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Phenomenological Impacts of the CP-odd Rephase-invariant Phase of the Chargino Mass Matrix in the Production of Light Chargino-Pair in e^+e^- Collisions

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Abstract

One CP–odd rephase-invariant phase appears in the chargino mass matrix in the minimal Supersymmetric Standard Model. We investigate in detail the phenomenological impacts of the CP-odd complex phase in the production of light charginos in e^+e^- annihilation. The values of the chargino masses and the mixing angles, determining the size of the wino and higgsino components in the chargino wave functions, are so sensitive to the CP-odd phase that the constraints on the supersymmetric parameters based on the conventional assumptions for the parameters are recommended to be re-evaluated including the CP-odd phase.

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1. Introduction

The Minimal Supersymmetric Standard Model (MSSM) [1] is a well-defined quantum theory of which the Lagrangian form is completely known, including the general R-parity preserving, soft supersymmetry (SUSY) breaking terms.

The full MSSM Lagrangian has 124 truly independent parameters - 79 real parameters and 45 CP-violating complex phases [2]. The number of parameters in MSSM is too large compared to 19 in the Standard Model (SM). Therefore, many studies on possible direct and indirect SUSY effects have been made by making several assumptions and investigating the variation of a few parameters [3, 4, 5]. Recently, it has, however, been shown [6] that limits on sparticle masses and couplings are very sensitive to the assumptions and need to be re-evaluated without making any of the simplifying assumptions that have been standard.

Despite the large number of phases in the model as a whole, just one CP-odd rephase-invariant phase [7], stemed from the chargino mass matrix, takes part in chargino production [8]. In light of this aspect, in the chargino system the analyses with the general parameter set are not so much more difficult than those with parameters assumed real. The CP-odd phase may be constrained indirectly by the electron or neutron electric dipole moment [7] and may be small, but the constraints on its actual size depends strongly on the assumptions in those analyses. As a matter of fact, the analysis of the electric dipole moments has not been done with a general phase structure, which will certainly give weaker constraints on the CP-odd phase in the chargino mass matrix. So, unless there exist any concrete demonstrations for a small value of the phase, it will be more reasonable to take the CP-odd phase as a free parameter.

Charginos are produced in e^+e^- collisions, either in diagonal or in mixed pairs. However, the second chargino $\tilde{\chi}_2^\pm$ is generally expected to be significantly heavier than the first state. At LEP2 [9], and potentially even in the first phase of e^+e^- linear colliders (see e.g. Ref. [10]), the chargino $\tilde{\chi}_1^\pm$ may be, for some time, the only chargino state that can be studied experimentally in detail. In the present note, we will focus on the diagonal pair production of the light chargino $\tilde{\chi}_1^\pm$ in e^+e^- collisions,

$$e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$$

and investigate in detail the phenomenological impacts of the CP-odd complex phase in the determinations of the relevant SUSY parameters in the production process.

The production of the light chargino-pair is completely described by the chargino mass $\tilde{\chi}_1^\pm$, two mixing angles determining the size of the wino and higgsino components in the charginos, and the sneutrino mass. So, first of all, in Section 2, we briefly recapitulate the elements of the mixing formalism and quantitatively discuss the dependence of the chargino masses and the mixing angles on the CP-odd phase. In Section 3 the cross section for chargino production along with the light chargino mass is mapped over the parameter space, especially for the gaugino mass M_2 and the higgsino mass parameter $|\mu|$, by varing the CP-odd phase. Then, we examine the phenomenological impacts of the CP-odd phase in constraining the parameter space. Conclusions are given in Section 4.

