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A Measurement of the Gluon Splitting Rate into $b\bar{b}$ Pairs in Hadronic Z Decays

The ALEPH Collaboration

Abstract

A measurement of the fraction of hadronic Z decays in which a gluon splits into a $b\bar{b}$ pair, $g_{b\bar{b}}$, is presented using data collected by ALEPH from 1992 to 1995 at the Z resonance. The selection is based on four-jet events. Events are selected by means of topological cuts and a lifetime tag. The result is $g_{b\bar{b}} = (2.77 \pm 0.42(\mathrm{stat}) \pm 0.57(\mathrm{syst})) \times 10^{-3}$.

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1 Introduction

The process of the splitting of a gluon into a heavy quark pair is poorly known, both theoretically and experimentally, despite the fact that this is one of the elementary processes in QCD.

The rate $g_{b\bar{b}}$ is defined as the fraction of hadronic events in which a gluon splits into a $b\bar{b}$ pair. At LEP I it is predicted to be almost one order of magnitude smaller than the corresponding one into a $c\bar{c}$ pair ($g_{c\bar{c}}$).

The values of $g_{b\bar{b}}$ and $g_{c\bar{c}}$ have been predicted by theory [1] within the framework of perturbative QCD. The result is exact to leading order in α_s, and large leading and next-to-leading logarithmic terms are resummed to all orders. For b quarks the uncertainty coming from neglected higher order terms is of the order of 20%. The rate is sensitive to the $\Lambda^5_{\overline{\mathrm{MS}}}$ parameter and to heavy quark masses [2]. The interference with diagrams where the b quarks are emitted directly from the Z is shown to be very small [1] and neglected. The theoretical predictions for e^+e^- colliders at the Z resonance are given in Table 1.

Table 1: Predicted values for the gluon splitting rates from Ref. [2]. The uncertainties are related to the assumed quark masses. For the c quark, the central mass is $m_c = 1.5$ GeV/c^2, the lower error is for $m_c = 1.8$ GeV/c^2 and the upper for $m_c = 1.2$ GeV/c^2. For the b quark, the central mass is $m_b = 4.75$ GeV/c^2, the lower error is for $m_b = 5$ GeV/c^2 and the upper for $m_b = 4.5$ GeV/c^2.

		$g_{c\bar{c}}$ (%)	$g_{b\bar{b}}$ (%)
$\Lambda^5_{\overline{\mathrm{MS}}} = 150\,\mathrm{MeV}$	$\alpha_S(M_{\mathrm{Z}}) = 0.112$	$1.35^{+0.48}_{-0.30}$	0.20 ± 0.02
$\Lambda^5_{\overline{\mathrm{MS}}} = 300\,\mathrm{MeV}$	$\alpha_S(M_{\mathrm{Z}}) = 0.125$	$1.85^{+0.69}_{-0.44}$	0.26 ± 0.03

Experimental measurements of $g_{c\bar{c}}$ have been performed by OPAL [3, 4] and suggest a higher value than the theoretical predictions. The only measurement of $g_{b\bar{b}}$ has been reported by DELPHI [5] and is in agreement with the prediction.

The limited accuracy of the $g_{b\bar{b}}$ prediction is one of the main sources of uncertainty in the measurement of the partial decay width of the Z to $b\bar{b}$ (R_b) [6]. In addition, about 50% of the b hadrons are produced via the gluon splitting process at the Tevatron, and a larger fraction is expected to contribute at the LHC. A better knowledge of this process can improve theoretical predictions of heavy flavour production at such colliders.

This paper describes a measurement of $g_{b\bar{b}}$ making use of a lifetime tag and topological properties of the events using the ALEPH experiment at LEP I.

2 The ALEPH Detector

The ALEPH detector and its performance are described in detail elsewhere [7, 8]; only a brief description will be given here. Charged tracks are reconstructed in a 1.5 Tesla axial magnetic field produced by a superconducting solenoid which surrounds the tracking detectors. At the core of the tracking system is a silicon strip vertex detector (VDET). It consists of two coaxial layers, at average radii of 6.5 and 11.3

cm, each providing measurements in both the r-ϕ and r-z projections. The spatial resolution for tracks at normal incidence is 12 μm in each projection. The angular coverage of the VDET is $|\cos\theta| < 0.85$ for the inner layer and $|\cos\theta| < 0.69$ for the outer layer. Surrounding the VDET is a cylindrical drift chamber (ITC), which measures up to eight coordinates per track in the r-ϕ projection, with a resolution of about 150 μm. The ITC is in turn enclosed in a large time projection chamber (TPC), lying between 30 and 180 cm radii. The TPC provides up to 21 three-dimensional coordinates per track, with resolutions of about 170 μm and 800 μm in r-ϕ and r-z projections, respectively. Tracks with two VDET coordinates have a transverse momentum resolution of $\Delta p_T/p_T = 6 \times 10^{-4} p_T \oplus 0.005$ (p_T in GeV/c). The impact parameter resolution is $(25+95/p)\mu$m (p in GeV/c) in both $r-\phi$ and $r-z$ projections.

Neutral particle energies and directions are measured in the electromagnetic (ECAL) and hadronic (HCAL) calorimeters, covering together almost the full solid angle. The ECAL consists of planes of proportional wire chambers interleaved with lead sheets, for a total of 22 radiation lengths for particles impinging at normal incidence. The ECAL is read out via projective towers typically $0.9° \times 0.9°$ wide. The energy resolution of the ECAL is $\sigma(E)/E = 0.18/\sqrt{E}+0.009$ (E in GeV). The HCAL uses the iron return yoke as absorber and has an average depth of 1.5 m. Hadronic showers are sampled by 23 planes of streamer tubes, which induce analog signals on copper pads, arranged in projective towers of about $3.7° \times 3.7°$. The HCAL energy resolution is $\sigma(E)/E = 0.85/\sqrt{E}$ (E in GeV). A double layer of streamer tubes surrounds the HCAL and forms the muon chambers.

