

# MASSIVE HIGHER SPIN STATES IN STRING THEORY AND THE PRINCIPLE OF EQUIVALENCE

Ioannis Giannakis<sup>a,b</sup>, James T. Liu<sup>a</sup> and Massimo Porrati<sup>a,b,c</sup> †

<sup>(a)</sup>*Physics Department, The Rockefeller University  
1230 York Avenue, New York, NY 10021-6399*

<sup>(b)</sup>*Department of Physics, New York University \*  
4 Washington Pl., New York, NY 10003*

<sup>(c)</sup>*Theory Division, CERN  
CH 1211 Geneva 23 (Switzerland)*

## Abstract

In this paper we study three point functions of the Type II superstring involving one graviton and two massive states, focusing in particular on the spin-7/2 fermions at the first mass level. Defining a gravitational quadrupole “ $h$ -factor”, we find that the non-minimal interactions of string states in general are parametrized by  $h \neq 1$ , in contrast to the preferred field theory value of  $h = 1$  (for tree-level unitarity). This difference arises from the fact that consistent gravitational interactions of strings are related to the presence of a complete tower of massive states, not present in the ordinary field theory case.

CERN-TH/98-304  
September 1998

---

† e-mail: giannak@theory.rockefeller.edu, jtliu@theory.rockefeller.edu,  
massimo.porrati@nyu.edu

\* Permanent address.

## 1. Introduction.

While the matter of consistent interactions of massive higher spin fields with gravity has been fairly well studied in the context of field theory [1], less is known in the case of string theory. Although for the latter, one may argue that strings must *a priori* present a consistent theory of gravity, it is nevertheless instructive to examine the nature of such interactions and to determine in particular how strings achieve such a consistency.

The simplest recipe for coupling massive fields to gravity (the “minimal coupling”) is inherently ambiguous. Indeed, by replacing ordinary derivatives with covariant derivatives one is still free to add to the action terms that vanish on flat space; the commutator of two covariant derivatives, for instance. In Ref. [2] a prescription was given for fixing some of the ambiguities. There, by imposing tree-level unitarity up to the Planck scale on forward “Compton” scattering amplitudes of a single massive high-spin field, many coefficients in its action were fixed unambiguously. In particular, it was shown that starting from spin-3/2, tree-level unitarity requires the presence of terms proportional to the Riemann tensor in the three-point vertex describing single-graviton emission and absorption by the fermions.

Tree level unitarity is really the statement that an interacting theory is weakly coupled up to a certain energy scale,  $M_{Planck}$  or  $(\alpha')^{-1/2}$ , for instance. The recipe of Ref. [2] is not immediately applicable to string theory. The spectrum of string theory, indeed, contains many particles of ever-increasing spin, some of which degenerate in mass. Thus, for instance, tree-level unitarity of a massive high-spin particle is achieved in string theory in part because of the three-point vertex, but also because an infinite number of particles of ever-increasing mass and spin propagate as intermediate states. To find which one-graviton vertex is selected by string theory, one must therefore resort to direct calculation.

The computation of the three-point vertex involving one graviton and two spin-7/2 fermions in superstring theory is described in Section 2, which also exhibits the effective action reproducing the string amplitudes. Section 3 focuses on the “gravitational quadrupole” term, that is the three-point vertex proportional to the Riemann tensor. In Section 3 we compare the results obtained from superstring theory with the predictions of tree-level unitarity. Section 4 extends the computation of the three-point vertex and effective action to states of arbitrary spin, and contains some comments on the implications of our results for tree-level unitarity in strings.

## 2. Massive three-point functions.

We begin by examining the simplest massive string interactions of the Type II string, namely three point functions at the first mass level. In this case, the (ten-dimensional) closed superstring states fall into the  $SO(9)$  spin representations<sup>1</sup>  $[44+84+128]_L \times [44+84+128]_R$ . Correspondingly, in four dimensions, such states carry spins up to four. Focusing on spacetime fermions (*i.e.* the R-NS sector), we now describe the vertex operator for emission or absorption of states in the  $128 \times 44$  of  $SO(9)$  and in the  $(q, \bar{q}) = (-\frac{1}{2}, -1)$  ghost picture.

Massive vertex operators have been discussed previously in [3]. Working in the R-NS sector, the vertex operator may be written as

$$V_F^{(-\frac{1}{2}, -1)}(z, \bar{z}) = u_{\mu\nu\rho}^\alpha(X) S_\alpha e^{-\frac{\phi}{2}} \partial X^\mu e^{-\bar{\phi}} \bar{\psi}^\nu \bar{\partial} X^\rho + v_{\mu\nu\rho}^{\dot{\alpha}}(X) (\gamma_\lambda)_{\dot{\alpha}}^\beta \psi^\mu \psi^\lambda S_\beta e^{-\frac{\phi}{2}} e^{-\bar{\phi}} \bar{\psi}^\nu \bar{\partial} X^\rho. \quad (1)$$

In this expression  $S_\alpha$  and  $e^{-\frac{\phi}{2}}$  represent the spin fields for the two-dimensional fermions  $\psi^\mu$  and the superconformal ghosts  $\beta, \gamma$  respectively. The wavefunctions  $u$  and  $v$  are now constrained by demanding that  $V_F^{(-\frac{1}{2}, -1)}(z, \bar{z})$  is BRST invariant, *i.e.* that  $[Q, V_F^{(-\frac{1}{2}, -1)}(z, \bar{z})]$  and similarly  $[\bar{Q}, V_F^{(-\frac{1}{2}, -1)}(z, \bar{z})]$  vanish up to a total derivative. The BRST charge for the Type II string theory is given by

$$Q = \int dz c(z) \left( -\frac{1}{2} \eta^{\mu\nu} \partial X_\mu \partial X_\nu - \frac{1}{2} \eta^{\mu\nu} \psi_\mu \partial \psi_\nu - \frac{3}{2} \beta \partial \gamma - \frac{1}{2} \partial \beta \gamma \right) (z) \\ + \int dz (bc \partial c + \frac{1}{2} \gamma \eta^{\mu\nu} \psi_\mu \partial X_\nu - \frac{1}{4} b \gamma^2) (z), \quad (2)$$

