where N is the total number of events in the sample; $f_{B_{d(s)}^0}$ is the fraction of events in which a $B_{d(s)}^0$ meson has been produced; $\varepsilon_2, \varepsilon_1$ are the tagging purities at the decay and production times respectively, defined as $\varepsilon = \frac{N_{\text{right}}}{N_{\text{right}} + N_{\text{wrong}}}$, where $N_{\text{right}}(N_{\text{wrong}})$ are the numbers of correctly (incorrectly) tagged events and σ_t is the proper time resolution given, approximately, as $\sigma_t = \sqrt{\left(\frac{m^2}{p^2}\right) \sigma_L^2 + \left(\frac{\sigma_p}{p}\right)^2 t^2}$, where σ_L and σ_p are the decay length and the momentum resolutions respectively.

Δm_d measurements

A lot of analyses have been performed since 1994. A typical time distribution is shown in Figure 1. The time dependence behaviour with frequency $\Delta m_d \sim 0.470 \text{ ps}^{-1}$, for the $B_d^0 - \bar{B}_d^0$ oscillation is clearly visible. This will be a textbook plot ! The present summary of the results on Δm_d , as given by [1], is shown in Figure 2. Combining LEP/CDF and SLD measurements it follows that:

$$\Delta m_d = (0.477 \pm 0.017) \text{ps}^{-1} \quad (4)$$

Δm_d is known with a precision of 3.4% relative error.

Analyses on Δm_s

Four types of analyses have been performed.

Table 1: The characteristics of the different analyses are given in terms of statistics (N), B_s^0 purity (f_{B_s}), tagging purity at the production and decay time ($\varepsilon_1, \varepsilon_2$) and time resolution in the first pico-second

Analysis	N(events)	f_{B_s}	ε_1	ε_2	$\sigma_t(t < 1\text{ps})$
Inclusive lepton	~ 50000	$\sim 10\%$	$\sim 70\%$	$\sim 90\%$	$\sim 0.25 \text{ ps}$
$D_s^\pm h^\mp$	~ 3000	$\sim 15\%$	$\sim 72\%$	$\sim 90\%$	$\sim 0.22 \text{ ps}$
$D_s^\pm \ell^\mp$	~ 400	$\sim 60\%$	$\sim 78\%$	$\sim 90\%$	$\sim 0.18 \text{ ps}$
Exclusive B_s^0	~ 25	$\sim 70\%$	$\sim 78\%$	$\sim 100\%$	$\sim 0.08 \text{ ps}$

For all of them, the latest analyses make use of the combined tag method for tagging a b or a \bar{b} at production time. At LEP, the produced b and \bar{b} quarks fragment independently and the events can be divided in two separate hemispheres. If the measurement of the proper time is performed in one of those (same hemisphere), the other (opposite hemisphere) can be used to determine if a b or a \bar{b} quark was produced in that hemisphere. Several variables are considered in the opposite hemisphere:

- $Q = \frac{\sum_{i=1}^n q_i (\vec{p}_i \cdot \vec{e}_S)^{0.6}}{\sum_{i=1}^n (\vec{p}_i \cdot \vec{e}_S)^{0.6}}$ the hemisphere charge, defined as the charge of all (n) charged tracks (q_i) present in the hemisphere, weighted by their momentum (p_i) projected along the thrust axis (\vec{e}_S) with a chosen value for the exponent (0.6),

- the hemisphere charge, considering only identified kaons,
- the charge of primary and secondary vertices,
- the presence of high p_t leptons.

The use of these variables allow to have a tagging purity of the order of 70%.

