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The NLO DGLAP extraction of α_s and higher twist terms from CCFR xF_3 and F_2 structure functions data for νN DIS

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Abstract

We performed the detailed NLO analysis of the combined CCFR xF_3 and F_2 structure functions data and extracted the value of α_s, parameters of distributions and higher-twist (HT) terms using the direct solution of the DGLAP equation. The value of $\alpha_s(M_Z) = 0.1222 \pm 0.0048(exp) \pm 0.0040(theor)$ was obtained. The result has larger central value and errors, than the original result of the CCFR collaboration, in view of the incorporation into the fits of the HT terms as the free model independent parameters. The x-shapes of the HT contributions to xF_3 and F_2 are in agreement with the results of other model-independent extractions and are in qualitative agreement with the predictions of the infrared renormalon model. We also argue that the low x CCFR data might have the defects, since their inclusion into the fits led to the following low x-behaviour of the gluon distribution $xG(x, 9\ GeV^2) \sim x^{0.092 \pm 0.0073}$, in contradiction with the results of its extraction from low x HERA data.

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1. The study of the possibility to separate power suppressed terms (namely higher-twist (HT) effects) from the perturbation theory logarithmic corrections in the analysis of scaling violation of the deep-inelastic scattering (DIS) processes has a rather long history (see e.g. Refs. [1, 2, 3] and Ref.[4] for the review). In the recent years the interest to this problem was renewed, mainly due to the consideration of the possibility to model HT terms in different processes using the infrared-renormalon (IRR) technique (see e.g. Refs.[5]-[10],[11] and especially Ref.[12] for the review).

On the other hand the experimentalists are improving the precision of their data and are achieving, sometimes, percent level of accuracy. For example the data on xF_3 and F_2 from the most precise νN DIS experiment, performed at Tevatron by CCFR collaboration, recently appeared [13, 14]. The CCFR data on xF_3 were analysed in Ref.[15] in the LO and and with inclusion of the NLO and approximate next-to-next-to-leading order (NNLO) corrections. For the latest,the NNLO QCD corrections to the coefficient function [16] were taken into account. The NNLO corrections to the anomalous dimensions of a limited set of even non-singlet moments [17] were also taken into account. The NNLO corrections to the anomalous dimensions of odd moments, which are not still explicitly calculated, were obtained using smooth interpolation procedure proposed in Ref.[18] and improved in Ref.[19]. The aim of the work of Ref.[15] was to make an attempt of the first NNLO determination of $\alpha_s(M_Z)$ from DIS and to extract the HT terms from the data on xF_3 within the framework of the IRR-model [7] and also by the model-independent way, similarly to the analysis of the combined SLAC/BCDMS data [20], performed in the NLO approximation. Theoretical uncertainties of this analysis were further estimated in Refs.[21, 22] at the N^{3}LO using the method of Padé approximants. It was found in Refs. [15, 21, 22] that the inclusion of the NNLO corrections leads to the decrease of the HT contribution value, so that at the NNLO its x-shape lies closer to zero.

In these analyses only statistical errors of data were taken into account. However, the systematic errors of the CCFR experiment are not small [14] and may even dominate in the determination of some parameters. In this paper we filled in this gap and performed the NLO analysis of the CCFR data with the help of QCD DGLAP evolution code, developed in Ref.[23]. (Remind, that the analyses of Refs.[15, 21, 22] were performed with the help of the Jacobi polynomial variant [24, 25, 26] of the DGLAP equation [27]). In addition, we included in our analysis the CCFR data on the singlet structure function F_2. It should be stressed that the code [23] was tested using the procedure proposed in Ref.[28] and demonstrated the accuracy at the level of $O(0.1\%)$ in the kinematic region covered by the analysed data. It was already applied for the nonsinglet DGLAP analysis of the combined SLAC/BCDMS SLAC data on F_2 [29].

