THEORETICAL WORK ON PLASMA ACCELERATORS AT UCLA

T Katsouleas, J M Dawson, W B Mori, J J Su, and S Wilks
University of California, Los Angeles, CA, USA

ABSTRACT
Theoretical research on plasma based accelerators performed at UCLA since 1985 is summarized This includes theory and computer simulations of the beat wave accelerator, plasma wakefield accelerator, and of plasma lens concepts

1 INTRODUCTION

This report briefly summarizes the theoretical progress made at UCLA on plasma based accelerators over the last two years As much of this work has recently been published, many of the major milestones are merely stated here for completeness, with references given to the appropriate publications. These milestones and references are listed in Table 1 Work that is new since the Madison conference is indicated by an asterisk in the table and is described in this report

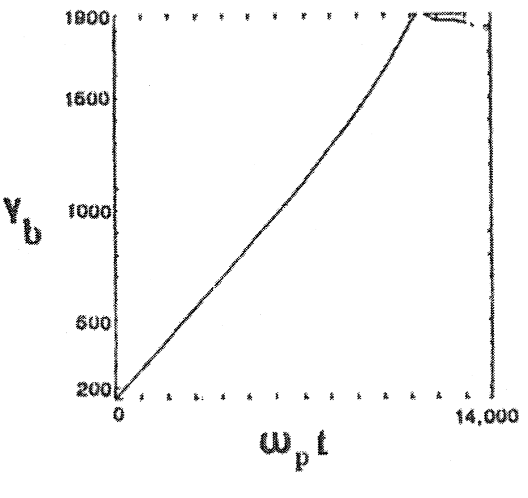

Fig 1 Test particle acceleration in a PIC simulation of plasma wakefield acceleration from 75 MeV to 1 GeV The use of a new simulation code which moves with the beam and test particles made this run possible with 5 hours of CRAY computer time The old (non moving) plasma codes would have required 10,000 hours

Table 1
Summary of theoretical progress since 1985

Topic	References
Beat Wave Physics:	
Calculations specific to UCLA experiment	Madison, WI Proceedings[1]; C Joshi, et al in these proceedings
Non linear frequency shift of plasma wave Resolution of controversy between Eulerian and Lagrangian fluid descriptions	Madison Proc ; W B Mori in Ref 2
Beat wave excitation including effects of plasma drifts, harmonics, pump rise time, frequency mismatch, damping, plasma inhomogeneities and two dimensions	W B Mori in Ref 3
Plasma wakefield Accelerator Physics:	
Mechanism for phase locking particles to plasma wave	Ref. 4
Development of simulation code moving with beam; modeling of test particle acceleration to 1 GeV	Madison Proceedings[1]; Fig 1
Driving beam instabilities and stabilization	J J Su, T Katsouleas, J Dawson, P Chen, M Jones, R Keinigs, in Ref 3,
Methods of overcoming the fundamental wakefield theorem *	
Plasma Wave Properties:	
Relativistic wavebreaking amplitude in a thermal plasma *	Ref. 5
Beam loading in plasma waves	Ref 6 and S Wilks in Ref 3
Plasma Lenses:	
2 D PIC Simulations	P Chen, J J Su, T Katsouleas, S. Wilks, J M Dawson in Ref 3, Ref 1
Pure ion lens *	

2 METHODS OF OVERCOMING THE FUNDAMENTAL WAKEFIELD THEOREM

In the plasma wakefield accelerator,[7] the plasma acts as a transformer to convert the energy of a low voltage (but still relativistic) driving (particle) beam to a trailing beam of fewer particles that are accelerated to high voltage (energy) Naturally, a primary requisite for such a scheme is that it provide a reasonably large transformer ratio (defined as the ratio of the energy gain of the trailing beam to the energy of the driving beam) A Chao, et al ,[8] showed that symmetric driving bunches produce transformer ratios no larger than two This has been called the fundamental wakefield theorem Although asymmetric bunches which are slowly ramped over N plasma wavelengths and sharply truncated can produce transformer ratios up to $2\pi N$ (see Fig 2),[9,4] the shaping of these bunches poses a major technological hurdle for realizing a plasma wakefield experiment Here we present two alternatives that enable a large transformer ratio with little or no pre shaping of the driving bunch These are self sharpening of the bunch tail within the plasma and the use of non linear wakes

