Kinematics and Resolution at LEP×LHC

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Abstract

Limitations due to resolution and kinematics are discussed of the (Q^2, x) range accessible in deep inelastic scattering of 50 GeV electrons from LEP and 2-8 TeV protons from LHC.

1 Introduction

The deep inelastic inclusive scattering cross section σ depends on two kinematic variables, besides the energy $s = 4E_lE_p$. In the experiment the polar angle $\theta_e(\theta_j)$ of the scattered electron e (the current jet j, resp.) and the secondary energies E_e (and E_j) are measured. The four-momentum transfer Q^2, the relative energy transfer y and the Bjorken variable x can be calculated from the measured quantities as follows:

$$\begin{aligned} Q^2 &= 4E_lE_e sin^2(\theta_e/2) \\ y &= 1 - E_e cos^2(\theta_e/2)/E_l \\ Q^2 &= E_j^2 sin^2(\theta_j)/(1-y) \\ y &= E_j sin^2(\theta_j/2)/E_e \end{aligned} \tag{1}$$

where x is given as Q^2/sy. Note that the angles θ are defined between the directions of the outgoing electron and the electron beam (θ_e) and between the jet and the proton beam (θ_j). In the tree approximation, both kinematic sets have to agree. This offers important cross calibration possibilities in the regions where both the electron and the hadronic jet measurements can be used. The accessible kinematic range is restricted due to angular coverage, finite detector resolution and calibration uncertainties. This study comes to conclusions which were similarly reached for HERA [1].

2 Accessible (x, Q^2) range

Lines of constant energy and angle of the scattered electron and the current jet are located differently in the (Q^2, x) plane because of the relations:

$$\begin{aligned} Q^2(x, E_e) &= sx(1 - E_e/E_l)/[1 - xE_p/E_l] \\ Q^2(x, E_j) &= sx(1 - E_j/xE_p)/[1 - E_l/(xE_p)] \\ Q^2\ (x, \theta_e) &= sx/[1 + xE_p cot^2(\theta_e/2)/E_l] \\ Q^2\ (x, \theta_j) &= sx/[1 + E_l cot^2(\theta_j/2)/xE_p] \end{aligned} \tag{2}$$

A first limitation of the kinematical range is introduced by the beam pipe which will exclude angles below a few degrees, i.e. about 3^o for the electrons. This introduces a cut at small momentum transfers, $Q^2\ (x, \theta_e) \simeq (2E_l tan^2(\theta_e/2))^2 \simeq 7GeV^2$, in most of the x region, except the extremely small x values. The jet measurement can be extended down to about 5^o provided single tracks are catched at even lower angles (down to 10mrad). One easily derives from (2) that this excludes the higher x region ($x \geq 0.01$), the borderline being linear in a double log plot, i.e. $Q^2\ (x, \theta_j) \simeq (2E_p x tan^2(\theta_j/2))^2$. Thus at $E_p = 8TeV$ the minimum Q^2 is about $5000x^2$ at a minimum angle of 5^o. Note that the extension of the kinematic range for the jet measurement towards larger x at LEPxLHC (and as well at HERA) relies only on lowering the proton-beam energy. In particular, it is independent of the electron-beam energy. It is not difficult to see the effect of varying the angular cut to lower or higher values, the principle conclusions, however, stay.

In addition to simple kinematics one has to study resolution effects in order to define the accessible region. A straightforward calculation yields the uncertainties in x and Q^2 given those for the polar angle and the secondary energy.

electrons:

$$\begin{aligned} \delta x/x &= 1/y \times \delta E_e/E_e \\ &\quad +[(1-y)/y \times tan(\theta/2) + cot(\theta/2)]\delta\theta \\ \delta Q^2/Q^2 &= \delta E_e/E_e + cot(\theta/2)\delta\theta \end{aligned} \tag{3}$$

jet:

$$\begin{aligned} \delta x/x &= 1/(1-y) \times \delta E_j/E_j \\ &\quad +[-2cot(\theta) \\ &\quad\ \ +(1-2y)/(1-y) \times cot(\theta/2)]\delta\theta \\ \delta Q^2/Q^2 &= (2-y)/(1-y) \times \delta E_j/E_j \\ &\quad +[2cot(\theta) + y/(1-y) \times cot(\theta/2)]\delta\theta \end{aligned} \tag{4}$$