2. Our fits were made in the NLO approximation within the $\overline{MS}$ factorization and renormalization schemes. The Q^2 dependence of the strong coupling constant α_s was defined from the following equation

$$\frac{1}{\alpha_s(Q)} - \frac{1}{\alpha_s(M_Z)} = \frac{\beta_0}{2\pi}\ln\Big(\frac{Q}{M_Z}\Big) + \beta\ln\Big[\frac{\beta + 1/\alpha_s(Q)}{\beta + 1/\alpha_s(M_Z)}\Big], \tag{1}$$

where $\beta = \frac{4\pi\beta_1}{\beta_0}$ and β_0 and β_1 are the coefficients of the QCD β-function, defined as

$$\beta(\alpha_s) = \frac{1}{4\pi}\mu\frac{\partial\alpha_s}{\partial\mu} = -2\sum_{i\geq 0}\beta_i(\frac{\alpha_s}{4\pi})^{i+2} \tag{2}$$

where $\beta_0 = 11 - (2/3)n_f$ and $\beta_1 = 102 - (38/3)n_f$. Note, that the explicit solution of Eq.(1) can be expressed through the Lambert function [30]. However, we did not use in our work this explicit representation and solved Eq.(1) numerically. The effective number of flavours n_f was chosen to be $n_f = 4$ for Q^2 less than the definite scale M_5^2 and increased to $n_f = 5$ at larger values of Q^2 keeping the continuity of α_s [31]. The value of the effective matching scale M_5 was varied from $M_5 = m_b$ to $M_5 = 6.5 m_b$. The last choice was advocated in Ref.[32] on the basis of the DIS sum rules consideration. The dependence of the results of the fits on the choice of the matching point gives one of the sources of theoretical uncertainties inherent to our analysis.

The leading twist term $xF_3^{LT}(x,Q)$ was obtained by direct integration of the DGLAP equation [27]

$$\frac{dxq^{NS}}{d\ln Q} = \frac{\alpha_s(Q)}{\pi}\int_x^1 dz P_{qq}^{NS}(z)\frac{x}{z}q^{NS}(x/z,Q), \tag{3}$$

where $P_{qq}^{NS}(x)$ denotes the NLO splitting function, taken from Ref.[34]. The function xF_3 is determined by the subsequent convolution with the NLO coefficient function $C_{3,q}(x)$:

$$xF_3^{LT}(x,Q) = \int_x^1 dz C_{3,q}(z)\frac{x}{z}q^{NS}(x/z,Q). \tag{4}$$

The boundary condition at the reference scale $Q_0^2 = 5\ GeV^2$ was chosen in the form analogous to the ones, used in Refs.[14, 15]

$$xq^{NS}(x,Q_0) = \eta_{NS}x^{b_{NS}}(1-x)^{c_{NS}}(1+\gamma x)\frac{3}{A_{NS}}, \tag{5}$$

where

$$A_{NS} = \int_0^1 x^{b_{NS}-1}(1-x)^{c_{NS}}(1+\gamma x)dx, \tag{6}$$

and η_{NS} is the measure of the deviation of the Gross-Llewellyn Smith integral [35] from its quark-parton value 3. The expression for the xF_3, which includes the HT contribution, looks as follows:

$$xF_3^{HT}(x,Q) = xF_3^{LT,TMC}(x,Q) + \frac{H_3(x)}{Q^2}, \tag{7}$$

where $F_3^{LT,TMC}(x,Q)$ is $F_3^{LT}(x,Q)$ with the target mass correction [36] applied.

At the first stage of this work, in order to perform the cross-checks of the code against the results of Refs.[14, 15], we fitted the data on xF_3 in the kinematical region $Q^2 > 5\ GeV^2$, $W^2 > 10\ GeV^2$, $x < 0.7$ (the number of data points is NDP=86). We made three fits with various ways of taking into account the HT effects. The first fit with no HT, i.e. $H_3(x) = 0$, was done to compare our results with the ones of Table 1 of Ref.[15], obtained using different method [24, 25, 26] and different computer code. The second fit with the HT chosen as one-half of the IRR model predictions [7], i.e.

