

COSMOLOGICAL FINE TUNING, SUPERSYMMETRY AND THE GAUGE HIERARCHY PROBLEM

Piotr H. Chankowski^{a)}, John Ellis^{b)}, Keith A. Olive^{c)} and Stefan Pokorski^{a)}

ABSTRACT

We study the extent to which the cosmological fine-tuning problem - why the relic density of neutralino cold dark matter particles χ is similar to that of baryons - is related to the fine-tuning aspect of the gauge hierarchy problem - how one arranges that $M_W \ll M_P$ without unnatural choices of MSSM parameters. Working in the minimal supergravity framework with universal soft supersymmetry-breaking parameters as inputs, we find that the hierarchical fine tuning is minimized for $\Omega_\chi h^2 \sim 0.1$. Conversely, imposing $\Omega_\chi h^2 < 1$ does not require small hierarchical fine tuning, but the exceptions to this rule are rather special, with parameters chosen such that $m_\chi \sim M_Z/2$ or $M_h/2$, or else $m_\chi \gtrsim m_t$. In the first two cases, neutralino annihilation receives a large contribution from a direct-channel pole, whereas in the third case it is enhanced by the large top Yukawa coupling.

^{a)} Institute of Theoretical Physics, Warsaw University, Poland

^{b)} Theory Division, CERN, Geneva, Switzerland

^{c)} Theoretical Physics Institute, School of Physics and Astronomy, University of Minnesota, Minneapolis, MN 55455, USA

One of the important philosophical issues to be addressed by any dark matter candidate is the extent to which a relic density of interest to astrophysicists and cosmologists, $0.01 < \Omega h^2 < 1$, is natural. Indeed, this question is often posed to proponents of favoured candidates such as a neutrino, the axion and the lightest supersymmetric particle, assumed to be the lightest neutralino χ [1]. In the case of a neutrino, the see-saw mechanism explains in a natural way why $m_\nu \ll m_q$ or m_ℓ , but does not lead inexorably to Ω_ν in the interesting range. In the case of the axion, experimental and astrophysical constraints restrict its relic density to the interesting range, but this is not yet predicted by any deeper theoretical argument, although such an argument may yet be found [2]. In the case of supersymmetry on the other hand, it is well known and will be emphasized below that there are indeed good physical reasons for expecting cosmologically significant relic densities [3].

The essential motivation for supersymmetric particles to appear below the TeV scale is provided by the gauge hierarchy problem. Supersymmetry by itself does not explain why $M_W \ll M_P$, but it does enable such a hierarchy to be stabilized against the effects of radiative corrections, averting the need for fine tuning and rendering the gauge hierarchy technically natural [4]. The appearance of supersymmetric particles at the TeV scale is also supported [5, 6] by the experimental value of $\sin^2 \theta_W$, in accord with supersymmetric grand unified theories, and by the indications that the Higgs boson may weigh around 100 GeV [7], in agreement with supersymmetric model calculations if sparticles weigh < 1 TeV [8].

The TeV scale also arises as a possible characteristic mass scale for a cold dark matter candidate. Particles that annihilate via conventional point-like interactions generically have $\Omega \sim 1$ if their masses $m \sim \sqrt{M_P \times T_{CMBR}}$, where the cosmic microwave background temperature $T_{CMBR} = 2.73$ K [9]. It so happens that $\sqrt{M_P \times T_{CMBR}} \sim 1$ TeV, making it plausible that any relic with mass around the electroweak scale might have a cosmological density of astrophysical interest.

In the case of the supersymmetric relic χ , detailed calculations have been performed [10], and it has often been observed that $0.01 < \Omega_\chi < 1$ is a generic feature of parameter choices in the minimal supersymmetric extension of the Standard Model (MSSM). Moreover, it has also often been argued that restricting $\Omega_\chi h^2 < 1$ suggests very strongly that m_χ is at most a few hundred GeV [11]. However, it is known that there are rays in parameter space along which $\Omega_\chi h^2$ may be kept small even though sparticle masses grow large. The purpose of this paper is to bring together and complete these observations, with the aim of clarifying the extent to which the supersymmetric resolution of the fine-tuning problem is related to the cosmological fine-tuning problem.

The criterion for hierarchical fine tuning that we use is that championed in [12, 13], namely the logarithmic sensitivity Δ_0 of M_Z to variations in MSSM input parameters a_i :

$$\Delta_0 \equiv \max |\Delta_{a_i}| : \quad \Delta_{a_i} = \frac{a_i}{M_Z} \frac{\partial M_Z}{\partial a_i} \quad (1)$$

The lower bounds imposed on Δ_0 by experiments at LEP and elsewhere have been discussed previously by several authors. There have been extensive discussions how this price varies as a function of $\tan \beta$, the ratio of Higgs vacuum expectation values (vev's), and how the price

may be reduced if some underlying theory imposes relations between some MSSM parameters [14, 15, 16]. Here we extend these discussions to include a cosmological dimension.

We study the extent to which the cosmological fine-tuning problem is related to the fine tuning of the gauge hierarchy, as measured by the quantity Δ_0 (1). Making an extensive search of the MSSM parameter space, we find that minimizing Δ_0 leads to $\Omega_\chi h^2 \sim 0.1$. Moreover, we find a general correlation between those parameter choices with larger $\Omega_\chi h^2$ and those with larger Δ_0 . On the other hand, imposing $\Omega_\chi h^2 < 1$ does not exclude all models with large Δ_0 . We can distinguish three regions in the neutralino mass m_χ (or, correspondingly, in the GUT scale gaugino mass parameter $M_{1/2}$) in which $\Omega_\chi h^2 < 1$ with large Δ_0 are possible. For $m_\chi \approx M_Z/2$ or $m_\chi \approx M_h/2$ (that is for $M_{1/2} \lesssim 150$ GeV) large pole-dominated s -channel annihilation cross sections give a low relic density in ways unrelated to the value of Δ_0 . In the intermediate region, $M_Z/2, M_h/2 \lesssim m_\chi \lesssim m_t$ neutralino annihilation proceeds mainly through slepton exchange, and $\Omega_\chi h^2 < 1$ then implies $m_0/M_{1/2} \lesssim \mathcal{O}(1)$ and small Δ_0 . For $m_\chi \gtrsim m_t$, $\Omega_\chi h^2 < 1$ is also possible in models with relatively light stop, so that the t -channel annihilation into $t\bar{t}$ pair is enhanced again leading to a lower relic density. Within the minimal SUGRA framework such models require large left-right mixing in the stop sector and hence, in addition to a large top Yukawa coupling, a large value of A_0 . Thus, models with $\Omega_\chi h^2 < 1$ and large fine-tuning are rather special: some have $m_\chi \approx M_Z/2$ or $M_h/2$ whilst others have $m_\chi \gtrsim m_t$ and relatively light stop. The former class of special solutions can be exhaustively explored by chargino searches at LEP 200, whilst the latter would be absent if $|A_0| \lesssim 1$ TeV. Our study does not find that the cosmological and hierarchical fine-tuning problems are equivalent, but it does confirm that an interesting cosmological relic density is indeed to be expected in supersymmetric models that do not exhibit extreme fine tuning.