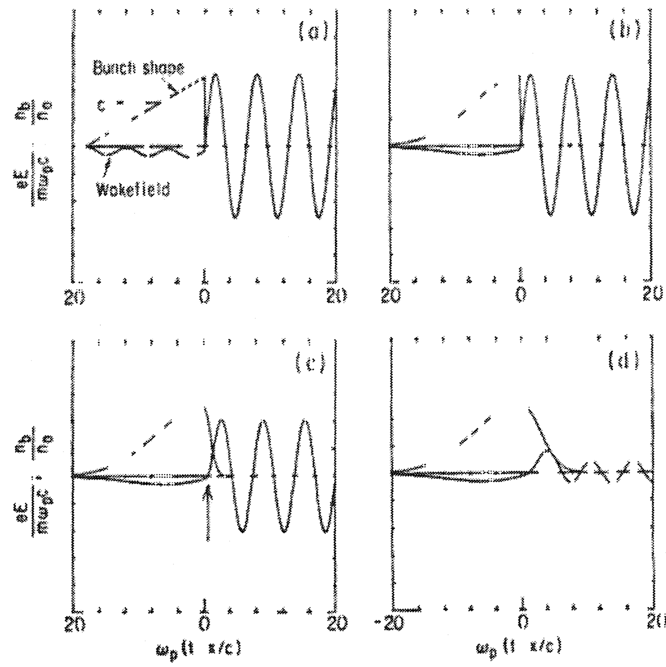

Fig 2 Numerical solutions of the cold plasma wakefield equations showing the longitudinal fields within and behind driving bunches of various shapes (from Ref 4) Note that the accelerating wakefield behind the bunch is significantly smaller if the tail of the bunch is cut off in a distance (time) longer than a few c/ω_p (ω_p^{-1})

2 1 Self sharpening.

Figure 2 illustrates that asymmetric bunches excite large accelerating wakefields behind the bunches It also shows the (retarding) wakefield acting on the particles within the bunch Due to the inertia of the plasma electrons the retarding field can vary or even change signs within the bunch as in Fig 2(c) and (d) Particles to the right of the arrow in Fig 2(c), for example, will experience an accelerating field which tends to make them catch up to the particles in the front of the bunch This will sharpen the tail of the bunch, increasing the accelerating wakefield and the transformer ratio This is illustrated in the self consistent PIC simulation below