$$H_3(x) = A_2'\int_x^1 dz C_2^{IRR}(z)\frac{x}{z}F_3^{LT}(x/z,Q), \tag{8}$$

where

$$C_2^{IRR}(z) = -\frac{4}{(1-z)_+} + 2(2+z+2z^2) - 5\delta(1-z) - \delta'(1-z) \tag{9}$$

and $A_2' = -0.1\ GeV^2$, advocated for the first time in Ref.[8]. The aim of this fit was to compare its outcomes with the results of Table 7.9 of Ref.[14], where the computer code,

Table 1: The results of the fits to data on xF_3 with statistical errors only I) without HT-terms, II) with HT accounted as one-half of the IRR model predictions, III) with model independent HT-contributions. $H_3^{(0),(2),(4),(6),(8)}$ are the values of $H_3(x)$ at $x = 0., 0.2, 0.4, 0.6, 0.8$.

	I	II	III
χ^2/NDP	88.5/86	81.9/86	70.3/86
b	0.789 ± 0.024	0.786 ± 0.024	0.805 ± 0.067
c	4.02 ± 0.11	4.00 ± 0.11	4.24 ± 0.21
γ	0.29 ± 0.30	0.26 ± 0.30	0.61 ± 0.71
η_{NS}	0.927 ± 0.014	0.949 ± 0.014	0.927 ± 0.030
$\alpha_s(M_Z)$	0.1193 ± 0.0025	0.1219 ± 0.0024	0.1216 ± 0.0066
$H_3^{(0)}$	–	–	0.18 ± 0.19
$H_3^{(2)}$	–	–	-0.26 ± 0.12
$H_3^{(4)}$	–	–	-0.21 ± 0.31
$H_3^{(6)}$	–	–	0.11 ± 0.26
$H_3^{(8)}$	–	–	0.90 ± 0.47

written by Duke and Owens [33] was used. In the third fit we used the model independent HT-expression, i.e. $H_3(x)$ parametrized at $x = 0., 0.2,\ 0.4, 0.6, 0.8$ with linear interpolation between these points. It was performed to compare our results with the ones presented in Table 3 of Ref. [15].

All results of these our fits are presented in Table 1. We observed a good agreement of our results on α_s with the both referenced papers. However, in the case of the values of x-shape parameters we found the certain discrepancy with the results of Ref.[15]. For example, the value of γ, as presented in column I of Table 1, is $\gamma = 0.26 \pm 0.30$, meanwhile the analogous parameter in Ref.[15] is $\gamma = 1.96 \pm 0.36$. At the same time our x-shape parameters are in agreement with the ones, extracted in Ref.[14] within errors. In addition, we made the fit, releasing parameter $A_2^{'}$ and obtained the value of $A_2^{'} = -0.12 \pm 0.05$ in agreement with the results of Ref.[15].

3. The next step of our analysis was to take into account the point-to-point correlations of the data due to systematic errors which, as we mentioned above, can be crucial for the estimation of full experimental errors of the parameters (see in particular [29], where the value $\alpha_s(M_Z) = 0.1180 \pm 0.0017$ $(stat + syst)$ was obtained as the result of the combined fits of the SLAC/BCDMS data with HT included). The systematic errors were taken into account analogously to the earlier works [23, 29]. The total number of the independent systematic errors sources for the analysed data is 18 and all of them were convoluted into the general correlation matrix, which was used for the construction of the minimized χ^2. The results of the fits to xF_3 data with the model independent HT and with the systematic errors taken into account are presented in the first column of Table 2. One can see that the account of systematic errors leads to the significant increase of the experimental uncertainties of the extracted HT contributions (compare the first column of Table 2 with the third column of Table 1). In addition, the central values of the HT parameters moved. However, even in this case there is definite agreement with the results of HT-behaviour of Ref.[15], obtained in NLO. Moreover, these results do not contradict to the IRR-model prediction of Ref.[7], since releasing $A_2^{'}$ we obtained $A_2^{'} = -0.10 \pm 0.09$.