Our study of these issues is based on the survey of MSSM parameter space made in [14, 16], which we review briefly here. As usual, we denote the Higgs mixing parameter by μ , and we assume the conventional minimal parameterization of soft supersymmetry breaking in the MSSM, via a universal scalar mass parameter m_0 , a universal gaugino mass parameter $M_{1/2}$, and a trilinear (bilinear) coupling A_0 (B_0). We assume that $\mu_0, m_0, M_{1/2}, A_0$ and B_0 are the appropriate inputs a_i at the GUT scale $M_{GUT} = 2 \times 10^{16}$ GeV which should be used in the hierarchical fine-tuning criterion (1). These parameters are renormalized down to the electroweak scale in the standard way, leading to an effective potential that breaks electroweak gauge symmetry spontaneously, with a calculable value of $\tan \beta$.

Our procedure for surveying the MSSM parameter space is, for reasons of convenience, to choose low-energy parameter sets that respect the experimental constraints and yield an appropriate electroweak vacuum, as determined using the full one-loop effective potential in the MSSM [17]. Specifically, using the measured value of M_Z , for each value of $\tan \beta$ and a given $sgn(\mu)$, we scan low-energy values of the left-handed doublet squark mass m_Q , the right-handed singlet up-squark mass m_U and the CP-odd MSSM Higgs mass M_A that are allowed by the experimental constraints [17]. We then use the renormalization-group equations to find the corresponding allowed values of the GUT input parameters.

As experimental constraints, we take into account the precision electroweak data published

at the Moriond conference [18], and require $\Delta\chi^2 < 4$ in a global MSSM fit. We also incorporate LEP lower limits on the masses of sparticles and Higgs bosons [19]. Another important accelerator constraint is provided by the recently measured $b \rightarrow s\gamma$ branching ratio $2 \times 10^{-4} < Br(B \rightarrow X_s\gamma) < 4.5 \times 10^{-4}$ [20], which we treat as described in [14]. We find that the GUT-scale parameters corresponding to successful choices with an upper cut of 1.2 TeV on the soft masses M_A and the squark masses, vary over the ranges $|\mu_0| \lesssim 2$ TeV, $m_0 \lesssim 1.7$ TeV, $M_{1/2} \lesssim 600$ GeV, $|A_0| \lesssim 5$ TeV and $|B_0| \lesssim 3$ TeV. Note, in particular, that the upper bound on $M_{1/2}$ (and, in consequence, on the chargino mass) visible on the plots follows simply from the cut imposed on the scanning procedure.

Finally, we note that the range of the relic neutralino density favoured by astrophysics and cosmology is $\Omega_\chi h^2 \sim 0.1$ (see, e.g., [21]), but we do not use this as a constraint in our analysis. Rather, our aim will be to explore the extent to which this is a natural outcome for successful MSSM parameter choices with small values of the hierarchical fine-tuning measure Δ_0 (1). To this end, within the context of the supergravity-based MSSM, we calculate the relic density for each of the models considered in the fine-tuning analysis of [14, 16]. Thus, for every allowed set of the GUT-scale parameters obtained by our scanning procedure, we have a calculation of the relic density $\Omega_\chi h^2$ in terms of all of the low-energy masses, which are determined from $M_{1/2}, m_0$ and A_0 for the same fixed values of $\tan\beta$ and $sgn(\mu)$.

We note that there are two recent refinements of the analysis of the MSSM parameter space and the dark matter density that have not been included in this survey. One is the latest implementation [22] of the requirement that our electroweak vacuum be stable against possible transitions to vacua that violate charge and/or color conservation. This requirement tends to exclude parameter choices with $m_0/M_{1/2} \lesssim 1/2$, which do not have exceptional values of either Δ_0 or $\Omega_\chi h^2$. We have also omitted the possibility of coannihilation [23] between χ and the $\tilde{\tau}_R$, which is the next-to-lightest supersymmetric particle in a generic domain of parameter space. This is important when $m_\chi \lesssim 1.1m_{\tilde{\tau}_R}$, which is the case only for a very small number of the parameter choices in our survey. When it is significant, it tends to enable points with larger hierarchical fine tuning Δ_0 to have a cosmologically interesting value of $\Omega_\chi h^2$.

Results for the case $\tan\beta = 2.5$ are shown in Fig. 1. The top left panel displays directly the correlation we find between the hierarchical fine-tuning parameter Δ_0 and the neutralino relic density. Here and elsewhere, the eight-pointed stars represent parameter choices where no specific direct-channel annihilation mechanism is dominant. The five-pointed stars represent parameter choices where $m_\chi \sim M_Z/2$, so that $\chi\chi$ annihilation via the direct-channel Z^0 pole is dominant. Because the width of the Z^0 is relatively large, the effect of s -channel annihilation through Z^0 's occurs only in a small patch of the parameter space. The open circles represent parameter choices where $m_\chi \sim M_h/2$, so that $\chi\chi$ annihilation via the direct-channel pole of the lightest MSSM Higgs boson h^0 is dominant. In this case, because of the very small width for h^0 , the suppression of the relic density actually begins even when $2m_\chi \sim 0.8M_h$ [24] and thus can cover a broader parameter volume. Finally, the dots represent parameter choices where $m_\chi \gtrsim m_t$, so that $\chi\chi \rightarrow \bar{t}t$ annihilation is important.

We see in the top left panel of Fig. 1 that (i) the minimum value of $\Delta_0 \sim 13$ is attained for

$\Omega_\chi h^2 \sim 0.1$, (ii) the minimum value of Δ_0 increases gradually for larger values of $\Omega_\chi h^2$, (iii) apart from a few exceptional points, mainly with $m_\chi > m_t$, all choices with $\Omega_\chi h^2 < 1$ have $\Delta_0 < 100$, and (iv) apart from choices where $\chi\chi \rightarrow \bar{t}t$ dominates, when $\Omega_\chi h^2 > 1$ there is a clear tendency for its value to be correlated with that of Δ_0 . The origins of the exceptional choices with low $\Omega_\chi h^2$ and large Δ_0 are seen in the top right panel of Fig. 1. They have values of $M_{1/2} \sim 100$ to 150 GeV, corresponding to $m_\chi \sim M_Z/2$ and/or $M_h/2$. We also see in this panel that the choices with annihilation into $\bar{t}t$ correspond to $M_{1/2} \gtrsim 400$ GeV, which are the largest in our sample. It is therefore not surprising that these correspond to some of the largest values of Δ_0 , as we see in the top left panel.

