$\Delta S = 2$ and $\Delta I = 3/2$ Matrix Elements in Quenched QCD *

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We present high-statistics lattice computations of B_K, $B_7^{3/2}$ and $B_8^{3/2}$. The calculations are performed at $\beta = 6.0$ and 6.2 in the quenched approximation, using mean-field-improved Sheikholeslami-Wohlert fermionic actions.

1. Introduction

We report on two, high-statistics, quenched lattice calculations of B_K, $B_7^{3/2}$ and $B_8^{3/2}$. B_K parametrizes the $K^0-\bar{K}^0$-oscillation contribution to CP-violation in K decays (ϵ), leading to a hyperbolic constraint on the summit of the unitarity triangle. $B_{7,8}^{3/2}$ are needed to compute the electro-penguin contribution to direct CP-violation (ϵ'), dominant in the $\Delta = 3/2$ channel. B_K and $B_{7,8}^{3/2}$ measure deviation from the vacuum saturation values of the following four-quark, matrix elements:

$$\langle\bar{K}^0|O_{\Delta S=2}|K^0\rangle = \frac{8}{3}|\langle 0|\bar{s}\gamma_\mu\gamma^5 d|K^0\rangle|^2\, B_K\ ,$$

with $O_{\Delta S=2} = (\bar{s}\gamma_\mu^L d)(\bar{s}\gamma_L^\mu d)$, where $\gamma_{R,L}^\mu = 1 \pm \gamma_5$, and in the chiral limit,

$$\langle\pi^+|O_7^{3/2}|K^+\rangle \to \frac{2}{3}\langle\pi^+|\bar{u}\gamma^5 d|0\rangle\langle 0|\bar{s}\gamma^5 u|K^+\rangle\, B_7^{3/2}$$

$$\langle\pi^+|O_8^{3/2}|K^+\rangle \to 2\langle\pi^+|\bar{u}\gamma^5 d|0\rangle\langle 0|\bar{s}\gamma^5 u|K^+\rangle\, B_8^{3/2},$$

where $O_7^{3/2}$ can be written as $(\bar{s}\gamma_\mu^L d)(\bar{u}\gamma_R^\mu u) + (\bar{s}\gamma_\mu^L u)(\bar{u}\gamma_R^\mu d)$ if one forbids penguin contractions and where $O_8^{3/2}$ is the corresponding color-mixed operator.

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2. Simulation Details

We describe quarks with the mean-field-improved, Sheikholeslami-Wohlert (SW) action

$$S_F^{SW} = S_F^W - ig\, c_{SW}\frac{\kappa}{2}\sum_{x,\mu,\nu}\bar{q}(x)\, P_{\mu\nu}\sigma_{\mu\nu}\, q(x)\ ,$$

with $c_{SW} = 1/u_0^3$ and $u_0 \equiv \langle\frac{1}{3}\mathrm{Tr}U_{pl}\rangle^{\frac{1}{4}}$ and where S_F^W is the Wilson fermion action, g, the bare gauge coupling, $P_{\mu\nu}$, a lattice definition of the field-strength tensor and κ, the appropriate quark hopping parameter. The parameters of the simulation are summarized in Table 1. We further perform full tadpole-improved, KLM rotation of quark fields. Though these normalization factors cancel in the calculation of B-parameters, they modify our renomalization constants.

3. Operator Matching

While the matching of quark bilinears is simple, the use of Wilson fermions induces mixing amongst four-quark operators of different chirality. To describe this mixing, we use the following complete basis of parity-conserving operators:

$$\begin{aligned} O_{1,2}^{lat} &= \gamma_\mu\times\gamma_\mu \pm \gamma_\mu\gamma_5\times\gamma_\mu\gamma_5\ ,\\ O_{3,4}^{lat} &= I\times I \pm \gamma_5\times\gamma_5\ ,\\ O_5^{lat} &= \sigma_{\mu\nu}\times\sigma_{\mu\nu}\ , \end{aligned}$$

with $\Gamma\times\Gamma \equiv (\bar{q}'\Gamma q)(\bar{q}'\Gamma q)^{lat}(a)$. Then, the matching can be written

$$O_{\Delta F=2}(\mu) \to \hat{Z}_{11}\ \hat{O}_1^{lat}(a)$$

Table 1
Simulation parameters. The masses below each κ are the corresponding pseudoscalar-meson masses.