2. Chargino Masses and Mixing Angles

In the MSSM, the spin–1/2 partners of the W boson and charged Higgs boson, $\tilde{W}^\pm$ and $\tilde{H}^\pm$, mix to form chargino mass eigenstates $\tilde{\chi}^\pm_{1,2}$. The mass eigenvalues $m_{\tilde{\chi}^\pm_{1,2}}$ and the mixing angles and phases are determined by the elements of the chargino mass matrix in the $(\tilde{W}^-, \tilde{H}^-)$ basis [1]

$$\mathcal{M}_C = \left(\begin{array}{cc} M_2 & \sqrt{2}m_W\cos\beta \\ \sqrt{2}m_W\sin\beta & \mu \end{array}\right), \tag{1}$$

which is built up by the fundamental supersymmetric parameters; the gaugino mass M_2, the Higgs mass parameter μ, and the ratio $\tan\beta = v_2/v_1$ of the vacuum expectation values of the two neutral Higgs fields which break the electroweak symmetry. In CP–noninvariant theories, the gaugino mass M_2 and the Higgs mass parameter μ can be complex. However, by reparametrizations of the fields, M_2 can be assumed real and positive without loss of generality [7] so that the only non–trivial invariant phase is attributed to μ:

$$\mu = |\mu| \mathrm{e}^{i\theta_\mu}. \tag{2}$$

In these theories the complex chargino mass matrix (1) is diagonalized by two unitary matrices U_L and U_R, which can be parameterized in the following way:

$$\begin{aligned} U_L &= \left(\begin{array}{cc} \cos\phi_L & \mathrm{e}^{-i\beta_L}\sin\phi_L \\ -\mathrm{e}^{i\beta_L}\sin\phi_L & \cos\phi_L \end{array}\right), \\ U_R &= \left(\begin{array}{cc} \mathrm{e}^{i\gamma_1} & 0 \\ 0 & \mathrm{e}^{i\gamma_2} \end{array}\right) \left(\begin{array}{cc} \cos\phi_R & \mathrm{e}^{-i\beta_R}\sin\phi_R \\ -\mathrm{e}^{i\beta_R}\sin\phi_R & \cos\phi_R \end{array}\right), \end{aligned} \tag{3}$$

and which render $U_R \mathcal{M}_C U_L^\dagger$ diagonal. The two chargino mass eigenvalues are given by

$$m^2_{\tilde{\chi}^\pm_{1,2}} = \frac{1}{2}\Big[M_2^2 + |\mu|^2 + 2m_W^2 \mp \Delta_C\Big], \tag{4}$$

with

$$\Delta_C = \sqrt{(M_2^2 - |\mu|^2)^2 + 4m_W^4\cos^2 2\beta + 4m_W^2(M_2^2 + |\mu|^2 + 2M_2|\mu|\sin 2\beta\cos\theta_\mu)}. \tag{5}$$

The mixing angles $\phi_{L,R}$ are given by the relations

$$\begin{aligned} \cos 2\phi_L &= -\frac{M_2^2 - |\mu|^2 - 2m_W^2\cos 2\beta}{\Delta_C}, \\ \sin 2\phi_L &= -\frac{2m_W\sqrt{M_2^2 + |\mu^2| + (M_2^2 - |\mu|^2)\cos 2\beta + 2M_2|\mu|\sin 2\beta\cos\theta_\mu}}{\Delta_C}, \end{aligned} \tag{6}$$

$$\begin{aligned} \cos 2\phi_R &= -\frac{M_2^2 - |\mu|^2 + 2m_W^2\cos 2\beta}{\Delta_C}, \\ \sin 2\phi_R &= -\frac{2m_W\sqrt{M_2^2 + |\mu^2| - (M_2^2 - |\mu|^2)\cos 2\beta + 2M_2|\mu|\sin 2\beta\cos\theta_\mu}}{\Delta_C}, \end{aligned} \tag{7}$$

The four nontrivial phase angles $\{\beta_L, \beta_R, \gamma_1, \gamma_2\}$ also depend on the invariant angle θ; for their expressions we refer to the appendix of the work [4].