The remaining panels of Fig. 1 show how the value of $\Omega_\chi h^2$ is correlated with the values of other MSSM parameters. In the middle left panel, we see a clear correlation between $\Omega_\chi h^2$ and the ratio $m_0/M_{1/2}$, again apart from a few choices where $\chi\chi \rightarrow Z^0$ or h^0 or $\chi\chi \rightarrow \bar{t}t$ dominates. Apart from these choices, we see that $\Omega_\chi h^2$ is minimized for $m_0/M_{1/2} \lesssim 1$. In the middle right panel, we see that $\Omega_\chi h^2 < 1$ is consistent with relatively small values of $|A_0|$ for the $m_\chi \approx M_Z/2$ or $M_h/2$ points and the points with no dominant annihilation channel, whereas large A_0 is required when $m_\chi > m_t$. This effect is even more pronounced for larger values of $\tan\beta$ (see Fig. 2 and related comments below).

It is worth emphasizing that the parameter choices dominated by $\chi\chi \rightarrow Z^0$ annihilation will soon be explored directly by LEP, as will to a large extent those parameters which lead to annihilation via the light Higgs pole. These choices have $m_{\chi^\pm} \lesssim 100$ GeV, and hence should reveal their secrets when the LEP center-of-mass energy is increased to ~ 200 GeV in the years 1999 and 2000.

We have carried out similar parameter studies for several larger values of $\tan\beta \leq 30$, as well as for $\tan\beta = 1.65$, and now discuss some similarities and differences in these cases. As a general rule, the cases with $\tan\beta > 2.5$ are qualitatively similar to the $\tan\beta = 2.5$ case. In particular, it is always true that minimizing Δ_0 favours $\Omega_\chi h^2 \sim 0.1$, as seen in Fig. 2 for $\tan\beta = 10$, for example. On the other hand there is no trend for the upper bound on Δ_0 to be improved if one selects $\Omega_\chi h^2 < 1$. We see again in the top left panel of Fig. 2 three distinctive sets of parameter choices which satisfy this condition. The one with $\chi\chi \rightarrow Z^0$ or h^0 annihilation, is unrelated to the values of Δ_0 , as seen in the top left panel. The case with no dominant annihilation channel has small Δ_0 . We also see there that the $\chi\chi \rightarrow \bar{t}t$ cases have distinctively larger values of Δ_0 . For $\chi\chi \rightarrow \bar{t}t$ cases the correlation of $\Omega_\chi h^2 < 1$ with large negative A_0 or very large positive A_0 is quite pronounced and, as explained earlier, reflects the necessity of a light stop, i.e., of a large left-right mixing. The asymmetry in the A_0 values follows from the renormalization-group running of the mixing parameter down to low energies, where it is related to its GUT-scale counterpart through the equation [25]:

$$A_t = A_0(1 - y) - \mathcal{O}(2)M_{1/2} \quad (2)$$

where y is the ratio of the top Yukawa coupling to its infra-red quasi-fixed point value. The correlation between $\Omega_\chi h^2$ and the ratio $m_0/M_{1/2}$ is less pronounced, and $m_0/M_{1/2} > 1$ is possible for $\Omega_\chi h^2 < 1$ even away from the exceptional cases mentioned above.

The situation is rather different for $\tan\beta = 1.65$, as seen in Fig. 3. Here, we see in the

top left panel that the tendency to favour low $\Omega_\chi h^2$ is less marked, though $\Omega_\chi h^2 \sim 1$ is still preferred. We again see that parameter choices with $\Omega_\chi h^2 < 1$ normally have $\Delta_0 < 200$, with exceptions provided by choices with important direct-channel Z^0 and h^0 poles, as seen in the top right panel of Fig. 3. We recall that the five-pointed star choices with $m_\chi \sim M_Z/2$ will be explored exhaustively by the LEP runs at $E_{CM} = 200$ GeV. We also see in the panel f) of Fig. 3 that LEP Higgs searches will be able to verify or exclude this possible value of $\tan\beta$: it predicts that $M_h \lesssim 96$ GeV, whereas LEP 200 should have a reach extending beyond $M_h = 100$ GeV.

The results presented so far have been obtained with a scan over m_Q , m_U and M_A up to 1.2 TeV. We see that the highest values of $M_{1/2}$ in this scan (and the corresponding values of m_χ) are consistent with $\Omega_\chi h^2 < 1$, but at the expense of increasing fine-tuning. It is interesting to extend the scan up to higher values of soft masses so that an absolute upper bound on $M_{1/2}$ (m_χ) is obtained from the requirement $\Omega_\chi h^2 < 1$. This bound is shown in Fig. 4, where for $\tan\beta = 10$ we show $\Omega_\chi h^2$ versus Δ_0 and $M_{1/2}$, scanning over m_Q , m_U and M_A values up to 8 TeV. In Fig. 5a we plot Δ_0 as a function of m_χ for models which give $\Omega_\chi h^2 < 1$. We observe that the bound for the heavier superpartner masses is weak, around 1 TeV, but is saturated only for large Δ_0 .

Let us now summarize the story so far. Minimizing the gauge hierarchy fine-tuning parameter Δ_0 favours values of $\Omega_\chi h^2$ close to the range favoured by astrophysicists and cosmologists. Conversely, restricting $\Omega_\chi h^2 < 1$ favours models with relatively low values of Δ_0 , with certain well-understood exceptions, some of which may soon be probed by experiments at LEP.

The question now arises how sensitive these observations are to the fine-tuning criterion we have used. In a recent paper [16], we have studied the consequences of postulating some linear relation between a pair of the MSSM input parameters, and we now discuss their possible implications for the cosmological fine-tuning problem. Fig. 6 displays the implications of assuming a linear relation between μ_0 and $M_{1/2}$, for the specific case $\tan\beta = 2.5$. We see that the global minimum of $\Delta_{M\mu}$ is significantly reduced, and that the preference for $\Omega_\chi h^2 \sim 0.1$ is maintained and even enhanced, as compared with Fig. 1 where no parameter relation was assumed. We also see that the patterns of correlations between values of $\Omega_\chi h^2$ and Δ_0 for three different sets of parameters are maintained, modulo an approximate overall rescaling in the values of Δ_0 . The picture if a linear relation between μ_0 and A_0 is assumed is somewhat different, as displayed in Fig. 7. The minimum of $\Delta_{A\mu}$ is again reduced, as compared to the case with no parameter relation, but there is no longer any preference for $\Omega_\chi h^2 < 1$: indeed, the favoured value is $\Omega_\chi h^2 > 10$. Finally, returning to Figs. 5b and 5c we see, for $\tan\beta = 10$, the values of $\Delta_{M\mu}$ and $\Delta_{A\mu}$ for points satisfying $\Omega_\chi h^2 < 1$ as functions of m_χ . The values $\Delta_{M\mu} \lesssim 10$ are now compatible with $m_\chi \lesssim 500$ GeV.