and a similar expression for  $\bar{Q}$ . We find that  $[Q, V_F^{(-\frac{1}{2}, -1)}(z, \bar{z})] = \partial(c(z) V_F^{(-\frac{1}{2}, -1)}(z, \bar{z}))$  and  $[\bar{Q}, V_F^{(-\frac{1}{2}, -1)}(z, \bar{z})] = \bar{\partial}(\bar{c}(\bar{z}) V_F^{(-\frac{1}{2}, -1)}(z, \bar{z}))$  when the wavefunctions  $u$  and  $v$  satisfy:

$$(\gamma^\lambda)_{\dot{\beta}}^{\dot{\alpha}} \partial_\lambda v_{\mu\nu\rho}^{\dot{\alpha}}(X) \delta_{\dot{\alpha}\dot{\beta}} + \frac{\sqrt{2}}{(D-2)} u_{\mu\nu\rho}^\alpha(X) \delta_{\alpha\beta} = 0, \quad \partial^\mu u_{\mu\nu\rho}^\alpha(X) = 0, \\ (\gamma^\lambda)_{\alpha\dot{\beta}} \partial_\lambda u_{\mu\nu\rho}^\alpha(X) + (D-2) v_{\mu\nu\rho}^{\dot{\alpha}}(X) \delta_{\dot{\alpha}\beta} = 0, \quad (\gamma^\mu)_{\dot{\alpha}}^\beta v_{\mu\nu\rho}^{\dot{\alpha}}(X) = 0, \\ (\gamma^\mu)_{\alpha\dot{\beta}} u_{\mu\nu\rho}^\alpha(X) - (D-2) \partial^\mu v_{\mu\nu\rho}^{\dot{\alpha}}(X) \delta_{\dot{\alpha}\beta} = 0, \quad \partial^\nu u_{\mu\nu\rho}^\alpha(X) = \partial^\nu v_{\mu\nu\rho}^{\dot{\alpha}}(X) = 0, \quad (3)$$

where  $D = 10$  is the space-time dimension. Conformal invariance thus provides equations of motion and gauge conditions for the wavefunctions  $u_{\mu\nu\rho}^\alpha(X)$  and  $v_{\mu\nu\rho}^{\dot{\alpha}}(X)$ . These

---

<sup>1</sup> Both IIA and IIB theories have identical massive spectra; their interactions, however, are in general distinct.

constraints can be written in a compact form if we combine  $u$  and  $v$  into a Dirac spinor  $\psi = \begin{pmatrix} u \\ (D-2)v \end{pmatrix}$ :

$$\begin{aligned} \gamma^\lambda \partial_\lambda \psi_{\mu\nu\rho}(X) + \sqrt{2} \psi_{\mu\nu\rho}(X) &= 0, \\ \gamma^\mu \psi_{\mu\nu\rho}(X) = \partial^\mu \psi_{\mu\nu\rho}(X) = \partial^\nu \psi_{\mu\nu\rho}(X) = \partial^\rho \psi_{\mu\nu\rho}(X) &= 0. \end{aligned} \quad (4)$$

We observe that  $\psi_{\mu\nu\rho}(X)$  obeys a massive Dirac equation and a set of gauge conditions. In order to extract the couplings of massive spin-7/2 particles to gravity we need to calculate the three point scattering amplitude of two spin-7/2 fermions and a graviton. To satisfy the superconformal ghost charge condition we consider the graviton vertex operator in the  $(q, \bar{q}) = (-1, 0)$  picture

$$V_G^{(-1,0)}(z, \bar{z}) = h_{\mu\nu}(X) \psi^\mu e^{-\phi} \bar{\partial} X^\nu + \partial_\lambda h_{\mu\nu}(X) \psi^\mu e^{-\phi} \bar{\psi}^\lambda \bar{\psi}^\nu, \quad (5)$$

where the graviton wavefunction obeys  $\square h^{\mu\nu}(X) = \partial_\mu h^{\mu\nu}(X) = 0$ . We proceed now to calculate  $\langle V_F^{(-\frac{1}{2}, -1)}(z_1, \bar{z}_1) V_G^{(-1,0)}(z_2, \bar{z}_2) V_F^{(-\frac{1}{2}, -1)}(z_3, \bar{z}_3) \rangle$ .

To calculate superstring three point functions involving spin fields we use the techniques developed in [4]. In particular, we note that three point functions always factorize into a product of a holomorphic and an antiholomorphic piece. We recall that the closed string graviton vertex operator, (5), is simply a product of separate left- and right-moving gauge boson vertices. Starting with the holomorphic Ramond sector, we now calculate the three point function of two massive fermions with a gauge boson. Dynamical issues of massive string states in open string theory were discussed in [5].

The vertex operators which describe massive spin-3/2 fermions and massless gauge bosons in the  $q = -\frac{1}{2}$  and  $q = -1$  ghost pictures respectively are given by

$$V_F^{(-\frac{1}{2})}(z, \bar{z}) = u_\mu^\alpha(X) S_\alpha e^{-\frac{\phi}{2}} \partial X^\mu + v_\mu^\dot{\alpha}(X) (\gamma_\lambda)^\beta_\alpha \psi^\mu \psi^\lambda S_\beta e^{-\frac{\phi}{2}}, \quad V_B^{(-1)}(z, \bar{z}) = A_\mu(X) \psi^\mu e^{-\phi}. \quad (6)$$

Conformal invariance implies that  $A_\mu$  and  $\psi = \begin{pmatrix} u \\ (D-2)v \end{pmatrix}$  satisfy

$$\gamma^\lambda \partial_\lambda \psi_\mu(X) + \sqrt{2} \psi_\mu(X) = 0, \quad \gamma^\mu \psi_\mu(X) = \partial^\mu \psi_\mu(X) = 0, \quad \square A^\mu(X) = \partial_\mu A^\mu(X) = 0. \quad (7)$$

The gauge conditions  $\gamma \cdot \psi = \partial \cdot \psi = 0$  eliminate the spin-1/2 components and thus the vector-spinor wavefunction  $\psi_\mu^\alpha$  describes a pure spin-3/2 massive open string state.