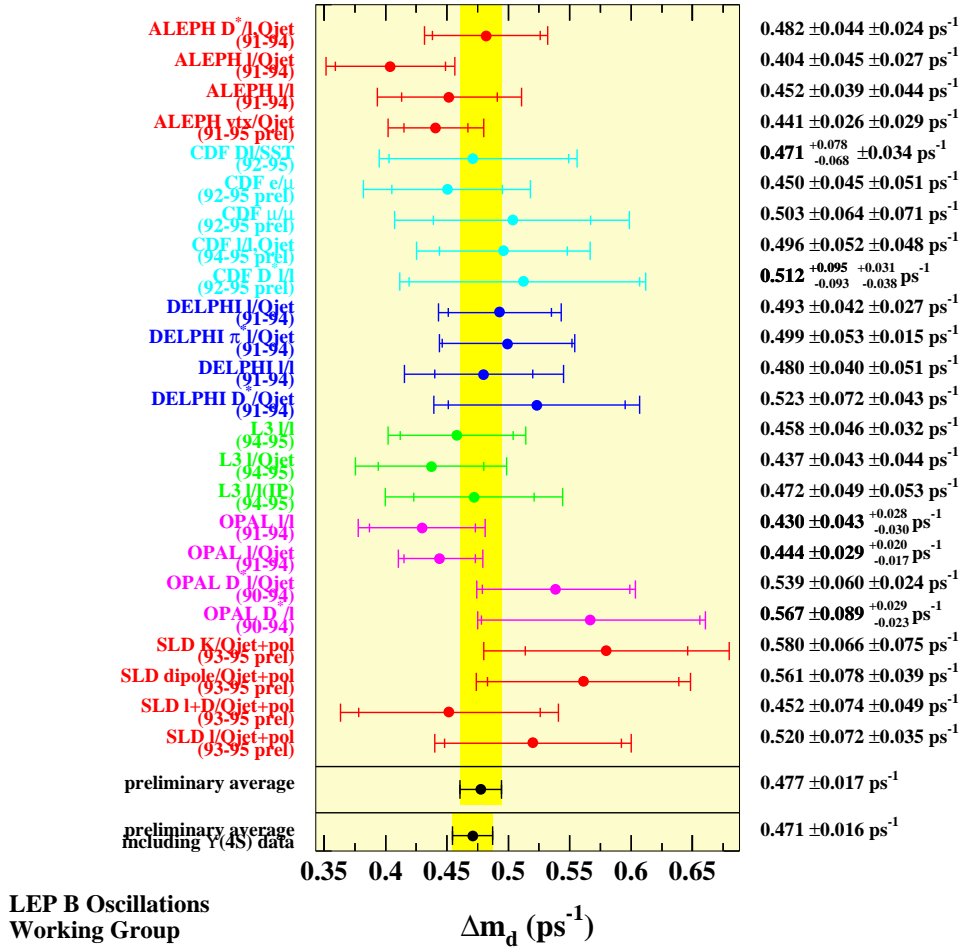


Figure 2: Summary of the Δm_d results from the LEP, SLD and CDF Collaborations are given. Details on how the different results have been combined are given in [1].

Tracks in the same hemisphere can be used also. This procedure is peculiarly clean if all the tracks from the B_s^0 have been reconstructed (as for $D_s^\pm \ell^\mp$ and exclusive B_s analyses). In this case, tracks from the B_s^0 decay can be removed and the others, coming from the primary vertex can be used. The addition of informations from the same hemisphere allows to reach a tagging purity of 74%. Finally the use of all these informations on an event by event basis gives a purity of 78%.

The tagging of a B or a \bar{B} meson at decay time depends on the specific analysis and will be given in the following. Before describing the different analyses, the method used to measure or put a limit on Δm_s is briefly discussed.

The amplitude method

The method used to measure or to put a limit on Δm_s consists in modifying equation 1 in the following way: $1 \pm \cos \Delta m_s t \rightarrow 1 \pm A \cos \Delta m_s t$. A and σ_A are measured at fixed values of Δm_s instead of Δm_s itself. In case of a clear oscillation signal, at given Δm_s , the value of the amplitude is compatible with $A = 1$ for this Δm_s and with $A = 0$ elsewhere. With this method it is also easy to set a limit. The values of Δm_s excluded at 95% C.L. are those satisfying the condition $A(\Delta m_s) + 1.645 \sigma_A(\Delta m_s) < 1$.

