

HEAVY MAJORANA NEUTRINOS AND BARYOGENESIS *

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The scenario of baryogenesis through leptogenesis is reviewed in models involving heavy Majorana neutrinos. The various mechanisms of CP violation occurring in the out-of-equilibrium lepton-number-violating decays of heavy Majorana neutrinos are studied within a resummation approach to unstable-particle mixing. It is explicitly demonstrated how the resummation approach preserves crucial field-theoretic properties such as unitarity and CPT invariance. Predictions of representative scenarios are presented after solving numerically the Boltzmann equations describing the thermodynamic evolution of the Universe. The phenomenological consequences of loop effects of heavy Majorana neutrinos on low-energy observables, such as lepton-flavour and/or lepton-number non-conservation in τ and Z -boson decays and electron electric dipole moment, are discussed.

1. Introduction

Present astronomical observations related to abundances of the light elements ${}^4\text{He}$ and ${}^4\overline{\text{He}}$, the content of protons versus antiprotons in cosmic rays, etc, lead to the conclusion^{1,2} that, before the nucleosynthesis epoch, the Universe must have possessed an excess in the baryon number B which is expressed by the small baryon-to-photon ratio of number densities

$$\frac{n_{\Delta B}}{n_\gamma} = (4 - 7) \times 10^{-10} . \quad (1.1)$$

This baryonic asymmetry, $n_{\Delta B} = n_B - n_{\bar{B}} \approx n_B$, should have survived until today if there had been no processes that violate the B number and/or modify the number density of photons n_γ . Sakharov, assuming that the Universe was created initially in a B -conserving symmetric state, was able to derive three necessary conditions³ to explain the baryon asymmetry in the Universe (BAU):

- (i) There must be B -violating interactions in nature, so that a net B number can in principle be generated.

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- (ii) The B -violating interactions should violate the discrete symmetries of charge conjugation (C) and that resulting from the combined action of charge and parity transformations (CP). In this way, an excess in baryons over antibaryons, ΔB , is produced.
- (iii) The B - and CP-violating processes must be out of thermal equilibrium,^{4,5} namely they should have an interaction rate smaller than the expansion rate of the Universe. This last requirement ensures that the produced ΔB is not washed out by the inverse processes.

Grand unified theories (GUT's) can in principle contain all the above necessary ingredients for baryogenesis.⁶ In such theories, out-of-equilibrium B - and CP-violating decays of super-heavy bosons with masses near to the grand unification scale $M_X \approx 10^{15}$ GeV can produce the BAU. However, this solution to the BAU has its own problems. The main difficulty is the generic feature that minimal GUT's predict very small CP violation, since it occurs at very high orders in perturbation theory. This problem may be avoided by augmenting GUT's with extra Higgs representations.¹ Also, GUT's must comply with limits obtained by experiments on the stability of the proton. Such experiments put tight constraints on the masses of the GUT bosons mediating B violation and their couplings to the matter. Another severe limitation to scenarios of baryogenesis arises from the anomalous $B + L$ -violating processes, also known as sphalerons,^{7,8,9} which are in thermal equilibrium for temperatures^{10,11} $200 \lesssim T \lesssim 10^{12}$ GeV. Unlike $B + L$, sphalerons preserve the quantum number $B - L$. Therefore, any primordial BAU generated at the GUT scale should not rely on $B + L$ -violating operators, which imposes a further non-trivial constraint on unified theories. In that vein, Kuzmin, Rubakov and Shaposhnikov⁹ suggested that the same anomalous $B + L$ -violating electroweak interactions may produce the observed excess in B during a first-order electroweak phase transition. Such a mechanism crucially depends on the Higgs-boson mass M_H ,^{12,13} and the experimental fact $M_H > 80$ GeV practically rules out this scenario of electroweak baryogenesis.¹⁴ Therefore, baryogenesis provides the strongest indication against the completeness of the SM, as well as poses limits on its possible new-physics extensions.

Among the many baryogenesis scenarios invoked in the literature, the most attractive one is due to Fukugita and Yanagida¹⁵, and our emphasis will be put on their scenario in this review article. In such a scenario, out-of-equilibrium L -violating decays of heavy Majorana neutrinos N_i , with masses $m_{N_i} \gg T_c$, produce an excess in the lepton number L which is converted into the desired excess in B by means of $B + L$ -violating sphaleron interactions, which are in thermal equilibrium above the critical temperature T_c . Over the last years, many authors have discussed such a scenario, also known as baryogenesis through leptogenesis.^{16,17,18,19,20,21} However, we should remark that isosinglet heavy neutrinos are not theoretically compelling in order to create an excess in the L number. Recently, Ma and Sarkar²² suggested a leptogenesis scenario based on a generalized Higgs-triplet model,^{23,24}

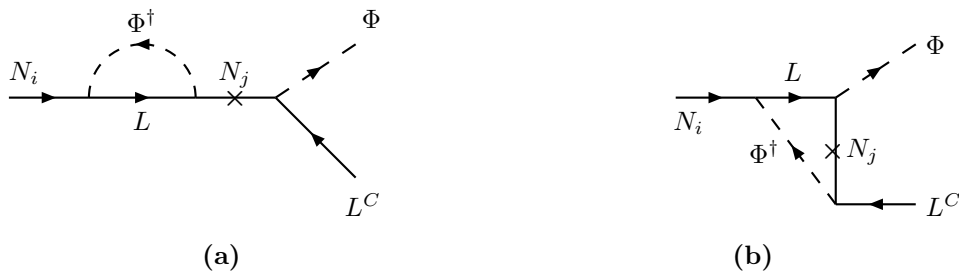


Fig. 1. One-loop (a) self-energy and (b) vertex graphs in heavy Majorana neutrino decays.

where the leptonic asymmetry is generated by out-of-equilibrium CP-violating decays of heavy doubly charged Higgs triplets into charged leptons. However, such alternatives seem to face the known gravitino problem if they are to be embedded in a supersymmetric theory.²⁵ The charged Higgs triplets or their supersymmetric partners may interact strongly with gravitinos and produce them in large abundances. The slow decay rate of gravitinos during the nucleosynthesis epoch distorts the abundances of the light elements at a level inconsistent with present observations.

Mechanisms that enhance CP violation play a decisive role in baryogenesis. Using the terminology known from the $K^0\bar{K}^0$ system,²⁶ one may distinguish the following two cases:

- (i) CP violation originating from the interference between the tree-level decay amplitude and the absorptive part of the one-loop vertex. Such a mechanism is usually called ε' -type CP violation.^{15,16,17,18}
- (ii) CP violation induced by the interference of the tree-level graph and the absorptive part of a one-loop self-energy transition. This mechanism is termed ε -type CP violation.^{27,28,9,19,20,21}

As can be seen from Fig. 1, both of the above two mechanisms of CP violation are present^{19,20,21} in the usual leptogenesis scenario¹⁵ of heavy Majorana neutrino decays. CP violation of the ε' type was extensively discussed in the literature.^{15,16,17,18} If all Yukawa couplings of the Higgs fields to N_i and the ordinary lepton isodoublets are of comparable order,^{16,18} then baryogenesis through the ε' -type mechanism requires very heavy Majorana neutrinos with masses of order 10^7 – 10^8 GeV. Such a high mass bound may be lifted if a strong hierarchy for Yukawa couplings and m_{N_i} is assumed.^{16,17} However, without the latter assumption,¹⁸ one obtains $\varepsilon' < 10^{-15}$ for $m_{N_i} \approx 1$ TeV, and hence very heavy neutrinos are needed to account for the BAU.

Recently, ε -type CP violation and its implications for the BAU has received much attention.^{20,21} In particular, it has been observed^{20,21} that CP violation can

be considerably enhanced through the mixing of two nearly degenerate heavy Majorana neutrinos. Such an analysis cannot be performed in the conventional field-theoretic framework, since finite-order perturbation theory breaks down in the limit of degenerate particles. To be specific, the wave-function amplitude that describes the CP-asymmetric mixing of two heavy Majorana neutrinos, N_1 and N_2 , say, is inversely proportional to the mass splitting $m_{N_1} - m_{N_2}$, and it becomes singular if degeneracy is exact. Solutions to this problem have been based on the Weisskopf and Wigner (WW)²⁹ approximation,²⁰ and the resummation approach.^{30,21} Both approaches lead to similar conclusions concerning the resonant enhancement of CP violation. Here, we shall follow the latter method, as the discussion of many crucial field-theoretic issues, such as renormalization, CPT invariance and unitarity, is conceptually more intuitive in this framework.

To describe the dynamics of CP violation through mixing of two unstable particles, one is compelled to rely on resummation approaches, which treat unstable particles in a consistent way.^{31,32} In fact, to any finite order in perturbation theory, physical amplitudes reflect the local gauge symmetry, respect unitarity, are invariant under the renormalization group, and satisfy the equivalence theorem. All of the above properties should also be present after resummation. Unfortunately, resummation methods often end up violating one or more of them. The reason is that subtle cancellations are distorted when certain parts of the amplitude are resummed to all orders in perturbation theory, whereas others, carrying important physical information, are only considered to a finite order. In this context, a novel diagrammatic resummation approach has been developed,³³ which is based on the pinch technique (PT)³⁴ and devoid of the above pathologies. In the PT resummation approach, basic field-theoretic requirements, such as analyticity, unitarity, gauge invariance and renormalizability,³³ are naturally satisfied. Apart from the great phenomenological importance of such a resummation formalism for the proper definition of the mass and the width of unstable particles, such as the W , the Z boson and the Higgs boson,^{33,35} this formalism may also be extended to the case of mixing between two intermediate resonant states in scattering processes^{36,30} retaining all the required field-theoretic properties mentioned above.

The afore-mentioned resummation formalism has been proved to be very successful in describing resonant transitions taking place in collider experiments. These are situations where the unstable particles are produced by given asymptotic states, e^+e^- , say, and their subsequent decay is observed by detecting some other asymptotic states in the final state, e.g. $e^+e^- \rightarrow Z^* \rightarrow \mu^+\mu^-$. However, in an expanding Universe, the unstable particles may undergo a huge number of collisions before they eventually decay. Each of these collisions contributes a Coulomb phase shift, and hence the mixed heavy particles are practically uncorrelated when they decay. To some extent, this thermodynamic phenomenon may be described by Boltzmann equations.⁴ In this context, a related formalism for decays has been developed,²¹ which effectively takes into account *decoherence* phenomena in the mixing and subsequent decay of heavy particles, namely heavy Majorana neutrinos in our case.

Specifically, it is shown that ε -type CP violation can even be of order unity.²¹ This is in agreement with earlier studies on resonant CP violation through mixing in scatterings involving top quarks, supersymmetric quarks or Higgs particles in the intermediate state.^{31,36,30} Finally, we must remark that alternative formulations of Boltzmann equations already exist in the recent literature³⁷ but they are expected not to alter drastically the existing conclusions^{20,21} as far as the resonant phenomenon of CP violation is concerned.

The organization of the review article is as follows: in Section 2 we briefly review the basic theoretical background concerning the $B + L$ anomaly in the Standard Model (SM), and the effect of sphaleron processes on the chemical potentials of SM particles. These considerations lead to a relation between the generated leptonic asymmetry and the observed baryonic asymmetry induced by sphaleron interactions. In Section 3 we discuss theories that naturally include heavy Majorana neutrinos. For illustration, we consider a minimal model with two isosinglet neutrinos and demonstrate how CP and L can simultaneously be violated in this model. In Section 4 we address the issue of renormalizability of the minimal isosinglet neutrino model. Section 5 discusses in detail the resummation approach and its effective extension to describe incoherent decays of heavy unstable fermions. In Section 6 we apply the effective approach to the decays of heavy Majorana neutrinos. In Section 7 we explicitly demonstrate how the resummation approach satisfies unitarity. In Section 8, we solve numerically the Boltzmann equations for representative leptogenesis scenarios, and give numerical estimates and comparisons for the BAU generated through ε - and/or ε' -type CP violation. Furthermore, we estimate the impact of finite-temperature effects on the resonant phenomenon of CP violation. Heavy Majorana neutrinos may also have important phenomenological implications for low-energy observables, as they can give rise to a non-vanishing electric dipole moment (EDM) of the electron at two loops or induce L -violating decays of the Z boson and the τ lepton. These new-physics effects are detailed in Section 9. Section 10 summarizes our conclusions.

2. $B + L$ anomaly and sphaleron processes

In the SM, the B and L numbers are only conserved in the classical action. After quantization, however, both baryonic and leptonic currents are violated by triangle anomalies, i.e.,

$$\partial_\mu J_B^\mu = \partial_\mu J_L^\mu = i \frac{N_F}{8\pi} \left(-\alpha_w W^{\mu\nu,a} \widetilde{W}_{\mu\nu}^a + \alpha_Y Y^{\mu\nu} \widetilde{Y}_{\mu\nu} \right), \quad (2.1)$$

where N_F is the number of flavours, and $\alpha_w = g_w^2/(4\pi)$, $\alpha_Y = g_Y^2/(4\pi)$, are the $SU(2)_L$ and $U(1)_Y$ fine-structure constants, respectively. Similarly, $W^{\mu\nu}$, $Y^{\mu\nu}$ are their respective field-strength tensors, and the antisymmetric tensors $\widetilde{W}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\rho} W^{\lambda\rho}$, $\widetilde{Y}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\rho} Y^{\lambda\rho}$ are their associated duals. Furthermore, baryonic

and leptonic currents are defined as

$$J_B^\mu = \frac{1}{3} \sum_{q,\alpha} \bar{q}^\alpha \gamma^\mu q^\alpha \quad (2.2)$$

$$J_L^\mu = \sum_{l,\nu_l} (\bar{l} \gamma^\mu l + \bar{\nu}_l \gamma^\mu \nu_l), \quad (2.3)$$

where q , l and ν_l denote quarks, charged leptons and neutrinos, respectively, and the index α indicates the colour degrees of freedom of the quarks. Since Eq. (2.1) also holds for individual lepton families, the actual anomaly-free charges are

$$\frac{1}{3}B - L_e, \quad \frac{1}{3}B - L_\mu, \quad \frac{1}{3}B - L_\tau. \quad (2.4)$$

It is then obvious that $B+L$ symmetry is anomalously broken at the quantum level.

The different gauge field configurations are characterized by different Chern-Simons numbers n_{CS} . The CS numbers label the infinitely many degenerate vacua of the system. The variation of $B+L$ number due to a quantum tunnelling from one vacuum state into another is given by

$$\Delta(B+L) = 2N_F \frac{\alpha_w}{8\pi} \int d^4x W^{\mu\nu,a} \widetilde{W}_{\mu\nu}^a = 2N_F \Delta n_{CS}. \quad (2.5)$$

At zero temperature, 't Hooft⁷ estimated the probability of B -violating processes, and found them to be extremely suppressed by a factor $\exp(-4\pi n_{CS}/\alpha_w) \approx \exp(-150n_{CS})$ relative to the B -conserving ones with $n_{CS} = 0$.

The situation changes drastically at finite temperatures. The effect of non-trivial topological instanton-type solutions, termed sphalerons,⁸ is amplified at high temperatures, thereby enhancing also the rate of the B -violating processes. To be precise, sphaleron interactions are in thermal equilibrium for temperatures in the interval

$$100 \text{ GeV} \lesssim T \lesssim 10^{12} \text{ GeV}. \quad (2.6)$$

Sphalerons may be thought of as the creation out of the vacuum of a state

$$\Pi_{i=1,N_F} (u_L d_L d_L \nu_L)_i. \quad (2.7)$$

Since these interactions violate $B+L$, any primordial baryonic asymmetry B should have a significant component in $B-L$ or in the charges stated in Eq. (2.4), which are preserved by sphalerons, whereas any $B+L$ component will be washed out. Decays of heavy Majorana neutrinos produce an excess in L , which can naively be written as a sum of an excess in $\frac{1}{2}(B+L)$ and in $\frac{1}{2}(B-L)$. Sphalerons will then erase the $B+L$ component but preserve the $B-L$, so one expects that about half of the leptonic asymmetry L will be converted into the baryonic asymmetry B , and also be preserved as $B-L$ asymmetry. As we will see below, a more careful analysis based on chemical potentials leads to the conclusion that sphalerons approximately convert one-third of an initial leptonic asymmetry L into the observed baryonic asymmetry B .

