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# Coherent Beam-Beam Effects in $e^+ e^-$ Colliding Rings with Dispersion at the Interaction Point

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## abstract

With the dispersion at the interaction point the synchrotron and betatron motions influence each other. This coupling could affect the coherent beam-beam phenomena in  $e^+ e^-$  colliding rings. We use the semi-analytical Soft Gaussian Model, to study the strong-strong effects, showing a flip-flop behaviour not significantly different from the one already known in absence of dispersion. The beam sizes, the bunch lengthening and the luminosity reduction confirm the structure of the resonances already found in the incoherent linear analysis.

## Contents

1	Introduction	2
2	Linear Motion in the Arc: One Turn Matrix from IP to IP	2
3	Radiation Effects	3
4	Soft Gaussian Model	4
5	Coherent Effects	5
6	Conclusions	7

# 1 Introduction

In the conventional colliders, the dispersion at the interaction point (IP) is designed to be zero and might have a small value due to machine errors. The effects of such a small dispersion at the IP have been studied regarding the dispersion as a small perturbation [1]. Recently, however, the monochromatization [2] has been considered seriously for future tau-charm factories [3], where a rather large dispersion exists at the IP with opposite signs for both beams. In this case, the dispersion effects cannot be discussed perturbatively.

To see the non-perturbative effects with large dispersion, the weak-strong simulation for the monochromatization scheme has been done on the basis of the 3D-symplectic beam-beam mapping [4], which showed the satisfactory performance of this scheme for the design parameters of the Beijing Tau-Charm factory [5]. On the other hand, with the simulation only, it is difficult to understand the general properties of the monochromatization scheme. In a previous paper [6] we discussed the effects of the dispersion at the IP paying enough attention to the mutual interaction between the betatron and the synchrotron degrees of freedom.

The aim of this paper is to get a deeper understanding of the influence of the dispersion at the IP, therefore we move to a more realistic assumptions, nonlinear beam-beam force and strong-strong approach, using a semi analytical method, the Soft Gaussian Model (SGM) [7].

In Sect. 2 we introduce the one turn matrix in the presence of a dispersion at the IP, in Sect. 3 the radiation effects, in Sect. 4 strong-strong model and in Sect. 5 the numerical results. Conclusions follow under Section 6.

## 2 Linear Motion in the Arc: One Turn Matrix from IP to IP

We will use the approach introduced in [8]. We consider the vertical and longitudinal motions only. The detail is shown in [6].

Let us define the physical variables of a particle for the betatron and synchrotron motions:  $\mathbf{x} = (y, p_y, z, \delta)$ , where  $y$  is the vertical coordinate,  $p_y$  is the vertical momentum normalized by the momentum  $p_0$  of the reference particle (a constant),  $z$  is the time advance relative to the reference particle and multiplied by the light velocity  $c$  and  $\varepsilon$  is the energy deviation  $E - E_0$  from the energy of the reference particle  $E_0$  and normalized by  $E_0$ .

The one turn matrix from the IP ( $s = 0$ ) to IP (excluding the beam-beam kick) can be put in the following form.

$$M_{arc} = M(0_-, 0_+) = H_0 B_0 \hat{M}_{arc} B_0^{-1} H_0^{-1}, \quad (1)$$

where  $\hat{M}_{arc} = \text{diag}(r(\mu_y^0), r(\mu_z^0))$  and  $B_0 = \text{diag}(b_y^0, b_z^0)$  with

$$r(\mu_{y,z}^0) = \begin{pmatrix} \cos \mu_{y,z}^0 & \sin \mu_{y,z}^0 \\ -\sin \mu_{y,z}^0 & \cos \mu_{y,z}^0 \end{pmatrix}, \quad b_{y,z}^0 = \begin{pmatrix} \sqrt{\beta_{y,z}^0} & 0 \\ 0 & 1/\sqrt{\beta_{y,z}^0} \end{pmatrix}, \quad (2)$$

and

$$H_0 = \begin{pmatrix} I & h_0 \\ \tilde{h}_0 & I \end{pmatrix}, \quad h_0 = \begin{pmatrix} 0 & D_0 \\ 0 & 0 \end{pmatrix}, \quad \tilde{h}_0 = \begin{pmatrix} 0 & D_0 \\ 0 & 0 \end{pmatrix}. \quad (3)$$

The nominal synchrotron tune  $\nu_z^0$  is negative for conventional electron machines with positive momentum compaction factor. We will however consider the both signs for  $\nu_z^0$  because the option with the negative momentum compaction factor[9] is being considered, which makes the  $\nu_z^0$  positive. We have assumed that there is only one IP and is a symmetric point with respect to betatron and synchrotron motions. We have also implicitly assumed that the dispersion does not exist in cavities.

