Ratio of productions of $\pi^+\pi^-$-atoms to free $\pi^+\pi^-$ pairs with account of the strong interaction in final states

L.AFANASYEV and O.VOSKRESENSKAYA
Joint Institute for Nuclear Research,
141980 Dubna, Moscow Region,
Russia

Abstract

The ratio of productions of $\pi^+\pi^-$-atoms to free $\pi^+\pi^-$ pairs is calculated with account of the strong interaction in final states. It is shown that this ratio is expressed via a squared ratio of the well-known Coulomb wave functions and thus can be calculated with a high accuracy.

A method of measurement of the $\pi^+\pi^-$-atom lifetime proposed by L.L.Nemenov [1] and which is under realization in the experiment DIRAC [2] at CERN Proton Synchrotron is essentially based on the assumption that the ratio $R(n\mathrm{s}/\mathrm{ps})$ of the $\pi^+\pi^-$-atom production rate in ns states in hadron-nuclear interactions at high energy to the production rate of so called "Coulomb" $\pi^+\pi^-$ pairs (i.e. free $\pi^+\pi^-$ pairs with the relative momentum less or of order of the Bohr momentum $\alpha m_\pi/2$ of the $\pi^+\pi^-$-atom) can be calculated with accuracy better than 1%. Here we consider the accuracy of this ratio with account of results of last papers [3, 4].

According to [1, 3] the ration can be written as:

$$R(n\mathrm{s}/\mathrm{ps}) = \frac{\left|\int \mathrm{M}(\vec{\mathrm{r}})\psi_{\mathrm{ns}}(\vec{\mathrm{r}})\,\mathrm{d}^3\mathrm{r}\right|^2}{\left|\int \mathrm{M}(\vec{\mathrm{r}})\psi_{\mathrm{ps}}(\vec{\mathrm{r}})\,\mathrm{d}^3\mathrm{r}\right|^2}\,, \tag{1}$$

where $\psi_{ns}(\vec{r})$ is the $\pi^+\pi^-$-atom wave function with the principal quantum number n and zero orbital one; $\psi_{ps}(\vec{r})$ is the wave function of $\pi^+\pi^-$ pair with the relative momentum p and with zero orbital momentum; $M(\vec{r})$ is the amplitude of $\pi^+\pi^-$-system production at the relative distance $\vec{r}$.

Calculations of $M(\vec{r})$ in the independent source model [2] allow to obtain [3] the following estimation:

$$\langle r\rangle_M = \frac{\int rM(r)d^3r}{\int M(r)d^3r} \sim 5 \div 10\,\mathrm{fm} \ll \mathrm{r_B} = 1/\mu\alpha = 387\,\mathrm{fm}\,, \tag{2}$$

here r_B is the pionum Bohr radius, $\mu = m_\pi/2$ is its reduced mass and α is the fine-structure constant. Thus the most significant contribution to the integrals in (1) comes from the short distance where an influence of the strong interaction on the wave function can be overruling [4].

To estimate the influence of the strong interaction on the considering ratio (1) the results of paper [4] on the pionium wave function calculation in the perturbation theory can be applied:

$$\psi_{\mathrm{s}}(\vec{r}) = \psi_{\mathrm{s}}^{(C)}(\vec{r}) + \frac{\mu}{2\pi}\int \frac{U(\vec{r'})}{|\vec{r}-\vec{r'}|}\,\psi_{\mathrm{s}}^{(C)}(\vec{r'})\,d^3r'\,. \tag{3}$$

Here $\psi_{\rm s}^{(C)}$ is the pure Coulomb wave function, i.e. the wave function without account of the strong interaction, indices s stand for s-wave functions of the discrete and continues spectra, $U(\vec{r})$ is the strong interaction potential of pions.

Then the ratio (1) can be rewritten as:

$$R(n{\rm s}/{\rm ps}) = \frac{\left| \int \widetilde{\rm M}(\vec{\rm r}) \psi_{\rm ns}^{(\rm C)}(\vec{\rm r})\, {\rm d}^3{\rm r} \right|^2}{\left| \int \widetilde{\rm M}(\vec{\rm r}) \psi_{\rm ps}^{(\rm C)}(\vec{\rm r})\, {\rm d}^3{\rm r} \right|^2}\,, \tag{4}$$

here

$$\widetilde{M}(\vec{r}) = M(\vec{r}) + \frac{\mu U(\vec{r})}{2\pi} \int \frac{M(\vec{r'})}{|\vec{r} - \vec{r'}|}\, d^3r'\,.$$

Obvious that the "renormalized" amplitude $\widetilde{M}(\vec{r})$ is short-range as initial $M(\vec{r})$:

$$\langle r \rangle_{\widetilde{M}} \sim \langle r \rangle_M \ll r_B\,.$$

It allows to use the power expansion of the wave functions $\psi_{\rm s}^{(C)}$ at calculation of the integrals in (4).

$$\psi_{ns}^{(C)}(r) = \psi_{ns}^{(C)}(0) \left[1 - \mu\alpha r + \frac{1}{6}\left(2 + \frac{1}{n^2}\right)(\mu\alpha r)^2 + O\left((\mu\alpha r)^3\right)\right] \tag{5}$$

$$\psi_{ps}^{(C)}(r) = \psi_{ps}^{(C)}(0) \left[1 - \mu\alpha r + \frac{1}{6}\left(2 - \frac{p^2}{(\mu\alpha)^2}\right)(\mu\alpha r)^2 + O\left((\mu\alpha r)^3\right)\right] \tag{6}$$

Finally the ratio (1) is written as:

$$R(n{\rm s}/{\rm ps}) = \frac{\left|\psi_{\rm ns}^{(\rm C)}(0)\right|^2}{\left|\psi_{\rm ps}^{(\rm C)}(0)\right|^2} \left(1 + {\rm O}(\mu^2\alpha^2 \langle {\rm r}^2 \rangle_{\widetilde{\rm M}})\right)\,, \tag{7}$$

where the value of O is of order 10^{-3}. So it reproduces the formula from the paper [3] which was obtained, basing on an erroneous results of [5, 6], for the pure Coulomb wave function of the $\pi^+\pi^-$-atom. Thus in spite of the significant influence of the strong interaction on the value of the pionium wave function at origin [4] the ratio of production rates of the pionium to Coulomb $\pi^+\pi^-$ pairs is expressed via the well-known Coulomb wave functions and in this way can be calculated with required accuracy.

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