Trying to minimize the errors of the parameters we added to the analysis the CCFR data

Table 2: The results of the fits with the account of systematic errors and model independent HT-effects. $H_{2,3}^{(0),(2),(4),(6),(8)}$ are the values of $H_2(x)$ and $H_3(x)$ $x = 0., 0.2, 0.4, 0.6, 0.8$. I) xF_3 with the cut $Q^2 > 5\ GeV^2$, $Q_0^2 = 5\ GeV^2$ II) $xF_3\&F_2$ with the cut $Q^2 > 5\ GeV^2$, $Q_0^2 = 9\ GeV^2$ III) $xF_3\&F_2$ with the cut $Q^2 > 1\ GeV^2$, $Q_0^2 = 9\ GeV^2$.

	I	II	III
χ^2/NDP	55.7/86	154.9/172	204.2/220
b_{NS}	0.797 ± 0.076	0.800 ± 0.016	0.782 ± 0.014
c_{NS}	4.24 ± 0.21	4.060 ± 0.068	4.131 ± 0.056
γ	0.75 ± 0.79	0.	0.
η_{NS}	0.945 ± 0.043	0.922 ± 0.027	0.920 ± 0.025
$\alpha_s(M_Z)$	0.1269 ± 0.0065	0.1248 ± 0.0048	0.1131 ± 0.0045
η_S	–	0.1785 ± 0.0077	0.1796 ± 0.0065
b_S	–	0.	-0.034 ± 0.023
c_S	–	8.37 ± 0.21	8.00 ± 0.29
b_G	–	0.	0.092 ± 0.073
c_G	–	7.5 ± 2.6	11.50 ± 0.90
η_G	–	0.69 ± 0.35	1.08 ± 0.19
$H_2^{(0)}$	–	-0.23 ± 0.56	0.09 ± 0.11
$H_2^{(2)}$	–	-0.28 ± 0.18	-0.239 ± 0.094
$H_2^{(4)}$	–	-0.14 ± 0.18	0.17 ± 0.13
$H_2^{(6)}$	–	-0.03 ± 0.13	0.204 ± 0.097
$H_2^{(8)}$	–	0.21 ± 0.18	0.14 ± 0.18
$H_3^{(0)}$	0.28 ± 0.21	0.34 ± 0.11	0.115 ± 0.031
$H_3^{(2)}$	-0.22 ± 0.19	-0.24 ± 0.16	-0.16 ± 0.16
$H_3^{(4)}$	-0.42 ± 0.35	-0.22 ± 0.22	0.28 ± 0.19
$H_3^{(6)}$	-0.09 ± 0.28	-0.05 ± 0.17	0.19 ± 0.15
$H_3^{(8)}$	1.21 ± 0.50	0.89 ± 0.44	0.88 ± 0.44

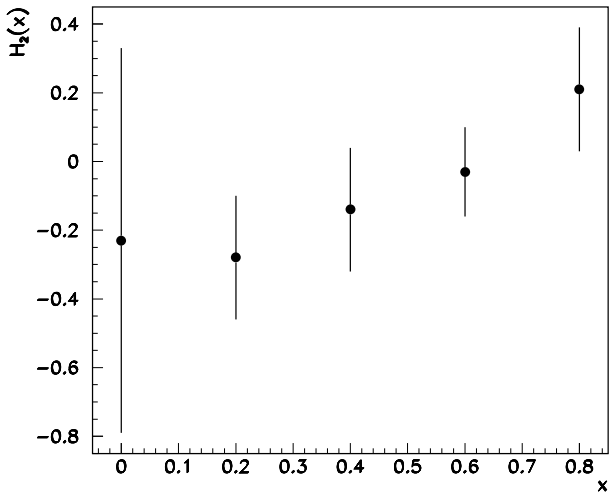

Figure 1: The high-twist contribution to F_2.

for the structure function F_2. To perform the QCD evolution of F_2 one is to involve into the analysis the singlet and gluon distributions:

$$F_2^{LT}(x,Q) = \int_x^1 dz\Big[C_{2,q}(z)\frac{x}{z}(q^{NS}(x/z,Q) + q^{PS}(x/z,Q)) + C_{2,G}(z)\frac{x}{z}G(x/z,Q)\Big], \qquad (10)$$