Note that $\cos\theta_\mu$ in eqs. (4), (6) and (7) for the chargino masses and the mixing angles appears along with a unique combination factor:

$$M_2|\mu|\sin 2\beta = M_2|\mu|\,\frac{2\tan\beta}{1+\tan^2\beta}.$$

This is a reflection of the fact that the CP-odd phase angle θ_μ can be absorbed by field re–definitions if at least one of the chargino mass matrix elements vanishes. In particular, when $\tan\beta$ is very small or very large, i.e. one of the two Higgs vacuum expectation values v_1 and v_2 is relatively very small [a], the effects of the phase angle θ_μ diminish. Keeping in mind that the CP-odd phase effects are very small for large $\tan\beta$, we present numerical analyses for a fixed value of $\tan\beta = 2$ in the following.

The light and heavy chargino masses are presented in Figs. 1(a) and (b) as a function of the cosine of the CP-violating phase angle θ_μ for a representative set of parameters. The parameters are chosen in the higgsino region $M_2 \gg |\mu|$, the gaugino region $M_2 \ll |\mu|$ and in the mixed region $M_2 \sim |\mu|$ for $\tan\beta = 2$ as

$$\begin{array}{lll} \text{gaugino region}: & (M_2,|\mu|) = (80\ \text{GeV}, & 200\ \text{GeV}),\\ \text{higgsino region}: & (M_2,|\mu|) = (210\,\text{GeV}, & 70\ \text{GeV}),\\ \text{mixed region}: & (M_2,|\mu|) = (90\ \text{GeV}, & 90\ \text{GeV}). \end{array} \tag{8}$$

The two masses are very sensitive to the phase angle θ_μ in all scenarios; the sensitivity is more prominent in the light chargino mass than in the heavy chargino mass, and it is most prominent in the mixed scenario. Figs. 1(c) and (d) exhibit $\cos 2\phi_L$ and $\cos 2\phi_R$ as a function of $\cos\theta_\mu$ [b]. Similarly, both of them depend more strongly on the CP-violating phase angle θ_μ in the mixed scenario than in the gaugino and higgsino scenarios.

3. Production Cross Section

The process $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$ is generated by the three mechanisms: s–channel γ and Z exchanges, and t–channel $\tilde{\nu}$ exchange. The transition matrix element, after a Fierz transformation of the $\tilde{\nu}$–exchange amplitude,

$$T\left(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-\right) = \frac{e^2}{s} Q_{\alpha\beta}\left[\bar{v}(e^+)\gamma_\mu P_\alpha u(e^-)\right]\left[\bar{u}(\tilde{\chi}_1^-)\gamma^\mu P_\beta v(\tilde{\chi}_1^+)\right], \tag{9}$$

can be expressed in terms of four bilinear charges, classified according to the chiralities $\alpha, \beta = L, R$ of the associated lepton and chargino currents

$$Q_{LL} = 1 + \frac{D_Z}{s_W^2 c_W^2}(s_W^2 - \frac{1}{2})\left(s_W^2 - \frac{3}{4} - \frac{1}{4}\cos 2\phi_L\right),$$

[a] Two vacuum expectation values v_1 and v_2 can not be larger than $v = \sqrt{v_1^2+v_2^2} \approx 250$ GeV.

[b] The two sines, $\{\sin 2\phi_L, \sin 2\phi_R\}$, and the four nontrivial phase angles $\{\beta_{L,R}, \gamma_{1,2}\}$ are not numerically presented in the present work because they are not involved in the diagonal pair production of the light or heavy charginos as shown in the next Section.

$$
\begin{aligned}
Q_{LR} &= 1 + \frac{D_Z}{s_W^2 c_W^2}(s_W^2 - \frac{1}{2})\left(s_W^2 - \frac{3}{4} - \frac{1}{4}\cos 2\phi_R\right) + \frac{D_{\tilde\nu}}{4s_W^2}(1+\cos 2\phi_R), \\
Q_{RL} &= 1 + \frac{D_Z}{c_W^2}\left(s_W^2 - \frac{3}{4} - \frac{1}{4}\cos 2\phi_L\right), \\
Q_{RR} &= 1 + \frac{D_Z}{c_W^2}\left(s_W^2 - \frac{3}{4} - \frac{1}{4}\cos 2\phi_R\right). \qquad (10)
\end{aligned}
$$

