

Neutral B Meson Mixing and Heavy-Light Decay Constants from Quenched Lattice QCD *

Laurent Lellouch ^{a †} and C.-J. David Lin ^{b ‡} (UKQCD Collaboration)

^aTheory Division, CERN, CH-1211 Geneva 23, Switzerland

^bDepartment of Physics & Astronomy, The University of Edinburgh, Edinburgh EH9 3JZ, Scotland

We present high-statistics results for neutral B -meson mixing and heavy-light-meson leptonic decays in the quenched approximation from tadpole-improved clover actions at $\beta = 6.0$ and $\beta = 6.2$. We consider quantities such as $B_{B_{d(s)}}$, $f_{D_{d(s)}}$, $f_{B_{d(s)}}$ and the full $\Delta B = 2$ matrix elements as well as the corresponding $SU(3)$ -breaking ratios. These quantities are important for determining the CKM matrix element $|V_{td}|$.

1. INTRODUCTION

The study of $B_d^0 - \bar{B}_d^0$ oscillations allows a clean extraction of the poorly known CKM matrix element $|V_{td}|$. However, the accuracy of this determination is currently limited by the theoretical uncertainty in the calculation of the matrix element,

$$\begin{aligned} \mathcal{M}_{bd} &= \langle \bar{B}_d^0 | \mathcal{O}_d^{\Delta B=2} | B_d^0 \rangle \\ &= \langle \bar{B}_d^0 | \bar{b} \gamma_\rho (1 - \gamma_5) d \bar{b} \gamma^\rho (1 - \gamma_5) d | B_d^0 \rangle, \end{aligned}$$

which is related to the mass difference of the two mass eigenstates of the $B_d^0 - \bar{B}_d^0$ system,

$$\begin{aligned} \Delta m_d &= \frac{G_F^2}{8\pi^2} M_W^2 |V_{td} V_{tb}^*|^2 S_0 \left(\frac{m_t^2}{M_W^2} \right) \eta_B C_B(\mu) \\ &\times \frac{1}{2M_{B_d}} |\mathcal{M}_{bd}(\mu)|, \end{aligned}$$

where G_F is the Fermi constant, M_W the W -boson mass, m_t the top-quark mass and μ the renormalisation scale. $S_0(m_t^2/M_W^2)$, η_B and $C_B(\mu)$ are perturbatively-calculated quantities.

An alternative approach, in which many theoretical uncertainties cancel, is to look at the ratio

$$\frac{\Delta m_s}{\Delta m_d} = \left| \frac{V_{ts}}{V_{td}} \right|^2 \frac{M_{B_d}}{M_{B_s}} \left| \frac{\mathcal{M}_{bs}}{\mathcal{M}_{bd}} \right| \equiv \left| \frac{V_{ts}}{V_{td}} \right|^2 \frac{M_{B_d}}{M_{B_s}} r_{sd}$$

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[†]On leave from: Centre de Physique Théorique, Case 907, CNRS Luminy, F-13288 Marseille Cedex 9, France.

[‡]Presenter.

where $M_{B_{d(s)}}$ is the meson mass. The matrix elements $\mathcal{M}_{bd(s)}$ can be parameterised as

$$\mathcal{M}_{bd(s)} = \frac{8}{3} f_{B_{d(s)}}^2 M_{B_{d(s)}}^2 B_{B_{d(s)}}$$

where $f_{B_{d(s)}}$ is the decay constant, and $B_{B_{d(s)}}$ the B -parameter of $B_{d(s)}^0$ mesons.

In this work, we obtain the ratio r_{sd} from the direct calculation of $\mathcal{M}_{bs}/\mathcal{M}_{bd}$ as well as from the calculations of f_{B_s}/f_{B_d} and B_{B_s}/B_{B_d} .

2. SIMULATION DETAILS

We use the tadpole-improved Sheikholeslami-Wohlert (SW) quark action,

$$S_F^{SW} = S_F^W - ig_0 c_{SW} \frac{\kappa}{2} \sum_{x,\mu,\nu} \bar{q}(x) P_{\mu\nu}(x) \sigma_{\mu\nu} q(x)$$

to perform simulations on a $24^3 \times 48$ lattice at $\beta = 6.2$ and a $16^3 \times 48$ lattice at $\beta = 6.0$. Here S_F^W is the standard Wilson action, g_0 the bare gauge coupling, c_{SW} the clover coefficient, κ the hopping parameter, and $P_{\mu\nu}$ a lattice definition of the gauge-field strength tensor. Table 1 gives the simulation parameters. We use KLM normalisation for the quark fields.