With this method, it is easy to combine different experiments and to treat systematic uncertainties in an usual way since, at each value of Δm_s , a value for A with a gaussian error σ_A , is measured. Furthermore, the sensitivity of the experiment can be defined as the value of Δm_s corresponding to $1.645 \sigma_A(\Delta m_s) = 1$ (for $A(\Delta m_s) = 0$, namely supposing that the “true” value of Δm_s is well above the measurable value of Δm_s). The sensitivity is the limit which would be reached in 50% of the experiments.

The inclusive lepton/combined tag analysis

This analysis uses high p_t leptons which are mainly coming from direct b semileptonic decays ($b \rightarrow \ell$). The sign of the lepton tags the B_s^0 at decay time. The initial sample consists in 80% leptons from B decays (and among those 90% $b \rightarrow \ell$ (direct) and 10% $b \rightarrow c \rightarrow \bar{\ell}$ (cascade)) and of 20% leptons from charm decays or misidentification. The events $b \rightarrow c \rightarrow \bar{\ell}$ give the wrong tag for the \bar{B}_s^0 meson at decay time.

To reconstruct a B decay proper time, algorithms have been developed which aim at identifying charged (neutral) tracks which are more likely to come from the B_s^0 decays. As result, in more than 50% of the cases, the error on the decay length is $\sigma_L \sim 250 \mu m$ and the relative error on the B energy is better than 10%, resulting in an error on the proper time of the order of 0.25 ps in the first pico-second.

A second crucial point for this analysis consists in trying to increase the B_s^0 purity of the sample (the natural B_s^0 purity of b events is around 10%) and to reduce the contribution from cascade decays. To enrich the sample in direct b semileptonic decays and, among those, in events coming from B_s^0 decays, several variables have been used as the momentum and transverse momentum of the lepton, the impact parameters of all tracks in the opposite hemisphere relative to the main event vertex, the kaons in primary and secondary vertices in the same hemisphere, and the charge of the secondary vertex.

The result of this procedure is to increase the B_s^0 purity by 30% and to reach more than 90% purity for the tagging at the decay time.

$D_s^\pm \ell^\mp$ /combined tag analysis

The use of events in which a reconstructed D_s is accompanied by a high p_t lepton with an electric charge opposite in sign allows to select a sample having 60% B_s purity. The

proper time resolution benefits also from the fact that the only missing particle is the neutrino: $\bar{B}_s^0 \rightarrow D_s^+ e^- \bar{\nu}_e$. In the first pico-second the time resolution is about 0.18 ps in more than 80% of the events.

The limiting factor is the available statistics because accessible D_s branching fractions are quite small (between $\sim 1\%$ and $\sim 5\%$). Several decay modes have to be selected. Figure 3 shows an example in which six hadronic and two semileptonic decay modes have been reconstructed.

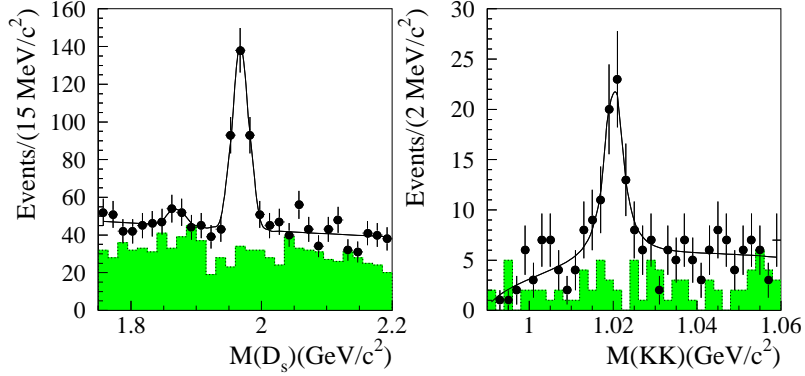


Figure 3: DELPHI $D_s^\pm \ell^\mp$ candidates. The figure on the left shows the D_s mass spectrum reconstructed from the following decay modes: $D_s^+ \rightarrow \phi\pi^+, \phi\pi^+\pi^0, \phi\pi^+\pi^-\pi^+, \bar{K}^{*0}K^+, \bar{K}^{*0}K^+$ and $K_S^0K^+$. The figure on the right shows the ϕ mass spectrum from the decays $D_s^+ \rightarrow \phi e^+ \nu_e$ and $\phi \mu^+ \bar{\nu}_\mu$. The sum of the two samples gives $230 \pm 18 B_s^0$ candidates.