The first index in $Q_{\alpha\beta}$ refers to the chirality of the $e^\pm$ current, the second index to the chirality of the $\tilde\chi_1^\pm$ current. The $\tilde\nu$ exchange affects only the LR chirality charge while all other amplitudes are built up by γ and Z exchanges. $D_{\tilde\nu}$ denotes the sneutrino propagator $D_{\tilde\nu} = s/(t - m_{\tilde\nu}^2)$, and D_Z the Z propagator $D_Z = s/(s - m_Z^2 + im_Z\Gamma_Z)$; the non–zero width can in general be neglected for the energies considered in the present analysis so that the charges are real.

The bilinear charges $\{Q_{RR}, Q_{RL}\}$ ($\{Q_{LR}, Q_{LL}\}$) with right (left) electron chirality depend only on $\cos 2\phi_R$ ($\cos 2\phi_L$) so that the dependence of the production amplitude on the CP-odd phase angle θ_μ is easily readable. Fig. 2 shows the total production cross section as a function of (a) $\cos\theta_\mu$ for a fixed sneutrino mass of 200 GeV and the parameter in eq. (8), and (b) the sneutrino mass in the mixed scenario of the parameter set (8) at a c.m. energy of 200 GeV. Interestingly, while the chargino masses and the mixing angles are most sensitive to the CP-violating phase angle in the mixed scenario, the production cross section itself is most sensitive to the phase in the gaugino scenario. This implies that the sensitivities to the phase angle θ_μ is strongly correlated with those to the sneutrino mass involved in the t-channel snuetrino exchange amplitude. Fig. 2(b) clearly shows that the total cross section grows up very sharply with $\cos\theta_\mu$ and the sneutrino mass for largest destructive interference in the production amplitude shifts to a lower value. Prior or simultaneous determination of $m_{\tilde\nu}$ will be therefore necessary to determine the phase and the other SUSY parameters.

The maximal $(M_2, |\mu|)$ parameter space, probed in the production of light charginos in e^+e^- collisions with a c.m. energy of 200 GeV, is presented in Fig. 3(a) for three different values of $\cos\theta_\mu$ with $\tan\beta = 2$. As $\cos\theta_\mu$ increases, the covered parameter space is enlarged. However, the maximal parameter space can not be fully covered in actual experiments, but its searchable regime relies on the number of produced charginos and the reconstruction efficiency of the chargino signals determined from their decay patterns [11]. In the case that the light chargino decays into the lightest neutralino, [usually considered to be the lightest supersymmetric particle (LSP)], a fermion and an anti-fermion, the difference between the chargino mass and the LSP mass plays a crucial role. Since the LSP mass also depends strongly on the CP-odd phase angle θ_μ as well as an extra phase angle [6], the determination of the upper limits of the chargino cross section, which can be excluded at high energy colliders such as LEPII, will be rather involved, but of course doable. In the present work, we simply take $\sigma_{tot} = 2$ pb as a reference value for the limit of the production cross section while its estimates based on the detailed investigations of the chargino decays will be touched on in our future work. Fig. 3(b) shows the contours for the total cross section $\sigma_{tot} = 2$ with $m_{\tilde\nu} = 200$ GeV at a c.m. energy of 200 GeV for three different values of $\cos\theta_\mu$. It is clear that the excluded region of the parameter space depend strongly on the phase angle θ_μ and its dependence is most prominent in the mixed scenario.