For the electrons, the main problem results from the coefficient $1/y$ for the x resolution.Thus the region of good cross-section measurements using the scattered electron is limited by a y cut around 0.05 and the line of minimum accessible scattering angle θ_e. This has been verified by performing a Monte-Carlo calculation of the smearing correction defined as the ratio of the generated cross section to the reconstructed one in a given bin. We have applied a Gaussian smearing to the electron energy and angle assuming

$$\delta E_e/E_e = 0.01 + 0.1/\sqrt{E_e/GeV} \qquad \delta\theta_e = 1mrad \tag{5}$$

The effect of finite angular resolution is smaller than the energy resolution effect. One should realize that the $1/y$ behaviour represents a severe limitation, i.e. if one wanted to extend the electron region one would be forced to think of a very accurate calorimetric measurement.

The hadronic jet measurement can be simulated similarly. We have assumed

$$\delta E_j/E_j = 0.02 + 0.4/\sqrt{E_j/GeV} \qquad \delta\theta_j = 10mrad \tag{6}$$

The smearing correction is large where the energy E_j becomes small resulting in curved bounds at lower Q^2 [2]. The uncertainty of the angular measurement should be kept below about 10 mrad in order to avoid a further increase of the minimum jet angle covered. Furthermore, it is clear from (4) that the jet measurement is less accurate at high y because the resolutions vary as $1/(1-y)$. The kinematic and resolution considerations are summarized in fig.1 showing the ranges accessible with electron (a) and current jet measurements (b). Note that we have combined the low and high energy options for LEP×LHC (i.e. 50×2000 and 50×8000 GeV^2) and HERA (30×820 and 15×300 GeV^2). Thus fig.1a gives a clear impression of how the ep colliders are going to extend the fixed target measurements with scattered leptons. As discussed above, the collider electron measurements are limited mainly by the beam-pipe cut at low Q^2 and the y cut at about 0.05.

Apart from early measurements of the x dependence in neutrino scattering at CERN and Fermilab there have been no deep inelastic structure function measurements based on the current jet. Fig.1b

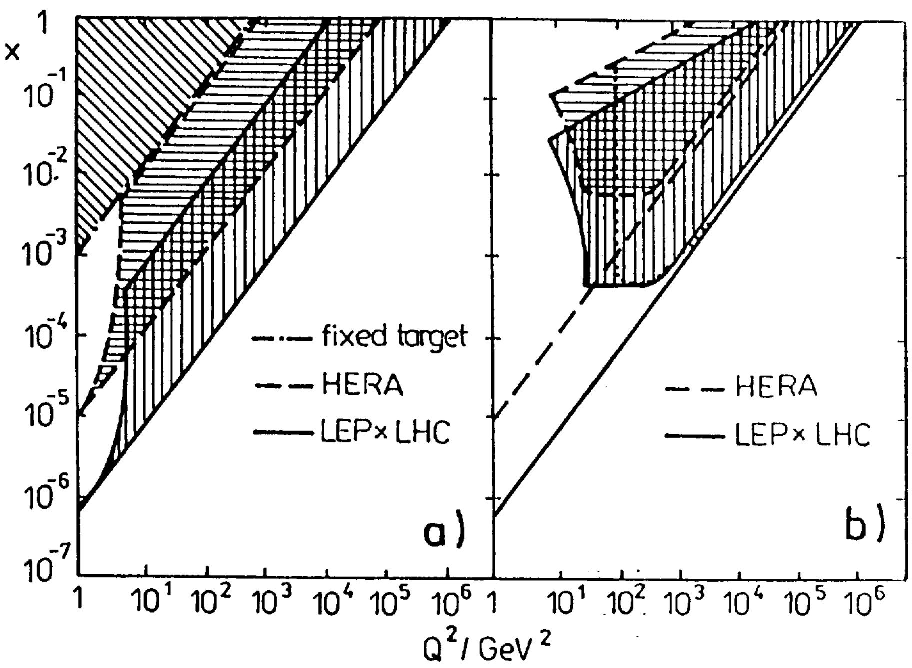

Figure 1: *Kinematical range accessible in fixed target and ep-collider experiments. a) lepton measurement; b) current jet measurement*

shows the (Q^2, x) range to be covered at HERA (dashed) and LEP×LHC. For the neutral current measurement the ranges are limited at high x due to the beam pipe (and angular resolution) and at low Q^2 and low x due to energy resolution as the jet energies become small in that region. Very roughly, the x_{min} value is somewhat below $x = E_l/E_p$ because the energies become small al lower x. Thus LEP×LHC extends the jet measurements of HERA towards smaller x.