3. OPERATOR MATCHING

Matching onto the $\overline{\text{MS}}$ scheme is performed at one-loop in perturbation theory using the coupling $\alpha_{\overline{\text{MS}}}(\mu)$ defined from the plaquette [1]. Since

Table 1

Simulation parameters. κ_Q and κ_q are the heavy- and light-quark hopping parameters.

β	# configs.	c_{SW}	κ_q			κ_Q				
6.0	498	1.48	0.13700	0.13810	0.13856	0.114	0.118	0.122	0.126	0.130
6.2	188	1.44	0.13640	0.13710	0.13745	0.120	0.123	0.126	0.129	0.132

the clover-leaf interaction term is proportional to g_0 , we can use the perturbative results obtained from a tree-level clover action [2] with modifications appropriate for tadpole-improvement and KLM normalisation.

For the matching of four-fermion operators, we use the basis

$$\begin{aligned}
\mathcal{O}_1^{lat} &= \gamma_\mu \times \gamma_\mu + \gamma_\mu \gamma_5 \times \gamma_\mu \gamma_5, \\
\mathcal{O}_2^{lat} &= \gamma_\mu \times \gamma_\mu - \gamma_\mu \gamma_5 \times \gamma_\mu \gamma_5, \\
\mathcal{O}_3^{lat} &= I \times I + \gamma_5 \times \gamma_5, \\
\mathcal{O}_4^{lat} &= I \times I - \gamma_5 \times \gamma_5, \\
\mathcal{O}_5^{lat} &= \sigma_{\mu\nu} \times \sigma_{\mu\nu}.
\end{aligned}$$

We set the coupling and matching scales to $\mu = \frac{1}{a}$ and, for consistency with the literature, run divergent operators to 5 GeV, using 2-loop continuum RG in the \overline{MS} scheme with the appropriate number of flavours.

To estimate the systematic error associated with the one-loop matching, we vary the scale μ in a range from $1/a$ to π/a . Decay constants are not affected since they are normalized by f_π and B -parameters change by about 3% (f_π varies by approximately 3%). Since we are mainly interested in $SU(3)$ -breaking ratios for which these effects are even smaller, we neglect these small variations in what follows.

4. ANALYSIS AND RESULTS

We determine κ_c and κ_s from pseudoscalar meson masses. We set the scale with M_ρ for spectral quantities and f_π for decay constants. In fact, these two quantities yield remarkably similar scales. (See Table 2.) We then linearly extrapolate and interpolate heavy-light decay constants, B -parameters and $\Delta B = 2$ matrix elements to κ_c and κ_s , keeping κ_c , κ_s , aM_ρ and af_π in the bootstrap loop. Fig. 1 shows examples of these extrapolations.

Table 2

Critical and strange hopping parameters and inverse lattice spacings.

β	6.0	6.2
κ_c	0.13924(1)	0.13793(1)
κ_s	0.13757(8)	0.13670(9)
$a^{-1}(M_\rho)$ (GeV)	1.96(5)	2.57(8)
$a^{-1}(f_\pi)$ (GeV)	1.92(4)	2.58(9)

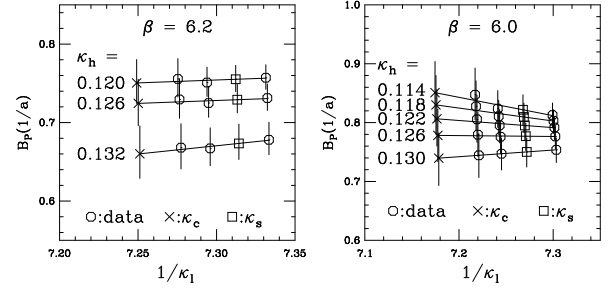


Figure 1. Light-quark-mass dependence of the heavy-light B -parameter, B_P , and extrapolation (interpolation) to $\kappa_l = \kappa_c$ (κ_s) at $\beta = 6.0$ and 6.2 .