The matrix  $H_0$  decouples the betatron and the synchrotron motions in a symplectic way. One can regard Eq.(1) as the definition of the dispersion,  $D_0$ .

### 3 Radiation Effects

The radiation effect was described in [6] in detail.

The one turn map for the second order moments  $\sigma_{\pm}$  of the  $e^{\pm}$  beam is as follows:

$$\sigma'_{\pm} = \bar{\Lambda} M_{arc}^{\pm} \sigma_{\pm} (\bar{\Lambda} M_{arc}^{\pm})^t + (I - \bar{\Lambda}^2) \bar{E}, \quad (4)$$

where

$$M_{arc} = H_{\pm} B_0 \hat{M}_{arc} B_0^{-1} H_{\pm}^{-1}, \quad \bar{\Lambda} = H_{\pm} B_0 \Lambda B_0^{-1} H_{\pm}^{-1}, \quad \bar{E} = H_{\pm} B_0 E B_0^t H_{\pm}^t, \quad (5)$$

$$\Lambda = \text{diag}(\lambda_y, \lambda_y, 1, \lambda_z^2), \quad E = \text{diag}(\epsilon_y^0, \epsilon_y^0, \epsilon_z^0, \epsilon_z^0), \quad (6)$$

$$H_{\pm} = \begin{pmatrix} I & \pm h_0 \\ \pm \tilde{h}_0 & I \end{pmatrix}. \quad (7)$$

Here

$$\sigma_{ij}^{\pm} = \langle (x_i - \bar{x}_i^{\pm})(x_j - \bar{x}_j^{\pm}) \rangle_{\pm}, \quad \bar{x}_i^{\pm} = \langle x_i \rangle_{\pm} \quad (8)$$

with

$$\langle \cdot \rangle_{\pm} = \int (\cdot) \psi_{\pm}(\vec{x}_{\pm}) d\vec{x}_{\pm}. \quad (9)$$

## 4 Soft Gaussian Model

We describe the strong-strong beam-beam interaction using a semi-analytical model, described in [7], called Soft Gaussian Model, based on the Gaussian approximation. We assume that the beam is Gaussian, even when the beam-beam force is present.

As further assumptions we have:

- the dipole moments  $\bar{x}_{\pm i}$  are stable, we let them vanish;
- the bunch is very small:  $\sigma_z^0 \ll \beta_{x,y}^0$  at the IP;
- Following [7] and [10] we use the flat beam limit  $\sigma_{\pm x}^0 \gg \sigma_{\pm y}^0$ , where therefore reasonably the horizontal particle motion is unaffected, then getting the nominal (incoherent) beam-beam parameter:

$$\xi_{\pm y}^0 = \frac{r_e}{2\pi\gamma_{\pm}} \frac{N_{\pm}\beta_{\pm y}^0}{\sigma_{\pm x}^0\sigma_{\pm y}^0}, \quad \sigma_{\pm y}^0 = \left[ \beta_{\pm y}^0\epsilon_{\pm y}^0 + D_0^2\epsilon_{\pm z}^0/\beta_{\pm z}^0 \right]^{1/2}, \quad (10)$$

where  $r_e$  is the classical electron radius,  $\gamma_{\pm}$  the relativistic factor,  $\sigma_{\pm x,y}^0$  the nominal horizontal (vertical) beam size,  $\epsilon_{\pm y}^0$  and  $\epsilon_{\pm z}^0 \equiv (\sigma_{\pm z}^0\sigma_{\pm \delta}^0)^{1/2}$  being the nominal vertical and longitudinal emittances. All quantities are evaluated at the IP.