We conclude by reiterating that there is a significant correlation between the amount of hierarchical fine tuning and the relic cold dark matter density in the MSSM. It is indeed “natural” that the supersymmetric relic particle have $\Omega_\chi h^2 \sim 0.1$ to 1, which we consider to be an attractive feature of this dark matter candidate, as compared to massive neutrinos or the axion, whose densities have no obvious reason to fall within this favoured range. However, this cannot be regarded as a hard prediction of the MSSM. Moreover, fine tuning is always

a subjective argument, rather than a hard-and-fast mathematical argument, and one would immediately embrace any experimental discovery of even an “unnatural” dark matter particle. Nevertheless, we find this correlation between cosmological and hierarchical fine tuning an interesting supplementary argument in favour of supersymmetric cold dark matter.

Acknowledgments.

We thank Marek Olechowski for collaboration at the early stage of this project. We would also like to thank Toby Falk for many helpful conversations. The work of P.H.Ch. and S.P. has been partly supported by the Polish State Committee for Scientific Research grant 2 P03B 040 12 (1998-1999). The work of K.A.O. was supported in part by DOE grant DE-FG02-94ER-40823 at the University of Minnesota.

References

- [1] J. Ellis, J.S. Hagelin, D.V. Nanopoulos, K.A. Olive and M. Srednicki, *Nucl. Phys.* **B238** (1984) 453;
M. Srednicki, for the Particle Data Group, *Eur. Phys. J.* **C3** (1998) 125.
- [2] G. Raffelt, *Phys. Rep.* **198** (1990) 1 and astro-ph/9707268;
M.S. Turner, *Phys. Rep.* **197** (1990) 67.
- [3] G. Jungman, M. Kamionkowski and K. Griest, *Phys. Rep.* **267** (1996) 195.
- [4] L. Maiani, *Proceedings Summer School on Particle Physics*, Gif-sur-Yvette, 1979 (IN2P3, Paris, 1980), p.3;
G. 't Hooft, in *Recent Developments in Field Theories*, eds. G. 't Hooft et al., (Plenum Press, New York, 1980);
E. Witten, *Nucl. Phys.* **B188** (1981) 513;
R.K. Kaul, *Phys. Lett.* **109B** (1982) 19.
- [5] J. Ellis, S. Kelley and D.V. Nanopoulos, *Phys. Lett.* **B249** (1990) 441 and **B260** (1991) 131;
C. Giunti, C.W. Kim and U.W. Lee, *Mod. Phys. Lett.* **A6** (1991) 1745;
U. Amaldi, W. de Boer and H. Fürstenau, *Phys. Lett.* **B260** (1991) 447;
P. Langacker and M. Luo, *Phys. Rev.* **D44** (1991) 817.
- [6] P.H. Chankowski, Z. Phuciennik and S. Pokorski, *Nucl. Phys.* **B439** (1995) 23.
- [7] M. Grünewald and D. Karlen, talks at XXIX *International Conference on High-Energy Physics* Vancouver, B.C., Canada, <http://www.cern.ch/LEPEWWG/misc/>.
- [8] M. Carena, M. Quiros and C.E.M. Wagner, *Nucl. Phys.* **B461** (1996) 407;
H.E. Haber, R. Hempfling and A.H. Hoang, *Zeit. für Phys.* **C75** (1997) 539.
- [9] S. Dimopoulos, *Phys. Lett.* **B246** (1990) 347.

- [10] J. McDonald, K.A. Olive and M. Srednicki, *Phys. Lett.* **B283** (1992) 80;
M. Drees and M.M. Nojiri, *Phys. Rev.* **D47** (1993) 376.
- [11] K.A. Olive and M. Srednicki, *Phys. Lett.* **B230** (1989) 78 and *Nucl. Phys.* **B355** (1991) 208;
K. Griest, M. Kamionkowski and M.S. Turner, *Phys. Rev.* **D41** (1990) 356;
R. Arnowitt and P. Nath, *Phys. Rev. Lett.* **70** (1993) 3696;
G. Kane, C. Kolda, L. Roszkowski and J. Wells, *Phys. Rev.* **D49** (1994) 6173;
T. Falk and K.A. Olive, *Phys. Lett.* **B375** (1996) 196.
- [12] J. Ellis, K. Enqvist, D.V. Nanopoulos and F. Zwirner, *Nucl. Phys.* **B276** (1986) 14.
- [13] R. Barbieri and G.-F. Giudice, *Nucl. Phys.* **B306** (1988) 63;
S. Dimopoulos and G.-F. Giudice, *Phys. Lett.* **B357** (1995) 573.
- [14] P.H. Chankowski, J. Ellis and S. Pokorski *Phys. Lett.* **B423** (1998) 327.
- [15] R. Barbieri and A. Strumia, *Phys. Lett.* **B433** (1998) 63.
- [16] P.H. Chankowski, J. Ellis, M. Olechowski and S. Pokorski, preprint CERN-TH-98-119 (hep-ph/9808275).
- [17] M. Olechowski and S. Pokorski, *Nucl. Phys.* **B404** (1993) 590.
- [18] D. Reid, talk at XXXIII *Rencontres de Moriond* (Electroweak Interactions and Unified Theories), Les Arcs, France, March 1998; LEP Electroweak Working Group, report LEPEWWG/98-01.
- [19] ALEPH Collaboration, ALEPH 98-029 CONF 98-017;
DELPHI Collaboration, talk by K. Moenig, LEPC Meeting, CERN, March 31, 1998;
M. Acciarri (L3 Collaboration) et al., preprint CERN-EP/98-052;
OPAL Physics Note PN340, March 1998;
V. Ruhlmann-Kleider, talk at the XXXIII *Rencontres de Moriond* (QCD and high energy interactions), Les Arcs, France, March 1998;
M. Felcini, talk at the 1st European Meeting *From Planck Scale to Electroweak Scale*, Kazimierz, Poland, May 1998.
- [20] CLEO Collaboration, preprint CLEO CONF 98-17 submitted to the XXIX *International Conference on High-Energy Physics* Vancouver, B.C., Canada, contributed paper ICHEP98 1011.
- [21] J. Ellis, T. Falk, K.A. Olive and M. Schmitt, *Phys. Lett.* **B413** (1997) 355.
- [22] S. Abel and T. Falk, CERN preprint TH/98-322 (hep-ph/9810297).
- [23] J. Ellis, T. Falk and K.A. Olive, CERN preprint TH/98-326 (hep-ph/9810360).
- [24] K. Griest and D. Seckel, *Phys. Rev.* **D43** (1991) 3191.