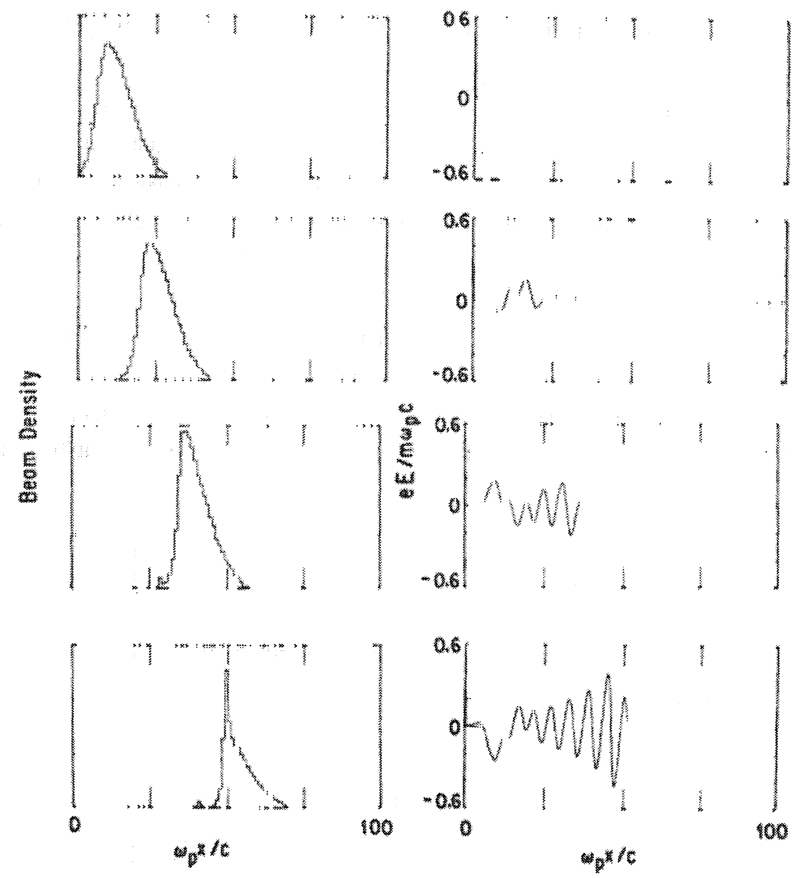

Fig 3 PIC simulation of self sharpining of a driving bunch and the increasing longitudinal wakefield excited by the bunch The initial bunch shape corresponds to Fig 2(d); gaussian rise [exp $(x^2/2\sigma^2)$] with $\sigma = 7\ c/\omega_p$ and gaussian fall with $\sigma = 3c/\omega_p$; the beam energy corresponded to $\gamma = 3$

Thus it may be possible to appropriately shape a (nearly) gaussian bunch by choosing the plasma density properly. For a gaussian rise of the form $e^{x^2/2\sigma^2}$ the plasma density should be chosen such that $\omega_p = 7c/\sigma$ or $n_0 = 1.4 \times 10^{15}\ cm^{-3} \times \sigma_{mm}^{-2}$, where σ_{mm} is expressed in millimeters

Since the distance required for the bunch to sharpen scales as γ^2, it may be desirable to use this scheme to shape a low γ beam, then accelerate the beam to the driving energy This possibility and the modeling of other initial bunch shapes are the subjects of continuing investigation

2 2 Non linear wakefields.

By using very dense driving beams ($n_b \approx n_0/2$) it is possible to excite very non linear wakefields Since the principle of linear superposition assumed in the derivation of the fundamental wakefield theorem no longer is valid, high transformer ratios become possible Solving the cold non linear plasma fluid equations[10] numerically we obtain the non-linear wake solutions to a symmetric gaussian beam in Fig. 4 In this case a transformer ratio (peak accelerating field divided by peak of retarding field) of 4 is obtained Note that the beam need not be short (or sharp) compared to c/ω_p in order to excite a large wake This enables the use of denser plasmas for a given bunch length and consequently gives higher acceleration gradients

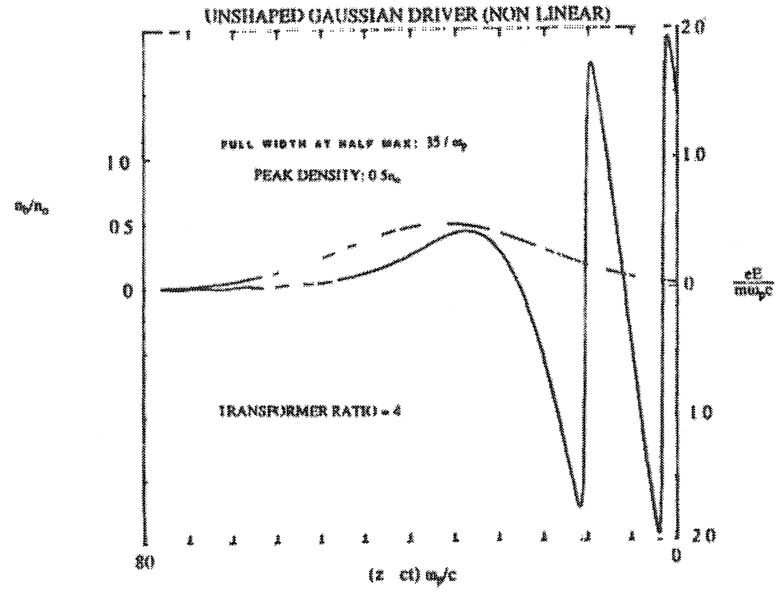

Fig. 4 Numerical solutions of the non linear wake excited by the gaussian bunch shown

A physical interpretation of the non linear excitation mechanism is the following. As the plasma motion becomes very non linear, the plasma frequency undergoes a large non linear frequency shift to lower frequency. Thus, the tail of the bunch appears to the plasma to be short (i.e., in units of inverse plasma periods). As measured in plasma periods, the bunch seems to be asymmetric so that a large wake is excited in analogy to the linear asymmetric cases (Fig. 2)