Taking into account momentum conservation and the conditions that different polarizations obey, we find

$$\begin{aligned} \mathcal{A}_{FF}^\gamma(\psi_{1\mu}, k_1; A_{2\sigma}, k_2; \psi_{3\mu}, k_3) &= \frac{1}{\sqrt{2}} \left[ \bar{\psi}_{1\mu} \gamma^\sigma A_{2\sigma} \psi_3^\mu - \bar{\psi}_{1\mu} \gamma^\sigma A_{2\sigma} \gamma_+ \psi_{3\nu} k_2^\mu k_2^\nu \right] \\ &+ i \left[ \bar{\psi}_{1\mu} \gamma_+ \psi_3^\nu - \bar{\psi}_{1\mu} \gamma_- \psi_3^\nu \right] k_2^\mu A_{2\nu}, \end{aligned} \quad (8)$$

where  $\gamma_+(\gamma_-)$  projects the Dirac spinor  $\psi$  onto its positive (negative) chirality components (in the ten-dimensional sense).

While this is meant to be viewed as the holomorphic component of the closed string amplitude, it also has the interpretation as an open string calculation of a single photon emission from a spin-3/2 particle. To this linear order in the gauge field, the corresponding effective field theoretic Lagrangian reproducing the three point function (8) is given by

$$\mathcal{L}_{(3/2)} = -\frac{i}{2}\bar{\psi}_\mu(\gamma^\lambda D_\lambda + m)\psi^\mu - \frac{1}{2m}\bar{\psi}_\mu F^{\mu\nu}\psi_\nu + \frac{1}{m^2}\bar{\psi}_\mu\partial^\mu F_{\nu\lambda}\gamma^\lambda\gamma_+\psi^\nu + \dots, \quad (9)$$

where  $m = \sqrt{2}$  and  $D_\mu = \partial_\mu + iA_\mu$ . The  $\dots$  represent both terms that possibly vanish on-shell and terms which are necessary to implement the physical state constraints (including the introduction of non-physical degrees of freedom). In the present letter we are mainly interested only in the form of the interaction between massive higher-spin string states and external gravitational or electromagnetic fields and not in the details of the additional terms necessary for the consistency of the effective Lagrangian<sup>2</sup>.

Completing the spin-7/2 calculation, we now examine the antiholomorphic sector, corresponding to gauge boson emission from a massive spin-2 boson. The calculation is straightforward, especially for the NS sector in the covariant formalism. The corresponding vertex operators in the ghost picture  $q = 0$  and  $q = -1$  read

$$V_B^{(0)}(z, \bar{z}) = A_\mu(X)\bar{\partial}X^\mu, \quad V_B^{(-1)}(z, \bar{z}) = \phi_{\mu\nu}(X)e^{-\bar{\phi}}\bar{\psi}^\mu\bar{\partial}X^\nu, \quad (10)$$

while the resulting amplitude is given by

$$\begin{aligned} \mathcal{A}_{BB}^\gamma(\phi_{1\mu\nu}, k_1; A_{2\sigma}, k_2; \phi_{3\mu\nu}, k_3) = & \phi_{1\mu\nu}\phi_{3\lambda\sigma}[\eta^{\mu\lambda}\eta^{\nu\sigma}A_2 \cdot k_3 + 4\eta^{\mu\lambda}k_2^{[\nu}A_2^{\sigma]} \\ & - \eta^{\mu\lambda}k_2^\nu k_2^\sigma A_2 \cdot k_3 - 2k_2^\mu k_2^\lambda k_2^{[\nu}A_2^{\sigma]}], \end{aligned} \quad (11)$$

with corresponding (open string) effective Lagrangian

$$\begin{aligned} \mathcal{L}_{(2)} = & -\frac{1}{2}D_\rho\phi_{\mu\nu}D^\rho\phi^{\mu\nu} - \frac{1}{2}m^2\phi_{\mu\nu}\phi^{\mu\nu} + 2i\phi_{\mu\nu}F^{\nu\sigma}\phi^\mu{}_\sigma \\ & - \frac{2i}{m^2}(\phi_{\mu\nu}\partial^\nu F_{\rho\sigma}\partial^\rho\phi^\mu{}_\sigma - \phi_{\mu\nu}\partial^\mu\partial^\lambda F_{\nu\sigma}\phi^\sigma{}_\lambda). \end{aligned} \quad (12)$$

---

<sup>2</sup> See *e.g.* [6] for a discussion about the need to introduce non-physical degrees of freedom for higher spin string states.

The three point function for the closed string states,  $\mathcal{A}_{FF}^h$ , follows by combining the holomorphic and antiholomorphic three point functions for the open string states, Eqns. (8), and (11). We find