For illustration, we shall assume that all SM particles are almost massless at temperatures above the critical temperature T_c . Actually, they have thermal masses but, to leading order, these are small and may be neglected. The number density of a particle α is given by

$$n_\alpha = g_\alpha \int \frac{d^3\vec{p}_\alpha}{2E_\alpha(2\pi)^3} \frac{1}{\exp[(E_\alpha - \mu_\alpha)/T] \pm 1}, \quad (2.8)$$

where g_α counts the internal degrees of freedom of α , \vec{p}_α and $E_\alpha = (|\vec{p}_\alpha|^2 + m_\alpha)^{1/2}$ are the three-momenta and the energy of the particle, respectively. The plus sign in Eq. (2.8) is for particles obeying the Fermi-Dirac statistics and the minus for particles governed by the Bose-Einstein statistics. The chemical potential for anti-particles, e.g. that of $\bar{\alpha}$, is opposite to that of the particles, i.e. $\mu_\alpha = -\mu_{\bar{\alpha}}$. The later relation is valid if particles and antiparticles have interaction rates with photons or other gauge particles much higher than the expansion rate of the Universe. This is almost the case for all SM particles. However, this is not generally true for non-SM particles, such as isosinglet or right-handed neutrinos, which do not have any tree-level coupling to the W - and Z - bosons; their couplings are suppressed by loops and small Yukawa couplings. Under these assumptions, the number-density asymmetry of a SM particle α versus its antiparticle $\bar{\alpha}$ is easily estimated by

$$n_{\Delta\alpha} = n_\alpha - n_{\bar{\alpha}} \approx \frac{g_\alpha}{\pi^2} T^3 \left(\frac{\mu_\alpha}{T} \right). \quad (2.9)$$

We shall now turn to an analysis of chemical potentials in the SM. Since FCNC interactions are sufficiently fast, we assign the same chemical potential for all different families of up and down quarks, i.e. $(\mu_{u_L}, \mu_{d_L}, \mu_{u_R}, \mu_{d_R})$. In contrast to quarks, individual leptons possess different chemical potentials, i.e. $(\mu_{l_L}, \mu_{\nu_{lL}}, \mu_{l_R})$, where $l = e, \mu, \tau$. Furthermore, the chemical potential of all neutral gauge bosons, such as gluons, photons, and Z bosons, vanish, and μ_W is the chemical potential of the W^- boson. Finally, the components of the Higgs doublet $[\chi^-, \phi^0 = (H - i\chi^0)/\sqrt{2}]$ have chemical potentials (μ_0, μ_-) . Many chemical potentials can be eliminated by means of chemical equilibrium reactions in the SM. More explicitly, we have

$$\begin{aligned} W^- &\leftrightarrow \bar{u}_L + d_L, & \mu_W &= -\mu_{u_L} + \mu_{d_L}, \\ W^- &\leftrightarrow \bar{\nu}_{lL} + l_L, & \mu_W &= -\mu_{\nu_{lL}} + \mu_{l_L}, \\ W^- &\leftrightarrow \chi^- + \phi^0, & \mu_W &= \mu_- + \mu_0, \\ \phi^0 &\leftrightarrow \bar{u}_L + u_R, & \mu_0 &= -\mu_{u_L} + \mu_{u_R}, \\ \phi^0 &\leftrightarrow \bar{d}_L + d_R, & \mu_0 &= -\mu_{d_L} + \mu_{d_R}, \\ \phi^0 &\leftrightarrow \bar{l}_L + l_R, & \mu_0 &= -\mu_{l_L} + \mu_{l_R}. \end{aligned} \quad (2.10)$$

As independent parameters, we consider $\mu_u = \mu_{u_L}$, $\mu = \sum_l \mu_{\nu_{lL}} = \sum_l \mu_{l_L}$, μ_0 and μ_W . In the SM with N_F families and N_H Higgs doublets, the baryon and lepton number B and L as well as the electric charge Q and hypercharge Q_3 may be expressed in terms of these quantities, as follows:

$$B = 4N_F\mu_u + 2N_F\mu_W,$$

$$\begin{aligned}
L &= 3\mu + 2N_F\mu_W - N_F\mu_0, \\
Q &= 2N_F\mu_u - 2\mu + 2(2N_F + N_H)\mu_0 - 2(2N_F + 2 + N_H)\mu_W, \\
Q_3 &= -(2N_F + 4 + 2N_H)\mu_W.
\end{aligned} \tag{2.11}$$

Furthermore, the sphaleron interactions in Eq. (2.7) give rise to the additional relation

$$N_F(3\mu_u + 2\mu_W) + \mu = 0. \tag{2.12}$$

Above the electroweak phase transition, both charges Q and Q_3 are conserved, i.e. $\langle Q \rangle = \langle Q_3 \rangle = 0$. Thus, we have: $\mu_W = 0$, $\mu = -3N_F\mu_u$, and $\mu_0 = -8N_F\mu_u/(4N_F + 2N_H)$. Using these relationships among the chemical potentials, it is not difficult to obtain^{38,39}

$$B(T > T_c) = \frac{8N_F + 4N_H}{22N_F + 13N_H} (B - L). \tag{2.13}$$

From Eq. (2.13), one concludes that almost independently of the number of generations and Higgs doublets, roughly one-third of the initial $B-L$ and/or L asymmetry will be reprocessed by sphalerons into an asymmetry in B . This amount of B asymmetry persists even after the electroweak phase transition.

3. Models with heavy Majorana neutrinos

GUT's such as $SO(10)$ ^{40,41} or E_6 ⁴² models naturally predict heavy Majorana neutrinos. These theories also contain several other particles, e.g. leptoquarks, additional charged and neutral gauge bosons, etc., which may have significant interactions with heavy Majorana neutrinos and so affect the number density of heavy neutrinos. To avoid excessive complication, we shall assume that these new particles are much heavier than the lightest heavy Majorana neutrino, and are therefore expected to decouple sufficiently fast from the process of leptogenesis.

As already mentioned, $SO(10)$ ^{40,41} and/or E_6 ⁴² models may naturally accommodate heavy Majorana neutrinos. Specifically, $SO(10)$ models can break down to the SM gauge group in the following schematic way:

$$\begin{aligned}
SO(10) &\rightarrow G_{422} = SU(4)_{PS} \otimes SU(2)_R \otimes SU(2)_L \\
&\rightarrow G_{3221} = SU(3)_c \otimes SU(2)_R \otimes SU(2)_L \otimes U(1)_{(B-L)} \\
&\rightarrow SM = G_{321} = SU(3) \otimes SU(2)_L \otimes U(1)_Y,
\end{aligned} \tag{3.1}$$

where the subscript PS characterizes the Pati-Salam gauge group.⁴³ The spinor representation of $SO(10)$ is 16-dimensional and its decomposition under G_{422} reads

$$G_{422} : \mathbf{16} \rightarrow (4, 1, 2) \oplus (\bar{4}, 2, 1). \tag{3.2}$$

Evidently, $SO(10)$ contains the left-right-symmetric gauge group $SU(2)_R \otimes SU(2)_L \otimes U(1)_{(B-L)}$, which necessitates the presence of right-handed neutral leptons. In this scenario, there can exist several Higgs-boson representations that may cause a breaking of the groups G_{422} and G_{3221} down to the SM gauge group G_{321} .^{41,44}

E_6 theories⁴² may also have a breaking pattern related to $SO(10)$ theories. In fact, the **27** spinor representation decomposes into $\mathbf{16} \oplus \mathbf{10} \oplus \mathbf{1}$ under $SO(10)$. This leads to four singlet neutrinos per SM family: one neutrino as isodoublet member in **16**, two neutrinos as isodoublet members in **10**, and one singlet neutrino in **1**. In these models, it is argued that depending on the representation of the E_6 Higgs multiplets, two of the four isosinglets can have Majorana masses of few TeV,⁴² whereas the other two may be very heavy with masses of the order of the unification scale.

We shall now discuss a generic subgroup that may be derived from $SO(10)$ and/or E_6 models. This generic subgroup may be realized in the usual SM augmented by a number n_R of right-handed neutrinos ν_{Ri} , with $i = 1, 2, \dots, n_R$. As we have discussed above, in E_6 theories the active isosinglet neutrinos may be more than three. In $SO(10)$ models, left-right symmetry is more naturally realized with one right-handed neutrino per family (for interesting alternatives, see Ref.⁴¹). For the sake of generality, we shall keep the number of iso-singlet neutrinos arbitrary. For definiteness, the quark sector of the minimal model has the SM form, while the leptonic sector consists of the fields:

$$\left(\begin{array}{c} \nu_{lL} \\ l_L \end{array} \right), \quad l_R, \quad \nu_{Ri},$$

where $l = e, \mu, \tau$. At $T \gg T_c \gtrsim v$, the vacuum expectation value (VEV) of the SM Higgs doublet Φ at temperature T (with $v = v(0)$) vanishes, $v(T) = 0$. At these high temperatures, all SM particles including Higgs fields are massless; they only acquire thermal masses. However, one may have Majorana masses in the Lagrangian given by

$$-\mathcal{L}_M = \frac{1}{2} \sum_{i,j=1}^{n_R} \left(\bar{\nu}_{Ri}^C M_{ij}^\nu \nu_{Rj} + \bar{\nu}_{Ri} M_{ij}^{\nu*} \nu_{Rj}^C \right). \quad (3.3)$$

where M^ν is an $n_R \times n_R$ -dimensional symmetric matrix, which is in general complex. In Eq. (3.3), the superscript C denotes the operation of charge conjugation, which acts on the four-component chiral spinors ψ_L and ψ_R as follows:

$$(\psi_L)^C = P_R C \bar{\psi}^T, \quad (\psi_R)^C = P_L C \bar{\psi}^T, \quad (3.4)$$

where $P_{L(R)} = [1 - (+)\gamma_5]/2$ is the chirality projection operator. The mass matrix M^ν can be diagonalized by means of a unitary transformation

$$U^T M^\nu U = \widehat{M}^\nu, \quad (3.5)$$

where U is an $n_R \times n_R$ -dimensional unitary matrix and the diagonal matrix \widehat{M}^ν contains the n_R heavy Majorana masses. Then, the respective n_R mass eigenstates N_i are related to the flavour states ν_{Ri} through

$$\nu_{Ri} = P_R \sum_{j=1}^{n_R} U_{ij} N_j, \quad \nu_{Ri}^C = P_L \sum_{j=1}^{n_R} U_{ij}^* N_j. \quad (3.6)$$

In the mass basis of heavy Majorana neutrinos, the Yukawa sector governing the interactions of the heavy neutrinos with the Higgs and lepton isodoublets is given by

$$\mathcal{L}_Y = - \sum_{l=1}^{n_L} \sum_{j=1}^{n_R} h_{lj} (\bar{\nu}_{lL}, \bar{l}_L) \begin{pmatrix} (H - i\chi^0)/\sqrt{2} \\ -\chi^- \end{pmatrix} N_j + \text{H.c.} \quad (3.7)$$

At high temperatures, the CP-even Higgs field H , the CP-odd Higgs scalar χ^0 and the charged Higgs scalars χ^\pm given in (3.7) are massless. In the low- T limit $T \ll T_c$, the field H becomes the massive SM Higgs boson, whereas χ^0 and χ^\pm are the would-be Goldstone bosons eaten by the longitudinal degrees of freedom of the gauge bosons Z and W^\pm , respectively. In the calculations in Section 6, we shall include the M_H dependence in the CP asymmetries.

Let us now consider a simple one-generation model where the standard fermionic content is extended by adding two isosinglet neutrinos, e.g. ν_R and S_L . Then, the most general Yukawa sector that preserves lepton number reads

$$-\mathcal{L} = \frac{1}{2} (\bar{S}_L, (\bar{\nu}_R)^C) \begin{pmatrix} 0 & M \\ M & 0 \end{pmatrix} \begin{pmatrix} (S_L)^C \\ \nu_R \end{pmatrix} + h_R (\bar{\nu}_L, \bar{l}_L) \tilde{\Phi} \nu_R + \text{H.c.}, \quad (3.8)$$

where $\tilde{\Phi} = i\sigma_2 \Phi$ is the isospin conjugate Higgs doublet and σ_2 is the usual Pauli matrix. The kinematic parameters M and h_R may in general be complex but their phases are not physical. One can make both real by appropriate redefinitions of the fermionic fields, i.e.

$$L_L^T \equiv (\nu_L, l_L) \rightarrow e^{i\phi_L} L_L^T, \quad \nu_R \rightarrow e^{i\phi_R} \nu_R, \quad S_L \rightarrow e^{i\phi_S} S_L. \quad (3.9)$$

One choice could be: $\phi_R = 0$, $\phi_S = \arg(M)$, and $\phi_L = \arg(h_R)$. Retaining the L -conserving structure of the isosinglet mass matrix, such scenarios require a non-trivial mixing among the generations to describe CP violation.⁴⁵ Furthermore, such scenarios do not produce the necessary leptonic asymmetry for baryogenesis; however, see Ref.⁴⁶ for an interesting variant based on individual lepton-flavour violation.

In order to break both L and CP symmetries of the Lagrangian in Eq. (3.8), one must consider at least two extensions in the model:

- (i) The inclusion of two complex L -violating Majorana masses $\mu_R \bar{\nu}_R \nu_R^C$ and $\mu_L \bar{S}_L S_L^C$.
- (ii) The addition of the L -violating coupling $h_L (\bar{\nu}_L, \bar{l}_L) \tilde{\Phi} (S_L)^C$ and one L -violating mass parameter, e.g. $\mu_R \bar{\nu}_R \nu_R^C$.

The two models are related by a unitary rotation and are therefore equivalent. The necessary conditions for CP invariance in these two scenarios may be found to be

$$\begin{aligned} \text{(i)} \quad & |h_R|^2 \text{Im}(M^{*2} \mu_L \mu_R) = 0, \\ \text{(ii)} \quad & \text{Im}(h_L h_R^* \mu_R M^*) = 0. \end{aligned} \quad (3.10)$$

It is now interesting to remark that μ_L and μ_R can be much smaller than M within E_6 scenarios.⁴² These parameters may be induced after integrating out high-dimensional operators involving ultra-heavy non-active neutrinos. One may think that the lepton number is somehow violated, at the GUT or Planck scale M_X , by these additional non-active isosinglet fields, and it is communicated to the active isosinglet sector where $M \ll M_X$. In this way, one can naturally obtain a see-saw-like relation for the sizes of μ_L and μ_R , i.e.

$$\mu_L, \mu_R \sim \frac{M^2}{M_X} \quad \text{or} \quad \frac{M^2}{M_S}, \quad (3.11)$$

where $M_S \approx 10^{-3} M_X$ could be some intermediate see-saw scale. In such generic mass models, the heavy Majorana neutrinos N_1 and N_2 have a very small mass splitting given by

$$x_N = \frac{m_{N_2}}{m_{N_1}} - 1 \sim \frac{\mu_L}{M} \quad \text{or} \quad \frac{\mu_R}{M}. \quad (3.12)$$

For instance, if $M = 10$ TeV and $\mu_L = \mu_R = M^2/M_X$, one then finds $x_N \approx 10^{-12} - 10^{-11}$. As we will see in Section 6, such small values of x_N can lead to a resonant enhancement of CP asymmetries in the heavy Majorana neutrino decays.

To obtain the sufficient and necessary conditions of CP invariance for any flavour structure of the one-generation model with two isosinglet neutrinos, one should use a more general approach, based on generalized CP transformations⁴⁷ for the fermionic fields:

$$L_L \rightarrow e^{i\phi_L} (L_L)^C, \quad \nu_{Ri} \rightarrow V_{ij} (\nu_{Rj})^C, \quad (3.13)$$

where V is a 2×2 dimensional unitary matrix. Notice that the transformations given by Eq. (3.13) satisfy the SM symmetry of the mass-independent, conformal invariant part of the Lagrangian; only M^ν breaks this symmetry softly. In such an approach,^{45,47} one looks for all possible weak-basis independent combinations that can be formed by Yukawa couplings and the neutrino mass matrix M^ν , and are simultaneously invariant under the transformations (3.13). In this way, we find the condition

$$\text{Im Tr}(h^\dagger h M^{\nu\dagger} M^\nu M^{\nu\dagger} h^T h^* M^\nu) = m_{N_1} m_{N_2} (m_{N_1}^2 - m_{N_2}^2) \text{Im}(h_{l1} h_{l2}^*)^2 = 0, \quad (3.14)$$

where $h = (h_{l1}, h_{l2})$ is a row vector that contains the Higgs Yukawa couplings defined in the mass basis of isosinglet neutrinos. From Eq. (3.14), one readily observes that CP invariance holds if $m_{N_1} = m_{N_2}$ and/or one of the isosinglet neutrinos is massless. These considerations may be extended to models with n_L weak isodoublets and n_R neutral isosinglets. In this case, there exist many conditions analogous to Eq. (3.14), which involve high-order terms in the Yukawa-coupling matrix h . However, not all of the conditions are sufficient and necessary for CP invariance. If we assume that Higgs triplets are not present in the theory, the total number of all non-trivial CP-violating phases is $\mathcal{N}_{CP} = n_L(n_R - 1)$.⁴⁸

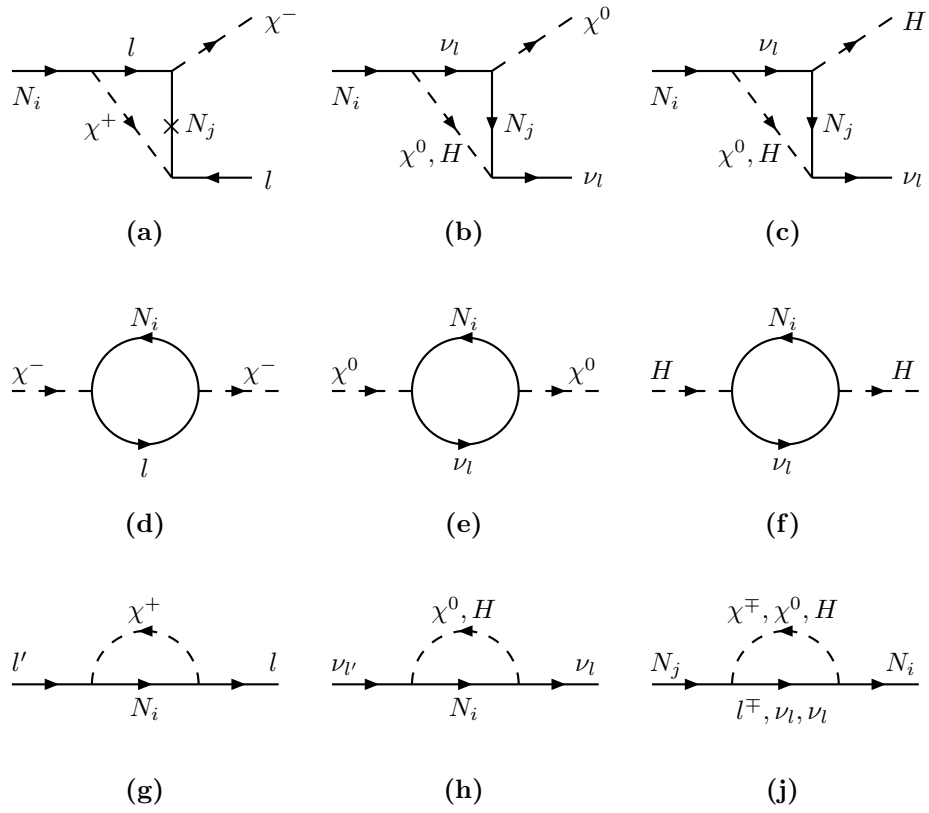


Fig. 2. One-loop graphs contributing to the renormalization of the couplings $\chi^- l N_i$, $\chi^0 \nu_l N_i$ and $H \nu_l N_i$.

4. Renormalization

At the tree level, CP violation in particle decays amounts to CPT violation and therefore vanishes identically. A non-vanishing contribution to CP asymmetries only arises at the one-loop level, considering the diagrams shown in Fig. 2. For this reason, it is important to discuss how one-loop renormalization applies to heavy Majorana-neutrino models⁴⁹ and its possible consequences on CP asymmetries.