- [25] M. Carena, M. Olechowski, S. Pokorski and C.E.M. Wagner, *Nucl. Phys.* **B419** (1994) 213.

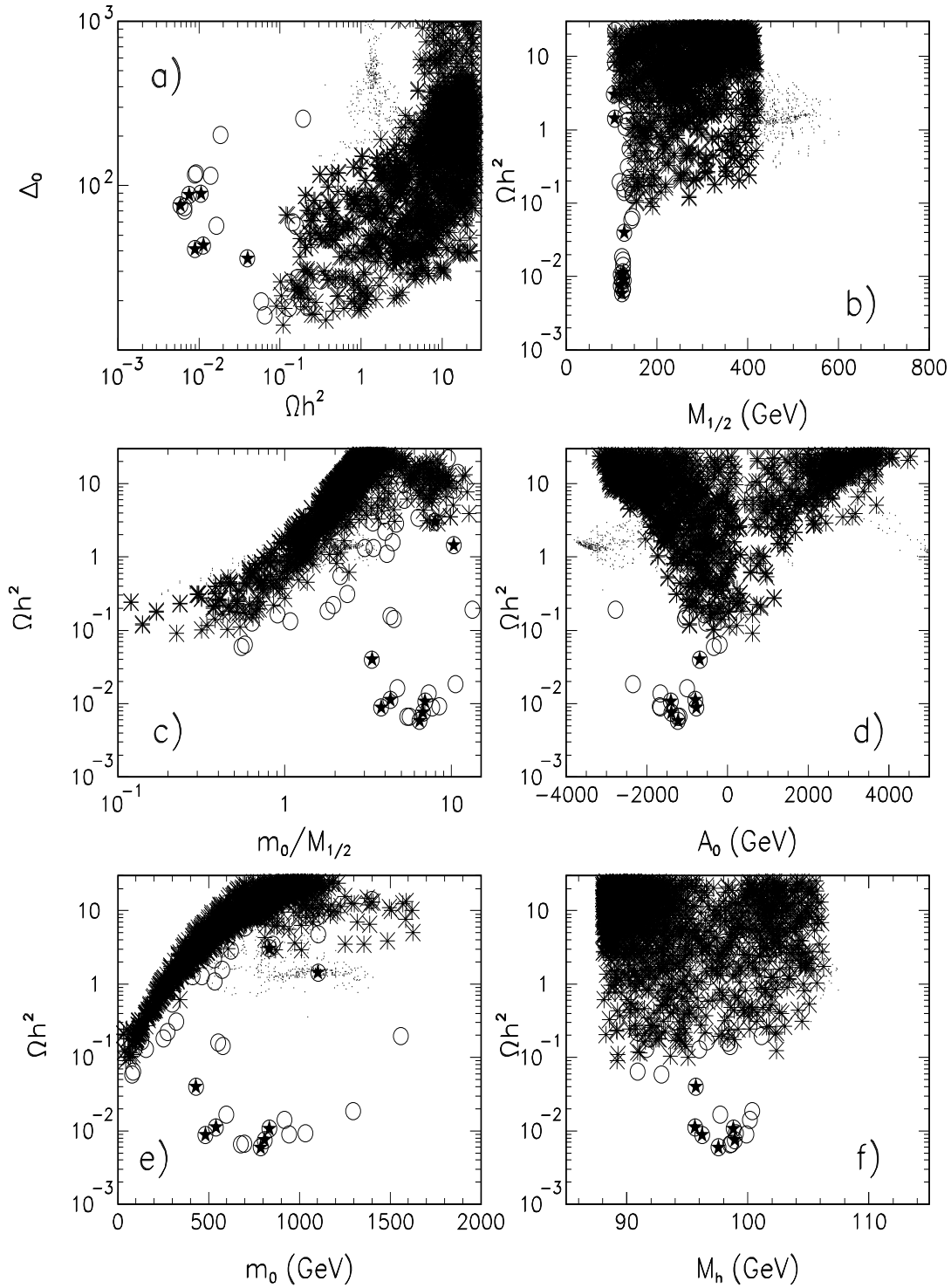


Figure 1: *The price of fine tuning and $\Omega_\chi h^2$ for $\tan \beta = 2.5$, as functions of various variables in the minimal supergravity model. The experimental constraints described in the text are included. The five-pointed stars (open circles) represent cases where $m_\chi \sim M_Z/2$ ($m_\chi \sim M_h/2$). The dots represent parameter choices where $m_\chi \gtrsim m_t$. Eight-pointed stars represent parameter choices where no specific direct-channel annihilation mechanism is dominant.*