$$\begin{aligned}
 \mathcal{A}_{FF}^h(\psi_{1\mu\nu\lambda}, k_1; h_{\mu\nu}, k_2; \psi_{3\mu\nu\lambda}, k_3) &= \mathcal{A}_{FF}^{\gamma(\text{left})} \times \mathcal{A}_{BB}^{\gamma(\text{right})} \\
 &= \bar{\psi}_{1\mu\nu\lambda} \gamma^\sigma \psi_3^{\mu\nu\lambda} h_{\sigma\rho} k_3^\rho + 2\bar{\psi}_{1\mu\nu\lambda} \gamma^\sigma \psi_3^{\mu\nu\rho} (h_{\rho\sigma} k_2^\lambda - h^\lambda{}_\sigma k_{2\rho}) + i\sqrt{2}(\bar{\psi}_{1\mu\nu\lambda} \gamma_+ \psi_{3\sigma}{}^{\nu\lambda} \\
 &\quad - \bar{\psi}_{1\sigma\nu\lambda} \gamma_- \psi_{3\mu}{}^{\nu\lambda}) h^{\mu\rho} k_2^\sigma k_{3\rho} + 2i\sqrt{2}(\bar{\psi}_{1\mu\nu\lambda} \gamma_+ \psi_{3\sigma}{}^{\nu\rho} - \bar{\psi}_{1\sigma\nu\lambda} \gamma_- \psi_{3\mu}{}^{\nu\rho}) (h^\mu{}_\rho k_2^\sigma k_2^\lambda - h^{\mu\lambda} k_2^\sigma k_{2\rho}) \\
 &\quad - \bar{\psi}_{1\mu\nu\lambda} \gamma^\sigma (1 + \gamma_+) \psi_{3\rho}{}^{\nu\lambda} h_{\sigma\delta} k_3^\delta k_2^\mu k_2^\rho - \bar{\psi}_{1\mu\nu\lambda} \gamma^\sigma (1 + 2\gamma_+) \psi_3^{\mu\rho\delta} (h_{\sigma\delta} k_2^\nu k_{2\rho} k_2^\lambda - h_\sigma{}^\lambda k_2^\nu k_{2\rho} k_{2\delta}) \\
 &\quad - i\sqrt{2}(\bar{\psi}_{1\mu\nu\lambda} \gamma_+ \psi_{3\sigma}{}^{\nu\rho} - \bar{\psi}_{1\sigma\nu\lambda} \gamma_- \psi_{3\mu}{}^{\nu\rho}) h^{\mu\delta} k_2^\sigma k_{3\delta} k_2^\lambda k_{2\rho} \\
 &\quad - i\sqrt{2}(\bar{\psi}_{1\mu\nu\lambda} \gamma_+ \psi_{3\sigma\rho\delta} - \bar{\psi}_{1\sigma\nu\lambda} \gamma_- \psi_{3\mu\rho\delta}) (h^{\mu\delta} k_2^\sigma k_2^\nu k_2^\lambda k_{2\rho} - h^{\mu\lambda} k_2^\sigma k_2^\nu k_2^\delta k_{2\rho}) \\
 &\quad + \bar{\psi}_{1\mu\nu\lambda} \gamma^\sigma \gamma_+ \psi_{3\rho}{}^{\nu\delta} h_{\sigma\epsilon} k_3^\epsilon k_2^\mu k_2^\lambda k_{2\delta} k_2^\rho + \bar{\psi}_{1\mu\nu\lambda} \gamma^\sigma \gamma_+ \psi_{3\rho\delta\epsilon} (h_{\sigma\epsilon} k_2^\nu k_2^\mu k_2^\lambda k_{2\delta} k_2^\rho - h_{\sigma\lambda} k_2^\nu k_2^\mu k_2^\epsilon k_{2\delta} k_2^\rho).
 \end{aligned} \tag{13}$$

As a result the effective Lagrangian that reproduces this particular three point function contains terms up to five derivatives. Up to terms that vanish on-shell (but that are nevertheless crucial for the consistency of such a massive higher-spin Lagrangian), we find

$$\begin{aligned}
 \mathcal{L}_{\text{eff}} &= \sqrt{-g} \left[ \frac{1}{2} \bar{\psi}_{\mu\nu\lambda} (\gamma^\rho \nabla_\rho + m) \psi^{\mu\nu\lambda} - \frac{1}{m} \bar{\psi}_{\mu\nu\lambda} (R^{\mu\alpha\nu\beta} - \frac{1}{8} R^{\mu\alpha\rho\sigma} \gamma_{\rho\sigma} \eta^{\nu\beta}) \psi_{\alpha\beta}{}^\lambda \right. \\
 &\quad - \frac{1}{m^2} (\bar{\psi}_{\mu\nu\lambda} R^{\sigma(\mu\alpha)\rho} \gamma_\sigma (1 + \gamma_+) \partial_\rho \psi_{\alpha}{}^{\nu\lambda} - \bar{\psi}_{\mu\nu\lambda} \partial^{(\mu} R^{\alpha)\sigma\nu\beta} \gamma_\sigma (1 + 2\gamma_+) \psi_{\alpha\beta}{}^\lambda) \\
 &\quad - \frac{1}{m^3} (\bar{\psi}_{\mu\nu\lambda} \partial^{(\nu} R^{\beta)\sigma\mu\alpha} \partial_\sigma \psi_{\alpha\beta}{}^\lambda + \bar{\psi}_{\mu\nu\lambda} \partial^\mu \partial^\alpha R^{\nu\beta\lambda\gamma} \psi_{\alpha\beta\gamma}) \\
 &\quad \left. - \frac{2}{m^4} (\bar{\psi}_{\mu\nu\lambda} \partial^\mu \partial^\alpha R^{\sigma(\beta\nu)\delta} \gamma_\sigma \gamma_+ \partial_\delta \psi_{\alpha\beta}{}^\lambda - \bar{\psi}_{\mu\nu\lambda} \partial^\nu \partial^\delta \partial^{(\lambda} R^{\gamma)\sigma\mu\rho} \gamma_\sigma \gamma_+ \psi_{\rho\delta\gamma}) \right].
 \end{aligned} \tag{14}$$

Several points are in order here. First of all, working with the three point function, we only obtain information up to linearized order in the graviton,  $h_{\mu\nu}$ . For this reason, at this order there is no distinction between bare and covariant derivatives of the Riemann tensor that appear in (14). Secondly, this effective Lagrangian is by no means unique, as we are always allowed to shift it by terms vanishing on-shell. In particular, we note that the  $\gamma$ -transverse condition  $\gamma \cdot \psi = 0$  allows use of the on-shell identity

$$\bar{\psi}_{\mu\nu\lambda} R^{\mu\alpha\rho\sigma} \gamma_{\rho\sigma} \eta^{\nu\beta} \psi_{\alpha\beta}{}^\lambda = 2\bar{\psi}_{\mu\nu\lambda} R^{\mu\alpha\nu\beta} \psi_{\alpha\beta}{}^\lambda - \bar{\psi}_{\mu\nu\lambda} R^{\mu\alpha\rho\sigma} \gamma_{\rho\sigma}{}^{\nu\beta} \psi_{\alpha\beta}{}^\lambda, \tag{15}$$

indicating that even the leading two-derivative term is by no means unique. Furthermore, note that the presence of the ten-dimensional chirality projection  $\gamma_+$  for spacetime fermions indicates that, even while massive, such states maintain chiral interactions with gravity. It is at this point where the difference between massive IIA and IIB string states shows up. The complete Type II spectrum includes in fact a pair of spin-7/2 states at the first mass level, one each from the R-NS and the NS-R sectors. In our conventions, Eqn. (14) corresponds to the state from the R-NS sector, while a similar equation with either  $\gamma_-$

or  $\gamma_+$  would describe the state from the NS-R sector for the Type IIA or IIB theory respectively.

