SUM RULES FOR TOTAL INTERACTION CROSS SECTIONS OF RELATIVISTIC ELEMENTARY ATOMS WITH ATOMS OF MATTER UP TO TERMS OF ORDER α^2

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Abstract

It is shown that α^2-term of sum rules for the total cross sections of interaction of elementary atoms with matter ones, obtained in ref.[2] is wrong. New sum rules valid up to terms of order α^2 are derived.

The most full qualitative analysis of the interaction of relativistic elementary atoms (EA) with matter in the Born approximation have been done by S. Mrowczynski in the ref.[1] In the ref.[2] the sum rules for the total cross sections of interaction of EA with target (matter) atoms (TA) have being derived from general results of ref.[1] under some assumption, validity of which will be discussed below.

Since the all elastic scattering amplitudes, calculated in the Born approximation, are real, one can't apply the optical theorem for the calculation of the corresponding total cross sections. Due to this reason in the Born approximation the total cross sections calculated by summing of all partial transition total sections σ_{if}

$$\sigma^{tot}(i) = \sum_f \sigma_{fi}\,. \tag{1}$$

In the considering case σ_{if} being the cross sections of transitions between some initial (i) and final (f) states of EA in the screened Coulomb field of TA, described by potential $U(r)$. According to the results of ref.[1] they are presented in the form

$$\sigma_{fi} = \int |A_{fi}(\vec{q})|^2\, d^2 q_\perp\,, \tag{2}$$

where

$$A_{fi}(\vec{q}) = U(Q)[\rho_{fi}(\vec{q}) - \vec{\beta}\vec{\gamma}_{fi}(\vec{q})]\,, \tag{3}$$

$$U(Q) = 4\pi \int U(r)\frac{\sin Qr}{Q} r dr\,, \tag{4}$$

$$Q = \sqrt{\vec{q}^{\,2} - q_0^2} = \sqrt{\vec{q}_\perp^{\,2} + q_L^2 - q_0^2} = \sqrt{\vec{q}_\perp^{\,2} + q_L^2(1-\beta^2)}\,, \tag{5}$$

$$q_0 = \varepsilon_f - \varepsilon_i + \frac{Q^2}{2M} = \beta q_L\,.$$

Here $\vec{q}_\perp$ and q_L are the transverse and longitudinal components of the vector $\vec{q}$ — three dimension momentum transverse to the target, M is mass of EA, β is it's velocity in the lab system.

The transition density $\rho_{fi}(\vec{q})$ and transition current $\vec{j}_{fi}(\vec{q})$ are expressed in terms of the wave functions ψ_i and ψ_f of initial state of EA with help of relations:

$$\rho_{fi}(\vec{q}) = \int \rho_{fi}(\vec{r}) \left(e^{i\vec{q}_1\vec{r}} - e^{-i\vec{q}_2\vec{r}} \right) d^3r, \tag{6}$$

$$\vec{j}_{fi}(\vec{q}) = \int \vec{j}_{fi}(\vec{r}) \left(\frac{\mu}{m_1} e^{i\vec{q}_1\vec{r}} + \frac{\mu}{m_2} e^{-i\vec{q}_2\vec{r}} \right) d^3r, \tag{7}$$

$$\rho_{fi}(\vec{r}) = \psi_f^*(\vec{r})\psi_i(\vec{r}) \,, \tag{8}$$

$$\vec{j}_{fi}(\vec{r}) = \frac{i}{2\mu} \left[\psi_i(\vec{r})\vec{\nabla}\psi_f^*(\vec{r}) - \psi_f^*(\vec{r})\vec{\nabla}\psi_i(\vec{r}) \right], \tag{9}$$

$$\vec{q}_1 = \frac{\mu}{m_1}\vec{q}, \quad \vec{q}_2 = \frac{\mu}{m_2}\vec{q}, \quad \mu = \frac{m_1 m_2}{M}, \quad M = m_1 + m_2;$$

where $m_{1,2}$ are the masses of elementary components of EA.