We start our discussion by expressing all bare quantities in terms of renormalized ones in the following way:

$$\begin{aligned} \nu_{iL}^0 &= \sum_{l'=1}^{n_L} \left(\delta_{il'} + \frac{1}{2} \delta Z_{il'}^\nu \right) \nu_{l'L}, & l_L^0 &= \sum_{l'=1}^{n_L} \left(\delta_{ll'} + \frac{1}{2} \delta Z_{ll'}^l \right) l_{l'}^0, \\ N_i^0 &= \sum_{j=1}^{n_R} \left(\delta_{ij} + \frac{1}{2} \delta Z_{ij}^N \right) N_j, & \tilde{\Phi} &= \left(1 + \frac{1}{2} \delta Z_\Phi \right) \tilde{\Phi}, & h_{lj}^0 &= h_{lj} + \delta h_{lj}, \end{aligned} \quad (4.1)$$

where unrenormalized kinematic parameters and fields are indicated by a superscript ‘0’. Note that δZ_Φ collectively represents the wave-function renormalization constants of all components of the Higgs doublet $\tilde{\Phi}$ (or Φ), i.e. the fields χ^\pm , χ^0 and H . In Appendix A, we give analytic expressions for Higgs and fermion self-energies. From these, one can easily see that the divergent part of the Higgs wave-function renormalization is exactly the same. In fact, δZ_Φ is universal in the limit $M_H \rightarrow 0$.

Let us now consider that all quantities in the Lagrangian (3.7) are bare and we can substitute Eqs. (4.1) into that bare Lagrangian. In addition to the renormalized Lagrangian, which has the same structural form as the bare one, we then find the counter-term (CT) Lagrangian

$$-\delta\mathcal{L}_Y = \frac{1}{2} \sum_{l=1}^{n_L} h_{lj} \sum_{j=1}^{n_R} \left(2 \frac{\delta h_{lj}}{h_{lj}} + \delta Z_\Phi + \sum_{l'=1}^{n_L} \delta Z_{l'l}^L + \sum_{k=1}^{n_R} \delta Z_{jk}^N \right) \bar{l}_{l'} \tilde{\Phi} N_k + \text{H.c.}, \quad (4.2)$$

where $L_l = (\nu_{lL}, l_L)^T$ and $\delta Z^L = (\delta Z^l, \delta Z^\nu)$. Owing to charge and hypercharge conservation on the vertices, it is not difficult to show by naive power-counting that the one-loop vertex corrections in Fig. 2(a)–(c) are ultra-violet (UV) finite (see also Appendix A).

Despite the fact that vertex corrections are UV finite by themselves, the wave-function renormalizations of the Higgs and lepton isodoublets and that of neutrino isosinglets contain UV divergences that do not cancel. In accordance with the CT Lagrangian (4.2), one may require that all UV terms are to be absorbed into the definition of h_{lj} , i.e.

$$\delta h_{lj} = -\frac{1}{2} \left(h_{lj} \delta Z_\Phi^{\text{div}} + \sum_{l'=1}^{n_L} h_{l'j} \delta Z_{l'l}^{L*} + \sum_{k=1}^{n_R} h_{lk} \delta Z_{kj}^N \right). \quad (4.3)$$

It is important to stress that one-loop renormalization involves the dispersive parts of self-energies and effectively leads to a redefinition of the kinematic parameters,

whereas all absorptive corrections remain unaffected. Even though there might be some high-order dependence due to the choice of different renormalization schemes, we carry out the mass renormalization in the on-shell (OS) scheme.⁵⁰ As we will see in Section 7, this scheme has some field-theoretic advantages over other schemes, when applied to the resummation approach describing the mixing of two unstable particles.

5. Resummation approach to unstable-particle mixing

The consistent description of unstable particles within the conventional framework of perturbative S-matrix theory is an issue related to a number of field-theoretic difficulties. Since unstable particles decay exponentially with time, they cannot appear as asymptotic *in* or *out* states in a process. Furthermore, finite-order perturbation theory breaks down. The usual propagator describing the unstable particle in the intermediate state of a given process displays a physical singularity when the particle comes on its mass shell. One is therefore compelled to use resummation methods that treat unstable particles and unstable-particle mixing in a consistent way; this is a rather subtle issue within the context of gauge theories.^{33,30}

In a simple scalar theory with one unstable particle, Veltman⁵¹ was able to show that, even if one removes the unstable particle from the initial and final states and substitutes it in terms of asymptotic states, the so-truncated S-matrix theory will still maintain the field-theoretic properties of unitarity and causality. Veltman's truncated S-matrix theory is rather useful to describe resonant processes in collider experiments where the initial and final states can be well prepared and detected. However, this formalism cannot directly be applied to the early Universe, as it does not take account of the many decoherential collisions that an unstable particle may undergo with the thermal background before it decays. Therefore, one must seek a method that isolates the *incoherent*⁵² part of an S-matrix amplitude. The new resummation method should include finite width effects in the mixing and decay of unstable particles. This will be done in an effective manner, by employing a procedure related to the Lehmann–Symanzik–Zimmermann formalism (LSZ).⁵³ Then, the *incoherent* decay amplitude derived with this method may equivalently be embedded into a transition element^{33,30} in line with Veltman's S-matrix formulation. As we will see in Section 9, the squared resummed decay amplitudes thus obtained will become the relevant collision terms entering the Boltzmann equations for the thermodynamic evolution of the Universe.

We shall now demonstrate the effective resummation approach to unstable particle mixing. Let us consider a theory with two neutral unstable scalars, e.g. S_1 and S_2 . The approach can then be extended to the case of unstable fermions such as heavy Majorana neutrinos. The bare (unrenormalized) fields S_i^0 and their respective masses M_i^0 may then be expressed in terms of renormalized fields S_i and masses M_i in the following way:

$$S_i^0 = Z_{ij}^{1/2} S_j = \left(\delta_{ij} + \frac{1}{2} \delta Z_{ij} \right) S_j, \quad (5.1)$$

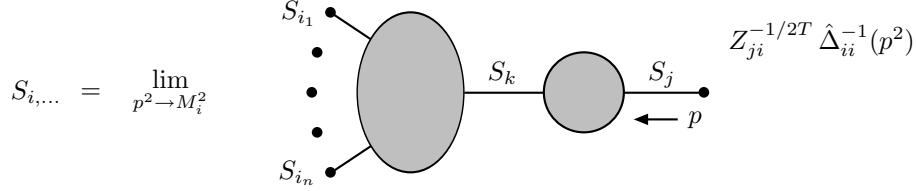


Fig. 3. Diagrammatic representation of the renormalized $n - 1$ -non-amputated amplitude, $S_{i,\dots}$, and the LSZ reduction formalism.

$$(M_i^0)^2 = M_i^2 + \delta M_i^2. \quad (5.2)$$

Here and henceforth, summation is understood over repeated indices that do not appear on both sides of an equation. In Eqs. (5.1) and (5.2), $Z_{ij}^{1/2}$ and δM_i are the wave-function and mass renormalization constants, respectively, which can be determined from renormalization conditions imposed on the two-point correlation functions, $\Pi_{ij}(p^2)$, for the transitions $S_j \rightarrow S_i$ in some physical scheme, such as the on-mass-shell (OS) renormalization scheme.⁵⁰ More details may be found in the appendix.

In order to include the mixing of the unstable scalars, we must first calculate all the $S_i S_j$ Green functions, with $i, j = 1, 2$. After summing up a geometric series of the self-energies $\Pi_{ij}(p^2)$, the full propagators may be obtained by inverting the following inverse propagator matrix:

$$\Delta_{ij}^{-1}(p^2) = \begin{bmatrix} p^2 - (M_1^0)^2 + \Pi_{11}(p^2) & \Pi_{12}(p^2) \\ \Pi_{21}(p^2) & p^2 - (M_2^0)^2 + \Pi_{22}(p^2) \end{bmatrix}. \quad (5.3)$$

The result of inverting the matrix in Eq. (5.3) may be given by

$$\Delta_{11}(p^2) = \left[p^2 - (M_1^0)^2 + \Pi_{11}(p^2) - \frac{\Pi_{12}^2(p^2)}{p^2 - (M_2^0)^2 + \Pi_{22}(p^2)} \right]^{-1}, \quad (5.4)$$

$$\Delta_{22}(p^2) = \left[p^2 - (M_2^0)^2 + \Pi_{22}(p^2) - \frac{\Pi_{12}^2(p^2)}{p^2 - (M_1^0)^2 + \Pi_{11}(p^2)} \right]^{-1}, \quad (5.5)$$

$$\begin{aligned} \Delta_{12}(p^2) &= \Delta_{21}(p^2) = -\Pi_{12}(p^2) \left[\left(p^2 - (M_2^0)^2 + \Pi_{22}(p^2) \right) \right. \\ &\quad \left. \times \left(p^2 - (M_1^0)^2 + \Pi_{11}(p^2) \right) - \Pi_{12}^2(p^2) \right]^{-1}. \end{aligned} \quad (5.6)$$

where $\Pi_{12}(p^2) = \Pi_{21}(p^2)$. Moreover, we observe the crucial factorization property for the off-diagonal ($i \neq j$) resummed scalar propagators

$$\Delta_{ij}(p^2) = -\Delta_{ii}(p^2) \frac{\Pi_{ij}(p^2)}{p^2 - (M_j^0)^2 + \Pi_{jj}(p^2)}$$

$$= - \frac{\Pi_{ij}(p^2)}{p^2 - (M_i^0)^2 + \Pi_{ii}(p^2)} \Delta_{jj}(p^2). \quad (5.7)$$

The resummed unrenormalized scalar propagators $\Delta_{ij}(p^2)$ are related to the respective renormalized ones $\hat{\Delta}_{ij}(p^2)$ through the expression

$$\Delta_{ij}(p^2) = Z_{im}^{1/2} \hat{\Delta}_{mn}(p^2) Z_{nj}^{1/2T}, \quad (5.8)$$

where $\hat{\Delta}_{ij}(p^2)$ may be obtained from Eqs. (5.4)–(5.6), just by replacing M_i^0 with M_i and $\Pi_{ij}(p^2)$ with $\hat{\Pi}_{ij}(p^2)$. Note that the property given in Eq. (5.7) will also hold true for the renormalized scalar propagators $\hat{\Delta}_{ij}(p^2)$.

Suppose that we wish to find the effective resummed decay amplitude $\hat{\mathcal{T}}_{S_i}$ for the decay S_i to n light stable scalars S_{i_1}, \dots, S_{i_n} . In analogy to the LSZ formalism, one starts with the Green function describing the transition shown in Fig. 3, and amputates the external legs by their inverse propagators. For the stable external lines S_{i_1}, \dots, S_{i_n} , the procedure is essentially the same with the usual LSZ formalism. This formalism may then be extended to the external line describing the $S_i S_j$ system. The intermediate steps of this procedure are given by

$$\begin{aligned} \hat{\mathcal{T}}_{i\dots} &= \lim_{p^2 \rightarrow M_i^2} \mathcal{T}_{k\dots}^{amp} Z_{km}^{1/2} \hat{\Delta}_{mn}(p^2) Z_{nj}^{1/2T} Z_{ji}^{-1/2T} \hat{\Delta}_{ii}^{-1}(p^2) \\ &= \lim_{p^2 \rightarrow M_i^2} \left[\mathcal{T}_{k\dots}^{amp} Z_{ki}^{1/2} - \mathcal{T}_{k\dots}^{amp} Z_{km}^{1/2} \frac{\hat{\Pi}_{mi}(p^2)(1 - \delta_{mi})}{p^2 - M_m^2 + \hat{\Pi}_{mm}(p^2)} \right] \\ &= \mathcal{T}_{i\dots} - \mathcal{T}_{j\dots} \frac{\hat{\Pi}_{ji}(M_i^2)(1 - \delta_{ij})}{M_i^2 - M_j^2 + \hat{\Pi}_{jj}(M_i^2)}, \end{aligned} \quad (5.9)$$

where $\mathcal{T}_{i\dots}$ and $\mathcal{T}_{j\dots}$ are the renormalized transition elements evaluated in the stable-particle approximation. One should bear in mind that the OS renormalized self-energies $\hat{\Pi}_{ji}(M_i^2)$ in Eq. (5.9) have no vanishing absorptive parts, as renormalization can only modify the dispersive (real) part of these self-energies. The reason is that the CT Lagrangian must be Hermitian as opposed to the absorptive parts which are anti-Hermitian. In fact, these additional width mixing effects are the ones we wish to include in our formalism for decay amplitudes and are absent in the conventional perturbation theory. It is also important to observe that our approach to decays is not singular, i.e. $\hat{\mathcal{S}}_{i\dots}$ displays an analytic behaviour in the degenerate limit $M_i^2 \rightarrow M_j^2$, because of the appearance of the imaginary term $i\text{Im}\hat{\Pi}_{jj}(M_i^2)$ in the denominator of the mixing factor present in the last equality of Eq. (5.9). Finally, we must stress that the inclusion of these phenomena has been performed in an effective manner. Since the decaying unstable particle cannot appear in the initial state,⁵¹ the resummed decay amplitude must be regarded as being a part which can effectively be embedded into a resummed S-matrix element.³³ This resummed S-matrix element describes the dynamics of the very same unstable particle, which is produced by some asymptotic states, resides in the intermediate state, and subsequently decays either directly or indirectly, through mixing, into the observed final states.

The resummation approach outlined above can now carry over to the mixing between two unstable fermions, call them f_1 and f_2 . As we did for the case of scalars, we express the bare left- and right-handed chiral fields, f_{Li}^0 and f_{Ri}^0 (with $i = 1, 2$), in terms of renormalized fields as follows:

$$f_{Li}^0 = Z_{Lij}^{1/2} f_{Lj}, \quad f_{Ri}^0 = Z_{Rij}^{1/2} f_{Rj}, \quad (5.10)$$

where $Z_{Lij}^{1/2}$ ($Z_{Rij}^{1/2}$) is the wave-function renormalization constant for the left (right)-handed chiral fields, which may be determined from the fermionic self-energy transitions $f_j \rightarrow f_i$, $\Sigma_{ij}(\not{p})$, e.g. in the OS renormalization scheme.⁴⁹ Analogously to Eq. (5.3), the resummed fermion propagator matrix may be obtained from

$$S_{ij}(\not{p}) = \left[\begin{array}{cc} \not{p} - m_1^0 + \Sigma_{11}(\not{p}) & \Sigma_{12}(\not{p}) \\ \Sigma_{21}(\not{p}) & \not{p} - m_2^0 + \Sigma_{22}(\not{p}) \end{array} \right]^{-1}, \quad (5.11)$$

where $m_{1,2}^0$ are the bare fermion masses, which can be decomposed into the OS renormalized masses $m_{1,2}$ and the CT mass terms $\delta m_{1,2}$ as $m_{1,2}^0 = m_{1,2} + \delta m_{1,2}$. Inverting the matrix-valued 2×2 matrix in Eq. (5.11) yields

$$S_{11}(\not{p}) = \left[\not{p} - m_1^0 + \Sigma_{11}(\not{p}) - \Sigma_{12}(\not{p}) \frac{1}{\not{p} - m_2^0 + \Sigma_{22}(\not{p})} \Sigma_{21}(\not{p}) \right]^{-1}, \quad (5.12)$$

$$S_{22}(\not{p}) = \left[\not{p} - m_2^0 + \Sigma_{22}(\not{p}) - \Sigma_{21}(\not{p}) \frac{1}{\not{p} - m_1^0 + \Sigma_{11}(\not{p})} \Sigma_{12}(\not{p}) \right]^{-1}, \quad (5.13)$$

$$\begin{aligned} S_{12}(\not{p}) &= -S_{11}(\not{p}) \Sigma_{12}(\not{p}) \left[\not{p} - m_2^0 + \Sigma_{22}(\not{p}) \right]^{-1} \\ &= - \left[\not{p} - m_1^0 + \Sigma_{11}(\not{p}) \right]^{-1} \Sigma_{12}(\not{p}) S_{22}(\not{p}), \end{aligned} \quad (5.14)$$

$$\begin{aligned} S_{21}(\not{p}) &= -S_{22}(\not{p}) \Sigma_{21}(\not{p}) \left[\not{p} - m_1^0 + \Sigma_{11}(\not{p}) \right]^{-1} \\ &= - \left[\not{p} - m_2^0 + \Sigma_{22}(\not{p}) \right]^{-1} \Sigma_{21}(\not{p}) S_{11}(\not{p}). \end{aligned} \quad (5.15)$$

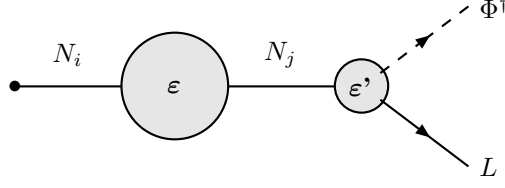
Equations (5.14) and (5.15) show that the resummed propagators $S_{12}(\not{p})$ and $S_{21}(\not{p})$ are endowed with a factorization property analogous to Eq. (5.7). Similarly, the renormalized and unrenormalized resummed propagators are related by

$$S_{ij}(\not{p}) = (Z_{Lim}^{1/2} P_L + Z_{Rim}^{1/2} P_R) \widehat{S}_{mn}(\not{p}) (Z_{Lnj}^{1/2\dagger} P_R + Z_{Rnj}^{1/2\dagger} P_L), \quad (5.16)$$

where the caret on $S_{ij}(\not{p})$ indicates that the resummed fermionic propagators have been renormalized in the OS scheme. Moreover, the renormalized propagators $\widehat{S}_{ij}(\not{p})$ may be obtained by $S_{ij}(\not{p})$ in Eqs. (5.12)–(5.15), if the obvious replacements $m_i^0 \rightarrow m_i$ and $\Sigma_{ij}(\not{p}) \rightarrow \widehat{\Sigma}_{ij}(\not{p})$ are made.