- At IP, each particle is kicked as

$$p_{\pm y} \rightarrow p_{\pm y} - \kappa_{\pm} \operatorname{erf} \left( y_{\pm} / \sqrt{2\sigma_{\mp y}} \right), \quad (11)$$

$$\kappa_{\pm} = \frac{2\pi^{3/2}\xi_{\pm y}^0}{\beta_{\pm y}^0} \sqrt{\beta_{\pm y}^0\epsilon_{\pm y}^0 + D_0^2\epsilon_{\pm z}^0/\beta_{\pm z}^0}. \quad (12)$$

The mapping for the second order moments due to nonlinear beam-beam kick is:

$$\begin{aligned} \sigma'_{\pm 22} &= \sigma_{\pm 22} + 4\kappa A(\rho)\sigma_{\pm 12}/\sqrt{\sigma_{\pm 11}} + 4\kappa^2 C(\rho) \\ \sigma'_{\pm 12} &= \sigma_{\pm 12} + 2\kappa A(\rho)\sqrt{\sigma_{\pm 11}} \\ \sigma'_{\pm 23} &= \sigma_{\pm 23} + 2\kappa A(\rho)\sigma_{\pm 13}/\sqrt{\sigma_{\pm 11}} \\ \sigma'_{\pm 24} &= \sigma_{\pm 24} + 2\kappa A(\rho)\sigma_{\pm 14}/\sqrt{\sigma_{\pm 11}} \end{aligned} \quad (13)$$

$$\rho = \frac{\sigma_{+y}}{\sigma_{-y}}, \quad C = \frac{1}{4} - \frac{1}{\pi} \arcsin \left( \sqrt{\frac{\rho}{2(1+\rho)}} \right), \quad A(\rho) = -\frac{1}{\sqrt{2\pi(1+\rho)}}. \quad (14)$$

Other envelope elements do not change. Therefore the complete one-turn map for  $\sigma_{\pm ij}$  is obtained by successive applications of Eq.(13) and Eq.(4).

## 5 Coherent Effects

In [11] we emphasized that, due to the beam-beam interaction and the presence of dispersion at IP, all incoherent parameters (twiss parameters, tunes, etc.) change after the collision. Here we want briefly discuss the evolution of few coherent parameters.

One of the most important observable quantities is the luminosity, expressed as:

$$L = \frac{f_r N_+ N_-}{2\pi \Sigma_x^0 \Sigma_y^0} \exp \left\{ -\frac{(\bar{x}_+ - \bar{x}_-)^2}{2(\Sigma_x^0)^2} - \frac{(\bar{y}_+ - \bar{y}_-)^2}{2(\Sigma_y^0)^2} \right\}, \quad (15)$$

where

$$\Sigma_x^0 = \sqrt{(\sigma_{+x}^0)^2 + (\sigma_{-x}^0)^2}, \quad \Sigma_y^0 = \sqrt{(\sigma_{+y}^0)^2 + (\sigma_{-y}^0)^2}. \quad (16)$$

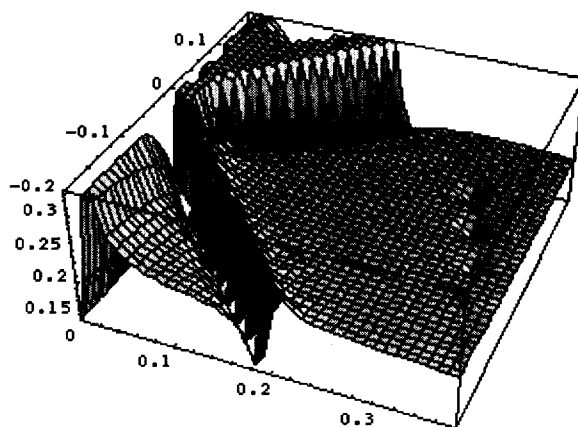


Figure 1: The luminosity reduction  $R_L$  as function of  $\nu_y^0$  and  $\nu_z^0$  with  $\xi_y^0 = .05$  and  $D_0 = .4$ .

Let us define the luminosity reduction factor  $R_L$  and the bunch size ratios:

$$R_L = \frac{L}{L_0} = \frac{\Sigma_y}{\Sigma_y^0}, \quad R_z = \frac{\sigma_{+z}}{\sigma_{-z}}, \quad R_y = \frac{\sigma_{+y}}{\sigma_{-y}}. \quad (17)$$

where  $L_0$  is the nominal value without collision effect. Figure 1 shows  $R_L(\nu_y^0, \nu_z^0)$ . There are some regions in the  $(\nu_y^0, \nu_z^0)$  plane corresponding to instabilities, where the effective luminosity  $L$  changes abruptly.