An energy flow algorithm [8] combines charged particle momenta and calorimetric energy measurements and provides a list of energy flow particles on which the analysis is based. The precision on the measurable total visible energy is $\sigma(E) = (0.6\sqrt{E}+0.6)$ GeV (E in GeV).

3 Monte Carlo and data Samples

The measurement uses 3.7 million events collected from 1992 to 1995 which pass the standard ALEPH hadronic event selection [9], with the requirement that the vertex detector was fully operational.

The analysis also uses 6 million fully simulated events produced with a generator based on JETSET 7.4 Parton Shower (PS) Model [10]. The JETSET and HERWIG [11] PS Monte Carlo models both provide a description of the parton shower cascade accurate to leading logarithmic order, together with matching to the first-order prediction for the production of a hard gluon. The rates predicted by these Monte Carlo generators are in reasonable agreement with the theoretical prediction [1].

Monte Carlo events are reweighted to take into account the latest LEP Heavy Flavour Working Group recommendations [12]. In particular, the rate of the gluon splitting in $c\bar{c}$ pairs is set to $g_{c\bar{c}} = (2.38 \pm 0.48)\%$ while the rate of splitting to $b\bar{b}$ is set to $g_{b\bar{b}} = (0.3 \pm 0.1)\%$. A dedicated Monte Carlo production of 35 000 gluon splitting to $b\bar{b}$, 90 000 to $c\bar{c}$ and about 3 million $\mathrm{Z} \to b\bar{b}$ events are used in order to better evaluate the efficiencies.

Besides the signal events, hereafter called $\mathcal{B}$, two categories of background events exist:

- events which do not contain any gluon splitting into heavy flavour at all, hereafter called $\mathcal{Q}$ events
- events in which a gluon splits to a charm quark pair, named $\mathcal{C}$ events.

4 Event Selection

The two b hadrons coming from the gluon tend to be produced in a particular topological configuration, which allows one to discriminate signal from background. Events with a four-jet topology are selected and the b jet candidates are tagged by a lifetime algorithm. The candidate jet pair is required to form a small angle. The other two jets originating from the hadronization of the two initial quarks are required to be at large angle to each other.

Jets are formed with energy flow particles , using the Durham algorithm [13] with $y_{\rm cut} = 0.006$. Events with five or more jets (0.5% of the total in background and about 2% for the signal) are forced to exactly four jets by increasing $y_{\rm cut}$ to y_{45}. The four-jet rates for the $\mathcal{B}, \mathcal{C}$, and $\mathcal{Q}$ events predicted by the simulation are about $50\%, 30\%$ and 10%, respectively. The four-jet rate in the data is $(9.56 \pm 0.02)\%$, where the error is statistical only. In the Monte Carlo simulation the rate is $(9.38 \pm 0.01 \pm 0.14)\%$ where the first error is statistical and the second is due to the uncertainty in the simulation of heavy quark hadron physics discussed in section 6. These efficiencies and those from subsequent cuts are summarized in Table 2.

Jets containing b hadron decay products are then searched for by making use of the information coming from the vertex detector, using the ALEPH b-tagging algorithm [14] based on track impact parameters. The four-jet events are retained only if each jet has at least one track with one VDET hit. The fraction of events that survive in data and in Monte Carlo simulation is $(4.98 \pm 0.01)\%$ and $(4.85 \pm 0.01 \pm 0.08)\%$, respectively.

For each jet, the confidence level CL that all the charged tracks in that jet are consistent with coming from the primary vertex is computed. The four jets are then ordered in decreasing $-\log(CL)$, so that the first jet is more likely to have tracks from b decay than the others.

The two jets with the largest $-\log(CL)$ are then considered as candidates for originating from the gluon splitting process $g \to b\bar{b}$. Figure 1 shows the $-\log(CL)$ distributions for signal and background events. The requirement

$$-\log(CL)_1 > 3.1 \quad , \quad -\log(CL)_2 > 1.7$$

selects events with two b-like jets; at this point about 6% of the selected events are estimated to come from signal.

After this requirement the background, which is dominated by $\mathrm{Z} \to b\bar{b}$, tends to have the b jets in different hemispheres, while in the signal events they are more likely to form a smaller angle. Figure 2 shows the angular separation of the candidate b jets

Table 2: Efficiencies for the $\mathcal{B},\mathcal{C}$, and $\mathcal{Q}$ events, Monte Carlo, data and Ratio data/MC as a function of increasing cuts. The first error is statistical, the second one is systematic. $g_{b\bar{b}}$ is fixed to 0.3% in the Monte Carlo.