By analogy, one can derive the resummed decay amplitude, $\widehat{\mathcal{T}}_{i\dots}$, of the unstable fermion $f_i \rightarrow X$, as we did for the scalar case. More explicitly, we have

$$\begin{aligned} \widehat{\mathcal{T}}_{i\dots} u_i(p) &= \mathcal{T}_{k\dots}^{amp} (Z_{Lkm}^{1/2} P_L + Z_{Rkm}^{1/2} P_R) \widehat{S}_{mn}(\not{p}) (Z_{Lnj}^{1/2\dagger} P_R + Z_{Rnj}^{1/2\dagger} P_L) \\ &\quad \times (Z_{Lji}^{-1/2\dagger} P_R + Z_{Rji}^{-1/2\dagger} P_L) \widehat{S}_{ii}^{-1}(\not{p}) u_i(p) \\ &= \mathcal{T}_{i\dots} u_i(p) - (1 - \delta_{ij}) \mathcal{T}_{j\dots} \widehat{\Sigma}_{ji}(\not{p}) \left[\not{p} - m_j + \widehat{\Sigma}_{jj}(\not{p}) \right]^{-1} u_i(p). \end{aligned} \quad (5.17)$$

Fig. 4. ε - and ε' -type CP violation in the decays of heavy Majorana neutrinos.

Again, $\mathcal{T}_{i\dots}$ represent the respective renormalized transition amplitudes evaluated in the stable-particle approximation. The amplitudes $\mathcal{T}_{i\dots}$ also include all high-order n -point functions, such as vertex corrections. Based on the formula (5.17), we shall calculate the CP asymmetries in the decays of heavy Majorana neutrinos in the next section.

6. CP asymmetries

The resummation approach presented in the previous section may be applied to describe ε - and ε' -type CP violation in heavy Majorana neutrino decays shown in Fig. 4. The same formalism may also be used to determine the collision terms for the inverse decays, which occur in the formulation of the Boltzmann equations (see also Section 8).

Let us consider the decay $N_1 \rightarrow l^- \chi^+$ in a model with two right-handed neutrinos. The inclusion of all other decay channels is then obvious. We shall first write down the transition amplitude responsible for ε -type CP violation, denoted as $\mathcal{T}_{N_1}^{(\varepsilon)}$, and then take CP-violating vertex corrections into account. Applying (5.17) to heavy Majorana neutrino decays, we obtain

$$\mathcal{T}_{N_1}^{(\varepsilon)} = h_{l1} \bar{u}_l P_R u_{N_1} - i h_{l2} \bar{u}_l P_R \left[\not{p} - m_{N_2} + i \Sigma_{22}^{abs}(\not{p}) \right]^{-1} \Sigma_{21}^{abs}(\not{p}) u_{N_1}, \quad (6.1)$$

where the absorptive part of the one-loop transitions $N_j \rightarrow N_i$, with $i, j = 1, 2$, has the general form

$$\Sigma_{ij}^{abs}(\not{p}) = A_{ij}(p^2) \not{p} P_L + A_{ij}^*(p^2) \not{p} P_R, \quad (6.2)$$

with

$$A_{ij}(p^2) = \frac{h_{l'i} h_{l'j}^*}{32\pi} \left[\frac{3}{2} + \frac{1}{2} \left(1 - \frac{M_H^2}{p^2} \right)^2 \right]. \quad (6.3)$$

In the limit $M_H \rightarrow 0$, Eq. (6.3) gives $A_{ij} = h_{l'i} h_{l'j}^*/(16\pi)$. The CP-transform resummed amplitude describing the decay $N_1 \rightarrow l^+ \chi^-$, $\overline{\mathcal{T}}_{N_1}^{(\varepsilon)}$, reads

$$\begin{aligned} \overline{\mathcal{T}}_{N_1}^{(\varepsilon)} &= h_{l1}^* \bar{v}_{N_1} P_L v_l - i h_{l2} \bar{v}_{N_1} \Sigma_{12}^{abs}(-\not{p}) \left[-\not{p} - m_{N_2} + i \Sigma_{22}^{abs}(-\not{p}) \right]^{-1} P_L v_l \\ &= h_{l1}^* \bar{u}_l P_L u_{N_1} - i h_{l2}^* \bar{u}_l P_L \left[\not{p} - m_{N_2} + i \overline{\Sigma}_{22}^{abs}(\not{p}) \right]^{-1} \overline{\Sigma}_{21}^{abs}(\not{p}) u_{N_1}, \end{aligned} \quad (6.4)$$

where

$$\bar{\Sigma}_{ij}^{abs}(\not{p}) = A_{ij}(p^2) \not{p}P_R + A_{ij}^*(p^2) \not{p}P_L \quad (6.5)$$

is the charge-conjugate absorptive self-energy. The last step of Eq. (6.4) is derived by making use of the identities

$$u(p, s) = C\bar{v}^T(p, s), \quad C\gamma_\mu C^{-1} = -\gamma_\mu^T. \quad (6.6)$$

The expressions in Eqs. (6.1) and (6.4) may be simplified even further, if the Dirac equation of motion is employed for the external spinors. Then, the two resummed decay amplitudes, $\mathcal{T}_{N_1}^{(\varepsilon)}$ and $\bar{\mathcal{T}}_{N_1}^{(\varepsilon)}$, take the simple form

$$\mathcal{T}_{N_1}^{(\varepsilon)} = \bar{u}_l P_R u_{N_1} \left[h_{l1} - ih_{l2} \frac{m_{N_1}^2 (1 + iA_{22}) A_{21}^* + m_{N_1} m_{N_2} A_{21}}{m_{N_1}^2 (1 + iA_{22})^2 - m_{N_2}^2} \right], \quad (6.7)$$

$$\bar{\mathcal{T}}_{N_1}^{(\varepsilon)} = \bar{u}_l P_L u_{N_1} \left[h_{l1}^* - ih_{l2}^* \frac{m_{N_1}^2 (1 + iA_{22}) A_{21} + m_{N_1} m_{N_2} A_{21}^*}{m_{N_1}^2 (1 + iA_{22})^2 - m_{N_2}^2} \right]. \quad (6.8)$$

In addition, the respective transition amplitudes involving the decays $N_2 \rightarrow l^- \chi^+$, $\mathcal{T}_{N_2}^{(\varepsilon)}$, and $N_2 \rightarrow l^+ \chi^-$, $\bar{\mathcal{T}}_{N_2}^{(\varepsilon)}$, may be obtained by interchanging the indices ‘1’ and ‘2’ everywhere in Eqs. (6.7) and (6.8).

In order to study the ε - and ε' -type mechanisms of CP violation in heavy Majorana neutrino decays, we define the following CP-violating quantities:

$$\varepsilon_{N_i} = \frac{|\mathcal{T}_{N_i}^{(\varepsilon)}|^2 - |\bar{\mathcal{T}}_{N_i}^{(\varepsilon)}|^2}{|\mathcal{T}_{N_i}^{(\varepsilon)}|^2 + |\bar{\mathcal{T}}_{N_i}^{(\varepsilon)}|^2}, \quad \text{for } i = 1, 2, \quad (6.9)$$

$$\varepsilon_N = \frac{|\mathcal{T}_{N_1}^{(\varepsilon)}|^2 + |\mathcal{T}_{N_2}^{(\varepsilon)}|^2 - |\bar{\mathcal{T}}_{N_1}^{(\varepsilon)}|^2 - |\bar{\mathcal{T}}_{N_2}^{(\varepsilon)}|^2}{|\mathcal{T}_{N_1}^{(\varepsilon)}|^2 + |\mathcal{T}_{N_2}^{(\varepsilon)}|^2 + |\bar{\mathcal{T}}_{N_1}^{(\varepsilon)}|^2 + |\bar{\mathcal{T}}_{N_2}^{(\varepsilon)}|^2}. \quad (6.10)$$

Correspondingly, the CP-violating parameters ε'_{N_i} and ε'_N may be defined by

$$\varepsilon'_{N_i} = \frac{|\mathcal{T}_{N_i}^{(\varepsilon')}|^2 - |\bar{\mathcal{T}}_{N_i}^{(\varepsilon')}|^2}{|\mathcal{T}_{N_i}^{(\varepsilon')}|^2 + |\bar{\mathcal{T}}_{N_i}^{(\varepsilon')}|^2}, \quad \text{for } i = 1, 2, \quad (6.11)$$

$$\varepsilon'_N = \frac{|\mathcal{T}_{N_1}^{(\varepsilon')}|^2 + |\mathcal{T}_{N_2}^{(\varepsilon')}|^2 - |\bar{\mathcal{T}}_{N_1}^{(\varepsilon')}|^2 - |\bar{\mathcal{T}}_{N_2}^{(\varepsilon')}|^2}{|\mathcal{T}_{N_1}^{(\varepsilon')}|^2 + |\mathcal{T}_{N_2}^{(\varepsilon')}|^2 + |\bar{\mathcal{T}}_{N_1}^{(\varepsilon')}|^2 + |\bar{\mathcal{T}}_{N_2}^{(\varepsilon')}|^2}. \quad (6.12)$$

The last parameters quantify CP violation coming exclusively from the one-loop irreducible vertices. In Eqs. (6.9) and (6.10), the parameters ε_{N_i} and ε_N share the common property that they do not depend on the final state that N_i decays, despite the fact that the individual squared matrix elements do. In general, both ε - and ε' -type contributions are not directly distinguishable in the decay widths $\Gamma(N_i \rightarrow l^\pm \chi^\pm)$, unless $\varepsilon_{N_i} \gg \varepsilon'_{N_i}$ and vice versa, for some range of the kinematic

parameters. Evidently, the physical CP asymmetries are given by

$$\delta_{N_i} = \frac{\Gamma(N_i \rightarrow L\Phi^\dagger) - \Gamma(N_i \rightarrow L^C\Phi)}{\Gamma(N_i \rightarrow L\Phi^\dagger) + \Gamma(N_i \rightarrow L^C\Phi)}, \quad \text{for } i = 1, 2, \quad (6.13)$$

$$\delta_N = \frac{\sum_{i=1}^2 \Gamma(N_i \rightarrow L\Phi^\dagger) - \sum_{i=1}^2 \Gamma(N_i \rightarrow L^C\Phi)}{\sum_{i=1}^2 \Gamma(N_i \rightarrow L\Phi^\dagger) + \sum_{i=1}^2 \Gamma(N_i \rightarrow L^C\Phi)}, \quad (6.14)$$

where L refers to all fermionic degrees of freedom of the leptonic isodoublet that heavy Majorana neutrinos can decay. Nevertheless, the parameters ε_{N_i} , ε_N , ε'_{N_i} and ε'_N defined above are very useful to determine the contributions due to the different mechanisms of CP violation.

We now turn to the calculation of the CP-violating contribution, which is entirely due to the heavy-neutrino self-energy effects. Substituting Eqs. (6.7) and (6.8) into (6.9), we arrive at the simple formulas²¹

$$\varepsilon_{N_1} \approx \frac{\text{Im}(h_{11}^* h_{12})^2}{|h_{11}|^2 |h_{12}|^2} \frac{\Delta m_N^2 m_{N_1} \Gamma_{N_2}}{(\Delta m_N^2)^2 + m_{N_1}^2 \Gamma_{N_2}^2}, \quad (6.15)$$

$$\varepsilon_{N_2} \approx \frac{\text{Im}(h_{11}^* h_{12})^2}{|h_{11}|^2 |h_{12}|^2} \frac{\Delta m_N^2 m_{N_2} \Gamma_{N_1}}{(\Delta m_N^2)^2 + m_{N_2}^2 \Gamma_{N_1}^2}, \quad (6.16)$$

where $\Delta m_N^2 = m_{N_1}^2 - m_{N_2}^2$ and

$$\Gamma_{N_i} = \frac{|h_{li}|^2}{8\pi} m_{N_i} \quad (6.17)$$

are the decay widths of the heavy Majorana neutrinos. Equations (6.15) and (6.16) are a very good approximation for any range of heavy-neutrino masses of interest. Both CP asymmetries ε_{N_1} and ε_{N_2} are of the same sign and go individually to zero when $\Delta m_N^2 \rightarrow 0$, as it should be on account of Eq. (3.14). In the conventional perturbation theory, the width terms $m_{N_1}^2 \Gamma_{N_2}^2$ and $m_{N_2}^2 \Gamma_{N_1}^2$ occurring in the last denominators on the RHS of Eqs. (6.15) and (6.16) are absent. This very last fact is precisely what causes a singular behaviour when the degeneracy between the two heavy Majorana neutrinos is exact. On physical grounds, however, the only natural parameter that can regulate such a singularity is the finite width of the heavy neutrinos, which is naturally implemented within the resummation approach.

From Eqs. (6.15) and (6.16), it is not difficult to derive the sufficient and necessary conditions for resonant enhancement of CP violation. To be specific, CP violation can be of order unity if and only if

$$(i) \quad m_{N_1} - m_{N_2} \sim \pm A_{22} m_{N_2} = \frac{\Gamma_{N_2}}{2} \quad \text{and/or} \quad A_{11} m_{N_1} = \frac{\Gamma_{N_1}}{2}, \quad (6.18)$$

$$(ii) \quad \delta_{CP} = \frac{|\text{Im}(h_{11}^* h_{12})^2|}{|h_{11}|^2 |h_{12}|^2} \approx 1. \quad (6.19)$$

Before we present numerical estimates of CP asymmetries, we calculate for completeness the contributions to CP violation arising entirely from vertex effects.

The ε' -type contributions can be significant for large differences of heavy neutrino masses, e.g. for $m_{N_1} - m_{N_2} \sim m_{N_1}$ or m_{N_2} . In this regime, both ε -type and ε' -type effects are of comparable order.¹⁹ It is useful to define first the function

$$F(x, \alpha) = \sqrt{x} \left[1 - \alpha - (1+x) \ln \left(\frac{1-\alpha+x}{x} \right) \right]. \quad (6.20)$$

With $\alpha = 0$, $F(x, \alpha)$ reduces to the Fukugita-Yanagida loop function $f(x) = \sqrt{x}[1 - (1+x) \ln(1+1/x)]$.¹⁵ Then, L -violating absorptive parts of the one-loop vertices $\chi^+ l N_i$, $\chi^0 \nu_l N_i$ and $H \nu_l N_i$, shown in Figs. 2(a)–(c), are given by

$$\mathcal{V}_{\chi^+ l N_i}^{abs}(\not{p}) = - \frac{h_{l'j}^* h_{l'j} h_{lj}}{16\pi\sqrt{p^2}} \not{p} P_L F\left(\frac{m_{N_j}^2}{p^2}, 0\right), \quad (6.21)$$

$$\begin{aligned} \mathcal{V}_{\chi^0 \nu_l N_i}^{abs}(\not{p}) &= \mathcal{V}_{H \nu_l N_i}^{abs}(\not{p}) \\ &= - \frac{h_{l'j}^* h_{l'j} h_{lj}}{32\pi\sqrt{p^2}} \not{p} P_L \left[F\left(\frac{m_{N_j}^2}{p^2}, 0\right) + F\left(\frac{m_{N_j}^2}{p^2}, \frac{M_H^2}{p^2}\right) \right]. \end{aligned} \quad (6.22)$$

Here, we have assumed that the external decaying heavy Majorana neutrinos are off-shell, whereas the leptons and Higgs fields are on their mass shell. The complete analytic expressions are calculated in the appendix. Using Eqs. (6.21) and (6.22) and neglecting wave-function contributions, we compute the ε' -type CP asymmetry in the conventional perturbation theory. Considering all decay channels for the decaying heavy Majorana neutrino, e.g. N_1 , we find

$$\begin{aligned} \varepsilon'_{N_1} &= \frac{\text{Im}(h_{l1}^* h_{l2})^2}{16\pi|h_{l1}|^2 \left[\frac{3}{4} + \frac{1}{4}(1 - M_H^2/m_{N_1}^2)^2 \right]} \left\{ \frac{5}{4} F\left(\frac{m_{N_2}^2}{m_{N_1}^2}, 0\right) + \frac{1}{4} F\left(\frac{m_{N_2}^2}{m_{N_1}^2}, \frac{M_H^2}{m_{N_1}^2}\right) \right. \\ &\quad \left. + \frac{1}{4} \left(1 - \frac{M_H^2}{m_{N_1}^2}\right)^2 \left[F\left(\frac{m_{N_2}^2}{m_{N_1}^2}, 0\right) + F\left(\frac{m_{N_2}^2}{m_{N_1}^2}, \frac{M_H^2}{m_{N_1}^2}\right) \right] \right\}. \end{aligned} \quad (6.23)$$

In the limit $M_H \rightarrow 0$, the last formula simplifies to the known result^{15,16,17,18}

$$\varepsilon'_{N_1} = \frac{\text{Im}(h_{l1}^* h_{l2})^2}{8\pi|h_{l1}|^2} f\left(\frac{m_{N_2}^2}{m_{N_1}^2}\right). \quad (6.24)$$

Unlike ε_{N_1} , ε'_{N_1} does not vanish in the degenerate limit of the two heavy Majorana neutrinos N_1 and N_2 . However, when the value of m_{N_1} approaches that of m_{N_2} , the ε' -type part of the transition amplitude squared for the N_1 decay becomes equal but opposite in sign to the respective one of the N_2 decay. As a result, these two ε' -type terms cancel one another, leading to the vanishing of the CP-violating parameter ε'_N defined in Eq. (6.12). Consequently, as opposed to ε effects, ε' effects cannot become resonant for any kinematic region of mass parameters.

Both ε and ε' contributions can be included into the resummed decay amplitudes. Considering Eqs. (6.21) and (6.22) into account, we obtain

$$\mathcal{T}_{N_1} = \bar{u}_l P_R \left\{ h_{l1} + i \mathcal{V}_{l1}^{abs}(\not{p}) - i \left[h_{l2} + i \mathcal{V}_{l2}^{abs}(\not{p}) \right] \right\}$$

$$\times \left[\not{p} - m_{N_2} + i\Sigma_{22}^{abs}(\not{p}) \right]^{-1} \Sigma_{21}^{abs}(\not{p}) \left. \vphantom{\left[\not{p} - m_{N_2} + i\Sigma_{22}^{abs}(\not{p}) \right]^{-1}} \right\} u_{N_1}, \quad (6.25)$$

$$\begin{aligned} \overline{\mathcal{T}}_{N_1} &= \bar{u}_l P_L \left\{ h_{l1}^* + i\overline{\mathcal{V}}_{l1}^{abs}(\not{p}) - i \left[h_{l2}^* + i\overline{\mathcal{V}}_{l2}^{abs}(\not{p}) \right] \right. \\ &\quad \left. \times \left[\not{p} - m_{N_2} + i\overline{\Sigma}_{22}^{abs}(\not{p}) \right]^{-1} \overline{\Sigma}_{21}^{abs}(\not{p}) \right\} u_{N_1}, \end{aligned} \quad (6.26)$$

where the notation of the off-shell one-loop vertices has been simplified to $\mathcal{V}_i^{abs}(\not{p})$. The vertex functions $\overline{\mathcal{V}}_i^{abs}(\not{p})$ are the charge conjugates of $\mathcal{V}_i^{abs}(\not{p})$ and may hence be recovered from Eqs. (6.21) and (6.22), by taking the complex conjugate for the Yukawa couplings and replacing P_R with P_L . Although the calculation of the CP-violating observables δ_{N_i} defined in Eq. (6.13) is quite straightforward from Eqs. (6.25) and (6.26), it is not very easy to present analytic expressions in a compact form.