Exclusive B_s^0 /combined tag analysis

At the 1998 Moriond Conference, the DELPHI Collaboration has proposed the use of exclusively reconstructed B_s^0 decays for Δm_s analyses. These events have an excellent proper time resolution $\sigma_t \sim 0.08$ ps and provide a gain in sensitivity at high values of Δm_s (equation 3). Figure 4 shows the B_s^0 mass spectrum using the decay modes: $B_s^0 \rightarrow D_s\pi$ (or a_1) and $B_s^0 \rightarrow D^0K\pi$ (or a_1). The D_s has been reconstructed in six hadronic decay modes, as in the $D_s^\pm \ell^\mp$ analysis, and the D^0 is observed using $K\pi$ and $K\pi\pi\pi$ decay modes. 17 ± 8 events have been reconstructed in the B_s^0 mass region. The combinatorial background is estimated to be 35%.

Summary of Δm_s analyses

The combined result of LEP/SLD/CDF [1] analyses is shown in Figure 5 and is:

$$\Delta m_s > 12.4 \text{ ps}^{-1} \text{ at } 95\% \text{ C.L.}$$

The sensitivity is at 13.8 ps^{-1} . LEP alone has a limit at 11.5 ps^{-1} at 95% C.L., with a sensitivity at 12.9 ps^{-1} . $\Delta m_s = 0$ is excluded between 14.5 ps^{-1} and 16.5 ps^{-1} with a 2σ significance at 15 ps^{-1} . The present summary of the results is given in Figure 5.

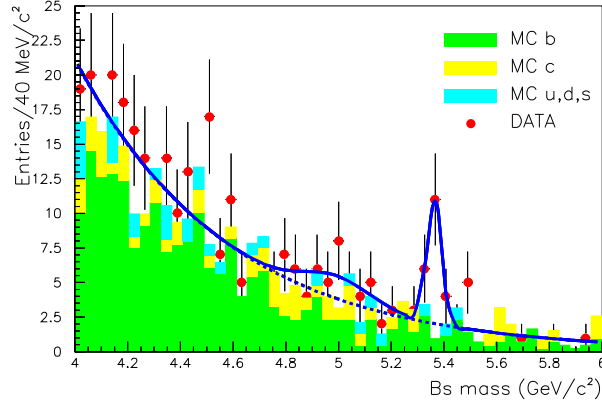


Figure 4: The B_s^0 mass spectrum obtained by the DELPHI Collaboration. The points with the error bars are the data with the fit superimposed. The contributions from non- B_s^0 decays, as given by the Monte Carlo simulation, are also shown.