Cut	MC efficiencies		data and MC comparison	
No. of jets = 4	$\epsilon_{\mathcal{B}}$	$(47.7 \pm 0.3 \pm 1.4)\%$	MC	$(9.38\pm0.01\pm0.14)\%$
	$\epsilon_{\mathcal{C}}$	$(28.8 \pm 0.1 \pm 1.4)\%$	data	$(9.56\pm0.02)\%$
	$\epsilon_{\mathcal{Q}}$	$(8.73 \pm 0.01 \pm 0.09)\%$	data/MC	$1.020\pm0.002\pm0.015$
No. of VDET hits ≥ 1	$\epsilon_{\mathcal{B}}$	$(27.8 \pm 0.3 \pm 0.9)\%$	MC	$(4.85\pm0.01\pm0.08)\%$
	$\epsilon_{\mathcal{C}}$	$(15.8 \pm 0.1 \pm 0.8)$	data	$(4.98\pm0.01)\%$
	$\epsilon_{\mathcal{Q}}$	$(4.51 \pm 0.01 \pm 0.05)\%$	data/MC	$1.03\pm0.003\pm0.017$
$-\log(CL)_2 > 1.7$	$\epsilon_{\mathcal{B}}$	$(6.4 \pm 0.1 \pm 0.2)\%$	MC	$(3.38\pm0.02\pm0.18)10^{-3}$
	$\epsilon_{\mathcal{C}}$	$(1.40 \pm 0.03 \pm 0.60)\%$	data	$(3.83\pm0.03)10^{-3}$
	$\epsilon_{\mathcal{Q}}$	$(2.93 \pm 0.03 \pm 0.11)10^{-3}$	data/MC	$1.13\pm0.01\pm0.06$
$-\log(CL)_1 > 3.1$	$\epsilon_{\mathcal{B}}$	$(4.39 \pm 0.12 \pm 0.17)\%$	MC	$(2.28\pm0.02\pm0.13)10^{-3}$
	$\epsilon_{\mathcal{C}}$	$(8.54 \pm 0.25 \pm 0.40)10^{-3}$	data	$(2.63\pm0.03)10^{-3}$
	$\epsilon_{\mathcal{Q}}$	$(2.00 \pm 0.02 \pm 0.09)10^{-3}$	data/MC	$1.13\pm0.01\pm0.06$
$\cos\theta_{12} > 0.2$	$\epsilon_{\mathcal{B}}$	$(1.58 \pm 0.08 \pm 0.08)\%$	MC	$(1.87\pm0.05\pm0.20)10^{-4}$
	$\epsilon_{\mathcal{C}}$	$(7.73 \pm 0.73 \pm 0.57)10^{-4}$	data	$(1.82\pm0.07)10^{-4}$
	$\epsilon_{\mathcal{Q}}$	$(1.24 \pm 0.04 \pm 0.11)10^{-4}$	data/MC	$0.98\pm0.05\pm0.10$
$\cos\theta_{34} < 0.1$	$\epsilon_{\mathcal{B}}$	$(1.09 \pm 0.06 \pm 0.05)\%$	MC	$(8.2\pm0.3\pm1.2)10^{-5}$
	$\epsilon_{\mathcal{C}}$	$(3.68 \pm 0.49 \pm 0.19)10^{-4}$	data	$(7.9\pm0.5)10^{-5}$
	$\epsilon_{\mathcal{Q}}$	$(4.14 \pm 0.21 \pm 0.35)10^{-5}$	data/MC	$0.96\pm0.07\pm0.14$
$p_1 < 27$ GeV/c	$\epsilon_{\mathcal{B}}$	$(1.04 \pm 0.06 \pm 0.04)\%$	MC	$(7.4\pm0.3\pm1.1)10^{-5}$
	$\epsilon_{\mathcal{C}}$	$(3.18 \pm 0.45 \pm 0.14)10^{-4}$	data	$(7.1\pm0.4)10^{-5}$
	$\epsilon_{\mathcal{Q}}$	$(3.60 \pm 0.20 \pm 0.31)10^{-5}$	data/MC	$0.97\pm0.07\pm0.14$
$p_4 > 11$ GeV/c	$\epsilon_{\mathcal{B}}$	$(9.58 \pm 0.55 \pm 0.42)10^{-3}$	MC	$(6.3\pm0.3\pm1.0)10^{-5}$
	$\epsilon_{\mathcal{C}}$	$(2.30 \pm 0.35 \pm 0.16)10^{-4}$	data	$(6.0\pm0.4)10^{-5}$
	$\epsilon_{\mathcal{Q}}$	$(2.91 \pm 0.19 \pm 0.25)10^{-5}$	data/MC	$0.97\pm0.08\pm0.15$