To gauge better the dependence of CP asymmetries on the heavy neutrino masses, we shall adopt two simple scenarios with two-right handed neutrinos that mix actively with one lepton family l only:

$$\begin{aligned} \text{I.} \quad & m_{N_1} = 10 \text{ TeV}, \quad h_{l1} = 10^{-6}, \quad h_{l2} = 10^{-6}(1+i), \\ \text{II.} \quad & m_{N_1} = 10^9 \text{ TeV}, \quad h_{l1} = 10^{-2}, \quad h_{l2} = 10^{-2}(1+i). \end{aligned} \quad (6.27)$$

We assume that N_2 is always heavier than N_1 , i.e. $m_{N_1} \leq m_{N_2}$. The above two scenarios comply qualitatively with Sakharov's third requirement of out-of-equilibrium condition (see also discussion in Section 8). In view of Eq. (6.19), both scenarios I and II given above represent maximal cases of CP violation with $\delta_{CP} = 1$. Therefore, results for any other model may readily be read off by multiplying the CP asymmetries with the appropriate model-dependent factor δ_{CP} .

Figure 5 exhibits the dependence of the CP asymmetries as a function of the parameter x_N for scenario I. The parameter x_N defined in Eq. (3.12) is a measure of mass degeneracy for the two heavy Majorana neutrinos N_1 and N_2 . We divide the range of values for the parameter x_N into two regions: the first region is plotted in Fig. 5(a) and pertains to the kinematic domain where resonant CP violation due to heavy-neutrino mixing occurs. The second one, shown in Fig. 5(b), represents the kinematic range far away from the resonant CP-violating phenomenon. The dotted line in Fig. 5(a) gives the prediction of ε_{N_1} , when Eq. (6.15) is calculated in the conventional finite-order perturbation theory. Obviously, $\varepsilon_{N_1}^{pert}$ diverges for sufficiently small values of x_N , e.g. $x_N < 10^{-13}$. If resummation of the relevant fermionic self-energy graphs is considered, the prediction for ε_{N_1} becomes analytic and is given by the dashed line in Fig. 5. The ε_{N_1} line shows a maximum for $x_N \approx 10^{-13}$. In agreement with the conditions in Eqs. (6.18) and (6.19), CP violation may resonantly increase up to order unity.^{36,30} The solid line in Fig. 5 displays the dependence of the CP-violating parameter δ_N in Eq. (6.14) on x_N , where ε' -type contributions are included. The latter are very small in this scenario, so as to account for the BAU, e.g. $\varepsilon'_{N_1} \approx 10^{-16}$. Finally, we comment on the fact that δ_N vanishes in the CP-invariant limit $x_N \rightarrow 0$, as it should be on account of Eq. (3.14).

Figures 6 and 7 give numerical estimates of CP asymmetries in scenario II. The difference of this model with that of scenario I is that the ε' -type effects may not be negligible in the off-resonant region, as can be seen from Figs. 6(a) and 7. In particular, for values of the parameter $x_N < 10^{-11}$ or $x_N > 1$, the individual ε'_{N_1} - and ε'_{N_2} -type contributions may prevail over the ε -type ones. Models with $x_N > 1$ have been extensively discussed in the literature.^{15,16,17,18} Numerical estimates for such models are displayed in Fig. 7. We first focus our attention on the domain with $x_N < 10^{-2}$. In Fig. 6(a), we observe that ε'_{N_1} and ε'_{N_2} , represented by the dotted lines, do not vanish in the CP-invariant limit $x_N \rightarrow 0$, as opposed to ε_{N_1} . As a consequence, the CP asymmetry δ_{N_1} in Eq. (6.13), in which both ε_{N_1} - and ε'_{N_1} -type terms are considered within our formalism, does not vanish either. The

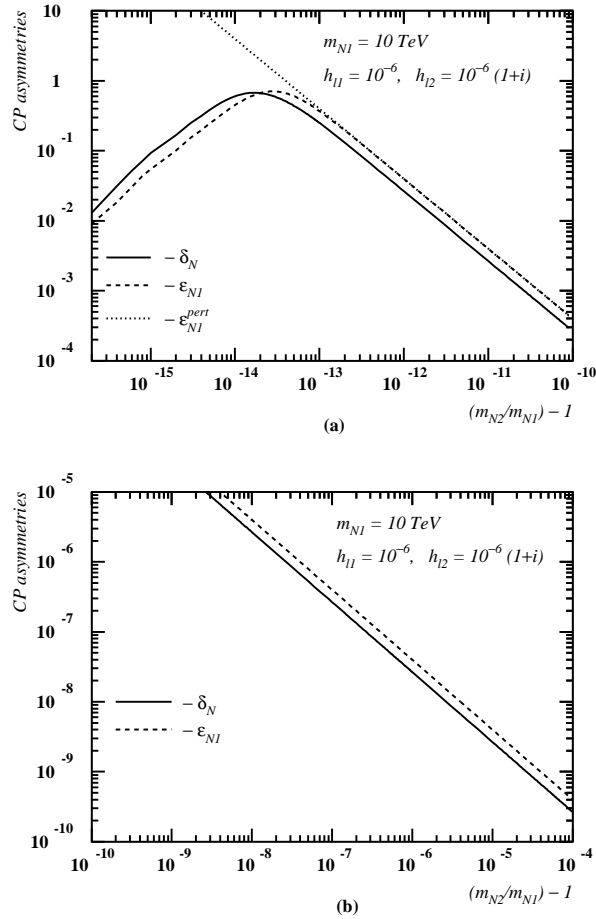
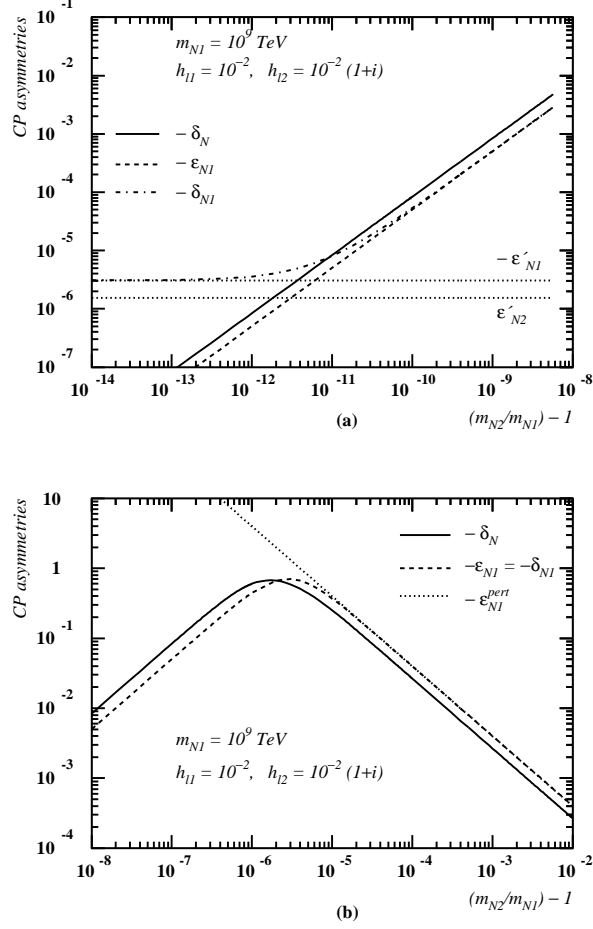


Fig. 5. Numerical estimates of CP asymmetries in scenario I.

Fig. 6. Numerical estimates of CP asymmetries versus $m_{N2}/m_{N1} - 1$ in scenario II.

reason is that the physical CP-violating parameter in this highly degenerate mass regime for N_1 and N_2 is the observable δ_N defined in Eq. (6.14). In fact, δ_N and ϵ_{N1} share the common feature that both tend consistently to zero as $x_N \rightarrow 0$. This fact must be considered to be one of the successes of the resummation approach. Again, CP violation is resonantly amplified, when the condition in Eq. (6.18) is satisfied, as can be seen from Fig. 6(b). Finally, we must remark that $-\delta_N$ flips sign and eventually becomes negative for $x_N \gg 1$, as can be seen from Fig. 7. However, in this kinematic range, we must consider a further refinement into the definition of δ_N . The effect of the different dissipative Boltzmann factors multiplying the decay rates of the heavy Majorana neutrinos N_1 and N_2 must also be included in δ_N . These phenomena will be taken into account in Section 8.

7. Unitarity and CPT invariance in the resummation approach

It is interesting to see how the resummation approach preserves CPT invariance and unitarity.³⁰ An immediate consequence of unitarity and CPT symmetry is that CP violation in the L -violating scattering process $L\Phi^\dagger \rightarrow L^C\Phi$ is absent to order h_{li}^6 .^{54,55} We will concentrate on the resonant part of the amplitude, as it is the dominant one.

Our aim is to show that to ‘one loop’,

$$\Delta_{\text{CP}} = \int d\text{LIPS} |\mathcal{T}_{L\Phi^\dagger \rightarrow L^C\Phi}^{\text{res}}|^2 - \int d\text{LIPS} |\overline{\mathcal{T}}_{L^C\Phi \rightarrow L\Phi^\dagger}^{\text{res}}|^2 = 0, \quad (7.1)$$

where LIPS stands for the two-body Lorentz-invariant phase space. For simplicity, we omit external spinors and absorb irrelevant constants in the definition of the Yukawa-coupling matrix $h = (h_{l1}, h_{l2})$. Using matrix notation, the resummed transition amplitudes are written

$$\mathcal{T}_{L\Phi^\dagger \rightarrow L^C\Phi}^{\text{res}} = h P_R S(\not{p}) P_R h^T, \quad \overline{\mathcal{T}}_{L^C\Phi \rightarrow L\Phi^\dagger}^{\text{res}} = h^* P_L \bar{S}(\not{p}) P_L h^\dagger, \quad (7.2)$$

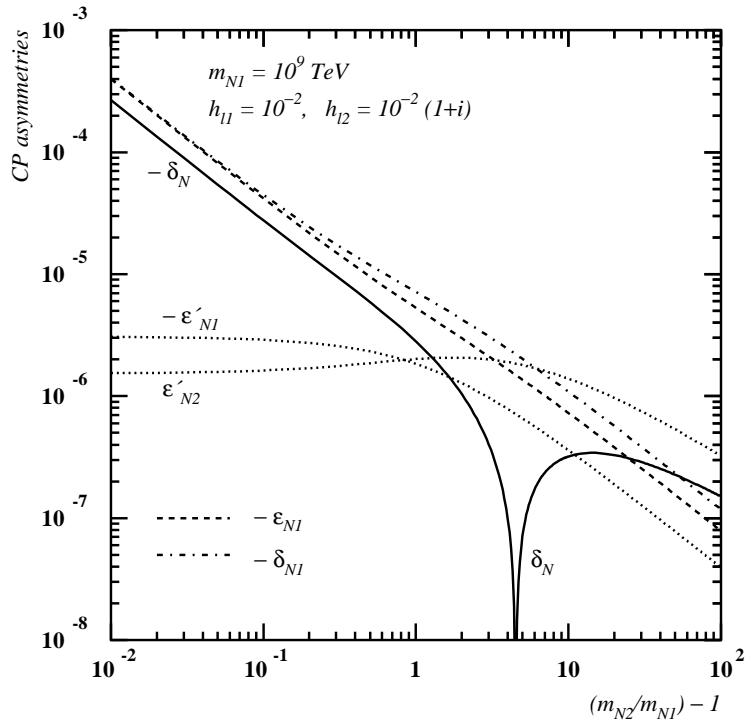


Fig. 7. Numerical estimates of CP asymmetries as a function of $m_{N2}/m_{N1} - 1$ in scenario II.

with $\bar{S}(\not{p}) = S^T(\not{p})$ being the CP/T-conjugate propagator matrix of $S(\not{p})$. In writing the CP/T-conjugate amplitude $\overline{\mathcal{T}}_{L^C\Phi \rightarrow L\Phi^\dagger}^{\text{res}}$, we have employed the identities (6.6) for spinor objects and made use of the rotational symmetry of the amplitude. The latter has the effect of reversing the spatial components of the four momenta. We also neglect possible P-odd spin correlations involving external leptons since they will be averaged away when forming the matrix element squared.

We start the proof by noticing that as a consequence of CPT invariance,

$$|\mathcal{T}_{L\Phi^\dagger \rightarrow L\Phi^\dagger}^{\text{res}}|^2 = |\mathcal{T}_{L^C\Phi \rightarrow L^C\Phi}^{\text{res}}|^2. \quad (7.3)$$

This equality is indeed valid, since

$$|hP_R S(\not{p}) P_L h^\dagger| = |h^* P_L S^T(\not{p}) P_R h^T| = |h^* P_L \bar{S}(\not{p}) P_R h^T|. \quad (7.4)$$

Unitarity of the theory prescribes the following relation governing the resummed propagators:

$$S^{-1}(\not{p}) - S^{-1\dagger}(\not{p}) = -i \int d\text{LIPS } \not{p}(h^T h^* P_L + h^\dagger h P_R). \quad (7.5)$$

This last relation is also known as the optical theorem. Based on the optical theorem, we can prove the equality

$$\int d\text{LIPS } |\mathcal{T}_{L\Phi^\dagger \rightarrow L\Phi^\dagger, L^C\Phi}^{\text{res}}|^2 = \int d\text{LIPS } |\overline{\mathcal{T}}_{L^C\Phi \rightarrow L\Phi^\dagger, L^C\Phi}^{\text{res}}|^2. \quad (7.6)$$

Indeed, using Eq. (7.5), we find

$$\begin{aligned} & \int d\text{LIPS } |\mathcal{T}_{L\Phi^\dagger \rightarrow L\Phi^\dagger, L^C\Phi}^{\text{res}}|^2 \\ &= \int d\text{LIPS } hP_R S(\not{p}) \not{p}(h^T h^* P_L + h^\dagger h P_R) S^\dagger(\not{p}) P_L h^\dagger \\ &= -i hP_R [S(\not{p}) - S^\dagger(\not{p})] P_L h^\dagger = 2hP_R \text{Im}S(\not{p}) P_L h^\dagger, \end{aligned} \quad (7.7)$$

and for the CP-conjugate total rate,

$$\begin{aligned} \int d\text{LIPS } |\mathcal{T}_{L^C\Phi \rightarrow L\Phi^\dagger, L^C\Phi}^{\text{res}}|^2 &= 2h^* P_L \text{Im}\bar{S}(\not{p}) P_R h^T \\ &= 2hP_R \text{Im}S(\not{p}) P_L h^\dagger. \end{aligned} \quad (7.8)$$

As the RHSs of Eqs. (7.7) and (7.8) are equal, the equality (7.6) is obvious. Subtracting Eq. (7.3) from Eq. (7.6), it is not difficult to show that Δ_{CP} vanishes at the one-loop resummed level. We should remark that the resummation approach³⁰ satisfies CPT and unitarity *exactly*, without recourse to any re-expansion of the resummed propagator. If we also include resummed amplitudes subleading in the Yukawa couplings, then residual CP-violating terms that are formally of order h_{li}^8 and higher occur in Δ_{CP} . These terms result from the interference of two resummed amplitudes containing one-loop vertex graphs. Because of unitarity, however, the

residual CP-violating terms of order h_{li}^8 and h_{li}^{10} will cancel at two loops together with respective CP-violating terms coming from one-loop $2 \rightarrow 4$ scatterings, and so on.

In the approach under consideration,³⁰ the physical transition amplitude is obtained by sandwiching the resummed propagators between matrix elements related to initial and final states of the resonant process. Therefore, diagonalization of $S(\not{p})$ is no longer necessary, thereby avoiding possible singularities emanating from non-diagonalizable (Jordan-like) effective Hamiltonians [or equivalently $S^{-1}(\not{p})$].³⁰ In fact, such effective Hamiltonians represent situations in which the CP-violating mixing between the two unstable particles reaches its maximum and physical CP asymmetries are therefore large. In such a case, the complex mass eigenvalues of the effective Hamiltonian are exactly equal.