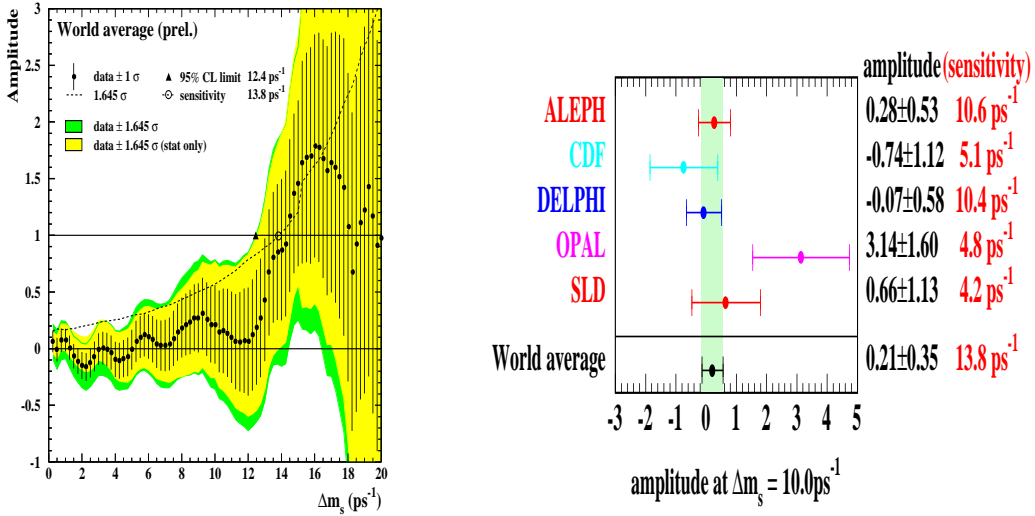


Figure 5: The plot on the left shows the combined Δm_s results from LEP/SLD/CDF analyses shown in an amplitude versus Δm_s plot. The point with error bars are the data; the lines show the 95% C.L. curves (in dark the systematics have been included). The dotted curve shows the sensitivity. The plot on the right shows the summary of the Δm_s results per experiment. The error are given at $\Delta m_s = 10 \text{ ps}^{-1}$ (the sensitivity is also given). The way in which the combined value is obtained is described in [1].

$|V_{ub}|/|V_{cb}|$ measurement

The presence of leptons above the kinematical limit for those produced in the decay $B \rightarrow D\ell\bar{\nu}_\ell$ ($b \rightarrow c$ transition proportional to the $|V_{cb}|$ CKM matrix element) is attributed to the transition $b \rightarrow u\ell\bar{\nu}_\ell$ (proportional to the $|V_{ub}|$ CKM matrix element).

The CLEO and ARGUS Collaborations have been pioneers in this measurement. Nevertheless, as only a small fraction of the energy spectrum of these leptons is measurable, the systematic uncertainties in the modelling of the $b \rightarrow u$ transition to evaluate the ratio $|V_{ub}|/|V_{cb}|$ are quite large (of the order of 20%-25% relative error). Recently LEP experiments have shown their capabilities of measuring $|V_{ub}|$ with a statistical precision similar to the one from CLEO and with reduced systematic uncertainties. They use several kinematical variables, in events with an identified high transverse momentum lepton, which have a distinctive power to discriminate between $b \rightarrow c$ and $b \rightarrow u$ transitions. The first measurement has been performed by the ALEPH Collaboration by means of a neural network discriminating method.

The DELPHI measurement is simpler. With respect to the ALEPH analysis the information from the presence of a secondary vertex from the D decay is used. In $b \rightarrow u$ transitions, all tracks are coming from the B decay vertex. The presence of kaons at the D meson vertex is also used. The method is based on the fact that the hadronic system recoiling against the lepton in $b \rightarrow u\ell\nu$ decays is expected to have an invariant mass lower than the charm mass [2]. The sample is finally divided into a $b \rightarrow u$ enriched and a $b \rightarrow u$ depleted components and the energy of the lepton in the B rest frame is calculated. The result is shown in Figure 6 together with the summary of the results on $|V_{ub}|$.