3 RELATIVISTIC WAVEBREAKING AMPLITUDE IN A WARM PLASMA

A question fundamental to plasma accelerators, and in particular the non linear scheme of the previous section, is how large the plasma wave field can be. The most often quoted limit is the cold non relativistic wavebreaking field due to Dawson[11] (1959):

$$\frac{eE}{m\,\omega_p c} = 1$$

This is the same estimate one obtains from Poisson's equation by estimating the maximum density perturbation associated with a plasma wave as the background plasma density (e.g., as in a 100% rarefaction). A relativistic cold plasma treatment gives a significantly larger wavebreaking field for high phase velocity (v_ϕ) waves[12]:

$$\frac{eE}{m\,\omega_p c} = \sqrt{2}\,(\gamma_\phi - 1)^{1/2} ,$$

where $\gamma_\phi = (1 - v_\phi^2/c^2)^{-1/2}$. However, the density compressions become so large that the plasma pressure becomes important even for fairly low temperature plasmas. Thus, it becomes necessary to include thermal terms in the non linear plasma field equations.

The details of the relativistic wavebreaking calculation for a warm plasma are given in the internal report cited in Ref. 5. In the limit that $v_\phi = c$ an analytic solution is possible. The result is

$$\frac{eE}{m\,\omega_p c} = \sqrt{2}\left(\frac{2}{5}\beta^{-1/4} + \frac{2}{3}\beta^{1/4} - \frac{1}{15}\beta - 1\right)^{1/2},$$

where $\beta \equiv k_B T/mc^2 \ll 1$, and T is the plasma temperature This is plotted in Fig 5 For $v_\phi \le c$, the wavebreaking field can be found numerically. As an example consider $\gamma_\phi = 100$ and $k_B T = 30$ eV Then the cold wavebreaking field would be $eE/m\,\omega_p c \approx 14$, but the thermal corrections reduce this to $eE/m\,\omega_p c \approx 2.2$.

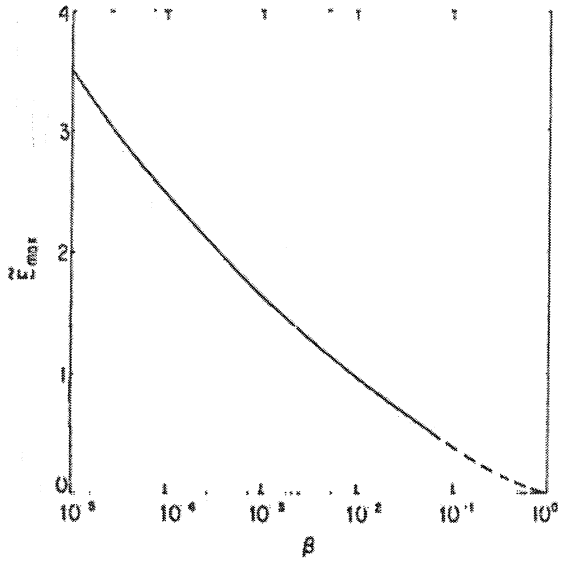

Fig 5 Normalized wavebreaking field vs β for $v_\phi = c$

4 BEAM LOADING IN PLASMAS

Beam loading in plasma waves is analyzed in Refs 6, 3, and 1 Here we briefly summarize the main results These were obtained by considering the linear superposition of the field of a plasma wave and the wakefields produced by a beam load

The theoretical maximum number of particles that can be loaded into a plasma wave is given by

$$N^{MAX} = 5 \times 10^5 \sqrt{n_0}\; A_{eff}\; \varepsilon\,,$$

where n_0 is the plasma density in cm^{-3}, ε is the normalized plasma wave amplitude $eE/m\,\omega_p c \approx n_1/n_0 < 1$ (for linear waves), and A_{eff} is the effective area of the beam in cm^2 A_{eff} equals the actual beam cross section for beam radius $r > c/\omega_p$ and $A_{eff} \approx c^2/\omega_p^2$ for $r < c/\omega_p$

If the beam load can be properly shaped, it is possible in principle to eliminate energy spread of the accelerated beam[1,3] (neglecting phase slippage) This is illustrated in Fig 6 which shows a uniform accelerating field on the beam