To see this point in more detail, let us consider the following effective Hamiltonian for the $N_1 N_2$ system:

$$\mathcal{H}(\not{p}) = \begin{bmatrix} m_1 - \widehat{\Sigma}_{11}(\not{p}) & -\widehat{\Sigma}_{12}(\not{p}) \\ -\widehat{\Sigma}_{21}(\not{p}) & m_2 - \widehat{\Sigma}_{22}(\not{p}) \end{bmatrix} \approx \begin{bmatrix} m_N + a - i|b| & -ib \\ -ib^* & m_N - a - i|b| \end{bmatrix}, \quad (7.9)$$

in the approximation $\not{p} \rightarrow m_N \approx m_1 \approx m_2$. In Eq. (7.9), the parameters a and b are real and complex, respectively, and $m_1 = m_N + a$, $m_2 = m_N - a$. The complex parameter b represents the absorptive part of the one-loop neutrino transitions $N_i \rightarrow N_j$. Unitarity requires that the determinant of the absorptive part of $\mathcal{H}(\not{p})$ be non-negative. For the effective Hamiltonian (7.9), the corresponding determinant is zero. One-generation models naturally lead to such an absorptive effective Hamiltonian. If $a = |b|$, the two complex mass eigenvalues of $\mathcal{H}(\not{p})$ are exactly degenerate and equal to $m_N - i|b|$. Then, the effective Hamiltonian cannot be diagonalized via a similarity transformation in this limit, i.e. the respective diagonalization matrices become singular.³⁰

An interesting question one may raise in this context is the following. Since models with non-diagonalizable effective Hamiltonians lead to an exact equality between their complex mass eigenvalues, how then can this fact be reconciled with the CP-invariance condition (3.14)? According to the condition (3.14), any effect of CP violation must vanish identically, and should not even be large! To resolve this paradox, one should notice that in the presence of a large particle mixing, the mass eigenstates of $S^{-1}(\not{p})$ are generally non-unitary among themselves, whereas the OS-renormalized mass eigenstates⁴⁹ form a well-defined unitary basis (or any other renormalization scheme that preserves orthonormality of the Hilbert space), upon which perturbation theory can be formulated order by order. Therefore, the field-theoretic OS renormalized masses are those that enter the condition of CP invariance given by Eq. (3.14). Consequently, if the two complex mass eigenvalues of the effective Hamiltonian are equal, this does not necessarily entail an equality between their respective OS renormalized masses, and therefore absence of CP violation as well.³⁰

8. Boltzmann equations

The thermodynamic evolution of the system in the radiation-dominated era of the Universe may be described by a set of coupled Boltzmann equations (BE's).^{4,5,2,38} These equations determine the time evolution of the lepton-number asymmetry which will be converted into the observed BAU by sphalerons. We shall solve the BE's numerically and present results for the expected BAU within the two different democratic-type scenarios I and II discussed in Section 6. Finally, we will give estimates of the finite-temperature effects, and discuss their impact on resonant CP violation through mixing.

Before solving numerically the BE's, it is instructive to discuss first the out-of-equilibrium constraints on heavy neutrino decays. Sakharov's third necessary condition requires that the expansion rate of the Universe be smaller than the decay rate of any L -violating process. The most dominant L -violating process is the decay of the heavy Majorana neutrinos themselves, Γ_{N_i} (cf. Eq. (6.17)). To a good approximation, we have the approximate inequality

$$\Gamma_{N_i}(T = m_{N_i}) \lesssim 2 K H(T = m_{N_i}), \quad (8.1)$$

where $K \approx 1$ –1000 is a factor quantifying the deviation of the decay rates from the expansion rate of the Universe, and $H(T)$ is the Hubble parameter

$$H(T) = 1.73 g_*^{1/2} \frac{T^2}{M_{\text{Planck}}}. \quad (8.2)$$

with $M_{\text{Planck}} = 1.2 \times 10^{19}$ GeV and $g_* \approx 100$ –400 being the number of active degrees of freedom in usual extensions of the SM. Then, the out-of-equilibrium constraint in Eq. (8.1) translates into the bound

$$|h_{li}|^2 \lesssim 7.2 K \times 10^{-14} \left(\frac{m_{N_i}}{1 \text{ TeV}} \right). \quad (8.3)$$

Although not mandatory, this very last constraint may be applied to all Yukawa couplings.

As we have discussed in Section 2 (see also Eq. (2.13)), above the electroweak phase transition the $B + L$ -sphaleron interactions convert approximately one-third of the lepton-to-entropy density ratio $Y_L = n_L/s$ into a baryon-to-entropy density ratio $Y_B = n_B/s$, i.e.^{11,38}

$$Y_B \approx -\frac{1}{3} Y_L \approx -\frac{1}{3K} \frac{\delta_{N_i}}{g_*}. \quad (8.4)$$

The last approximate equality represents the asymptotic solution of the relevant BE's.^{16,21} From Eq. (8.4), we see that Y_B can be in the observed ballpark, i.e. $Y_B \approx 10^{-10}$, if $|\delta_{N_i}|/K$ are of order 10^{-7} – 10^{-6} . Clearly, CP asymmetries of order unity allow for very large values of K . As a consequence, the thermal plasma can then be rather dense and the conditions of kinetic equilibrium in BE's can comfortably be satisfied even within the minimal leptogenesis scenario under study.

We now turn to the discussion of BE's. The lepton asymmetry for a system with two heavy Majorana neutrinos is determined by the coupled system of BE's^{2,16}

$$\frac{dn_{N_i}}{dt} + 3Hn_{N_i} = - \left(\frac{n_{N_i}}{n_{N_i}^{eq}} - 1 \right) \gamma_{N_i} \quad (8.5)$$

$$\frac{dn_L}{dt} + 3Hn_L = \sum_{i=1}^2 \left[\delta_{N_i} \left(\frac{n_{N_i}}{n_{N_i}^{eq}} - 1 \right) - \frac{n_L}{2n_l^{eq}} \right] \gamma_{N_i} - \frac{n_L}{n_l^{eq}} \gamma_\sigma, \quad (8.6)$$

where n_{N_i} , $n_L = n_l - n_{\bar{l}}$ are the densities of the number of N_i and the lepton-number asymmetry, respectively, and $n_{N_i}^{eq}$ and n_l^{eq} are their values in thermal equilibrium. The Hubble parameter $H = (dR/dt)/R$ determines the expansion rate of the Universe and also depends on the temperature T , through the relation in Eq. (8.2). In Eqs. (8.5) and (8.6), γ_{N_i} and γ_σ are the decay and scattering collision terms, respectively:

$$\gamma_{N_i} = n_{N_i}^{eq} \frac{K_1(m_{N_i}^2/T)}{K_2(m_{N_i}^2/T)} \Gamma_{N_i}, \quad (8.7)$$

$$\gamma_\sigma = \frac{T}{8\pi^4} \int_{s_{\text{thr}}}^{\infty} ds s^{3/2} K_1(\sqrt{s}/T) \hat{\sigma}(s). \quad (8.8)$$

Here, s_{thr} is the threshold of a generic process $a + b \rightarrow c + d$, and

$$\hat{\sigma}(s) = \frac{1}{4} \theta(\sqrt{s} - m_a - m_b) \lambda \left(1, \frac{m_a^2}{s}, \frac{m_b^2}{s} \right) \hat{\sigma}(s) \quad (8.9)$$

with $\lambda(x, y, z) = (x - y - z)^2 - 4yz$. In Eq. (8.7), $K_1(z)$ and $K_2(z)$ are the modified Bessel functions defined in Ref.⁵⁶ The cross section $\hat{\sigma}(s)$ mainly comprises the scatterings $L^C \Phi \rightarrow L \Phi^\dagger$ and its CP-conjugate process $L \Phi^\dagger \rightarrow L^C \Phi$, and is evaluated at $T = 0$ by subtracting all those real intermediate contributions that have already been taken into account in the direct and inverse decays of heavy Majorana neutrinos.⁵ The collision term γ_σ acts as a CP-conserving depletion term, which is formally of order $\gamma_{N_i}^2$ at $T \approx m_{N_i}$.² There is also the $\Delta L = 2$ reaction $\Phi \Phi \leftrightarrow LL$, which is much weaker than the latter, as long as the out-of-equilibrium constraint on the Yukawa couplings in Eq. (8.3) is imposed. Finally, there exist additional contributions to the BE's,¹⁶ coming from processes such as $N_i L \leftrightarrow Q_i \bar{t}_R$, $N_i Q_i \leftrightarrow L t_R$. These contributions are quite strong at very high temperatures, $T \gg m_{N_i}$, and lead to a decoherence phenomenon between the heavy neutrinos N_1 and N_2 . At the crucial leptogenesis epoch, when $T \approx m_{N_i}$, the rates of the latter processes are kinematically suppressed and smaller than the decay rates of the heavy Majorana neutrinos.²

Many applicable assumptions are involved in BE's (8.5) and (8.6). More details may be found in Ref.² First, we have considered the Friedmann-Robertson-Walker model in the non-relativistic limit. Second, we have adopted the Maxwell-Boltzmann statistics, which is a good approximation in the absence of effects that originate from Bose condensates or arise from a degeneracy of many Fermi degrees

of freedom. Third, we have assumed that the lepton and Higgs weak isodoublets, L and Φ , are practically in thermal equilibrium, and neglected high orders in n_L/n_l^{eq} and δ_{N_i} . In this context, it has also been assumed that the different particle species are in kinetic equilibrium, i.e. that the particles may rapidly change their kinetic energy through elastic scatterings but the processes responsible for a change of the number of particles are out of equilibrium. These out-of-equilibrium reactions are described by the BE's (8.5) and (8.6).

To solve these BE's numerically, it proves useful to make the following change of variables:

$$x = \frac{m_{N_1}}{T}, \quad t = \frac{1}{2H(T)} = \frac{x^2}{2H(x=1)}. \quad (8.10)$$

Such an ansatz is also valid for the radiation-dominated phase of the Universe while baryogenesis takes place. Then, we define the parameters

$$K = \frac{K_1(x)}{K_2(x)} \frac{\Gamma_{N_1}}{H(x=1)}, \quad \gamma = \frac{K_2(x)K_1(\xi x)}{K_1(x)K_2(\xi x)} \frac{\Gamma_{N_2}}{\Gamma_{N_1}}, \quad (8.11)$$

with $\xi = m_{N_2}/m_{N_1} \geq 1$. In addition, we introduce the quantities $Y_{N_i} = n_{N_i}/s$ and $Y_L = n_L/s$, where s is the entropy density. In an isentropically expanded Universe, the entropy density has the time dependence $s(t) = \text{const.} \times R^{-3}(t)$ and may be related to the number density of photons, n_γ , as $s = g_* n_\gamma$, where g_* is given after Eq. (8.2). Employing the above definitions and relations among the parameters, we obtain the BE's in terms of the new quantities Y_{N_1} , Y_{N_2} and Y_L :

$$\frac{dY_{N_1}}{dx} = -(Y_{N_1} - Y_{N_1}^{eq})Kx^2, \quad (8.12)$$

$$\frac{dY_{N_2}}{dx} = -(Y_{N_2} - Y_{N_2}^{eq})\gamma Kx^2, \quad (8.13)$$

$$\begin{aligned} \frac{dY_L}{dx} = & \left[(Y_{N_1} - Y_{N_1}^{eq})\delta_{N_1} + (Y_{N_2} - Y_{N_2}^{eq})\gamma\delta_{N_2} - \frac{1}{2}g_*Y_L(Y_{N_1}^{eq} + \gamma Y_{N_2}^{eq}) \right. \\ & \left. - g_*Y_L Y_{N_1}^{eq} \frac{\gamma\sigma}{\gamma_{N_1}} \right] Kx^2. \end{aligned} \quad (8.14)$$

The heavy-neutrino number-to-entropy densities in equilibrium $Y_{N_i}^{eq}(x)$ are given by⁵

$$Y_{N_1}^{eq}(x) = \frac{3}{8g_*} \int_x^\infty dz z \sqrt{z^2 - x^2} e^{-z} = \frac{3}{8g_*} x^2 K_2(x), \quad (8.15)$$

and $Y_{N_2}^{eq}(x) = Y_{N_1}^{eq}(\xi x)$. The differential equations (8.12)–(8.14) are solved numerically, using the initial conditions

$$Y_{N_1}(0) = Y_{N_2}(0) = Y_{N_1}^{eq}(0) = Y_{N_2}^{eq}(0) \quad \text{and} \quad Y_L(0) = 0. \quad (8.16)$$

These initial conditions merely reflect the fact that our Universe starts evolving from a lepton-symmetric state, in which the heavy Majorana neutrinos are originally in thermal equilibrium. Here, we should remark that the low-temperature limit of the numerical predictions does not strongly depend on the initial conditions (8.16), if L -violating interactions are in thermal equilibrium at $T \gg m_{N_i}$. The reason is that at

very high temperatures, the BE's (8.12)–(8.14) exhibit a running independent fixed point, and any initial value for Y_{N_1} , Y_{N_2} and Y_L is rapidly driven to the thermal-equilibrium values given by Eq. (8.16).⁵⁷ After the evolution of the Universe to temperatures much below m_{N_1} , a net lepton asymmetry has been created. This lepton asymmetry will then be converted into the BAU via the sphalerons. During a first order electroweak phase transition, the produced excess in L is also encoded as an excess in B , which is given by Eq. (2.13).^{11,38} The observed BAU is $Y_B^{obs} = (0.6 - 1) \times 10^{-10}$,² which corresponds to an excess of leptons $-Y_L^{obs} \approx 10^{-9} - 10^{-10}$. In the latter estimate, we have included the possibility of generating the BAU via an individual lepton asymmetry.³⁹

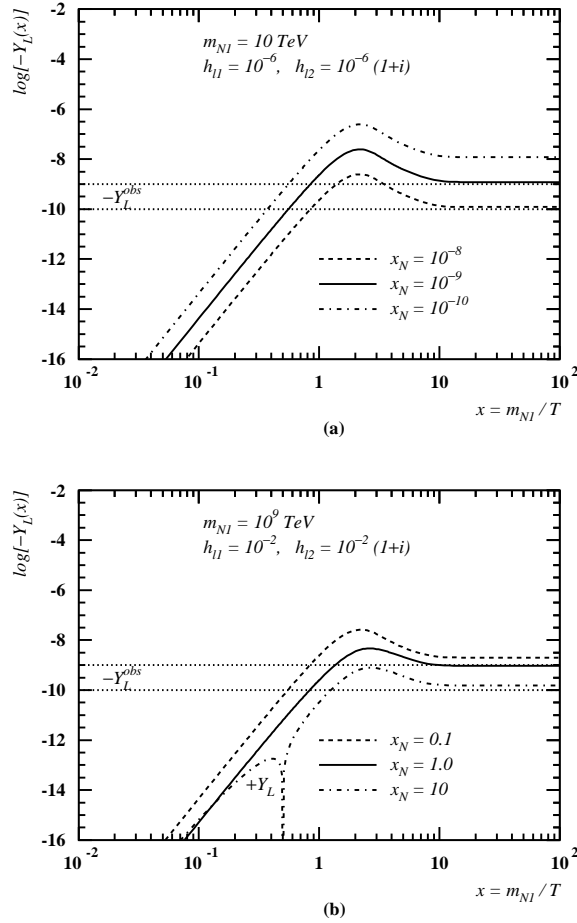


Fig. 8. Lepton asymmetries for selected heavy Majorana neutrino scenarios.

Figure 8 shows the dependence of $Y_L(x)$ on $x = m_{N_1}/T$ for two representative

scenarios defined in Eq. (6.27) for different values of $x_N = m_{N_2}/m_{N_1} - 1$.²¹ The observed range for Y_L, Y_L^{obs} , is indicated with two confining horizontal dotted lines. In scenario I (Fig. 8(a)), a heavy-neutrino mass splitting x_N of order 10^{-9} – 10^{-8} is sufficient to account for the BAU. For comparison, it is worth mentioning that the degree of mass degeneracy between K_L and K_S is of order 10^{-15} , which is by far smaller than the one considered here. We find that the ε -type CP violation is dominant, whereas ε' -type effect are extremely suppressed. Numerical estimates for the second scenario are displayed in Fig. 8(b). This scenario is closer to the traditional one considered in Ref.¹⁵ Here, it is not necessary to have a high degree of degeneracy for N_1 and N_2 to get sufficient CP violation for the BAU. In this case, both ε - and ε' -type mechanisms of CP violation are equally important. Therefore, the main consequence of ε -type CP violation is that the leptogenesis scale may be as low as 1 TeV, even for models with universal Yukawa couplings.²¹

In the scenario of leptogenesis induced by mixing of heavy Majorana neutrinos, one may have to worry about effects, which could affect the resonant condition of CP violation in Eq. (6.18). For instance, there may be broadening effects at high temperatures due to collisions among particles. Such effects will contribute terms of order $h_{l_i}^4$ to the N_i widths and are small in general.^{5,58} On the other hand, finite temperature effects on the $T = 0$ masses of particles may be significant. Because of the SM gauge interactions, the leptons and the Higgs fields receive appreciable thermal masses,^{59,60,61} i.e.

$$\frac{m_L^2(T)}{T^2} = \frac{1}{32} (3g^2 + g'^2) \approx 0.044, \quad (8.17)$$

where g and g' are the $SU(2)_L$ and $U(1)_Y$ gauge couplings at the running scale M_Z . The isosinglet heavy neutrinos also acquire thermal masses through Yukawa interactions⁵⁹, i.e.

$$\frac{m_{N_i}^2(T) - m_{N_i}^2(0)}{T^2} = \frac{1}{16} |h_{l_i}|^2. \quad (8.18)$$

Such a T -dependent mass shift is small and comparable to the N_i widths at $T \approx m_{N_i}$. Therefore, it is easy to see that the condition for resonant CP violation through mixing in Eq. (6.18) is qualitatively satisfied. Finally, the Higgs field also receives appreciable thermal contributions. The authors of Ref.⁶¹ have computed the one-loop Higgs thermal mass, and found that $M_\Phi(T)/T \lesssim 0.6$ for values of the Higgs-boson mass favoured by LEP2, i.e. $M_H < 200$. In this range of Higgs masses, the thermal widths $\Gamma_{N_i}(T)$ will be reduced with respect to $\Gamma_{N_i}(0)$ by a factor of 2 or 3 due to sizeable phase-space corrections. Nevertheless, the influence on the resonant phenomenon of CP violation through mixing is not dramatic when the latter effects are included, and therefore large leptonic CP asymmetries are still conceivable.

9. Low-energy phenomenology of heavy Majorana neutrinos

Whether heavy Majorana neutrinos can lead to interesting phenomenology at collider and lower energies is an issue that strongly depends on the out-of-equilibrium constraint given by Eq. (8.3). If this constraint is applied to all lepton families, heavy Majorana neutrinos have very little impact on collider phenomenology. However, this very last statement is rather model dependent. One can imagine, for example, a scenario in which ΔL_e -violating operators exist and are out-of-equilibrium, and the μ and τ sectors do not communicate any interaction to the electron sector, i.e. $\Delta(L_e - L_\mu) = \Delta(L_e - L_\tau) = 0$. Since sphalerons conserve the individual quantum number $B/3 - L_e$ (see also Section 2), the observed baryonic asymmetry can be preserved in an excess of L_e , independently of whether ΔL_μ - and/or ΔL_τ -non-conserving operators are in thermal equilibrium or not. As we will discuss below, such scenarios with strong mixing in the muon and tau sectors only can give rise to a variety of new-physics phenomena in a strength that can be probed in laboratory experiments.