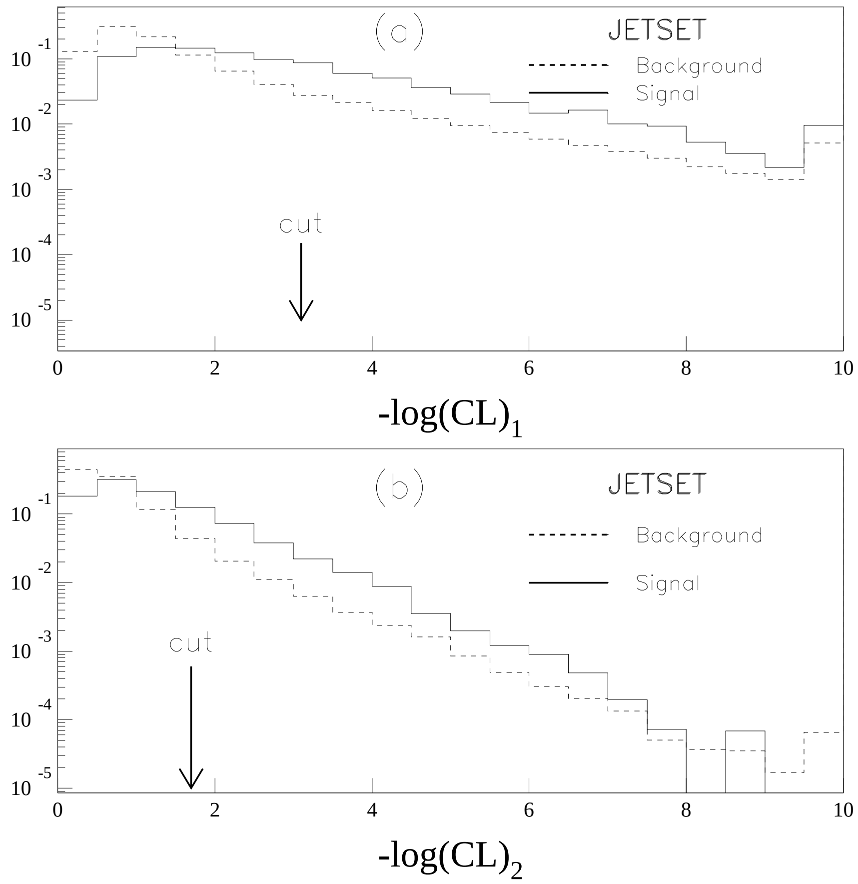

Figure 1: $-\log(CL)$ distributions for (a) jet 1 and (b) jet 2 in signal and background events. All distributions are normalized to unity.

(θ_{12}) for signal and background events. The requirement $\cos\theta_{12} > 0.2$ is imposed to select signal events.

In the remaining signal events the angle θ_{34} between the third and the fourth jet tends to be distributed at large values, while the background events populate the small angle region, as shown in Fig. 3. The cut $\cos\theta_{34} < 0.1$ is chosen.

Finally, the b jets coming from a gluon tend to have a softer momentum than the other two. By requiring the momentum of the fourth jet to be greater than 11 GeV/c and the momentum of the first jet to be lower than 27 GeV/c the purity of the signal is increased to about 45%.

At this stage 70% of the background comes from Z $\rightarrow b\bar{b}$ events, 9% from Z $\rightarrow c\bar{c}$ events, 6% from Z $\rightarrow q\bar{q}$ $(q \neq b, c)$ events and the remaining 15% from $\mathcal{C}$ events.

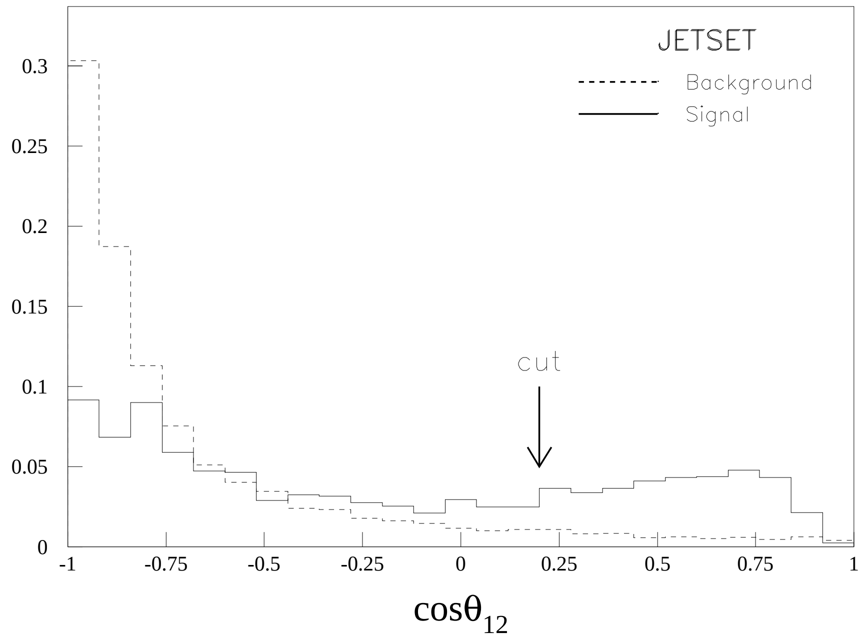

Figure 2: Normalized distributions of the cosine of the angle between the first two jets for signal and background after the $-\log(CL)_{1,2}$ cuts.

Table 3: Efficiencies after all cuts for the three categories. Errors are statistical only.

Source	Efficiency (%)
$\mathcal{B}$	0.958 ± 0.055
$\mathcal{C}$	0.023 ± 0.003
$\mathcal{Q}$	0.0029 ± 0.0002

5 Results

After all the above mentioned cuts 222 events are selected in the data. Table 3 shows the tagging efficiencies for the three categories of events, where the errors are statistical only. From these efficiencies and the fraction of events selected in the data $f_d = (6.04 \pm 0.41) \times 10^{-5}$, one can extract the value of $g_{b\bar{b}}$:

$$g_{b\bar{b}} = \frac{f_d - (1 - g_{c\bar{c}})\epsilon_{\mathcal{Q}} - g_{c\bar{c}}\epsilon_{\mathcal{C}}}{\epsilon_{\mathcal{B}} - \epsilon_{\mathcal{Q}}}.$$

The measured value of the gluon splitting rate into $b\bar{b}$ pairs is

$$g_{b\bar{b}} = (2.77 \pm 0.42) \times 10^{-3},$$

where the error is due to data statistics only.