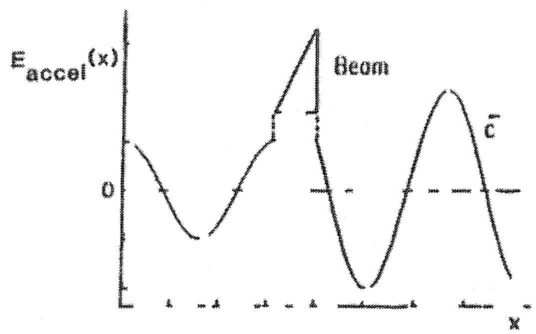

Fig 6 Numerical solution of the total field (plasma wave plus beam wake) for the indicated beam shape In this case $N = \frac{3}{4} N^{MAX}$, the accelerating gradient is half the wave amplitude, and the energy extraction efficiency is 75%

Finite width plasma waves have large radial electric fields which can lead to unacceptable emittance growth Two ways to beam load efficiently while preserving a low emittance are (a) to use wide plasma waves ($\gg c/\omega_p$) with uniform radial profiles and correspondingly wide beam leads, or (b) to use very narrow ($\ll c/\omega_p$) beam loads in waves of order c/ω_p wide The latter option is illustrated in Fig 7 Here the beam can be seen to absorb more than 50% of the wave energy even though it represents only 1% of the cross sectional area of the wave This is because the beam's wakefields extend to a distance of order c/ω_p regardless of how narrow is the beam

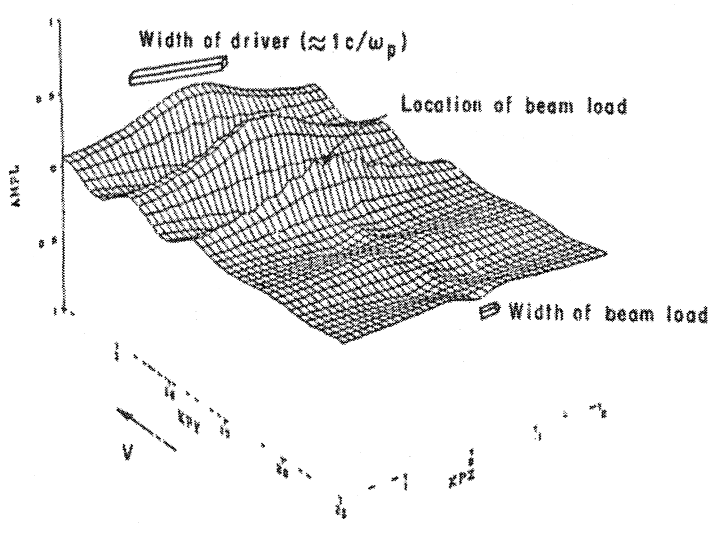

Fig 7 Numerical solution of total longitudinal electric field produced by a driving beam c/ω_p wide and a beam load 0 1 c/ω_p wide placed on the third peak of the wave Absorption of the wave energy behind the beam load is apparent

A non optimized example of beam loading that is consistent with the very small emittance (e g, 10^{-12} cm rad) desired for future colliders is as follows. We assume for the plasma density and wave $n_0 = 10^{16}$ cm^{-3}, eE/m $\omega_p c = 0.5$, wave radius $\approx c/\omega_p \approx 50\mu$ We assume a beam to be shaped as in Fig 6 over a length of 25μ. Now if the matched beam emittance is to be below 10^{-12} cm rad, we find that the accelerated beam radius should be less than 0.2μ The remaining quantities follow from the above assumptions: gradient 2.5 GeV/m, number of accelerated particles $\approx 3\times10^8$, energy spread $\approx 10\%$ (due to radial variation of accelerating field on the bunch), and beam loading efficiency $\approx 20\%$

5 STABILITY OF WAKEFIELD DRIVERS

The stability of the wakefield driving beam in the plasma has been analyzed by J J Su, et al., in Ref 3 The sample simulation below shows the stabilization of the Weibel (filamentation) instability by introducing a small amount of transverse thermal energy on the driving beam (25 keV, independent of γ).