4. Conclusions

We have analyzed how the parameters of the chargino system, the masses of the charginos $\tilde{\chi}^\pm$ and the size of the wino and higgsino components in the chargino wave functions, parametrized in terms of the two angles ϕ_L and ϕ_R, are affected by the CP-violating rephase-invariant angle θ_μ in the chargino mass matrix. In addition, we have studied the dependence of the production cross section of the light chargino-pair in e^+e^- collisions on the angle θ_μ in detail.

The chargino masses and the production cross section of the light chargino-pair in e^+e^- collisions are very sensitive to the CP-violating angle θ_μ. The sensitivities are most prominent around $\tan\beta = 1$ and in the mixed scenario with $M_2 \sim |\mu|$, where the limits on the light chargino mass might be much weaker than has been reported [11] and need to be re-evaluated. It is then essential to probe the parameter space including the possible complex phase angle θ_μ in the search of the SUSY particles at high energy colliders.

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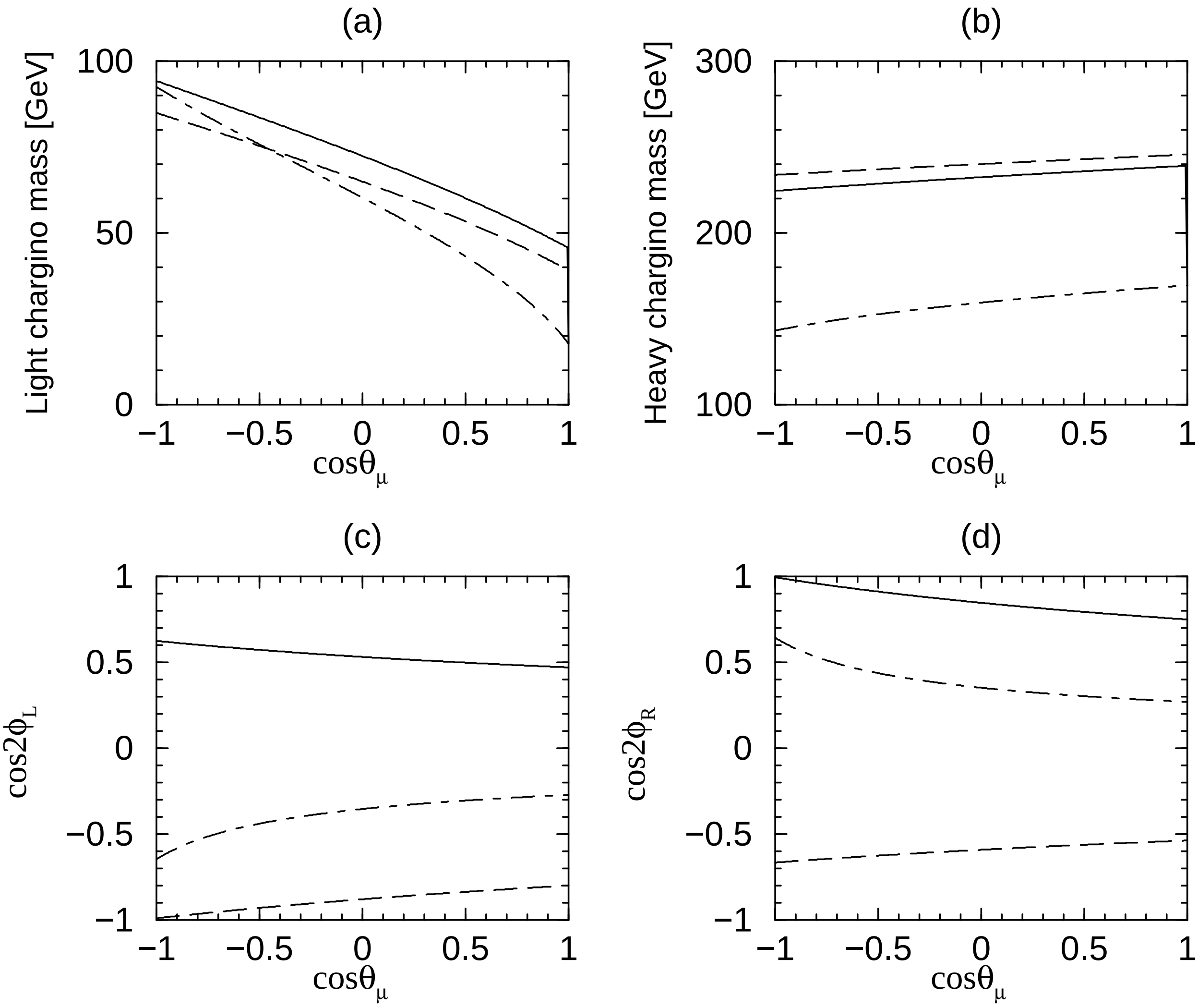