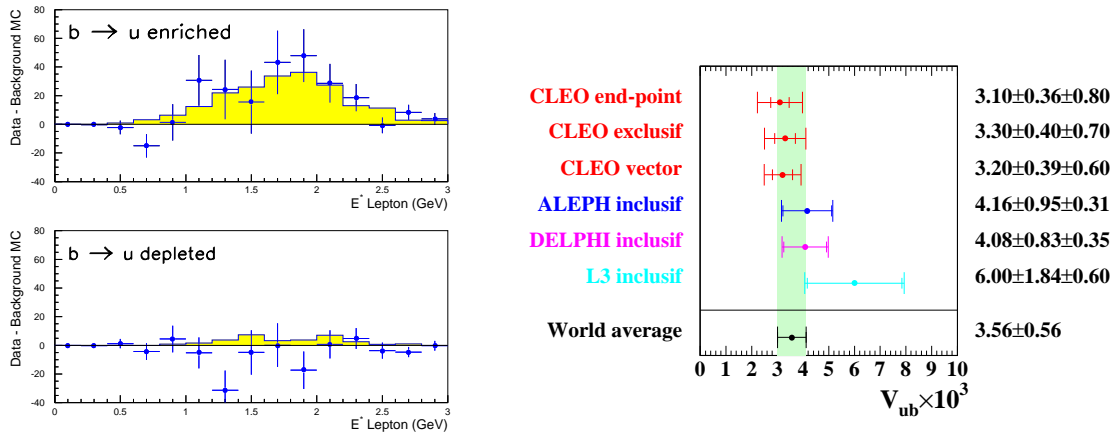


Figure 6: The plots on the left show the energy of the lepton in the B rest frame after the background subtraction for the $b \rightarrow u$ enriched and $b \rightarrow u$ depleted samples. On the right the summary of $|V_{ub}|$ results is given.

Status of the V_{CKM} matrix

Table 2: The four constraints which allow, at present, to define the accessible region for the ρ and η parameters are listed in the first column. In the second column the dependence of these constraints relative to the different parameters is given. The last column gives the explicit dependence in terms of $\bar{\rho}$ and $\bar{\eta}$.

Measurement	$V_{\text{CKM}} \times \text{other}$	Constraint
$b \rightarrow u / b \rightarrow c$	$(V_{ub} / V_{cb})^2$	$\bar{\rho}^2 + \bar{\eta}^2$
Δm_d	$ V_{td} ^2 f_{B_d}^2 B_{B_d} f(m_t)$	$(1 - \bar{\rho})^2 + \bar{\eta}^2$
$\Delta m_d / \Delta m_s$	$\left \frac{V_{td}}{V_{ts}} \right ^2 \frac{f_{B_d}^2 B_{B_d}}{f_{B_s}^2 B_{B_s}}$	$(1 - \bar{\rho})^2 + \bar{\eta}^2$
ε_K	$f(A, \bar{\eta}, \bar{\rho}, B_K)$	$\sim \bar{\eta}(1 - \bar{\rho})$

The V_{CKM} matrix can be parametrized in terms of four parameters: λ , A , ρ and η (the Wolfenstein parametrization [3]). The Standard Model predicts relations between the different processes which depend on these parameters. The unitarity of the V_{CKM} matrix can be visualized as a triangle in the $\rho - \eta$ plane. Several quantities which depend on ρ and η have to be measured and, if Standard Model is correct, they must define compatible values for the two parameters inside measurement errors and theoretical uncertainties. The measurement of $b \rightarrow u / b \rightarrow c$ transitions gives a constraint of the form $\bar{\rho}^2 + \bar{\eta}^2$.² The measurement of Δm_d gives a constraint of the form $(1 - \bar{\rho})^2 + \bar{\eta}^2$. A measurement of the ratio $\Delta m_d / \Delta m_s$ gives the same type of constraint in the $\bar{\rho} - \bar{\eta}$ plane, as a measurement of Δm_d , but this ratio is expected to have smaller theoretical uncertainties since the ratio $f_{B_s}^2 B_{B_s} / f_{B_d}^2 B_{B_d}$ is better known than the absolute value $f_{B_d}^2 B_{B_d}$.

All details of the analysis presented here can be found in [4]. Using the available and most recent measurements and up to date theoretical calculations [4] the allowed region in the $\bar{\rho} - \bar{\eta}$ plane can be determined. It is shown in Figure 7 and corresponds to:

$$\bar{\rho} = 0.189 \pm 0.074 ; \bar{\eta} = 0.354 \pm 0.045$$

It is of interest to determine the central values and the uncertainties on the quantities $\sin 2\alpha$, $\sin 2\beta$ and γ which will be directly measured at future B-factories or LHC experiments. The result is shown in Figure 8 and is:

$$\sin 2\beta = 0.73 \pm 0.08 ; \sin 2\alpha = -0.15 \pm 0.30 ; \gamma = (62 \pm 10)^{\circ} \quad (5)$$

The value of $\sin 2\beta$ is rather precisely determined with an accuracy already at the level expected after the first years of running at B factories. Finally it is possible to remove from the calculation the information of one of the constraint and to obtain its probability density function. The result for Δm_s and $|V_{ub}|/|V_{cb}|$ is shown in Figure 9 and summarized in Table 3.