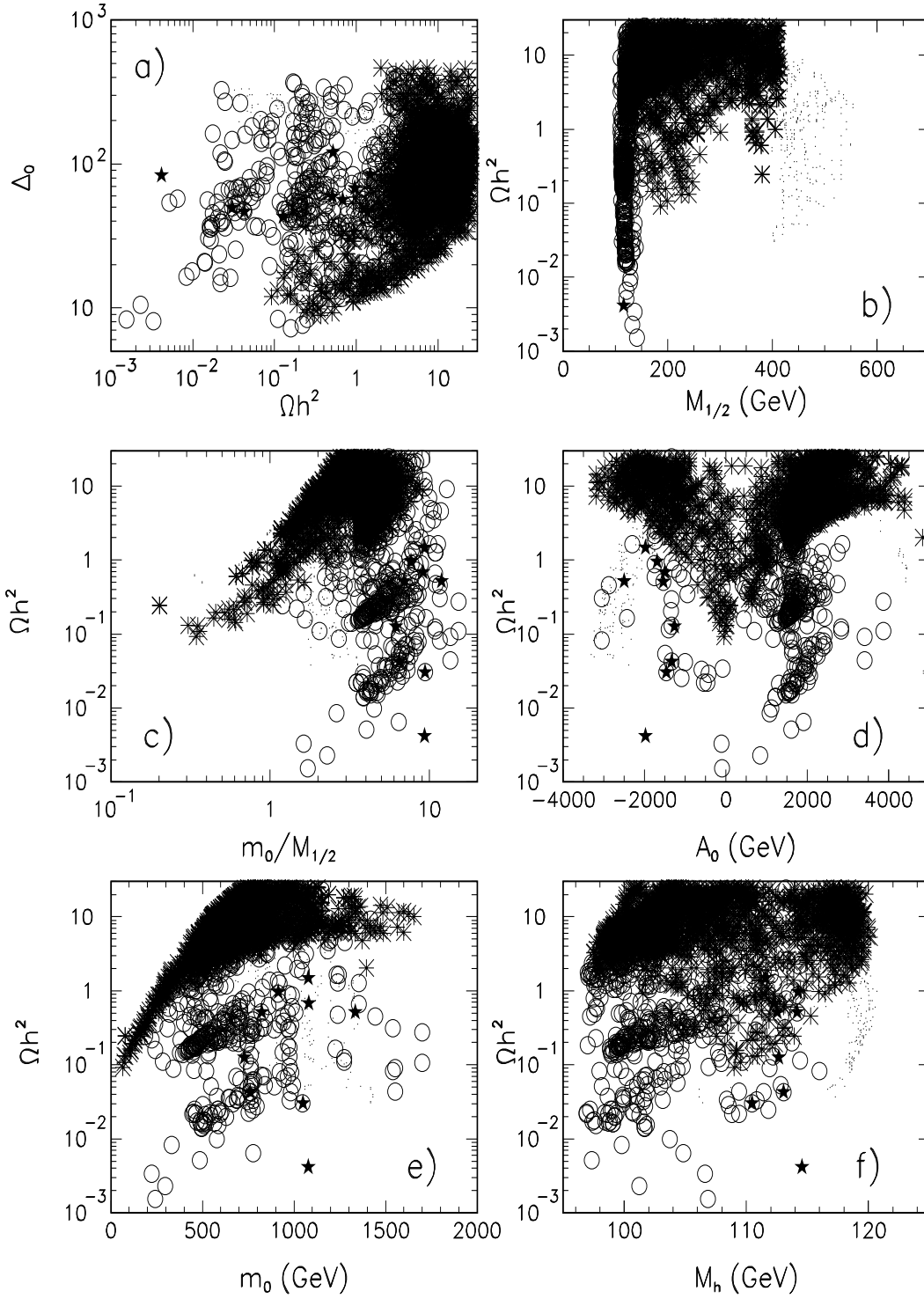


Figure 2: As in Fig. 1, but for $\tan \beta = 10$.

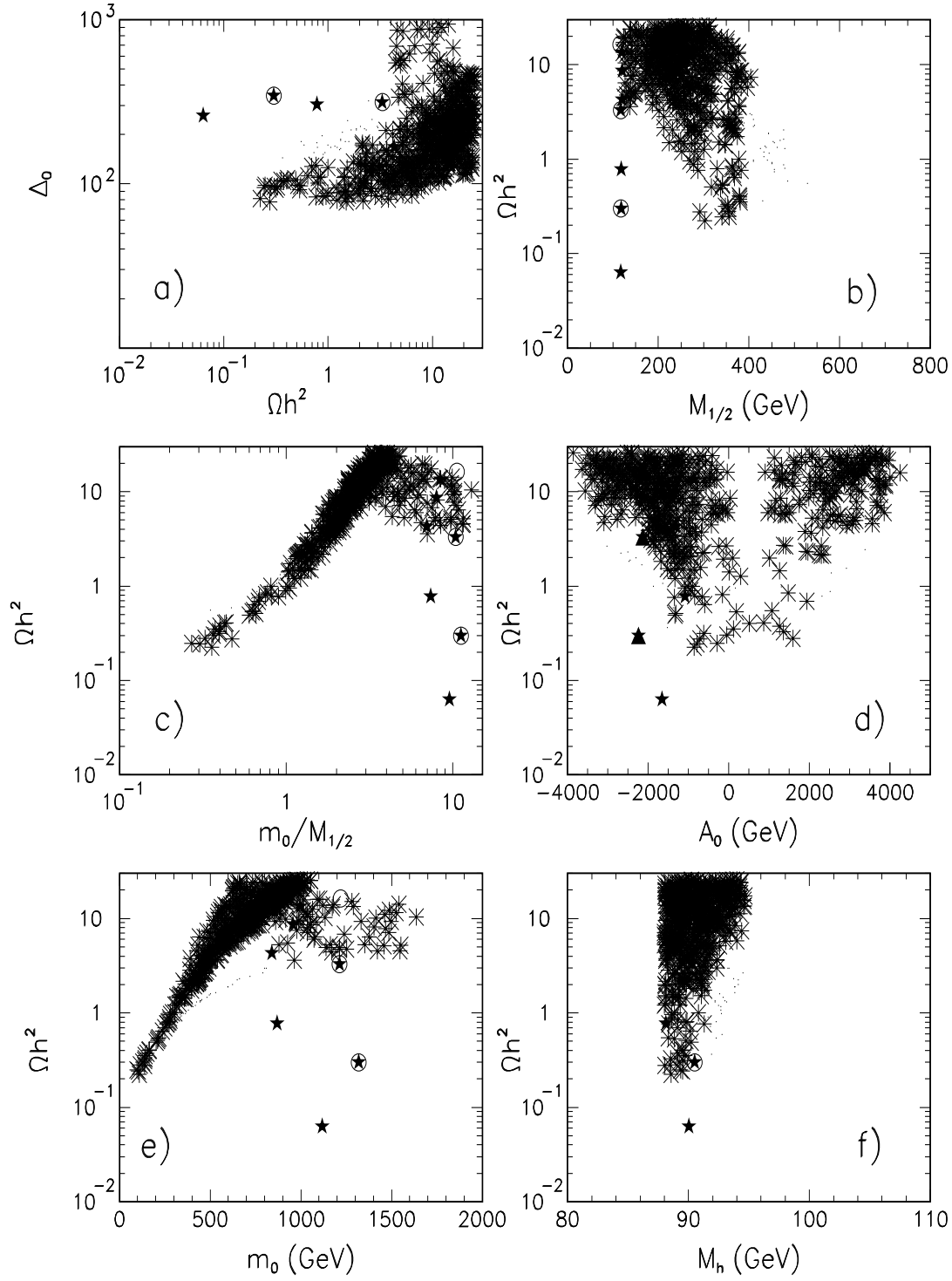


Figure 3: As in Fig. 1, but for $\tan \beta = 1.65$.