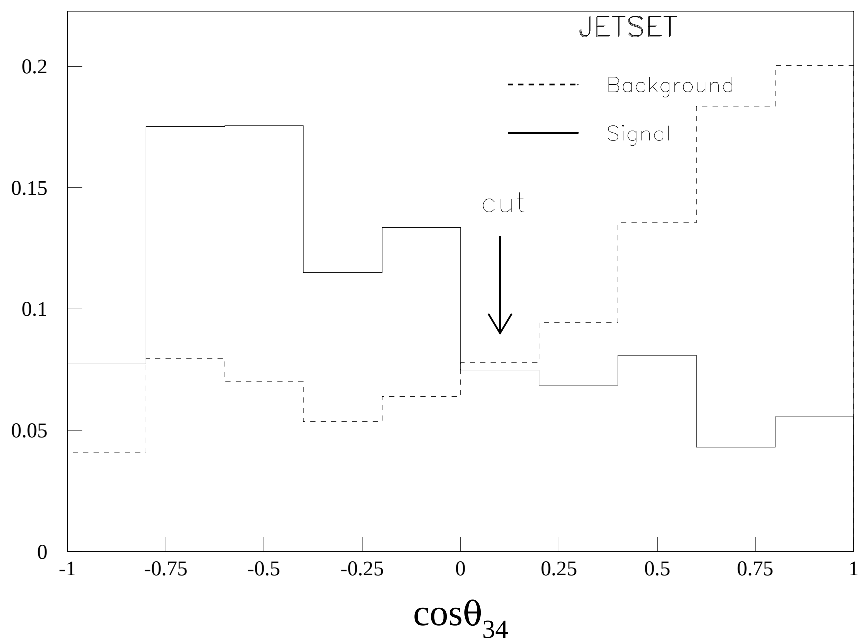

Figure 3: Normalized distribution of the cosine of the angle between the third and fourth jets in signal and background after the cuts in $-\log(CL)$ and in $\cos\theta_{12}$.

6 Systematic errors

The efficiencies for the three event categories are evaluated by Monte Carlo simulation. The inadequacy of the simulation in estimating these efficiencies leads to an uncertainty on the result.

The error due to the limited Monte Carlo statistics in the efficiency evaluation is $\Delta g_{b\bar{b}} = \pm 0.26 \times 10^{-3}$. This uncertainty comes mainly from the efficiency to tag $\mathcal{Q}$ events.

A large fraction of events remaining after the selection cuts contain b and c hadrons. The uncertainty in the knowledge of the physical processes in the simulation of heavy flavour production and decays constitutes a source of systematic error. All the physical simulation parameters are varied within their allowed experimental ranges, as is done in [6]. In particular, the b and c hadron lifetimes as well as production rates are varied, following the latest recommendations of the LEP Heavy Flavour Working Group [12]. The charm topological branching ratios and their rate into K^0_S are also varied within the experimental limits [15]. Two additional sources of systematic uncertainties are the average charged multiplicity of b hadron decays (5.46 ± 0.09 [16]) and the rate of hard gluon events, in which the two original b hadrons are pushed into the same hemisphere by a very energetic gluon $(2.2 \pm 0.4)\%$. The central value for the rate of this type of events is the JETSET parton shower prediction, while the error is taken as the difference between the parton shower and matrix element models. Table 4 summarizes the main sources of physics uncertainties.

Table 4: Systematic errors in $g_{b\bar{b}}$ due to bottom and charm physics uncertainties.

Source	$\Delta g_{b\bar{b}} (10^{-3})$
b hadron lifetimes	± 0.12
b hadron production	± 0.07
b hadron fragmentation	± 0.10
c hadron lifetimes	± 0.09
c hadron production	± 0.01
c hadron fragmentation	± 0.03
BR$(D \to K_S^0)$	± 0.03
D^+ charged multiplicity	± 0.03
D^0 charged multiplicity	± 0.03
D_s charged multiplicity	± 0.03
b hadron charged multiplicity	± 0.05
Hard Gluon	± 0.24
Total	± 0.32

The effect of the possibly different angular distributions in data and simulation has been investigated. The distributions of the jet separation angle in data and Monte Carlo simulation, before applying the $-\log(CL)$ cuts, have been checked. Jets were ordered in decreasing energy, in order not to be biased by any $-\log(CL)$ ordering discrepancy. A comparison was made in the angle between the two hardest jets (α_{12}) and the two softest jets (α_{34}) in data and simulation, as shown in Fig. 4(a). The α_{12} distribution is reproduced within 5%, while a disagreement of about 10% is found for α_{34} due to missing higher order terms in the PS simulation [17]. Notice the better agreement for the θ_{12} and θ_{34} angles, shown in Fig. 4(b).

In order to evaluate the impact of this discrepancy on the analysis, the Monte Carlo background distribution in (α_{12},α_{34}) was reweighted to match the data before the cuts in $-\log(CL)$, and the analysis was repeated. The difference in the result was $\Delta g_{b\bar{b}} = 0.05 \times 10^{-3}$, which is taken as a systematic uncertainty due to the topological cuts.

The simulation of the signal events is based on JETSET PS Monte Carlo, which is in good agreement with the theoretical predictions [1]. In order to estimate the uncertainty on this assumption we have produced 25,000 $g \to b\bar{b}$ events using a modified version of HERWIG Monte Carlo[1](HERWIG5.9) [18] at the generator level. The signal tagging efficiency mainly depends on the description of the split gluon: its energy E_g, its mass m_g and the decay angle θ^* of the two b hadrons in their centre-of-mass frame. This efficiency function, computed with JETSET, is reweighted by the ratio of HERWIG to JETSET initial distributions to obtain the average $\mathcal{B}$ efficiency. A systematic error of

[1]The treatment of the angular distribution of b quarks from gluon splitting is not properly handled in the standard HERWIG5.9 generator. The problem has been cured in version 5.91, given to us by courtesy of the author.