9.1. Lepton-flavour and/or number processes

Heavy Majorana neutrinos with masses in the range 0.2 – 1 TeV may be produced directly at high-energy ee ,⁶² ep ,⁶³ and pp colliders,⁶⁴ whose subsequent decays can give rise to distinct like-sign dilepton signals. If heavy Majorana neutrinos are not accessible at high-energy colliders, they can still induce lepton-flavour-violating decays of the Z boson,^{65,66} the Higgs particle,⁶⁷ and the τ and μ leptons.^{68,69} As we will see, non-decoupling quantum effects due to potentially large $SU(2)_L$ -breaking masses play a key role in these flavour-changing-neutral-current (FCNC) phenomena.⁶⁸ Heavy Majorana neutrinos may cause breaking of universality in leptonic diagonal Z -boson⁷⁰ and π decays⁴⁹ or influence⁷¹ the size of the electroweak oblique parameters S , T and U .⁷² In fact, there exist many observables summarized in Ref.⁷³ to which heavy Majorana neutrinos may have sizeable contributions. These observables include τ -polarization asymmetries, neutrino-counting experiments at the CERN Large Electron Positron Collider (LEP1) or at the Stanford Linear Accelerator (SLC), etc.

In the following we shall show that high $SU(2)_L$ -breaking masses in a class of neutrino models can lead to large FCNC effects. Because these effects are not correlated with light neutrino masses, one can overcome the see-saw suppression relations that usually accompany such new-physics phenomena.⁷⁴ Let us consider a two-generation model of the kind. The model is similar to the one discussed in Section 3. It has two isosinglet neutrinos ν'_R and S'_L . In the weak basis $((\nu_{\mu L})^C, (\nu_{\tau L})^C, (S'_L)^C, \nu'_R)$ the neutrino mass matrix then takes the form

$$\mathcal{M}^\nu = \begin{pmatrix} 0 & 0 & 0 & m_1 \\ 0 & 0 & 0 & m_2 \\ 0 & 0 & 0 & M \\ m_1 & m_2 & M & \mu \end{pmatrix}. \quad (9.1)$$

Diagonalization of \mathcal{M}^ν yields two zero eigenvalues, which would correspond to massless μ and τ neutrinos. If $\mu \neq 0$ and $\mu/M \ll 1$, they receive small radiative masses at high orders.⁷⁵ The other two states are very heavy, of order $M \pm \mu$. In contrast to the usual seesaw scenario, the ratios m_1/M and m_2/M remain fully unconstrained. Global analyses of low-energy data⁷³ restrict their values to $m_1/M, m_2/M \lesssim 0.1$. For reader's convenience, we define the parameters

$$(s_L^{\nu\mu})^2 \simeq \frac{m_1^2}{M^2}, \quad (s_L^{\nu\tau})^2 \simeq \frac{m_2^2}{M^2}, \quad (9.2)$$

The newly introduced parameters quantify neutrino mixings between the light and heavy Majorana states. They also parameterize the deviation of the modified $Wl\nu_l$ coupling from the SM one. An extensive discussion is given in Ref.^{73,68} Including renormalization effects into the definition of light-heavy neutrino mixings,⁴⁹ one may tolerate the following upper limits

$$(s_L^{\nu\mu})^2 < 0.010, \quad (s_L^{\nu\tau})^2 < 0.035, \quad \text{and} \quad (s_L^{\nu e})^2 < 1 \times 10^{-8}. \quad (9.3)$$

The last limit comes from the requirement that only the electron family is responsible for baryogenesis. Of course, electron and muon families may interchange their roles in Eq. (9.3).

Heavy Majorana neutrinos can induce sizeable FCNC decays of the type $Z \rightarrow \mu\tau$, $\tau \rightarrow \mu^-e^-e^+$ or $\tau \rightarrow \mu^-\mu^-\mu^+$ through quantum corrections presented in Figs. 9 and 10. Thus, the matrix element relevant for the generic decay $\tau(p_\tau) \rightarrow l(p_l)l_1(p_1)\bar{l}_2(p_2)$ acquires contributions from γ - and Z -mediated graphs as well as from graphs with box diagrams. The respective transition elements are given by

$$\begin{aligned} i\mathcal{T}_\gamma(\tau \rightarrow ll_1\bar{l}_2) &= \frac{\alpha_w^2 s_w^2}{4M_W^2} \delta_{l_1 l_2} \bar{u}_{l_1} \gamma^\mu v_{l_2} \bar{u}_l \left[F_\gamma^{\tau l} (\gamma_\mu - \frac{q_\mu \not{q}}{q^2}) (1 - \gamma_5) \right. \\ &\quad \left. - iG_\gamma^{\tau l} \sigma_{\mu\nu} \frac{q^\nu}{q^2} (m_\tau (1 + \gamma_5) + m_l (1 - \gamma_5)) \right] u_\tau, \end{aligned} \quad (9.4)$$

$$i\mathcal{T}_Z(\tau \rightarrow ll_1\bar{l}_2) = \frac{\alpha_w^2}{16M_W^2} \delta_{l_1 l_2} F_Z^{\tau l} \bar{u}_l \gamma_\mu (1 - \gamma_5) u_\tau \bar{u}_{l_1} \gamma^\mu (1 - 4s_w^2 - \gamma_5) v_{l_2}, \quad (9.5)$$

$$i\mathcal{T}_{\text{box}}(\tau \rightarrow ll_1\bar{l}_2) = \frac{\alpha_w^2}{16M_W^2} F_{\text{box}}^{\tau ll_1 l_2} \bar{u}_l \gamma_\mu (1 - \gamma_5) u_\tau \bar{u}_{l_1} \gamma^\mu (1 - \gamma_5) v_{l_2}, \quad (9.6)$$

where $q = p_1 + p_2$, $s_w^2 = 1 - M_W^2/M_Z^2$, and $F_\gamma^{\tau l}$, $G_\gamma^{\tau l}$, $F_Z^{\tau l}$, $F_{\text{box}}^{\tau ll_1 l_2}$ are certain composite form factors whose analytic form is given in Ref.⁶⁸ Nevertheless, it is useful to examine the asymptotic behaviour of the composite form factors for large values of $\lambda_{N_1} = m_{N_1}^2/M_W^2$ and $\lambda_{N_2} = m_{N_2}^2/M_W^2$ in the two-generation model under discussion. For simplicity, we consider $\lambda_{N_1} \sim \lambda_{N_2} \sim \lambda_N \gg 1$. In this limit, we find

$$F_\gamma^{\tau l} \rightarrow -\frac{1}{6} s_L^{\nu\tau} s_L^{\nu l} \ln \lambda_N, \quad (9.7)$$

$$G_\gamma^{\tau l} \rightarrow \frac{1}{2} s_L^{\nu\tau} s_L^{\nu l}, \quad (9.8)$$

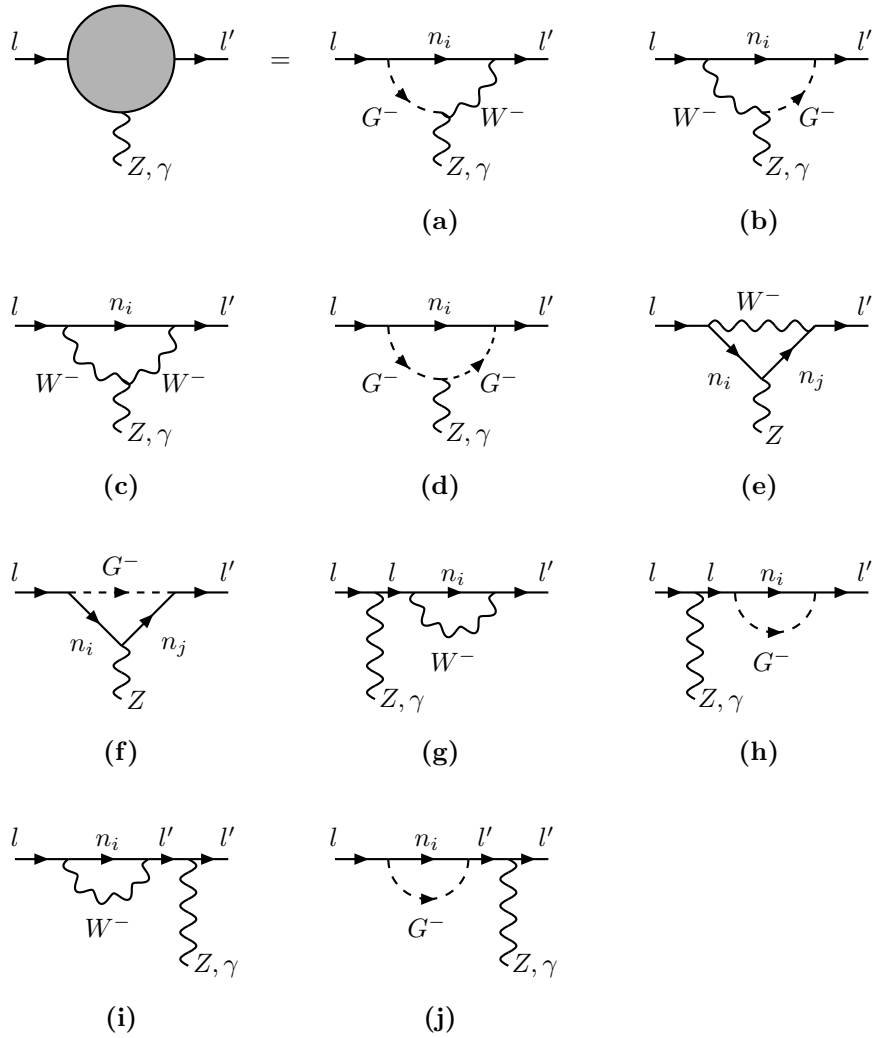


Fig. 9. Feynman graphs pertaining to the decay $Z \rightarrow ll'$. Graphs related to the effective $\gamma ll'$ vertex are also displayed.

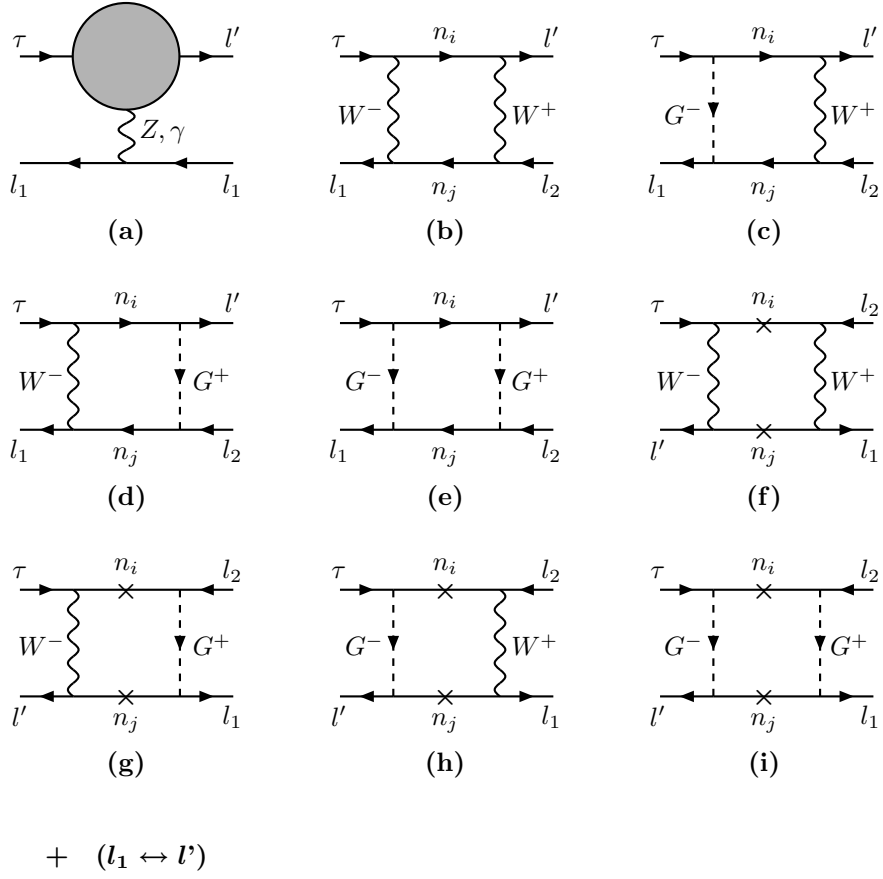


Fig. 10. Feynman graphs pertaining to the decay $\tau \rightarrow l' l_1 l_2$.

$$F_Z^{\tau l} \rightarrow -\frac{3}{2}s_L^{\nu_\tau} s_L^{\nu_l} \ln \lambda_N - \frac{1}{2}s_L^{\nu_\tau} s_L^{\nu_l} \sum_{i=1}^3 (s_L^{\nu_i})^2 \lambda_N, \quad (9.9)$$

$$F_{Box}^{\tau ll_1 l_2} \rightarrow -(s_L^{\nu_\tau} s_L^{\nu_l} \delta_{l_1 l_2} + s_L^{\nu_\tau} s_L^{\nu_{l_1}} \delta_{ll_2}) + \frac{1}{2}s_L^{\nu_\tau} s_L^{\nu_l} s_L^{\nu_{l_1}} s_L^{\nu_{l_2}} \lambda_N. \quad (9.10)$$

If all light-heavy neutrino mixings $s_L^{\nu_l}$ are held fixed to a constant, the one-loop functions $F_\gamma^{\tau l}$, $G_\gamma^{\tau l}$, $F_Z^{\tau l}$, and $F_{Box}^{\tau ll_1 l_2}$ in Eqs. (9.7)–(9.10) do not vanish in the heavy neutrino limit, and therefore seem to violate the decoupling theorem due to Appelquist and Carazzone.⁷⁶ However, it is known that the decoupling theorem does not apply to theories based on the spontaneous or dynamical breaking of gauge symmetries. Since we hold $s_L^{\nu_l}$ fixed but increase the heavy-neutrino masses, the violation of the decoupling theorem originates from the $SU(2)_L$ -breaking Dirac mass terms m_1 and m_2 .^{75,67} The expected decoupling of the isosinglet mass M ⁷⁷ can also be seen in Eqs. (9.7)–(9.10). This time we keep the Dirac masses m_1 and m_2 fixed and increase M . Taking Eq. (9.2) into account, it is then easy to show that all composite form factors vanish for large values of $M \approx m_{N_1}$, m_{N_2} . Consequently, there is a competitive loop effect of two scales, namely Dirac versus Majorana scale. High Dirac masses lead to non-decoupling loop effects while large Majorana masses give rise to a screening and reduce the strength of the effective FCNC coupling. Nevertheless, extensive analyses have shown that a non-decoupling ‘window’ confined by the two mass scales exists, within which FCNC and other new-physics phenomena come out to be rather large at a level that may be probed in next-round experiments.^{67,68,69}

As examples, we calculate neutrinoless tau-lepton decays and flavour-violating Z -boson decays. Taking the dominant non-decoupling parts of the composite form factors into account, we arrive at the simple expressions for the branching ratios

$$\begin{aligned} B(\tau^- \rightarrow \mu^- e^- e^+) &\simeq \frac{\alpha_w^4}{24576\pi^3} \frac{m_\tau^4}{M_W^4} \frac{m_\tau}{\Gamma_\tau} \left[|F_{\text{box}}^{\tau\mu ee}|^2 \right. \\ &\quad \left. + 2(1 - 2s_w^2) \text{Re}[F_Z^{\tau\mu} F_{\text{box}}^{\tau\mu ee*}] + 8s_w^4 |F_Z^{\tau\mu}|^2 \right] \\ &\simeq \frac{\alpha_w^4}{98304\pi^3} \frac{m_\tau^4}{M_W^4} \frac{m_\tau}{\Gamma_\tau} \frac{m_N^4}{M_W^4} (s_L^{\nu_\tau})^2 (s_L^{\nu_\mu})^2 \left\{ (s_L^{\nu_e})^4 \right. \\ &\quad \left. + 2(1 - 2s_w^2) (s_L^{\nu_e})^2 \sum_i (s_L^{\nu_i})^2 + 8s_w^4 \left[\sum_i (s_L^{\nu_i})^2 \right]^2 \right\}, \quad (9.11) \end{aligned}$$

$$\begin{aligned} B(\tau^- \rightarrow \mu^- \mu^- \mu^+) &\simeq \frac{\alpha_w^4}{24576\pi^3} \frac{m_\tau^4}{M_W^4} \frac{m_\tau}{\Gamma_\tau} \left[\frac{1}{2} |F_{\text{box}}^{\tau\mu\mu\mu}|^2 \right. \\ &\quad \left. + 2(1 - 2s_w^2) \text{Re}[F_Z^{\tau\mu} F_{\text{box}}^{\tau\mu\mu\mu*}] + 12s_w^4 |F_Z^{\tau\mu}|^2 \right] \\ &\simeq \frac{\alpha_w^4}{98304\pi^3} \frac{m_\tau^4}{M_W^4} \frac{m_\tau}{\Gamma_\tau} \frac{m_N^4}{M_W^4} (s_L^{\nu_\tau})^2 (s_L^{\nu_\mu})^2 \left\{ \frac{1}{2} (s_L^{\nu_\mu})^4 \right. \\ &\quad \left. + 2(1 - 2s_w^2) (s_L^{\nu_\mu})^2 \sum_i (s_L^{\nu_i})^2 + 12s_w^4 \left[\sum_i (s_L^{\nu_i})^2 \right]^2 \right\}, \quad (9.12) \end{aligned}$$

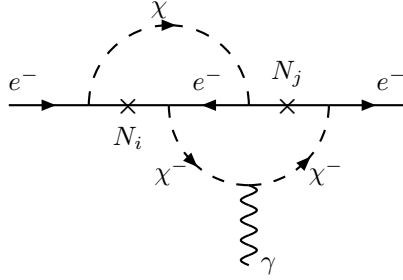


Fig. 11. Typical two-loop diagram contributing to the EDM of electron.

$$\begin{aligned}
 B(Z \rightarrow \tau^- \mu^+ + \mu^- \tau^+) &= \frac{\alpha_w^3}{48\pi^2 c_w^3} \frac{M_W}{\Gamma_Z} |\mathcal{F}_Z^{\tau\mu}(M_Z^2)|^2 \\
 &\simeq \frac{\alpha_w^3}{768\pi^2 c_w^3} \frac{M_W}{\Gamma_Z} \frac{m_N^4}{M_W^4} (s_L^{\nu\mu})^2 (s_L^{\nu\tau})^2 \left[\sum_i (s_L^{\nu_i})^2 \right]^2, \quad (9.13)
 \end{aligned}$$

where $\Gamma_\tau = 2.16 \times 10^{-12}$ GeV and $\Gamma_Z = 2.49$ GeV are respectively the total widths of the τ lepton and the Z boson known from experiment,⁷⁸ and $\mathcal{F}_Z^{\tau\mu}(0) = F_Z^{\tau\mu}/2$. The complete analytic results of the branching ratios in Eqs. (9.11)–(9.13) are presented in Ref.⁶⁸

To give an estimate of the size of the FCNC effects, we take the maximally allowed values $(s_L^{\nu\tau})^2 = 0.035$ and $(s_L^{\nu\mu})^2 = 0.010$ given by Eq. (9.3). These light-heavy neutrino mixings lead to the branching ratios

$$\begin{aligned}
 B(\tau^- \rightarrow \mu^- \mu^- \mu^+) &\lesssim 2 \times 10^{-6}, & B(\tau^- \rightarrow \mu^- e^- e^+) &\lesssim 1 \times 10^{-6}, \\
 B(Z \rightarrow \mu\tau) &\lesssim 1.1 \times 10^{-6}. \quad (9.14)
 \end{aligned}$$

In Eq. (9.14), the upper limits are estimated by using the heavy-neutrino mass $m_N \simeq 4$ TeV, which results from the requirement that perturbative unitarity be valid. The theoretical predictions of the branching ratios must be contrasted with the present experimental upper limits on these decays⁷⁸

$$\begin{aligned}
 B(\tau^- \rightarrow \mu^- \mu^- \mu^+), B(\tau^- \rightarrow \mu^- e^- e^+) &< 1.4 \times 10^{-5}, \\
 B(Z \rightarrow \mu\tau) &< 1.3 \times 10^{-5}, \quad (9.15)
 \end{aligned}$$

at the 90% confidence level. Future high-luminosity colliders and higher-precision experiments are capable of improving the above upper limits by one order of magnitude and so probe possible new-physics effects due to heavy Majorana neutrinos.