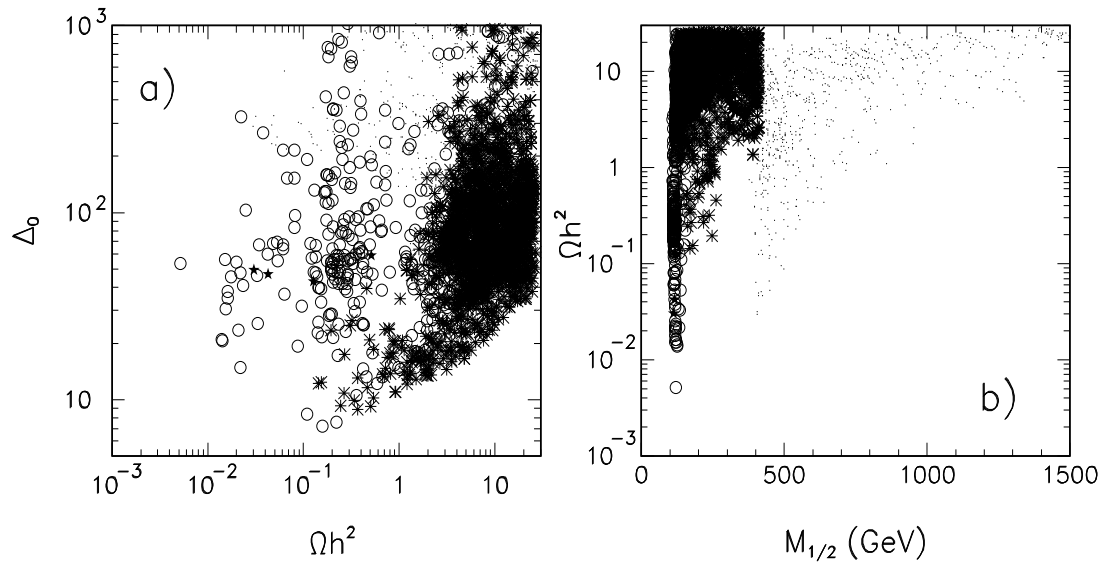


Figure 4: *The price of fine tuning and $\Omega_\chi h^2$ for $\tan\beta = 10$, with the scan over m_Q , m_U and M_A extended up to 8 TeV.*

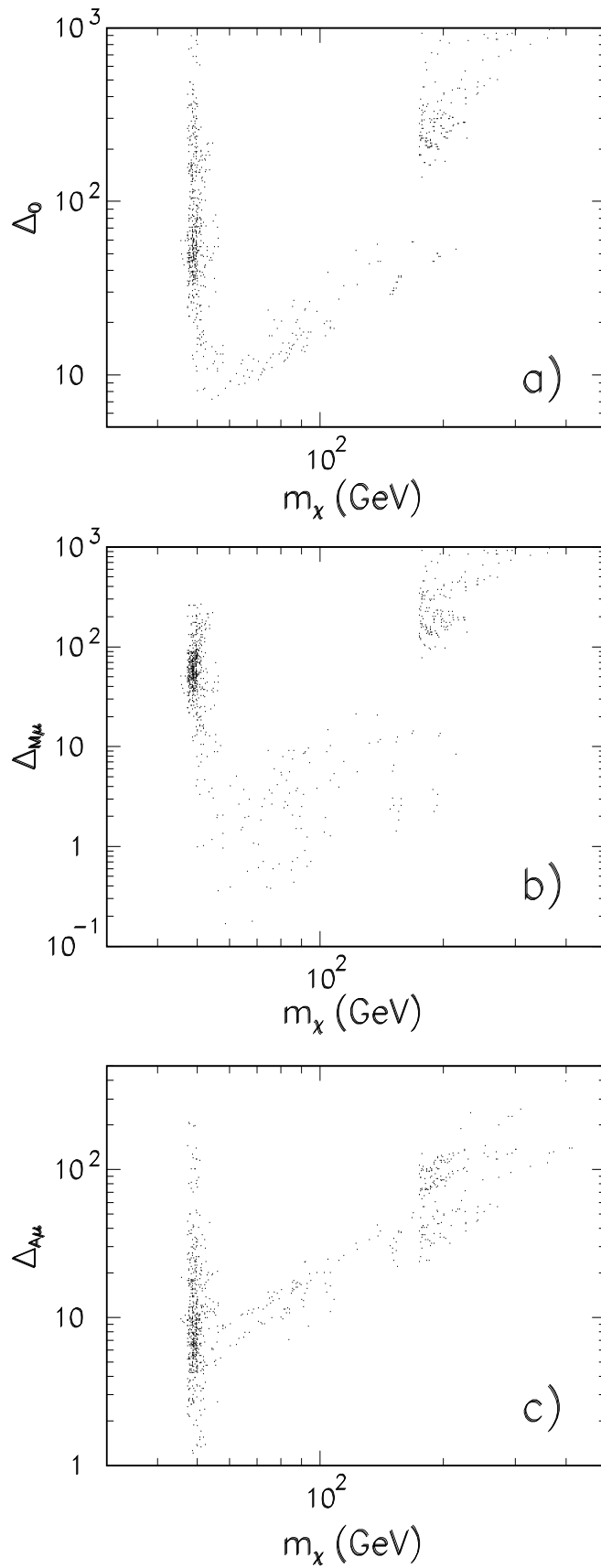


Figure 5: The fine-tuning measures Δ_0 , $\Delta_{M\mu}$ and $\Delta_{A\mu}$ as functions of the lightest neutralino mass for $\tan\beta = 10$. Only points satisfying $\Omega_\chi h^2 < 1$ are shown.