Finally, the effective gravitational interactions of string states at higher mass levels have correspondingly higher derivative couplings. Physically, this corresponds to the intuitive notion that highly excited strings are spread out, and hence feel tidal effects arising from the curvature of spacetime. This departure from the minimal coupling prescription leads to violation of the strong equivalence principle [7]. This is just a fact of life; in relativistic quantum field theory particles have an intrinsic size: their Compton wavelength. This makes them behave in some respect as extended objects, sensitive to tidal forces.

### 3. Non-minimal coupling and the gravitational quadrupole.

Non-minimal couplings to the Riemann tensor were discussed in Ref. [2] in the context of point particle field theory, where it was shown that tree-level unitarity for particles of spin  $> 2$  demands the presence of just such a non-minimal term. The authors of [2] give a general expression for the required non-minimal addition to the action for both integer and half-integer spins. For the former, the on-shell Lagrangian (also ignoring auxiliary fields)<sup>3</sup> for a boson of spin  $s$  takes the form

$$\mathcal{L} = \frac{1}{2}\phi^{(s)}(\nabla_\mu\nabla^\mu - m^2)\phi^{(s)} + \frac{s(s-1)}{2}\phi_{\mu\nu\dots}^{(s)}R^{\mu\alpha\nu\beta}\phi_{\alpha\beta}^{(s)}\dots + \dots \quad (16)$$

The resulting equation of motion for  $\phi^{(s)}$  may then be expressed as

$$(\nabla_\mu\nabla^\mu - m^2)\phi^{(s)} + [R_{\mu\nu\lambda\sigma}\frac{1}{2}\Sigma^{\mu\nu}\frac{1}{2}\Sigma^{\lambda\sigma}]\phi^{(s)} + \dots = 0, \quad (17)$$

where  $\dots$  indicate terms vanishing on-shell, and  $\Sigma^{\mu\nu}$  are the Lorentz generators in the spin- $s$  representation,

$$(\Sigma^{\mu\nu})_{\{\alpha\}}^{\{\beta\}} = 2s\delta_{[\alpha_1}^{\mu}\eta^{\nu][\beta_1}\delta_{\alpha_2\dots\alpha_s]}^{\beta_2\dots\beta_s]}, \quad (18)$$

where all symbols are antisymmetric with weight one.

On the other hand, for spin- $(n + \frac{1}{2})$  fermions in four dimensions, the non-minimal Lagrangian is

$$\mathcal{L} = \bar{\psi}^{(n)}(\gamma^\mu\nabla_\mu - m)\psi^{(n)} + \frac{n(n-1)}{2m}\bar{\psi}_{\mu\nu\dots}^{(n)}R^{+\mu\alpha\nu\beta}\psi_{\alpha\beta}^{(n)}\dots + \dots \quad (19)$$

---

<sup>3</sup> See *e.g.* [8] for an explicit form of the massive higher spin Lagrangian.

(again only up to terms vanishing on-shell). Here  $R_{\mu\nu\lambda\sigma}^+ = R_{\mu\nu\lambda\sigma} + \frac{1}{2}R_{\mu\nu\alpha\beta}\gamma^{\alpha\beta}{}_{\lambda\sigma}$  is a feature of the four-dimensional theory. Taking the first order equation of motion from (19) and multiplying by  $(\gamma^\mu\nabla_\mu + m)$ , we obtain the second order equation

$$(\nabla_\mu\nabla^\mu - m^2)\psi^{(n)} + [R_{\mu\nu\lambda\sigma}\frac{1}{2}\Sigma^{\mu\nu}\frac{1}{2}\Sigma^{\lambda\sigma}]\psi^{(n)} + \dots = 0, \quad (20)$$

where this time

$$(\Sigma^{\mu\nu})_{\{\alpha\}}^{\{\beta\}} = \frac{1}{2}\gamma^{\mu\nu}\delta_{[\alpha_1\dots\alpha_n]}^{[\beta_1\dots\beta_n]} + 2n\delta_{[\alpha_1}^{\mu}\eta^{\nu][\beta_1}\delta_{\alpha_2\dots\alpha_n]}^{\beta_2\dots\beta_n]} \quad (21)$$

is the Lorentz generator in the spin- $(n + \frac{1}{2})$  representation.

We now see from (17) and (20) that both integer and half-integer spin fields have identical forms for the preferred non-minimal coupling resulting from tree-level unitarity concerns. Furthermore, since the Riemann coupling has the form of a gravitational quadrupole moment, these results of Ref. [2] are suggestive of a gravitational version of the corresponding statement of “g=2” as a natural value for the gyromagnetic ratio for electromagnetic couplings [9].

The above discussion suggests the definition of a gravitational quadrupole “ $h$ -factor” that may be determined from the equations of motion according to

$$(\nabla_\mu\nabla^\mu - m^2)\varphi + h[R_{\mu\nu\lambda\sigma}\frac{1}{2}\Sigma^{\mu\nu}\frac{1}{2}\Sigma^{\lambda\sigma}]\varphi + \dots = 0. \quad (22)$$

While the  $h$ -factor may equally well be defined in terms of the non-minimal coupling of  $\varphi$  to the Riemann tensor in the Lagrangian, such a definition is complicated by the fact that there is an inherent ambiguity in the minimal coupling prescription itself (which is not present for the equations of motion). Using this definition, the results of Ref. [2] may be concisely summarized by the statement that  $h = 1$  is the preferred value of the  $h$ -factor based on the above field theory considerations.