9.2. Electric dipole moment of the electron

In general, CP-violating new-physics interactions may give rise to a large contribution to the EDM of the electron. This results in an interaction in the Lagrangian of the form

$$\mathcal{L}_d = ie \left(\frac{d_e}{e} \right) \bar{e} \sigma_{\mu\nu} \gamma_5 e \partial^\mu A^\nu. \quad (9.16)$$

The experimental upper bound on electron EDM is very strict: $(d_e/e) < 10^{-26}$ cm.⁷⁸ This bound is very crucial, as heavy Majorana neutrinos can induce an EDM of the electron at two loops.⁷⁹ A typical diagram is shown in Fig. 11. A simple estimate of this contribution based on a naive dimensional analysis for $m_{N_2}, m_{N_1} \gg M_W$ gives²¹

$$\frac{d_e}{e} \sim (10^{-24} \text{ cm}) \times \text{Im}(h_{1e} h_{2e}^*)^2 \frac{m_{N_1} m_{N_2} (m_{N_1}^2 - m_{N_2}^2)}{(m_{N_1}^2 + m_{N_2}^2)^2} \ln \left(\frac{m_{N_1}}{M_W} \right). \quad (9.17)$$

In Eq. (9.17) the factor depending on m_{N_i} is always much smaller than unity. Clearly, the above EDM limit could be important for $|h_{li}| \gg 0.1$ and/or ultra-heavy Majorana neutrinos with $m_{N_i} > 10^{11}$ TeV. In this prediction, one should bear in mind that stability of the Higgs potential under radiative corrections requires $|h_{li}| = \mathcal{O}(1)$.⁸⁰ Nevertheless, the EDM contribution is several orders of magnitude below the experimental bound for $x_N < 10^{-3}$ and/or $|h_{l1}|, |h_{l2}| \lesssim 10^{-2}$. In this context, it is interesting to notice that leptogenesis models with nearly degenerate heavy Majorana neutrinos can naturally evade possible EDM constraints.

10. Conclusions

We have reviewed many recent developments that have been taking place in the scenario of baryogenesis through leptogenesis, and discussed the implications that heavy Majorana neutrinos may have for laboratory experiments. In the standard leptogenesis scenario,¹⁵ L -violating decays of heavy Majorana neutrinos, which are out of thermal equilibrium, produce an excess in L that is converted into the observed BAU through $B + L$ -violating interactions mediated by sphalerons. We have paid more attention to different kinds of mechanisms of CP violation involved in the N_i decays. One has two generic types: (a) CP violation originates from the interference between a tree-level graph and the absorptive part of the one-loop vertex (ε' -type CP violation) and (b) CP violation comes from the interference between a tree-level graph and the absorptive part of the one-loop N_i self-energy (ε -type CP violation).

Recently, there has been renewed interest in ε -type CP violation in models with mixed heavy Majorana neutrinos. If the masses of two heavy neutrinos become degenerate, then finite-order perturbation theory does no longer apply, and various methods have been invoked to cope with this problem in the literature.²⁰ Here, we have discussed the whole issue based on an effective resummation approach to unstable particle mixing.²¹ One then finds that ε -type CP violation is resonantly

enhanced if the mass splitting of the heavy Majorana neutrinos is comparable to their widths (cf. Eq. (6.18)), and if the parameter δ_{CP} defined in Eq. (6.19) has a value close to 1. These two conditions turn out to be necessary and sufficient for resonant CP violation of order unity.²¹ As a consequence, the scale of leptogenesis may be lowered up to TeV energies. In fact, E_6 -motivated scenarios with nearly degenerate heavy Majorana neutrinos of order 1 TeV and universal Yukawa couplings can still be responsible for the BAU. This last observation receives firm support after solving numerically the BE's.²¹ In this kinematic range, the ε' -type contributions are extremely suppressed. Also, finite-temperature effects on masses of heavy neutrinos and on decay widths do not spoil the above conditions for resonant CP asymmetries. Finally, constraints due to electron EDM are still too weak to play a role in leptogenesis.

The fact that the isosinglet-neutrino scale can be lowered to TeV energies has a number of virtues. If one has to appeal to (local) supersymmetry in order to maintain the flatness of the inflaton potential, one then has to worry about the cosmological consequences of the gravitino during the nucleosynthesis epoch. Since the weakly interacting gravitinos are at most of the order of a few TeV, their slow decay rate will lead to an overproduction of D and ^3He unless the number density-to-entropy ratio at the time of reheating after inflation is less than about 10^{-10} .⁵ This leads to quite low reheating temperatures $T_{\text{RH}} \lesssim 10^{10}$ GeV, after which the radiation-dominated era starts and baryogenesis or leptogenesis can in principle occur. The latter causes a major problem for GUT-scale baryogenesis and GUT's as well, especially if $T_{\text{RH}} \sim M_{\text{GUT}}$. In this context, it is important to remark that supersymmetric extensions of models with isosinglet neutrinos in the multi-TeV range as the ones discussed here can comfortably evade the known gravitino problem mentioned above. In such scenarios, the heavy inflatons with a mass of order 10^{12} GeV can now decay abundantly to the relatively lighter heavy Majorana neutrinos yielding equilibrated (incoherent) thermal distributions for the latter particles.

Heavy Majorana neutrinos may also induce sizeable FCNC effects such as $Z \rightarrow \tau\mu$ and $\tau \rightarrow \mu\mu\mu$ at a level that can be probed in near-future experiments. Non-decoupling loop effects of high $SU(2)_L$ -breaking masses play a significant role in increasing drastically the strength of these new-physics phenomena.^{75,67,68,69} However, these heavy Majorana neutrinos cannot account for the BAU at the same time. Depending on the model, they can coexist with the heavy Majorana neutrinos responsible for leptogenesis without destroying baryogenesis. At the LHC,⁶⁴ the viability of such models can be tested directly by looking for like-sign dilepton signals. Such signals will then strongly point towards the scenario of baryogenesis through leptogenesis as the underlying mechanism for understanding the baryonic asymmetry in nature.

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Appendix A

We shall now list useful analytic expressions for one-loop self-energies of the Higgs and fermion fields as well as for one-loop vertex couplings $\chi^+ l N_i$, $\chi^0 \nu_l N_i$ and $H \nu_l N_i$. We present relations between wave-function CT's and unrenormalized self-energies in the OS renormalization scheme. The analytic results are expressed in terms of standard loop integrals presented in Ref.⁸¹ Instead, we adopt the signature for the Minkowskian metric $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$.

The Feynman rules pertaining to the minimal model may be read off from the Lagrangian (3.7). We first give the the Higgs self-energies $\chi^- \chi^-$, $\chi^0 \chi^0$ and HH , shown in Fig. 2(d)–(f)

$$\begin{aligned} \Pi_{\chi^- \chi^-}(p^2) &= \Pi_{\chi^0 \chi^0}(p^2) = \Pi_{HH}(p^2) \\ &= \sum_{l=1}^{n_L} \sum_{i=1}^{n_R} \frac{|h_{li}|^2}{8\pi^2} \left[m_{N_i}^2 B_0(p^2, m_{N_i}^2, 0) + p^2 B_1(p^2, m_{N_i}^2, 0) \right]. \end{aligned} \quad (\text{A.1})$$

From Eq. (A.1), the universality of the divergent parts of the wave functions δZ_{χ^-} , δZ_{χ^0} and δZ_H is evident, since $\delta Z_{\Phi}^{div} = -\text{Re}\Pi_{\Phi}^{div}(0)$ for all field components of the Higgs doublet Φ .

Figures 2(g), (h) and (j) show the individual contributions to the one-loop fermionic transitions, $l' \rightarrow l$, $\nu_{l'} \rightarrow \nu_l$ and $N_j \rightarrow N_i$, respectively. Explicit calculation of these self-energy transitions gives

$$\Sigma_{ll'}(\not{p}) = - \sum_{i=1}^{n_R} \frac{h_{li} h_{l'i}^*}{16\pi^2} \not{p} P_L B_1(p^2, m_{N_i}^2, 0), \quad (\text{A.2})$$

$$\begin{aligned} \Sigma_{\nu_l \nu_{l'}}(\not{p}) &= - \sum_{i=1}^{n_R} \frac{1}{32\pi^2} \left[(h_{li}^* h_{l'i} \not{p} P_R + h_{li} h_{l'i}^* \not{p} P_L) \left(B_1(p^2, m_{N_i}^2, 0) \right. \right. \\ &\quad \left. \left. + B_1(p^2, m_{N_i}^2, M_H^2) \right) + m_{N_i} (h_{li}^* h_{l'i}^* P_R + h_{li} h_{l'i} P_L) \right. \\ &\quad \left. \times \left(B_1(p^2, m_{N_i}^2, 0) - B_1(p^2, m_{N_i}^2, M_H^2) \right) \right], \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} \Sigma_{N_i N_j}(\not{p}) &= - \sum_{l=1}^{n_L} \frac{1}{16\pi^2} (h_{li}^* h_{lj} \not{p} P_R + h_{li} h_{lj}^* \not{p} P_L) \left(\frac{3}{2} B_1(p^2, 0, 0) \right. \\ &\quad \left. + \frac{1}{2} B_1(p^2, 0, M_H^2) \right). \end{aligned} \quad (\text{A.4})$$

Note that the light-neutrino self-energies in Eq. (A.3) contain non-zero masses in the limit $\not{p} \rightarrow 0$ if $M_H \neq 0$. Therefore, at $T = 0$, small radiative neutrino masses can be generated.⁷⁵ However, these contributions are suppressed by small Yukawa coupling, and therefore do not invalidate any experimental or cosmological limit.

Before we express the wave-function renormalization factors in terms of unrenormalized self-energies,⁴⁹ we first notice that the one-loop fermionic transitions $f_j \rightarrow f_i$ (with $f_i = l, \nu_l, N_i$) given by Eqs. (A.2)–(A.4) have the generic form

$$\Sigma_{ij}(\not{p}) = \Sigma_{ij}^L(p^2) \not{p} P_L + \Sigma_{ij}^R(p^2) \not{p} P_R + \Sigma_{ij}^M(p^2) P_L + \Sigma_{ij}^{M*}(p^2) P_R, \quad (\text{A.5})$$

where only dispersive parts are considered. If the transitions involve Majorana fermions only, one then has the additional properties $\Sigma_{ij}^L(p^2) = \Sigma_{ij}^{R*}(p^2)$ and $\Sigma_{ij}^M(p^2) = \Sigma_{ji}^M(p^2)$. Following Ref.⁴⁹, the wave-function CT's are given by

$$\begin{aligned} \delta Z_{ii}^f &= -\Sigma_{ii}^L(m_i^2) - 2m_i^2 \Sigma_{ii}^{L'}(m_i^2) - m_i \left[\Sigma_{ii}^{M'}(m_i^2) + \Sigma_{ii}^{M*'}(m_i^2) \right] \\ &\quad + \frac{1}{2m_i} \left[\Sigma_{ii}^M(m_i^2) - \Sigma_{ii}^{M*}(m_i^2) \right], \end{aligned} \quad (\text{A.6})$$

and, for $i \neq j$,

$$\delta Z_{ij}^f = \frac{2}{m_i^2 - m_j^2} \left[m_j^2 \Sigma_{ij}^L(m_j^2) + m_i m_j \Sigma_{ij}^{L*}(m_j^2) + m_i \Sigma_{ij}^M(m_j^2) + m_j \Sigma_{ij}^{M*}(m_j^2) \right]. \quad (\text{A.7})$$

The wave-function renormalization of charged leptons may be obtained by Eqs. (A.6) and (A.7), if all terms depending on $\Sigma_{ij}^M(p^2)$ and its derivative are neglected. At this point, it is important to remark that there will be additional contributions to our wave-function CT's of the Higgs and fermion fields coming from loops related to SM gauge particles. As we have seen in Section 8, the same kind of loops can induce non-zero thermal masses to the Higgs and fermion particles.^{59,60} These new contributions are universal and therefore pose no problem to the Yukawa-coupling renormalization discussed in Section 4.

Finally, one-loop corrections to the vertices $\chi^{\pm l \mp} N_i$, $\chi^0 \nu_l N_i$ and $H \nu_l N_i$ shown in Figs. 2(a)–(c) have been calculated in Ref.²¹ by keeping the complete functional dependence on the Higgs-boson mass M_H . Their analytic expressions are given by

$$\begin{aligned} i\mathcal{V}_{\chi^+ l^- N_i} &= -i \bar{u}_l P_R u_{N_i} \sum_{l'=1}^{n_L} \sum_{j=1}^{n_R} \frac{m_{N_i} m_{N_j}}{16\pi^2} h_{l'i}^* h_{l'j} h_{lj} \\ &\quad \times \left[C_0(0, 0, m_{N_i}^2, 0, m_{N_j}^2, 0) + C_{12}(0, 0, m_{N_i}^2, 0, m_{N_j}^2, 0) \right], \end{aligned} \quad (\text{A.8})$$

$$\begin{aligned} \mathcal{V}_{\chi^0 \nu_l N_i} &= -\bar{u}_l P_R u_{N_i} \sum_{l'=1}^{n_L} \sum_{j=1}^{n_R} \left\{ \frac{m_{N_i} m_{N_j}}{32\sqrt{2}\pi^2} h_{l'i}^* h_{l'j} h_{lj} \right. \\ &\quad \times \left[C_0(0, 0, m_{N_i}^2, 0, m_{N_j}^2, 0) + C_0(0, 0, m_{N_i}^2, M_H^2, m_{N_j}^2, 0) \right. \\ &\quad \left. + C_{12}(0, 0, m_{N_i}^2, 0, m_{N_j}^2, 0) + C_{12}(0, 0, m_{N_i}^2, M_H^2, m_{N_j}^2, 0) \right] \\ &\quad + \frac{1}{32\sqrt{2}\pi^2} h_{l'i} h_{l'j}^* h_{lj} \\ &\quad \times \left[m_{N_i}^2 \left(C_{12}(0, 0, m_{N_i}^2, 0, m_{N_j}^2, 0) - C_{12}(0, 0, m_{N_i}^2, M_H^2, m_{N_j}^2, 0) \right) \right. \\ &\quad \left. - M_H^2 C_0(0, 0, m_{N_i}^2, M_H^2, m_{N_j}^2, 0) \right] \left. \right\}, \end{aligned} \quad (\text{A.9})$$

$$\begin{aligned} -i\mathcal{V}_{H \nu_l N_i} &= i \bar{u}_l P_R u_{N_i} \sum_{l'=1}^{n_L} \sum_{j=1}^{n_R} \left\{ \frac{m_{N_i} m_{N_j}}{32\sqrt{2}\pi^2} h_{l'i}^* h_{l'j} h_{lj} \right. \\ &\quad \times \left[C_0(0, 0, m_{N_i}^2, 0, m_{N_j}^2, 0) + C_0(0, 0, m_{N_i}^2, M_H^2, m_{N_j}^2, 0) \right] \end{aligned}$$

$$\begin{aligned}
& + C_{12}(0, 0, m_{N_i}^2, 0, m_{N_j}^2, 0) + C_{12}(0, 0, m_{N_i}^2, M_H^2, m_{N_j}^2, 0) \Big] \\
& + \frac{1}{32\sqrt{2}\pi^2} h_{\nu_i} h_{\nu_j}^* h_{lj} \\
& \times \left[m_{N_i}^2 \left(C_{12}(0, 0, m_{N_i}^2, 0, m_{N_j}^2, 0) - C_{12}(0, 0, m_{N_i}^2, M_H^2, m_{N_j}^2, 0) \right) \right. \\
& \left. - M_H^2 C_0(0, 0, m_{N_i}^2, M_H^2, m_{N_j}^2, 0) \right] \Big\}. \tag{A.10}
\end{aligned}$$

From Eqs. (A.9) and (A.10) it is easy to see that the L -conserving part of the couplings $\chi^0 \nu_l N_i$ and $H \nu_l N_i$ is UV finite and vanishes identically for $M_H \rightarrow 0$.