β	size	# cfs.	c_{SW}	κ		
6.2	$24^3 \times 48$	188	1.442	0.13640	0.13710	0.13745
	$a^{-1}(m_\rho) = 2.56^{+8}_{-8}$ GeV			780 MeV	570 MeV	438 MeV
6.0	$16^3 \times 48$	498	1.479	0.13700	0.13810	0.13856
	$a^{-1}(m_\rho) = 1.96^{+6}_{-5}$ GeV			811 MeV	575 MeV	445 MeV

and

$$\left(\begin{array}{c} O_7^{3/2}(\mu) \\ -O_8^{3/2}(\mu)/2 \end{array} \right) \rightarrow \left(\begin{array}{cc} \hat{Z}_{22} & \hat{Z}_{24} \\ \hat{Z}_{42} & \hat{Z}_{44} \end{array} \right) \left(\begin{array}{c} \hat{O}_2^{lat}(a) \\ \hat{O}_4^{lat}(a) \end{array} \right)$$

in terms of the chirally subtracted operators $\hat{O}_1^{lat}(a) \equiv O_1^{lat}(a) + \sum_{i=2}^5 Z_{1i}\, O_i^{lat}(a)$ and $\hat{O}_i^{lat}(a) \equiv O_i^{lat}(a) + \sum_{j=1,3,5} Z_{ij}\, O_i^{lat}(a)$, i=2,4.

We perform matching to the $\overline{\rm MS}$-NDR scheme at one loop. Since the clover term is $\mathcal{O}(g)$, we can use the tree-level-clover-action results of [1] with modifications appropriate for tadpole-improvement and KLM normalization. For the coupling, we choose $\alpha_s^{\overline{\rm MS}}(\mu)$ defined from the plaquette [2], identifying the scale μ with that of the matching. To estimate the systematic error associated with our matching procedure, we vary μ in the range $1/a \rightarrow \pi/a$.

4. Analysis and Results

To obtain the desired B-parameters, we consider ratios of 3-point to two 2-point functions. In the limit that the three points are well separated in time, these ratios reduce to:

$$\begin{aligned} R_{\Delta F=2} &\rightarrow \frac{1}{Z^2_{\gamma_\mu\gamma_5}} \frac{\langle \bar{P}(\vec{q})|O^{(NDR)}_{\Delta F=2}|P(\vec{p})\rangle}{|\langle 0|P^\dagger|P\rangle|^2} \\ R_7^{3/2} &\rightarrow \frac{3}{2\,Z^2_{\gamma_5}} \frac{\langle P^+(\vec{q})|O_7^{3/2(NDR)}|P^+(\vec{p})\rangle}{\langle P^+|P^\dagger|0\rangle\langle 0|P|P^+\rangle} \\ R_8^{3/2} &\rightarrow \frac{1}{2\,Z^2_{\gamma_5}} \frac{\langle P^+(\vec{q})|O_8^{3/2(NDR)}|P^+(\vec{p})\rangle}{\langle P^+|P^\dagger|0\rangle\langle 0|P|P^+\rangle}, \end{aligned}$$

where P is a light-light, pseudoscalar meson. We calculate them for the momenta $\vec{p} \rightarrow \vec{q} = 0 \rightarrow 0$, $0 \rightarrow 1$, $1 \rightarrow 1_\perp$ and $1 \rightarrow 1_\parallel$ and for all hopping parameter combinations taken from Table 1.

To study their chiral behavior, we follow [3] and define the mass and recoil variables

$$\begin{aligned} X &= \frac{8}{3} \frac{f_P^2 M_P^2}{|\langle 0|P^\dagger|P\rangle|^2} \\ Y &= \frac{p\cdot q}{M_P^2} X. \end{aligned}$$

We then fit the ratios to

$$R(X,Y) = a_{00} + a_{10}\, X + a_{01}\, Y + \cdots\ , \qquad (1)$$

where we neglect both chiral logarithms, which are difficult to distinguish from the terms we include, and $SU(3)_f$ breaking terms, which appear to be small for the quark masses we consider.

We find that $R_{\Delta F=2}$ is well described by a linear form in X and Y as shown in Fig. 1 for $\mu = 1/a$. At $\beta = 6.2$, we further find that a_{00} and a_{10} are very small and consistent with zero, as chiral symmetry requires, and remain so as μ is increased up to π/a. B_K is thus simply a_{01}. At $\beta = 6.0$, a_{00} and a_{10} are less than 3 and 2σ away from zero and smaller than the values obtained in [4] with a tree-level SW action and boosted, one-loop matching. Taken in conjunction with other Wilson results obtained from less improved actions, our findings suggest that the relative weight of perturbative and discretization errors in the traditionally observed deviation from good chiral behavior may be different from that suggested in [3,5].