For charged currents one has another limitation of the range, as the charged current events have to be distinguished from the many NC events. This requires a minimum transverse energy cut. As a dotted curve we have shown in fig.1b the line $p_\perp = 10$ GeV, $(p_\perp^2 = (1-y)Q^2)$, indicating the narrowing of the CC jet measurement relatively to the NC case.

3 Calibration

Besides kinematics and resolution effects, severe limitations are caused by energy calibration uncertainties. Using the following formulae one can calculate the effect of systematic energy shifts on the cross section:

$$\begin{aligned}
\hat{x}_e &= \frac{x}{1-\epsilon_e/(1+\epsilon_e)y} & \hat{x}_j &= x\,\frac{(1+\epsilon_j)(1-y)}{1-y\,(1+\epsilon_j)} \\
\hat{Q}_e^2 &= Q^2(1+\epsilon_e) & \hat{Q}_j^2 &= Q^2\frac{(1+\epsilon_j)^2(1-y)}{1-y\,(1+\epsilon_j)} \qquad (7) \\
\hat{y}_e &= y - \epsilon_e(1-y) & \hat{y}_j &= y(1+\epsilon_j)
\end{aligned}$$

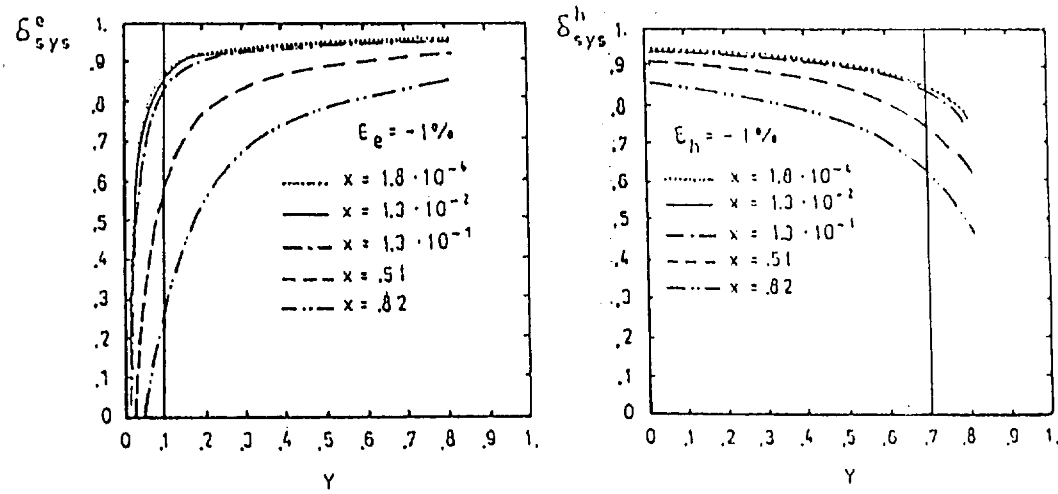

Figure 2: *Systematic shifts of the neutral current cross sections for the electron and hadron flow measurement*

It follows, as before, that $\hat{x}_e/x$ becomes large for $y \to 0$ whereas $\hat{x}_j/x$ and $\hat{Q}_j^2/Q^2$ grow with $y \to 1$. Fig.2 shows a calculation of the systematic shifts of the cross section by a 1% change of the scattered electron and jet energy, resp. As for HERA, the absolute energy calibration has to be managed at the per cent level. Since the electron and jet measurements are sensitive to miscalibrations in different regions of y, one has the possibility to reduce the energy uncertainty effect by cross calibrating the electron and jet measurements in the regions of overlap, see fig. 1. In a very wide region, the Q^2 as determined from the electrons is very precise, see (3). This may be used to replace or cross check the Q^2 measurement with the hadronic jets.

4 Conclusion

LEP × LHC will extend the kinematic domain compared to HERA by about one order of magnitude towards higher Q^2 and lower x. It will have enough overlap with the HERA range if the low E_p option of the LHC will be realized. As for HERA, the calorimetric measurements are very demanding, i.e. they have to cover the smallest possible angles and be calibrated to better than 1%.

References

[1] J.Feltesse, *Proceedings of the DESY Workshop on Physics at HERA*, DESY, Hamburg, 1988,Vol.**1**, p.33.

[2] J.Blümlein, M.Klein PHE 90-19 (1990)