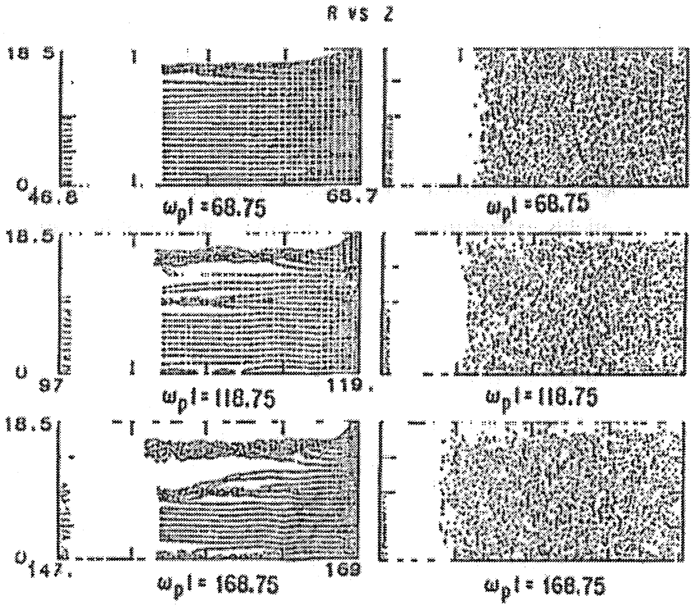

Fig 8 Real space plots of cold (left) and warm driving beams The Weibel instability manifests itself in the formation of filaments (left), but can be stabilized by perpendicular temperature (right) or by an axial magnetic field

6 PLASMA LENSES

In the plasma lens concept a beam of particles traversing a plasma experiences a radial focusing force due to the wakefields of the beam itself (for beams long compared to c/ω_p), or the wakefields excited by a precursor pulse (heat wave or particle beam), or a combination of the two The wakefield analysis of the process[14] has now been verified with 2 D PIC simulations in Ref. 15.

The plasma lens is promising for providing very strong focusing gradients ($F_r/r \approx 2\pi n_b e^2 \approx 100$ MG/cm, where n_b is the beam density) However, two limitations are that for long beams the focusing strength varies with the beam density along the beam and that the beam focuses to its own axis of symmetry rather than to any externally alignable lens axis Both problems could be circumvented by using a non neutral plasma lens (e g , a pure ion plasma lens[15])

A relativistic electron beam passing through a short column of pure ions of density n_i will experience a focusing force (by Gauss' law) $F_r \approx 2\pi n_i e^2 r$, independent of the beam's density This is illustrated in Fig 9 The beam is focused to the axis of the ion column Thus the final focus alignment is governed by the method of ion column creation

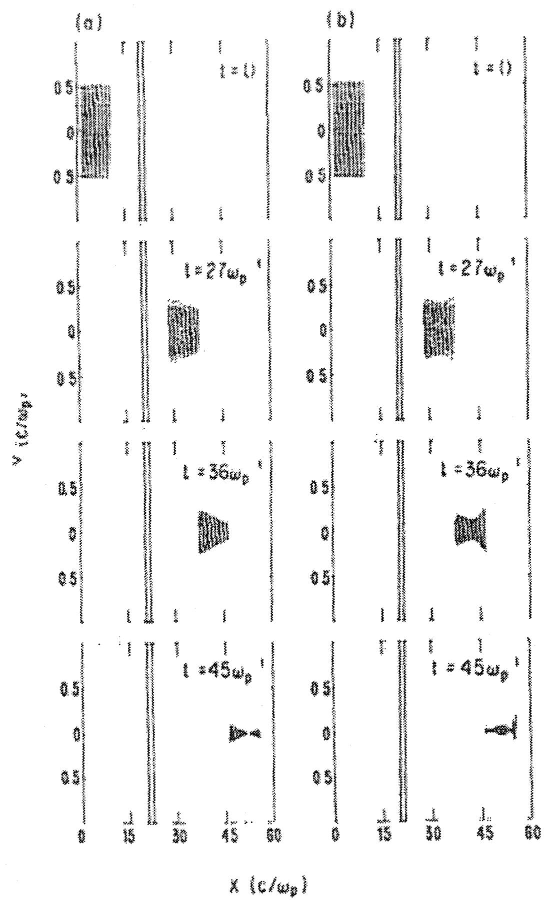

Fig 9 PIC simulation of the focusing of electron beams by a preformed thin ion disk (from Ref 15) In (a) $n_b = 0.008\ n_i$ and in (b) $n_b = 0.8\ n_i$; $\gamma = 30$ and the disk thickness is 1 c/ω_p, where $\omega_p^2 \equiv 4\pi n_i e^2/m$