² $\bar{\rho}(\bar{\eta}) = \rho(\eta)(1 - \lambda^2/2)$

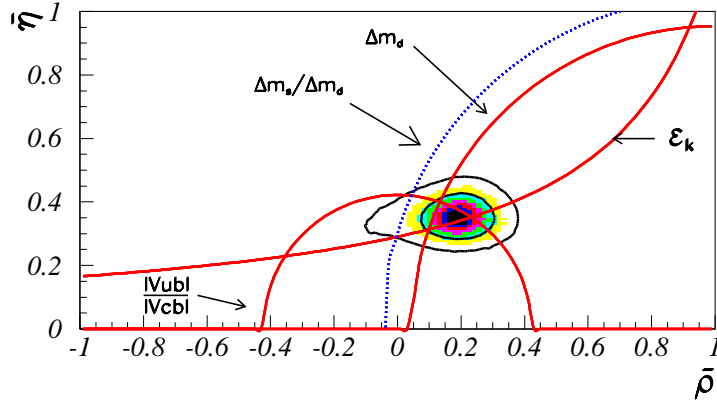


Figure 7: The $\bar{\rho} - \bar{\eta}$ allowed region. The contours at 68% and 95% C.L. are shown. The continuous lines correspond to the constraints obtained from the measurements of $\frac{|V_{ub}|}{|V_{cb}|}$, Δm_d , and ε_K . The dotted curve corresponds to the 95% C.L. limit obtained from the experimental limit on Δm_s .

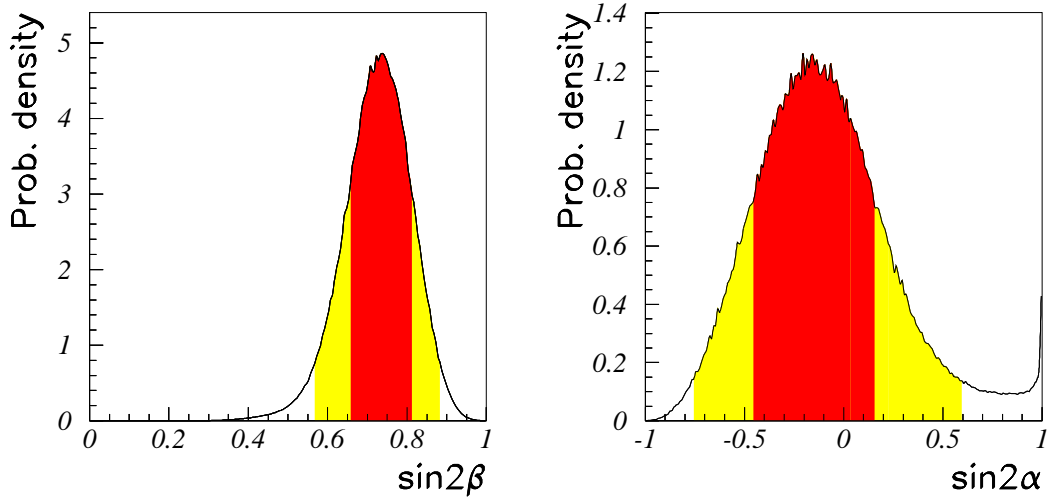


Figure 8: The $\sin 2\beta$ and $\sin 2\alpha$ probability density distributions. The dark-shaded and the clear shaded intervals correspond to 68% and 95% C.L. regions respectively.