Since $q_L \approx \varepsilon_f - \varepsilon_i \sim \mu\alpha^2 \sim \alpha < q_\perp >$, where $< q_\perp > \sim \mu\alpha$ is typical value of $q_\perp$ in the problem under consideration, on the first sight it is natural to neglect the q_L-dependence of the transition amplitude and to put

$$A_{fi}(\vec{q}) \approx A_{fi}(\vec{q}_\perp) \,. \tag{10}$$

Just this approximation have been made by authors[2] in their derivation of the following sum rules for the total EA — TA cross sections:

$$\sigma^{tot} = \sigma^{elect} + \sigma^{magnet} \,, \tag{11}$$

$$\sigma^{elect} = 2 \int U^2(q_\perp)[1 - S(q_\perp)] d^2q_\perp \,, \tag{12}$$

$$S(q_\perp) = \int |\psi(\vec{r})|^2 e^{i\vec{q}_\perp\vec{r}} d^3r \,; \tag{13}$$

$$\sigma^{magnet} = \int U^2(q_\perp) K(q_\perp) d^2q_\perp \,, \tag{14}$$

$$K(\vec{q}_\perp) = \int \left(\frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{2}{m_1 m_2} e^{i\vec{q}\vec{r}} \right) \left| \vec{\beta}\ \vec{\nabla}\ \psi_i(\vec{r}) \right|^2 d^3r. \tag{15}$$

Putting for the sake of qualitative estimations

$$U(r) = \frac{Z\alpha}{r} e^{-\lambda r}, \quad \lambda \sim m_e \alpha Z^{1/3}, \quad \alpha = \frac{1}{137}\,;$$

it is easily to derive from (12 – 15)

$$\sigma^{elect} \sim \frac{Z^2}{\mu^2} \ln\!\left(\frac{\mu}{Z^{1/3} m_e} \right), \tag{16}$$

$$\sigma^{magnet} \sim \frac{Z^{4/3}\alpha^2}{m_e^2}\,; \tag{17}$$

where m_e is electron mass.

Thus, although σ^{magnet} is proportional to α^2, squared electron mass in the denominator of the eq. (17) make the relative contribution of "magnetic" term not negligible small compared to electric one, especially for the case of EA, composed from the heavy hadrons. This result seems physically unreasonable, that forces us to reanalysis the problem.

If is easily to verify that the subsequent members of expansion of transition density and current over powers of small quantity q_L

$$\rho_{fi} = \sum_{n=0}^{\infty} \rho_{fi}^{(n)}\,, \quad \rho_{fi}^{(n)} = \frac{q_L^n}{n!}\left(\frac{d^n}{dq_L^n}\rho_{fi}\right)\Big|_{q_L=0}\,, \tag{18}$$

$$\vec{j}_{fi} = \sum_{n=0}^{\infty} \vec{j}_{fi}^{(n)}\,, \quad \vec{j}_{fi}^{(n)} = \frac{q_L^n}{n!}\left(\frac{d^n}{dq_L^n}\vec{j}_{fi}\right)\Big|_{q_L=0}\,; \tag{19}$$

obey the following relations:

$$\rho_{fi}^{(n)} \sim \alpha^n\,, \quad j_{fi}^{(n)} \sim \alpha^{n+1}\,. \tag{20}$$

Thus, neglecting the q_L-dependence of transition amplitude, the authors[2] have taking into account only one term of order α, namely $\vec{\beta}\cdot\vec{j}_{fi}{}^{(0)}$ and have neglected another one $\rho_{fi}^{(1)}$.

The detail analysis of the problem shows that the contribution of terms $\rho_{fi}^{(n+1)}$ and $\vec{\beta}\cdot\vec{j}_{fi}{}^{(n)}$ to the total cross section are strongly destructively interfere, that results in negligible contribution to it of all terms besides of $\rho_{fi}^{(0)}$. With help of simple but enough cumbersome calculations one can derive the following sum rules:

$$\sigma^{tot} = 2\int U(q_\perp)^2\,[1 - S(q_\perp)]\,d^2q_\perp - \int U(q_\perp)^2 W(q_\perp) d^2q_\perp + O(\alpha^4)\,, \tag{21}$$

$$W(q_\perp) = \frac{1}{M^2}\int \left(\vec{\beta}\vec{r}\right)^2 [q^4\psi_i^2(\vec{r}) + \left(2\vec{q}_\perp\vec{\nabla}\psi_i(\vec{r})\right)^2] e^{i\vec{q}_\perp\vec{r}} d^3r\,.$$

Only the first term in the relation (21) is numerically significant. The second one, being of order α^2, don't contain any enhancement factor and may be neglected in the practical applications. This justifies the usage of the simple relation

$$\sigma^{tot} = 2\int U(q_\perp)^2\,[1 - S(q_\perp)]\,d^2q_\perp \tag{22}$$

in the paper of authors[3].

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References

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