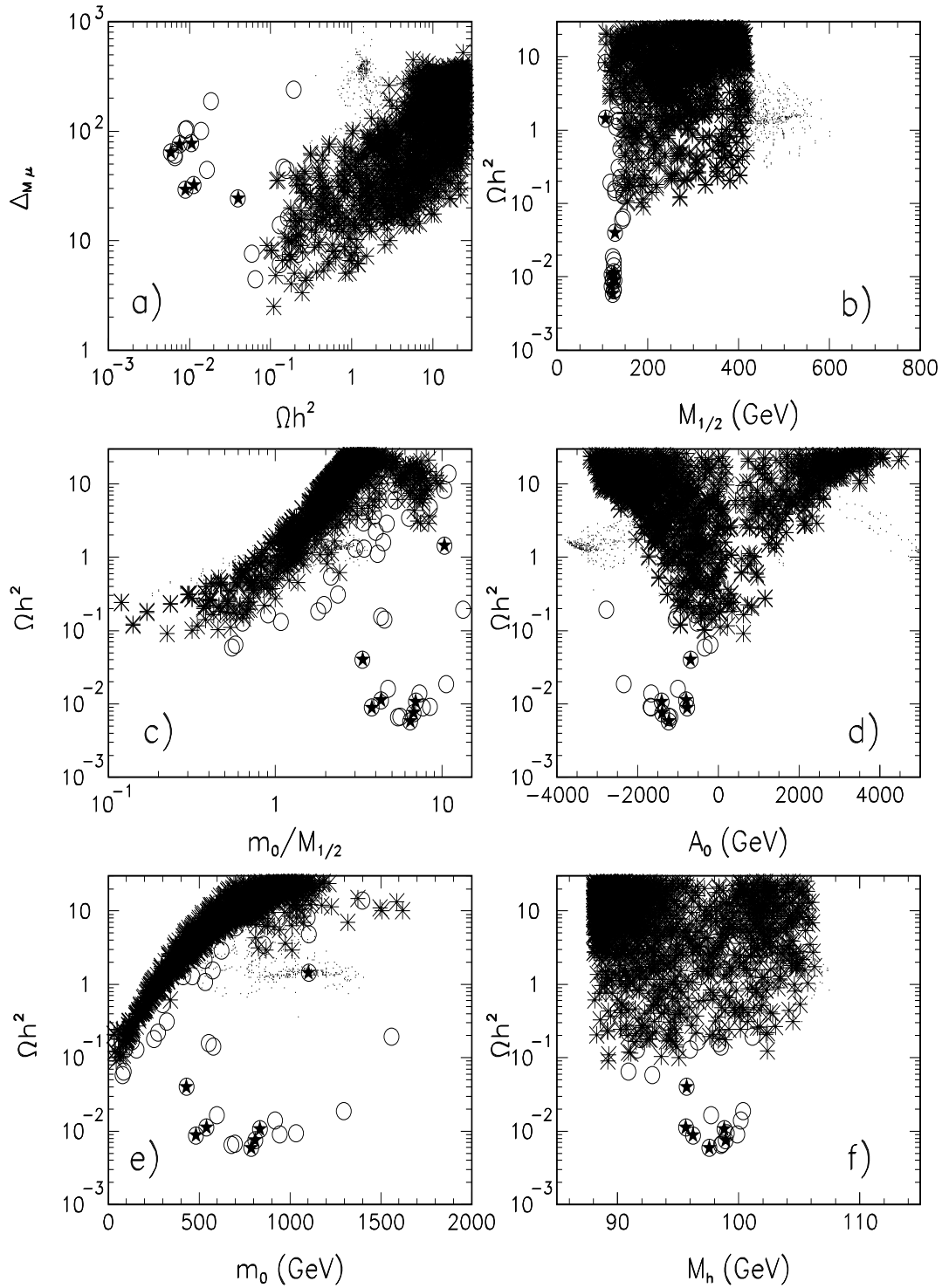


Figure 6: As in Fig. 1 ($\tan\beta = 2.5$), but assuming a linear correlation between $M_{1/2}$ and μ_0 . We show only points with $\Delta_{M\mu} < \Delta_0$.

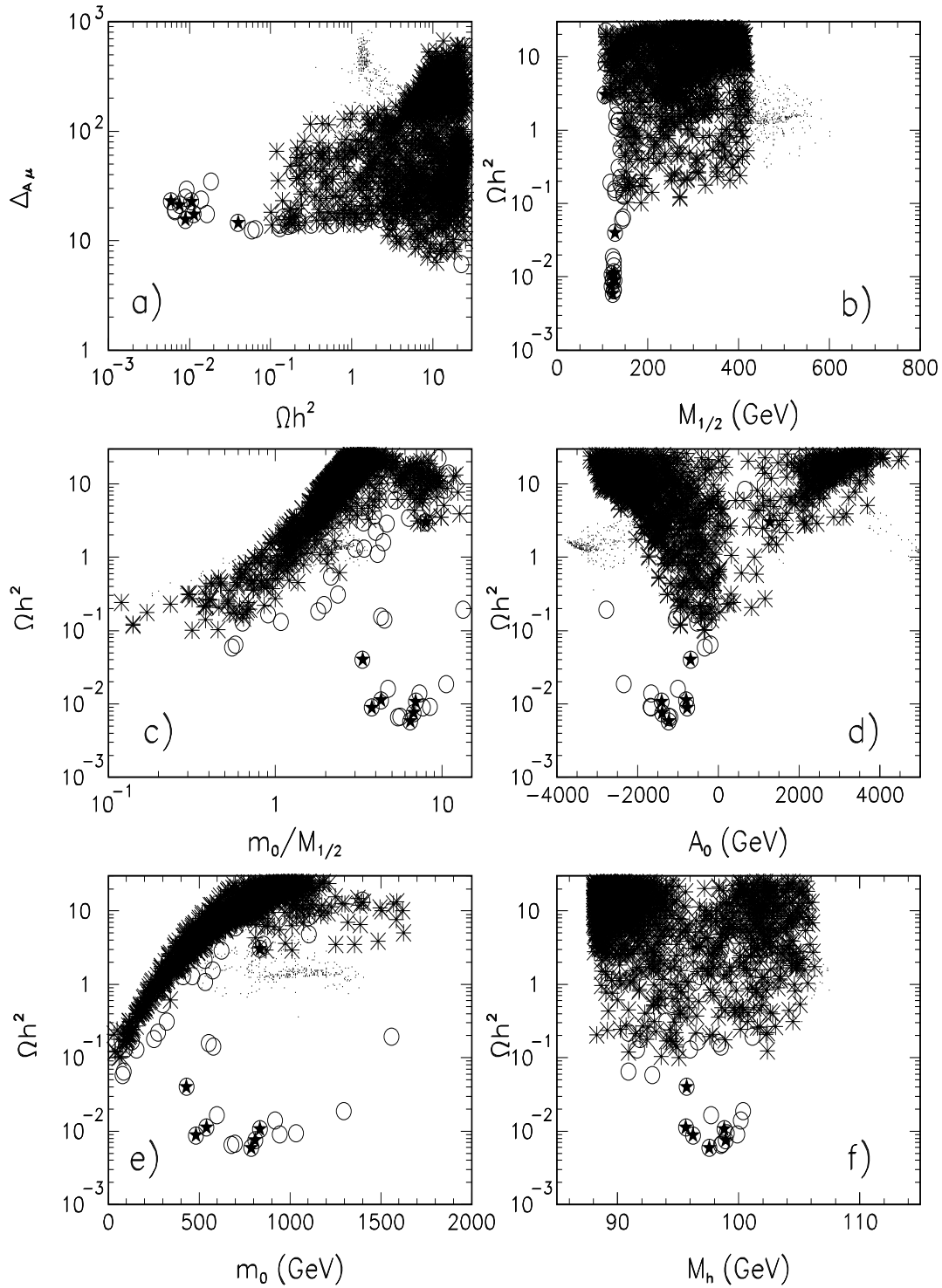


Figure 7: As in Fig. 1 ($\tan \beta = 2.5$), but instead assuming a linear correlation between A_0 and μ_0 . We show only points with $\Delta_{A\mu} < \Delta_0$.