One possible method of ion channel creation is to use an intense laser to ionize a neutral gas within conducting walls If the ponderomotive force of the laser is large enough to blow the electrons to the walls, an ion channel will remain for the short time required for an electron beam to pass through it and be focused This is similar to the ion focusing described by R Briggs (see Ref 3) in which the ion column is formed by a charging electron beam rather than a laser For typical ion column densities of 5×10^{13} cm^{-3}, the focusing gradient would be of order 100 kG/cm Thus the appeal of ion column focusing would be greatly increased if methods of producing denser ion colums and negative ion columns (for positron focusing) can be found

7. SUMMARY

Recent progress at UCLA on plasma accelerator theory has been outlined Since 1985, analytic models of beat wave excitation and non-linear plasma waves have been refined to include many effects expected in real plasmas including temperature, inhomogeneity, and damping Work on wakefield acceleration and plasma lenses has produced a number of new ideas The ability to realistically model these and other plasma accelerator schemes has been enhanced by the development of a new simulation code that moves with the accelerating particles and driving pulses. Theoretical work continues on three fronts - direct support of experiment, assessment of issues important to realization of a plasma accelerator, and exploration for improved concepts

Work supported by USDOE, NSF and ONR

* * *

REFERENCES

1) C Joshi, et al , in *Proc Advanced Accelerator Concepts Workshop, Madison, WI, 1986*, F E Mills, ed , AIP Conf Proc No 156 (AIP, NY, 1987)

2) *Rep Workshop on Interactions and Transport in Laser Plasmas*, M G Haines, ed (CECAM, Orsay, France), 1985, p 124

3) IEEE Trans Plasma Sci , 15 (2) 1987, Special Issue on Plasma Based High Energy Accelerators, T Katsouleas, guest ed

4) T Katsouleas, Phys Rev. A 33, 2056 (1986)

5) T. Katsouleas and W B. Mori, UCLA PPG 1040 (1987)

6) T Katsouleas, S Wilks, P Chen, J M Dawson, J J Su, Particle Accelerators 22, 81 99 (1987)

7) P Chen, J M Dawson, R W Huff, T Katsouleas, Phys Rev Lett 54, 693 (1985); R D Ruth, A Chao, P L Morton, P B Wilson, Particle Accelerators 17, 171 (1985)

8) A Chao, in *Physics of High Energy Particle Accelerators* (SLAC Summer School, 1982), AIP Conf Proc No 105, M Month, ed (AIP, NY, 1983)

9) K L F Bane, P Chen, and P. B Wilson, IEEE Trans Nucl Sci NS 32, 3524 (1985)

10) J Rosenzweig, in Ref 3

11) J M Dawson, Phys Rev 113, 383 (1959)

12) A I Akhiezer and R V Polovin, Sov Phys JETP 3, 696 (1956)

13) S Van der Meer, CLIC Note No 3, CERN/PS/85 65(AA) (Geneva, 1985)

14) P Chen, Particle Accelerators 20, 171 (1986)

15) P Chen, J J Su, T Katsouleas, S Wilks, and J M Dawson, in Ref 3

16) S Wilks, J M Dawson, and T Katsouleas, UCLA PPG 1064 (April, 1987)

Discussion

U. Amaldi, CERN

I remember that in theory Van der Meer was worried because in the scheme with a whole bunch, the trailing transversely small bunch was focused too strongly. This is a worrying effect, and I would like to know if there are hopes on how to cure it.

Reply

I believe that your concern stems from a CERN report of about two years ago in which synchrotron losses were calculated arising from the focusing wake produced by a narrow driving beam. Fortunately, that calculation was not correct because it made use of a linear plasma wakefield expression where it does not apply. Namely, the expression used for the transverse force blows up as the beam radius goes to zero. However, when the beam shrinks to the point that the beam density is greater than the plasma density, the actual focusing force asymptotes to a constant value. In this case, the focusing force does not appear to be too strong.