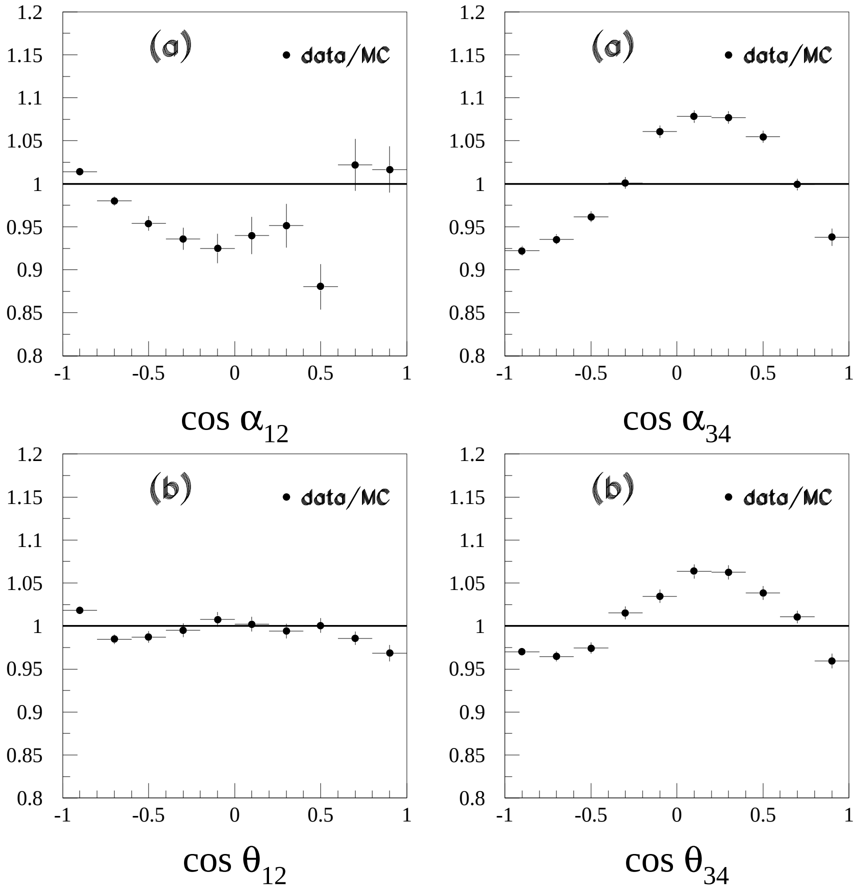

Figure 4: Angular distributions for $\cos\alpha_{12}$ and $\cos\alpha_{34}$ (upper plot) and $\cos\theta_{12}$ and $\cos\theta_{34}$ (lower plot) plotted as the ratio between data and Monte Carlo simulation.

$\Delta g_{b\bar{b}} = \pm 0.31 \times 10^{-3}$ is estimated from the difference in $\epsilon_{\mathcal{B}}$ from the two Monte Carlo models. In Fig. 5 are shown the different initial behavior for those three variables in JETSET and HERWIG, together with the differential efficiencies.

The dependence of the $\mathcal{B}$ efficiency on the b quark mass has also been investigated at the generator level. Events were generated using the WPHACT Monte Carlo [19], which is based on a matrix element calculation including b quark masses. The variation of the $\mathcal{B}$ efficiency is computed as done for HERWIG, using the WPHACT spectrum for b quark masses from 4.7 and 5.3 GeV/c^2. The efficiency varied by 6%, giving an uncertainty of $\Delta g_{b\bar{b}} = \pm 0.17 \times 10^{-3}$.

The uncertainty in the rate of the $g \to c\bar{c}$ background events, $\Delta g_{c\bar{c}} = \pm 0.48\%$, gives an error $\Delta g_{b\bar{b}} = \pm 0.11 \times 10^{-3}$.

Charged Monte Carlo tracks used by the lifetime tag are smeared to better reproduce distribution in data of the negative impact parameter significance [14]. Uncertainties in the efficiencies due to this smearing are assessed by evaluating the MC efficiencies without the smearing algorithm. Half of the difference in the $g_{b\bar{b}}$ result