Figure 1: *(a) the light chargino mass, (b) the heavy chargino mass, (c)* $\cos 2\phi_L$*, and (d)* $\cos 2\phi_R$ *as a function of the cosine of the CP-violating phase angle* θ_μ *for the representative set of SUSY parameters in eq. (8): solid line for the gaugino case, dashed line for the higgsino case, and dot-dashed line for the mixed case.*

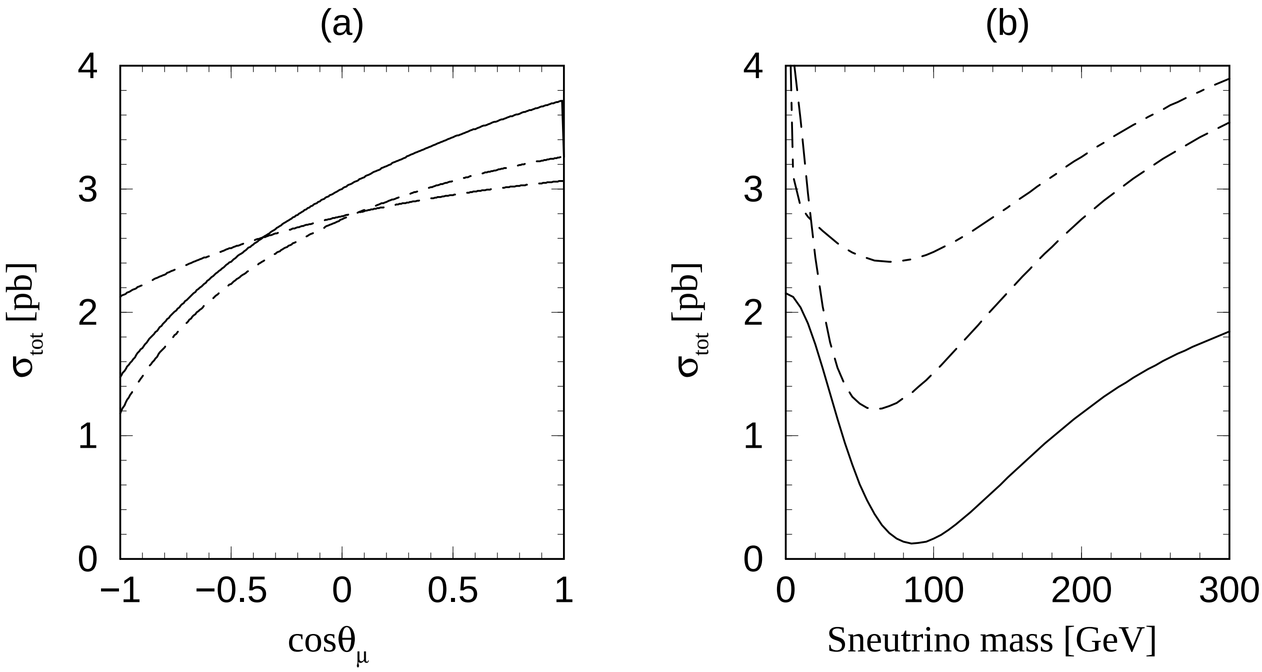

Figure 2: *The cross section for the production of light charginos (a) as a function of* $\cos\theta_\mu$ *with* $m_{\tilde{\nu}} = 200$ *GeV for the parameters set in eq. (8), and (b) as a function of the sneutrino mass in the mixed scenario at* $\sqrt{s} = 200$ *GeV; in (a) the solid