In the chiral limit, $B_{7,8}^{3/2}$ correspond to the leading term, a_{00}, in the expansion of Eq. (1). Since their determination does not require knowledge of the Y-dependence of $R_{7,8}^{3/2}$, we consider only the $\vec{p} = \vec{q} = \vec{0}$ ratios so as not to introduce unnecessary momentum-dependent discretization errors. We find that the description of the chiral behavior of $R_{7,8}^{3/2}$ requires a quadratic term in X.

To compare results for B_K and $B_{7,8}^{3/2}$ obtained at different μ and/or β, we must run them to

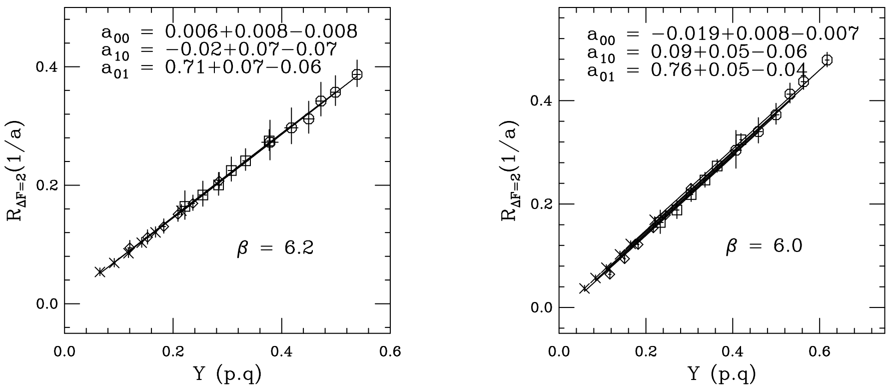

Figure 1. *Chiral behavior of $R_{\Delta F=2}$ for $\mu = 1/a$. The different symbols correspond to different $\vec{p} \to \vec{q}$ while the different points within a given set correspond to different pseudoscalar meson masses.*

Table 2
B-parameters at 2 GeV in the $\overline{\mathrm{MS}}$-NDR scheme as a function of β and the matching scale, μ.

β	μ	B_K	$B_7^{3/2}$	$B_8^{3/2}$
6.2	π/a	0.71^{+7}_{-6}	0.60^{+4}_{-5}	0.81^{+8}_{-8}
	$2/a$	0.71^{+8}_{-6}	0.58^{+4}_{-5}	0.80^{+8}_{-8}
	$1/a$	0.72^{+7}_{-6}	0.50^{+4}_{-4}	0.76^{+8}_{-7}
6.0	π/a	0.75^{+5}_{-4}	0.55^{+3}_{-3}	0.74^{+3}_{-3}
	$2/a$	0.75^{+5}_{-3}	0.53^{+3}_{-3}	0.73^{+4}_{-3}
	$1/a$	0.76^{+5}_{-4}	0.43^{+3}_{-2}	0.68^{+3}_{-2}

a common reference scale which we take to be 2 GeV. Running is performed at two loops with n_f=0. (See Table 2.)

We find that B_K is almost independent of μ in the range explored. a-dependence is also found to be small though it cannot be excluded given the statistical errors and the small deviations of a_{00} and a_{10} from zero at 6.0. Two values of the lattice spacing are not sufficient for a proper continuum-limit extrapolation and we quote as our final number the $\mu = 1/a$ result at $\beta = 6.2$, with the comment that residual discretization errors may be small.

The μ-dependence of $B_8^{3/2}$ and especially $B_7^{3/2}$ is significantly stronger than that of B_K. This is a result of the rather large matching constants and anomalous dimensions. We favor the results obtained at larger values of μ because at $\mu = 1/a$ some of the matching corrections are dangerously big while the two-loop running from π/a to 2 GeV is reasonable. The a-dependence, though not significant statistically at fixed $a\mu$, may be stronger than for B_K. Once again we cannot extrapolate to the continuum limit, so we quote as our final number the $\mu = 2/a$ result at $\beta = 6.2$, this time with the warning that residual discretization errors could still be somewhat important. We also add a systematic error to account for the observed μ-dependence.

Table 3
Preliminary results for B-parameters at 2 GeV in the $\overline{\mathrm{MS}}$-NDR scheme (see text).

B_K	$B_7^{3/2}$	$B_8^{3/2}$
0.72^{+7}_{-6}	0.58^{+4+2}_{-5-8}	0.80^{+8+1}_{-8-4}

Our results are summarized in Table 3. Discussion of additional uncertainties, such as quenching, must be postponed for lack of space. For further comparisons with other recent results, see[6].

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