Turning to the spin-7/2 Lagrangian, (14), it is clear that it cannot be written in the “preferred” form of (19), even through the use of the on-shell manipulation (15)<sup>4</sup>. As a result this provides evidence that  $h \neq 1$  for massive string states in general. In order to determine the spin-7/2  $h$ -factor, we first make use of (15) and work in four dimensions to note that the second order equation of motion arising from (14) has the form

$$(\nabla_\rho\nabla^\rho - m^2)\psi^{\mu\nu\lambda} + 3(R^{\mu\alpha\nu\beta} + \frac{1}{2}R^{\mu\alpha\rho\sigma}\gamma_{\rho\sigma}\eta^{\nu\beta})\psi_{\alpha\beta}{}^\lambda + \dots = 0. \quad (23)$$

On the other hand, for spin 7/2, (22) gives instead

$$(\nabla_\rho\nabla^\rho - m^2)\psi^{\mu\nu\lambda} + 6h(R^{\mu\alpha\nu\beta} + \frac{1}{4}R^{\mu\alpha\rho\sigma}\gamma_{\rho\sigma}\eta^{\nu\beta})\psi_{\alpha\beta}{}^\lambda + \dots = 0, \quad (24)$$

---

<sup>4</sup> The factor of  $\frac{1}{2}$  arises in (14) because there  $\psi_{\mu\nu\lambda}$  is a Majorana spinor, while in (19)  $\psi^{(n)}$  is a Dirac spinor.

which clearly has a different Lorentz structure. What this indicates is that, even when restricted to on-shell interactions, there are in fact *two* possible distinct Lorentz-invariant and parity conserving interactions that may be written in terms of the Riemann coupling. Thus a single “ $h$ -factor” is insufficient, and in fact two parameters are necessary to fully characterize this lowest order non-minimal interaction.

On the other hand, working in the Newtonian limit, we find that both  $R^{\mu\alpha\nu\beta}\psi_{\alpha\beta\dots}$  and  $(R^{\mu\alpha\rho\sigma}\gamma_{\rho\sigma}\eta^{\nu\beta})\psi_{\alpha\beta\dots}$  reduce to the same form, related to the (non-relativistic) quadrupole moment  $Q^{ij}$ . In particular, in four dimensions, the components of the Riemann tensor are given in terms of the Newtonian potential  $\phi$  as

$$\begin{aligned} R_{0i0j} &= \partial_i\partial_j\phi, \\ R_{ijkl} &= \delta_{ik}\partial_j\partial_l\phi + \delta_{jl}\partial_i\partial_k\phi - \delta_{il}\partial_j\partial_k\phi - \delta_{jk}\partial_i\partial_l\phi. \end{aligned} \quad (25)$$

Furthermore, non-relativistically, the transverse and  $\gamma$ -transverse conditions on  $\psi_{\mu\nu\dots}$  give both  $\psi_{0\dots} = O(p/M)\psi_{i\dots} \ll 1$  and  $\gamma^i\psi_{i\dots} \ll 1$ . Thus we find

$$R^{\mu\alpha\nu\beta}\psi_{\alpha\beta\dots} \rightarrow R_{ikjl}\psi^{kl\dots} = -2\partial_{(i}\partial^k\phi\psi_{j)}^{k\dots} + \delta_{ij}\partial_k\partial_l\phi\psi^{kl\dots}, \quad (26)$$

where the last term, having a trace form, does not contribute diagonally to the leading spin-7/2 quadrupole interaction, but instead gives an off-diagonal interaction between spins 7/2 and 3/2. For the other possibility, we find instead

$$(R^{\mu\alpha\rho\sigma}\gamma_{\rho\sigma}\eta^{\nu\beta})\psi_{\alpha\beta\dots} \rightarrow R_{iklm}\gamma^{lm}\psi^{k\dots} = -4\partial_i\partial_k\phi\psi^{k\dots} + 2\gamma^i(\partial_k\partial_l\phi\gamma^k\psi_l\dots). \quad (27)$$

This time the last term (having a  $\gamma$ -trace form) corresponds to an off-diagonal interaction between spins 7/2 and 5/2.

With the above in mind, in practice we define the “ $h$ -factor” of (22) only *in the Newtonian limit* (in other words focusing on angular momentum and not Lorentz generators). As a result, the diagonal spin-7/2 equation of motion, (23), reduces to

$$(\nabla_\rho\nabla^\rho - m^2)\psi^{ijk} - 6\partial^{(i}\partial_l\phi\psi_l^{jk)} + \dots = 0, \quad (28)$$

where  $\dots$  now also include off-diagonal interactions with lower-spin states (that are always present and fall on the sub-leading Regge trajectories). Contrasting this with

$$(\nabla_\rho\nabla^\rho - m^2)\psi^{ijk} - 9h\partial^{(i}\partial_l\phi\psi_l^{jk)} + \dots = 0, \quad (29)$$

which follows from (24), finally allows us to determine that  $h = 2/3$  for this particular massive spin-7/2 string state.

At this point we must clarify an apparent paradox. The non-relativistic formula for the quadrupole moment is, in space-time dimension  $D$ :

$$Q^{ij} = \int d^{D-1}x [(D-1)x^i x^j - \delta^{ij}x^2] T^{00}, \quad i, j = 1, \dots, D-1. \quad (30)$$

The action of particles of spin  $s > 1/2$  can have a non-minimal coupling proportional to the Einstein tensor:

$$\Delta S = \int d^D x B^{\mu\nu} G_{\mu\nu}, \quad (31)$$

where  $B^{\mu\nu}$  is a bilinear in the spin- $s$  field. To linear order in the gravitational field, this induces the following change in the stress energy tensor:

$$T^{00} = \partial_i \partial_j B^{ij} + \dots, \quad T_i^i = (3 - D) \partial_i \partial_j B^{ij} + \dots. \quad (32)$$

Here  $\dots$  stands for terms that do not contribute to the gravitational quadrupole. Substituting this equation into formula (30) we obtain a non-zero change of the quadrupole, induced by terms that vanish on the Einstein shell!<sup>5</sup> The solution to this paradox is that equation (30) is a good definition of the quadrupole only for non-relativistic matter. In more general cases this definition is wrong, *even in the Newtonian limit*. The correct definition is obtained by computing the energy of the particle in a static, slowly-varying external Newtonian potential  $\phi$ . The change in energy due to the quadrupole is:

$$\Delta H = \frac{1}{2(D-1)} Q^{ij} \partial_i \partial_j \phi. \quad (33)$$

This formula is insensitive by construction to all terms that vanish on the Einstein shell. In the Newtonian limit, the metric is  $g_{00} = -1 - 2\phi$ ,  $g_{ij} = (1 - \frac{2}{D-3}\phi)\delta_{ij}$ , all other terms vanish. This induces the following change in the Energy:

$$\Delta H = \int d^{D-1} x \phi \left( T^{00} + \frac{1}{D-3} T^{ii} \right). \quad (34)$$

Thus, the correct expression for the quadrupole is obtained by replacing  $T^{00}$  with  $T^{00} + (D-3)^{-1} T_i^i$  in Eqn. (30). By substituting Eqn. (32) into this new expression we find that the contribution to the quadrupole due to non-minimal coupling to the Ricci tensor vanishes, as it should. Eqn. (30) can be used in the Newtonian limit *only* when  $T_i^i \approx 0$ , as it holds, for instance, in macroscopic non-relativistic matter.