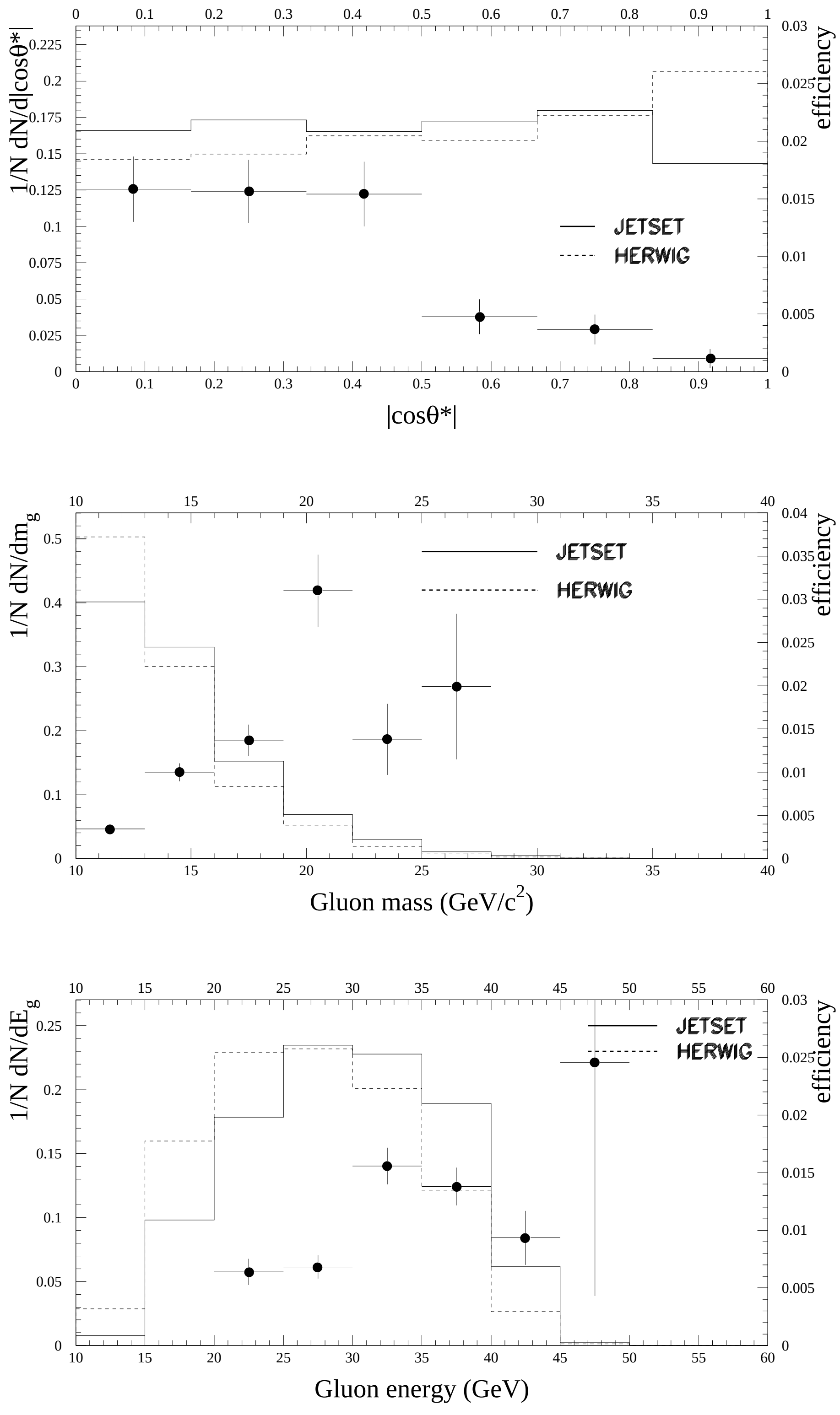

Figure 5: Initial signal distributions for the variables used in the reweighting. The points show the differential signal efficiencies (right ordinate).

Table 5: Systematic uncertainties on $g_{b\bar{b}}$.

Source	$\Delta g_{b\bar{b}}(10^{-3})$
Monte Carlo statistics	± 0.26
Bottom and Charm physics	± 0.32
Background topological cuts uncertainty	± 0.05
JETSET – HERWIG comparison	± 0.31
b quark mass	± 0.17
$g_{c\bar{c}} = (2.38 \pm 0.48)\%$	± 0.11
Tracking uncertainty	± 0.10
Total	± 0.57

is then taken as systematic error. The error on $g_{b\bar{b}}$ due to the tracking is then estimated as $\Delta g_{b\bar{b}} = \pm 0.10 \times 10^{-3}$.

The fraction of events that pass the cuts is well reproduced by the simulation, as shown in Table 2.

Table 5 summarizes the different sources of systematic error on $g_{b\bar{b}}$.

The final result is

$$g_{b\bar{b}} = (2.77 \pm 0.42(\mathrm{stat}) \pm 0.57(\mathrm{syst})) \times 10^{-3}.$$

7 Systematic checks

The jet tagging efficiency can be checked using data following the same procedure as in the R_b analysis [6]. A double tag was performed by using a cut in $-\log(CL) > 2.5$ for the two most energetic jets in 4 jet events. The average jet tag efficiency in the data was found to be $(28.44 \pm 0.35)\%$. The Monte Carlo efficiency is predicted to be $(27.67 \pm 0.14 \pm 2.7)\%$ (with a b purity of about 80%), where the first error is statistical and the second comes from uncertainties in the knowledge of the b hadron physics.

The tag efficiency in data is well reproduced, giving confidence on the simulation.

Figure 6 shows the angular separation of the third and fourth jets in data and simulation, together with the expected composition from Monte Carlo simulation. The Monte Carlo value for the $g_{b\bar{b}}$ is set to 2.77×10^{-3}. The distribution of the data is well reproduced by including the $\mathcal{B}$ event contribution in Monte Carlo simulation. The level of the background dominates the right hand side and is correctly reproduced.

The standard result is stable against variations of the $-\log(CL)$ and $\cos\theta_{12}$ and $\cos\theta_{34}$ cuts.

To further test the dependence on Monte Carlo models, the efficiency has been computed using the ARIADNE Monte Carlo [20], as it has been done for HERWIG in section 6. The process of the splitting of a gluon into a $b\bar{b}$ pairs is not straight forward to introduce into color dipole model, on which ARIADNE is based. In the default version of ARIADNE the rate of this process is larger than the theoretical calculation. As suggested in [1] and implemented as an option inside ARIADNE, the phase space of the gluon splitting process can be limited, so improving the agreement with the