The distributions $q^{PS}(x,Q)$ and $G(x,Q)$ were obtained by integrating the system

$$\frac{dxq^{PS}}{d\ln Q} = \frac{\alpha_s(Q)}{\pi}\int_x^1 dz\Big[P_{qq}^{PS}(z)\frac{x}{z}q^{PS}(x/z,Q) + P_{qG}(z)\frac{x}{z}G(x/z,Q)\Big] \qquad (11)$$

$$\frac{dxG}{d\ln Q} = \frac{\alpha_s(Q)}{\pi}\int_x^1 dz\Big[P_{Gq}(z)\frac{x}{z}q^{PS}(x/z,Q) + P_{GG}(z)\frac{x}{z}G(x/z,Q)\Big] \qquad (12)$$

with the boundary conditions

$$xq^{PS}(x,Q_0) = \eta_S x^{b_S}(1-x)^{c_S}/A_S, \qquad (13)$$

$$xG(x,Q_0) = \eta_G x^{b_G}(1-x)^{c_G}/A_G, \qquad (14)$$

where

$$A_S = \int_0^1 x^{b_S}(1-x)^{c_S}dx, \qquad (15)$$

$$A_G = \frac{1- < xQ(x) >}{\int_0^1 x^{b_G}(1-x)^{c_G}dx}. \qquad (16)$$

and $< xQ(x) >$ is the total momentum carried by quarks.

In order to provide the straightforward way for the comparison of our results with the analysis of Ref.[23], the initial reference scale Q_0^2=9 GeV^2 was chosen. In addition to the point-to-point correlation of the data due to systematic errors, the statistical correlations between F_2 and xF_3 were also taken into account. Performing the trial fits we convinced that adding the factor $(1+\gamma x)$ to the reference expressions for the the gluon and singlet distributions do not improve the quality of the fit. Also we fixed parameters γ_{NS}, b_S and b_G

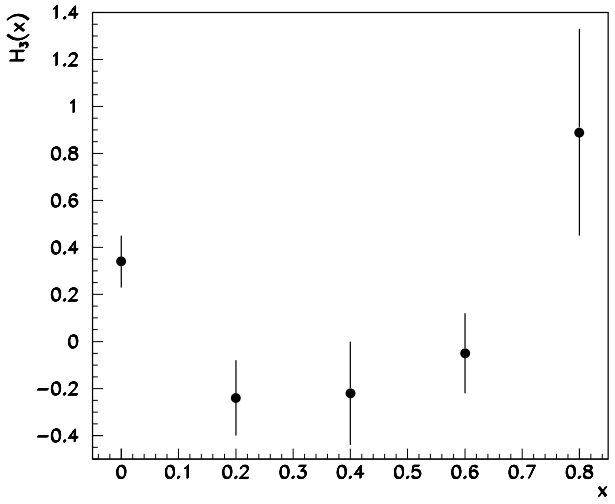

Figure 2: The high-twist contribution to xF_3.

at zero because this increased the value of χ^2 by few units only while χ^2/NDP remained less than unity. The HT contribution to F_2 was accounted analogously to xF_3 as:

$$F_2^{HT}(x,Q) = F_2^{LT,TMC}(x,Q) + \frac{H_2(x)}{Q^2}$$

where $H_2(x)$ was parametrized in the model independent form. The results of the fits of the parameters $H_2(x)$ and $H_3(x)$ are presented in the second column of Table 2 and are depicted in Fig.1 and Fig.2. One can note, that, comparing with the fit to xF_3 data only, the HT parameters errors are decreasing. Within the errors, the parameters, which describe the boundary distributions, are compatible with the outcomes of the similar fits from Ref.[14]. The coefficients of $H_3(x)$ are in agreement with the NLO results of Ref.[15] and the behaviour of $H_2(x)$ qualitatively reproduce the HT contribution to F_2, obtained from the combined fits of the SLAC/BCDMS data on F_2, performed in Refs.[20, 29].