#### 4. Generalization to higher-spin.

The results for the massive spin-7/2 state are easily generalized to arbitrary massive higher spin string states interacting with a graviton. As usual, the three point function factorizes into separate holomorphic and antiholomorphic parts. For simplicity we focus on states on the leading Regge trajectory, namely states of spin  $n + \frac{1}{2}$  and  $n + 1$  respectively for the R and NS sectors at mass level  $n$ .

---

<sup>5</sup> Terms proportional to the scalar curvature tensor do not contribute to the quadrupole.

In the Ramond sector, the spin- $(n + \frac{1}{2})$  vertex operator is given by

$$V_F^{(-\frac{1}{2})}(z, \bar{z}) = u_{\mu_1 \dots \mu_n}^\alpha(X) S_\alpha e^{-\frac{\phi}{2}} \partial X^{\mu_1} \dots \partial X^{\mu_n} + v_{\mu_1 \dots \mu_n}^{\dot{\alpha}}(X) (\gamma_\lambda)_{\dot{\alpha}}^\beta \psi^{\mu_1} \psi^\lambda S_\beta e^{-\frac{\phi}{2}} \partial X^{\mu_2} \dots \partial X^{\mu_n}. \quad (35)$$

The resulting three point function,  $\mathcal{A}_{FF}^\gamma$ , contains terms up to  $O((\alpha' k^2)^n)$ . To lowest order, we find

$$\begin{aligned} \mathcal{A}_{FF}^\gamma(\psi_1, k_1; A_{2\sigma}, k_2; \psi_3, k_3) &= \frac{n!}{\sqrt{2}} [\bar{\psi}_{1\mu_1 \dots \mu_n} A_{2\sigma} \gamma^\sigma \psi_3^{\mu_1 \dots \mu_n} \\ &\quad - i \frac{2n}{m} (\bar{\psi}_{1\alpha\mu_2 \dots \mu_n} \gamma_- \psi_{3\beta}^{\mu_2 \dots \mu_n} - \bar{\psi}_{1\beta\mu_2 \dots \mu_n} \gamma_+ \psi_{3\alpha}^{\mu_2 \dots \mu_n}) k_2^\alpha k_2^\beta + \dots], \end{aligned} \quad (36)$$

where  $m = \sqrt{2n}$  is the mass of the  $n^{\text{th}}$  excited level.

Prior to examining the closed string graviton amplitude, we note that the effective field theoretic Lagrangian which reproduces this three point function for the open string may be written as

$$\mathcal{L}_{n+\frac{1}{2}}^{(\text{open})} = -\frac{i}{2} \bar{\psi}_{\mu_1 \dots \mu_n} (\gamma^\sigma D_\sigma + m) \psi^{\mu_1 \dots \mu_n} - \frac{1}{2m} \sum_{i=1}^n \bar{\psi}_{\mu_1 \dots \mu_n} F^{\mu_i \nu_i} \psi_{\nu_i}^{\mu_1 \dots \mu_n} + \dots. \quad (37)$$

From the form of the Lagrangian we can verify that the gyromagnetic ratio  $g$  for all such massive fermionic open string states on the leading Regge trajectory is equal to 2 [9].

For the spin- $(n + 1)$  state in the NS sector, we find on the other hand the leading behavior for the three point function

$$\begin{aligned} \mathcal{A}_{BB}^\gamma(\xi_1, k_1; \zeta_2, k_2; \xi_3, k_3) &= n! \xi_{1\alpha\mu_1\mu_2 \dots \mu_n} \xi_{3\beta\nu_1}^{\mu_2 \dots \mu_n} \zeta_{2\lambda} [(\eta^{\alpha\beta} k_3^\lambda - \eta^{\alpha\lambda} k_2^\beta + \eta^{\beta\lambda} k_2^\alpha) \eta^{\mu_1\nu_1} \\ &\quad + n(\eta^{\nu_1\lambda} k_2^{\mu_1} - \eta^{\mu_1\lambda} k_2^{\nu_1}) \eta^{\alpha\beta} + \dots], \end{aligned} \quad (38)$$

where once again  $m = \sqrt{2n}$ . In general, as for the fermions, the complete expression contains terms up to  $O((\alpha' k^2)^n)$ . Viewed as an open string amplitude, the corresponding effective Lagrangian has the form

$$\begin{aligned} \mathcal{L}_{n+1}^{(\text{open})} &= -\frac{1}{2} D_\lambda \phi_{\mu_0 \dots \mu_n} D^\lambda \phi^{\mu_0 \dots \mu_n} - \frac{1}{2} m^2 \phi_{\mu_0 \dots \mu_n} \phi^{\mu_0 \dots \mu_n} \\ &\quad + i \sum_{i=0}^n \phi_{\mu_0 \dots \mu_n} F^{\mu_i \nu_i} \phi_{\nu_i}^{\mu_0 \dots \mu_n} + \dots. \end{aligned} \quad (39)$$

As expected, this indicates that the non-minimal electromagnetic coupling to  $F_{\mu\nu}$  gives precisely  $g = 2$  for the open string.