For heavy-quark (HQ) extrapolations, we define (M_P is the heavy-light meson mass)

$$\begin{aligned}
\Phi_f(M_P) &= \frac{af_P}{Z_A} \sqrt{\frac{M_P}{M_\rho}} \left\{ \frac{\alpha_s(M_P)}{\alpha_s(M_B)} \right\}^{\frac{2}{11}} \\
\Phi_{\Delta F=2}(M_P) &= \frac{a^4 \mathcal{M} M_\rho}{M_P} \left\{ \frac{\alpha_s(M_P)}{\alpha_s(M_B)} \right\}^{\frac{4}{11}}.
\end{aligned}$$

Then for $X(M_P) = \Phi_f(M_P), \Phi_{\Delta F=2}(M_P), B(M_P)$ and $SU(3)$ -breaking ratios, HQET predicts

$$X(M_P) = A_X \left\{ 1 + B_X \left(\frac{M_\rho}{M_P} \right) + C_X \left(\frac{M_\rho}{M_P} \right)^2 + \dots \right\}.$$

Fig. 2 shows examples of the HQ extrapolations.

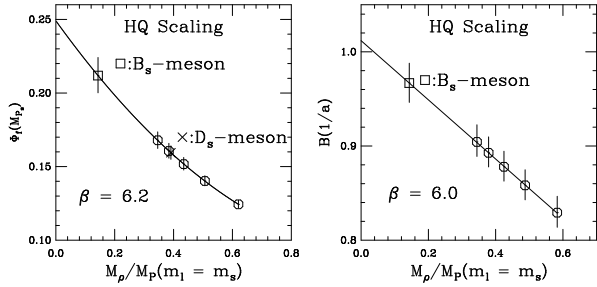
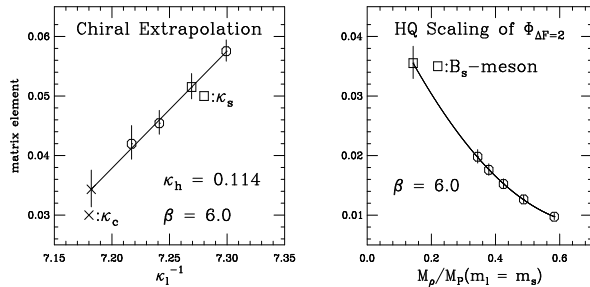
Figure 2. HQ scaling of Φ_f and B -parameter.

Figure 3. Chiral and HQ extrapolation of the matrix element.

For $SU(3)$ -breaking ratios, we find that taking the ratio before or after the HQ extrapolation leads to nearly indistinguishable results. We use the former for our final results since $SU(3)$ -breaking ratios have milder HQ-mass dependences.

Our main results are summarised in Table 3. We obtain r_{sd} from the direct calculation of $\mathcal{M}_{bs}/\mathcal{M}_{bd}$ as well as from f_{B_s}/f_{B_d} and B_{B_s}/B_{B_d} . Our results for the direct calculation are consistent with those of [3], obtained with propagating Wilson quarks, and, at $\beta = 6.0$, with the static result of [4]. However, as Fig. 3 suggests, it is more difficult to control the chiral and HQ extrapolations of the matrix elements in the direct calculation because these extrapolations are more pronounced.

Because we have results at only two values of the lattice spacing, we cannot extrapolate to

Table 3

Summary of results. Errors are statistical only.

β	6.0	6.2
f_{D_s} (MeV)	239(6)	221(9)
f_D (MeV)	213(6)	193(10)
f_{B_s} (MeV)	221(7)	190(12)
f_B (MeV)	191(10)	161(16)
$\frac{f_{D_s}}{f_D}$	1.12(1)	1.15(4)
$\frac{f_D}{f_{B_s}}$	1.15(4)	1.18(8)
$B_B^{\text{nl}_0}(5\text{GeV})$	0.86(2)	0.85(2)
$B_{B_s}^{\text{nl}_0}(5\text{GeV})$	0.83(4)	0.85(3)
$\frac{B_{B_s}}{B_B}$	1.03(3)	0.99(3)
$(\frac{M_{B_s}}{M_B} \frac{f_{B_s}}{f_B})^2 \frac{B_{B_s}}{B_B}$	1.38(7)	1.37(13)
$\frac{\mathcal{M}_{bs}}{\mathcal{M}_{bd}}$	1.52(19)	1.70(28)

the continuum limit. We therefore consider the $\beta = 6.2$ results to be our best, noting that decay constants may still suffer from relatively large discretisation errors (roughly a 2σ effect between 6.0 and 6.2) while $SU(3)$ -breaking ratios and B -parameters are consistent within errors at the two β values.

Although formally one need not include the $a\partial_\mu P$ correction to the axial current when using a mean-field improved, tree-level clover action, it would be interesting to investigate its effect on our results in view of understanding how non-perturbatively, $O(a)$ -improved decay constants may behave. We plan to do so in the future.

For a comparison of our results with other recent results, we refer the reader to [5].

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