In Figure 2a we show the contour plot of  $R_z$  in the  $(\nu_y^0, \nu_z^0)$  plane, showing the occurrence of some instabilities:  $\nu_y^0 \sim$  half integers (betatron instability),  $\nu_z^0 \sim$  half integers (synchrotron instability),  $\nu_z^0 + \nu_y^0 \sim$  integers (synchro-betatron instability), the same found in the linear incoherent analysis [6].

In Figure 2b we show the contour plots of  $R_y$  in the  $(\xi_y^0, D_0)$  plane with  $\nu_z^0 = .008$  and  $\nu_y^0 = .1$ . A flip-flop behaviour occurs: one beam blows up above certain current. We can clearly see the threshold line for the flip-flop occurrence: below this line the symmetry

is preserved,  $R_y = 1$ , and above it is not,  $R_y > 1$ . This phenomenon has been already described in [7] in absence of dispersion, for  $D_0 = 0$ ,  $\xi_y^0 = .023$  as shown in Fig. 2b.

Another interesting observable quantity is the effective (coherent) beam-beam parameter:

$$\Xi_{\pm y}^0 = \frac{r_e}{2\pi\gamma_{\pm}} \frac{N_{\pm}\beta_{\pm y}^0}{\Sigma_y^0(\Sigma_x^0 + \Sigma_y^0)} \quad \Sigma_x^0 \gg \Sigma_y^0 \quad \frac{r_e}{2\pi\gamma_{\pm}} \frac{N_{\pm}\beta_{\pm y}^0}{\Sigma_x^0\Sigma_y^0}. \quad (18)$$

Its evolution versus  $\xi_{\pm y}^{00} = N_{\pm}r_e\beta_{\pm y}^0/2\pi\gamma_{\pm}\sqrt{\beta_{\pm y}^0\epsilon_y^0\beta_{\pm x}^0\epsilon_x^0}$ , the nominal beam-beam parameter in absence of dispersion, shows saturation as found in [7].

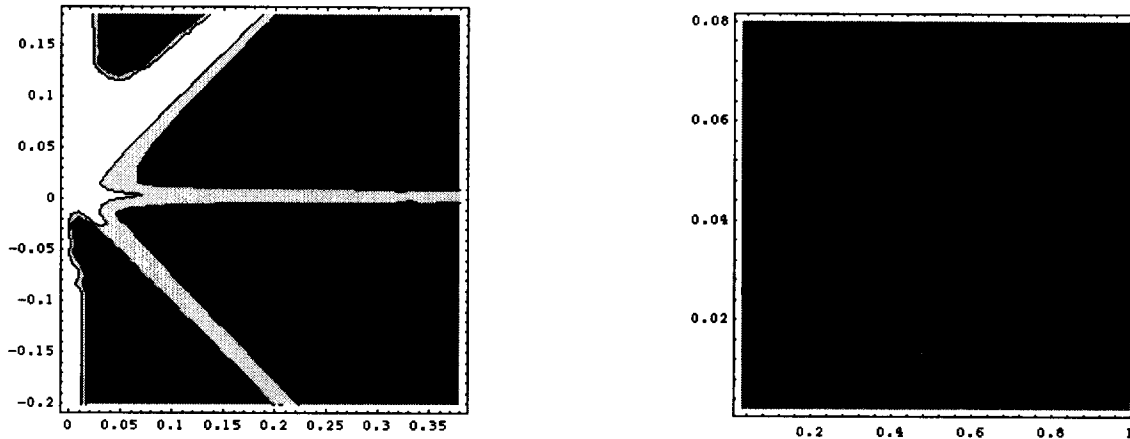


Figure 2: (Left) The contour plot of  $R_z$  (levels: 1, 1.2, 1.5) in the  $(\nu_y^0, \nu_z^0)$  plane with  $\xi_y^0 = .05$  and  $D_0 = .4$ . The darker contours correspond to bigger values. (Right) The contour plots of  $R_y$  in the  $(D_0, \xi_y^0)$  plane with  $\nu_z^0 = .008$  and  $\nu_y^0 = .1$ . The dark line corresponds to flip-flop thresholds.

## 6 Conclusions

We studied some coherent effects due the presence of dispersion at IP. We considered a strong-strong interaction and a nonlinear beam-beam force for short bunches and flat beams. We used the Soft Gaussian Model and tracked the evolution of the envelope and found effects in the luminosity rate, bunch length and beam size in parameters space. We found a flip-flop behaviour, not significantly different from the one already known in absence of dispersion. A study of the tune plane reveals an instability structure similar to the one found in the incoherent linear analysis.



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