Turning now to the closed string, combining (36) and (38) and symmetrizing on the vector indices (corresponding to the maximal spin state at mass level  $n$ ) gives the following on-shell form of the effective Lagrangian:

$$\begin{aligned} \mathcal{L}_{2n+\frac{3}{2}} = & \sqrt{-g} \left[ \frac{1}{2} \bar{\psi}^{(2n+1)} (\gamma^\mu \nabla_\mu + m) \psi^{(2n+1)} \right. \\ & \left. - \frac{n(n+1)}{2m} \bar{\psi}_{\mu_0 \mu_1 \dots \mu_{2n}} \left( R^{\mu_0 \nu_0 \mu_1 \nu_1} - \frac{1}{4(n+1)} R^{\mu_0 \nu_0 \lambda \sigma} \gamma_{\lambda \sigma} \eta^{\mu_1 \nu_1} \right) \psi_{\nu_0 \nu_1}^{\mu_2 \dots \mu_{2n}} + \dots \right]. \end{aligned} \quad (40)$$

The resulting second order equation for  $\psi$  takes the form

$$\begin{aligned} (\nabla_\rho \nabla^\rho - m^2) \psi_{\mu_0 \mu_1 \dots} + & \left[ 2(S_L - \frac{1}{2})(S_R - \frac{1}{2}) R^{\mu_0 \nu_0 \mu_1 \nu_1} \right. \\ & \left. + \frac{1}{2}(S - \frac{1}{2}) R^{\mu_0 \nu_0 \lambda \sigma} \gamma_{\lambda \sigma} \eta^{\mu_1 \nu_1} \right] \psi_{\nu_0 \nu_1 \dots} + \dots = 0, \end{aligned} \quad (41)$$

where  $S_L = n + 1$  and  $S_R = n + \frac{1}{2}$  are the components of the spin contributed by left- and right-movers on the worldsheet respectively, and  $S = S_L + S_R (= 2n + \frac{3}{2})$  is the total (spacetime) spin. Using this suggestive form of the non-minimal interaction, we find the corresponding  $h$ -factor to be

$$h = \frac{2S_L S_R}{(S - \frac{1}{2})^2} \quad (\text{half-integer spins}). \quad (42)$$

A similar calculation in the NS-NS sector (also on the leading Regge trajectory) gives similarly

$$h = \frac{2S_L S_R}{S(S-1)} \quad (\text{integer spins}). \quad (43)$$

Note the resemblance to the string  $g$ -factor result[10]

$$g_L = 2 \frac{\langle S_z^R \rangle}{S_z}, \quad g_R = 2 \frac{\langle S_z^L \rangle}{S_z}. \quad (44)$$

Based on the factorization of the graviton three point function in terms of holomorphic and antiholomorphic gauge boson amplitudes and the  $g = 1$  result for *all* massive open string states, it is now apparent that the  $h$ -factor result, (42) and (43), is equally valid for all massive string states and is not restricted to those on the leading Regge trajectory.

## 5. Conclusions.

In this letter we have examined the three-point couplings of massive higher-spin string states with gravity. Focusing on the spin-7/2 state at the first mass level of the Type II string, we have obtained an effective Lagrangian reproducing all *on-shell* interactions to linearized order in the graviton. In particular, this effective Lagrangian contains a non-minimal two-derivative coupling of the form  $(\bar{\psi} R \psi)$ , which was examined by the authors

of [2] in the context of tree-level unitarity. Since this has the form of a gravitational quadrupole interaction, we have defined the gravitational “ $h$ -factor” (in analogy with the electromagnetic  $g$ -factor) and demonstrated that Ref. [2] gives  $h = 1$  as a preferred value of the  $h$ -factor in field theory.

On the other hand, in a string theory,  $h$  is determined from a combination of left- and right-moving components of the spin for massive closed string states. Although generically  $h \neq 1$  in string theory in contrast to the field theoretic result, this is certainly not a disaster for string theory. In field theory the tree-level unitarity results hold for a single massive higher spin particle interacting with gravity, while in string theory tree-level unitarity is achieved not only by the three-point interaction but also because a whole tower of states of arbitrarily large masses and spins propagate as intermediate states.

### Acknowledgments.

I.G. would like to thank S. Samuel for useful discussions. M.P. would like to thank S. Deser for illuminating notes on the correct definition of the gravitational quadrupole, that lay too long in a drawer. This work was supported in part by the Department of Energy under Contract Number DE-FG02-91ER40651-TASK B, and by NSF under grant PHY-9722083.

### References.

- [1] C. Aragone and S. Deser, Phys. Lett. **B86** (1979) 161; M. Duff and S. M. Christensen, Nucl. Phys. **B154** (1979) 301.
- [2] M. Porrati, Phys. Lett. **B304** (1993) 77; A. Cucchieri, M. Porrati and S. Deser, Phys. Rev. **D51** (1995) 4543.
- [3] I. G. Koh, W. Troost and A. Van Proeyen, Nucl. Phys. **B292** (1987) 201; Y. Tanii and Y. Watabiki, Nucl. Phys. **B316** (1989) 171; S. Weinberg, Phys. Lett. **B156** (1985) 309.
- [4] V. A. Kostelecky, O. Lechtenfeld, S. Samuel and S. Watamura, Nucl. Phys. **B288** 173 (1987); S. Samuel, Nucl. Phys. **B294** (1987) 365.
- [5] J. M. F. Labastida and M. Vozmediano, Nucl. Phys. **B312** (1989) 308; P. C. Argyres and C. R. Nappi, Phys. Lett. **B224** (1989) 89; I. L. Buchbinder, V. A. Krykhtin and V. D. Pershin, Phys. Lett. **B348** (1995) 63.
- [6] M. Evans and I. Giannakis, Phys. Rev. **D44** (1991) 2467.
- [7] F. A. Berends and R. Gastmans, Ann. Phys. (NY) **98** (1976) 225; K. A. Milton, Phys. Rev. **D15** (1977) 2149; I. T. Drummonds and S. J. Hathrell, Phys. Rev. **D22** (1980) 343; R. D. Daniels and G. M. Shore, Nucl. Phys. **B425** (1994) 634; P. F. Mende, in *String Quantum Gravity and Physics at the Planck Scale*, Proceedings of the Erice Workshop, 1992, edited by N. Sanchez (World Scientific, Singapore, 1993); R. Lafrance and R. C. Myers, Phys. Rev. **D51** (1995) 2584.
- [8] L. P. S. Singh and C. R. Hagen, Phys. Rev. **D9** (1974) 898, 910.

- [9] S. Ferrara, M. Porrati and V. Telegdi, Phys. Rev. **D46** (1992) 3529.
- [10] J. G. Russo and L. Susskind, Nucl. Phys. **B437** (1995) 611; A. Sen, Nucl. Phys. **B440** (1995) 421.