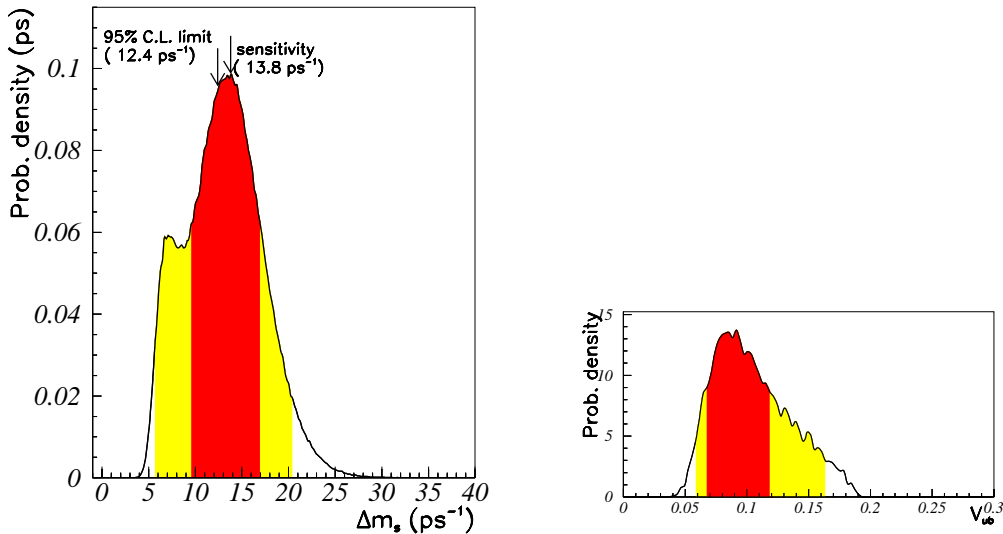


Figure 9: The left and the right plots show the probability density distributions for Δm_s and $|V_{ub}|/|V_{cb}|$ respectively. The dark-shaded and the clear shaded intervals correspond to 68% and 95% C.L. regions respectively.

Table 3: The Δm_s and $|V_{ub}|/|V_{cb}|$ measured values are compared with those obtained using the fitting procedure after having removed them from the fit.

Quantity	Measured value	Fitted value
Δm_s	$> 12.4 \text{ ps}^{-1}$ at 95% C.L.	$[9.5 - 17] \text{ ps}^{-1}$ 68% C.L.
$ V_{ub} / V_{cb} $	0.093 ± 0.014	$0.085^{+0.037}_{-0.023}$

From these results the important impact of these two measurements in the determination of the allowed region for ρ and η is clear. Furthermore the expected probability distribution for Δm_s shows that present analyses are exploring the one sigma region.

Conclusions

Important improvements have been obtained in the last two years in the analyses of $B^0 - \bar{B}^0$ oscillations. Combining LEP results with those from SLD and CDF, Δm_d frequency is presently known with a 3.4% relative error ($\Delta m_d = 0.477 \pm 0.017 \text{ ps}^{-1}$). The sensitivity on Δm_s is at 13.8 ps^{-1} and, the actual LEP/SLD/CDF combined limit, of 12.4 ps^{-1} at 95% of C.L., is exploring the region where Δm_s is expected to be according to the analysis [4]. The measurement of Δm_s is still a challenge for LEP collaborations, $|V_{ub}|$ has been

measured at LEP with about the same experimental precision as the one obtained by CLEO and with a reduced dependence on theoretical models.

The phenomenological analysis presented in this paper gives:

$$\bar{\rho} = 0.189 \pm 0.074 ; \bar{\eta} = 0.354 \pm 0.045$$

and, in an indirect way:

$$\sin 2\beta = 0.73 \pm 0.08 ; \sin 2\alpha = -0.15 \pm 0.30 ; \gamma = (62 \pm 10)^0$$

The situation will still be improved, at least until the next summer '99, before the starting of B-factories.

Acknowledgement

I would like to thank the organisers of HQ98 for the warm and nice atmosphere during the conference and for the unforgettable banquet at the Shedd Aquarium. Many thanks to Fabrizio Parodi and Patrick Roudeau for their help in the preparation and redaction of this contribution. Finally a grand merci to Jocelyne Brosselard, kind and efficient as usual in the preparation of this manuscript.

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