When the matching scale M_5 was changed from m_b to $6.5m_b$, the value of $\alpha_s(M_Z)$ shifted down by 0.0052 and then we ascribe to $\alpha_s(M_Z)$ the theoretical error of 0.0026 due to uncertainty of b-quark threshold matching. This uncertainty is in agreement with the results of the NLO Jacobi-polynomial fits of the CCFR data obtained within so-called spline $\overline{MS}$ prescription [37]. One more source of the theoretical uncertainty, which is due to the truncation of higher QCD orders, was evaluated following the way, proposed in Ref. [20]. In accordance with their procedure one can introduce renormalization scale k_R into QCD evolution equations in the way, illustrated on the example of NS evolution:

$$\frac{dxq^{NS}}{d\ln Q} = \frac{\alpha_s(k_R Q)}{\pi}\int_x^1 dz\Big\{P_{qq}^{NS,(0)}(z)+$$

$$+\frac{\alpha_s(k_R Q)}{2\pi}\Big[P_{qq}^{NS,(1)}(z) + \beta_0 P_{qq}^{NS,(0)}(z)\ln(k_R)\Big]\Big\}\frac{x}{z}q^{NS}(x/z,Q), \qquad (17)$$

where $P^{NS,(0)}$ and $P^{NS,(1)}$ denote the LO and the NLO parts of the splitting function P^{NS}. The dependence of the results on k_R would signal an incomplete account of the perturbation theory effects. The shift of $\alpha_s(M_Z)$ resulting from the reasonable variation of k_R leads to an additional error of over 0.003 due to the renormalization scale uncertainty. Taking

$Q_0^2 = 20\ GeV^2$ as the initial scale, we have checked that our results are rather stable to the variation of the factorization point.

The NLO value we fare presenting as the main result is thus

$$\alpha_s(M_Z) = 0.1222 \pm 0.0048(stat + syst) \pm 0.0040(thresh + ren.scale) \tag{18}$$

It differs a bit from the NLO value $\alpha_s(M_Z) = 0.119 \pm 0.002(stat + syst) \pm 0.004(theory)$, obtained in the CCFR analysis [13]. The increase of the experimental errors is due to the fact that while CCFR group used model-dependent form of the HT contributions, we are considering them as the additional free parameters and are extracting them from the fits.

In order to try to decrease further the errors, we repeated the fits of the combined xF_3 and F_2 data, using the less stringent cut $Q^2 > 1\ GeV^2$. The obtained results are presented in the third column of Table 2. In these fits the parameters b_s and b_G were released since their values turned out to be statistically different from zero. We found, that the values of α_s and b_G are correlated (the correlation coefficient is equal to –0.65). When we fixed $b_G = 0$, the value $\alpha_s(M_Z) = 0.1172 \pm 0.0029$ was obtained, and when we kept b_G as the free parameter, we obtained low value of $\alpha_s(M_Z) = 0.1131 \pm 0.0045$. The analogous effect of correlations was observed for the fit with the cut $Q^2 > 5\ GeV^2$, although with less statistical significance. It should be underlined, that when we released b_G in the fit with the cut $Q^2 > 1\ GeV^2$ we faced another problem: its value turned out to be

$$b_G = 0.092 \pm 0.073, \tag{19}$$

which is in the evident contradiction with the results, obtained in the analysis of HERA data (for example the combined analysis of DIS from HERA and CERN-SPS data results in the value $b_G = -0.267 \pm 0.043$ [23], while in the framework of MRST parametrization the value $b_G = -1.08$[38] was obtained).This problem might be related to the well-known discrepancy between CCFR and NMC/BCDMS data at small x.

Conclusion

In conclusion, we would like to stress that in order to perform similar analysis at the NNLO level it is necessary to calculate the yet unknown Altarelli-Parisi kernels to the corresponding DGLAP equations. Therefore, we are unable to obtain the results, similar to the NNLO ones of Refs.[15, 22]. We hope that future progress of theoretical calculations will allow us to generalize our results to the NNLO approximation.

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