line for the gaugino case, the dashed line for the higgsino case, and the dot-dashed line for the mixed case, and in (b) the solid line for* $\cos\theta_\mu = -1$*, the dashed line for* $\cos\theta_\mu = 0$*, and the dot-dashed line for* $\cos\theta_\mu = 1$.

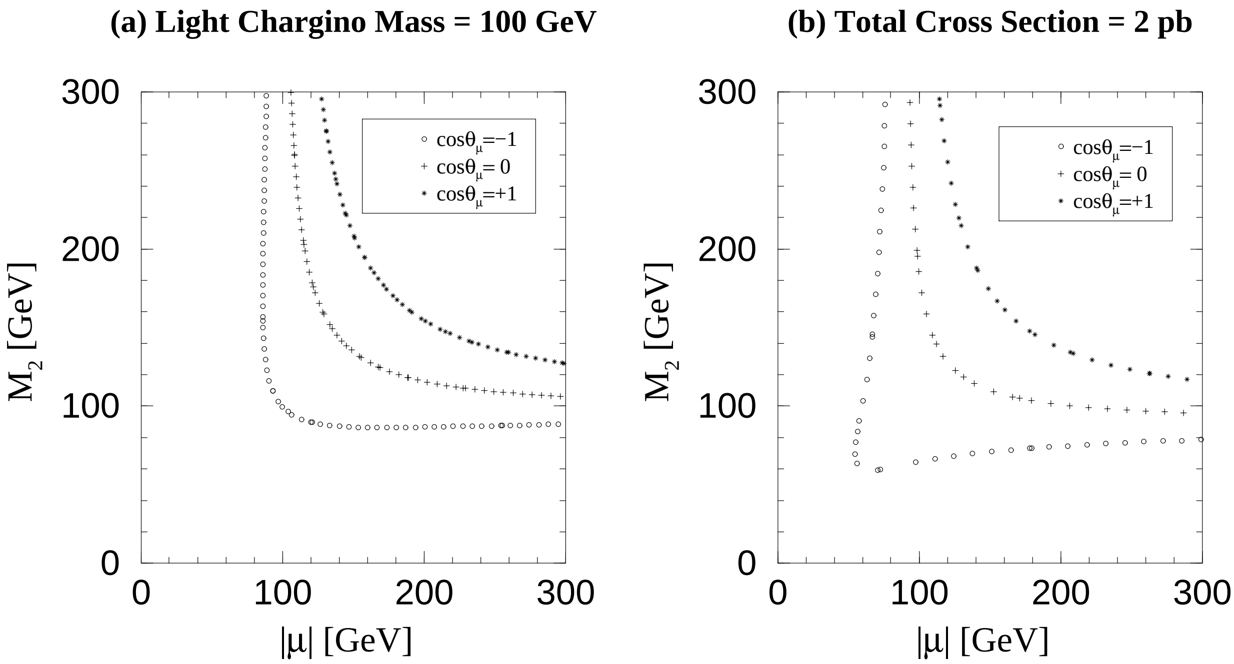

Figure 3: *Contours for (a) the light chargino mass* $m_{\tilde{\chi}_1^\pm} = 100$ *GeV and (b) the total cross section* $\sigma_{tot} = 2$ *pb with* $m_{\tilde{\nu}} = 200$ *GeV at* $\sqrt{s} = 200$ *GeV; the line with circle symbols for* $\cos\theta_\mu = -1$*, the line with plus symbols for* $\cos\theta_\mu = 0$ *and the line with star symbols for* $\cos\theta_\mu = +1$*.*