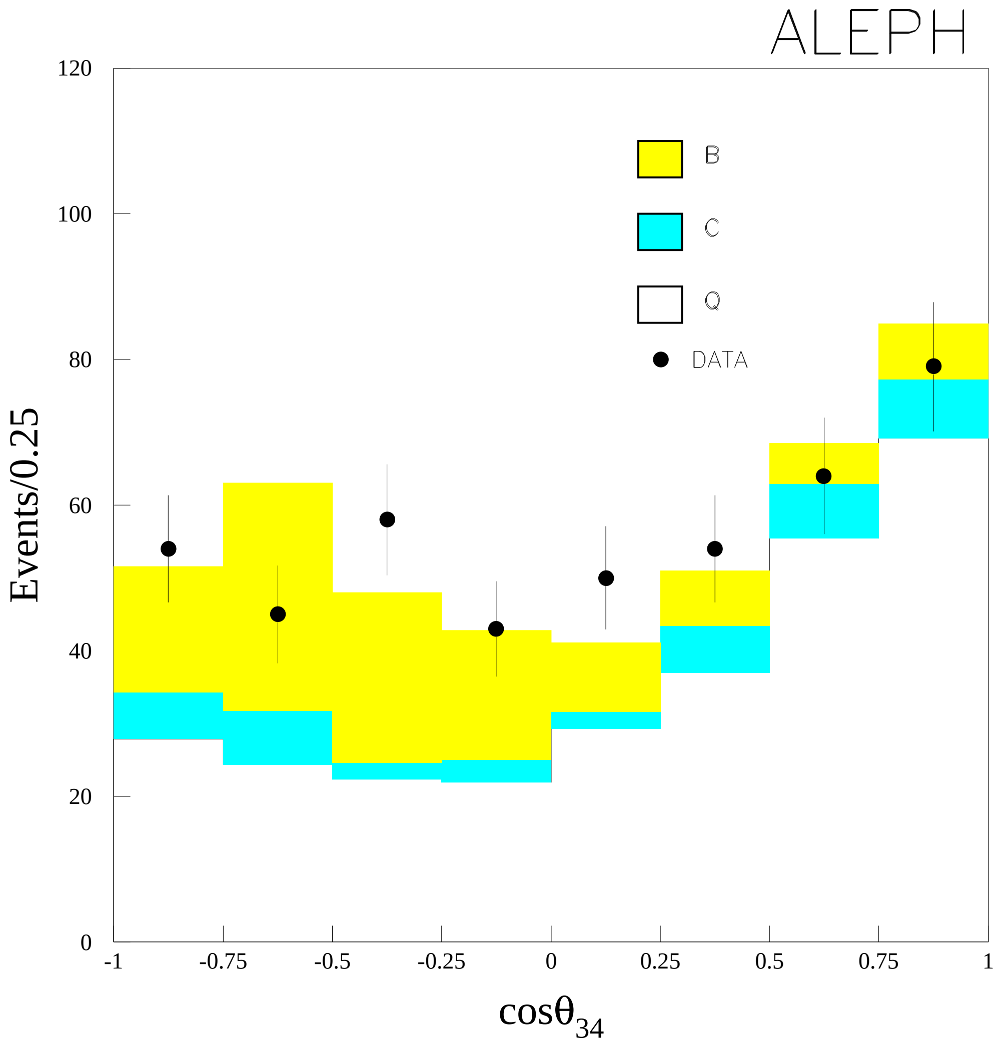

Figure 6: Angular distribution for the third and fourth jets in data and simulation, together with the expected background and measured $g_{b\bar{b}}$.

analytical calculations of [1]. Using this option, the efficiency $\epsilon_{\mathcal{B}}$ is only 6% smaller than the one found in JETSET.

A particularly interesting class of events in the signal is that with four b hadrons in the final state. To select these events cuts on $\log(CL)$ are applied for the first three jets:

$$-\log(CL)_1 > 3.1 \quad , \quad -\log(CL)_2 > 2.1 \quad , \quad -\log(CL)_3 > 1.0.$$

In addition, to suppress the largest background coming from Z $\rightarrow b\bar{b}$ events, the following cuts are imposed:

$$\cos\theta_{12} > -0.5 \quad , \quad \cos\theta_{34} < 0.2.$$

The number of selected events in the data is 259, of which 32 are in common with the standard selection. The cuts select four-b events with a signal purity of 65%. The result is $g_{b\bar{b}} = (3.6 \pm 0.6(\mathrm{stat}) \pm 0.8(\mathrm{syst})) \times 10^{-3}$, and the difference with the standard selection is $\Delta g_{b\bar{b}} = (0.8 \pm 0.9) \times 10^{-3}$, where correlations are taken into account in the calculation of the uncertainty. The statistical error is comparable to that obtained in the standard analysis, but the systematic error is much larger, mainly due to the uncertainties of hard gluon and b-decay charged multiplicities.

The requirement of at least one track per jet with a VDET hit was replaced with the condition that each jet should have $|\cos\theta_{\rm jet}| < 0.85$. The resulting change in $g_{b\bar{b}}$ is $\Delta g_{b\bar{b}} = (-0.15 \pm 0.31) \times 10^{-3}$.

Finally the analysis was repeated with a different value of the clustering parameter, $y_{\rm cut} = 0.01$. The difference in the result is $\Delta g_{b\bar{b}} = (+0.03 \pm 0.39) \times 10^{-3}$, where the error takes into account the uncorrelated data and simulation statistics.

8 Conclusions

A measurement of the gluon splitting rate to a $b\bar{b}$ pair in hadronic Z decays collected by ALEPH has been presented. The result is

$$g_{b\bar{b}} = (2.77 \pm 0.42(\mathrm{stat}) \pm 0.57(\mathrm{syst})) \times 10^{-3}.$$

The result is stable within errors with respect to the selection cuts. A consistent result has been obtained by selecting four b hadrons in the final state.

This result is also compatible with the theoretical expectations [2] and with the DELPHI measurement [5]: $g_{b\bar{b}} = (2.1 \pm 1.1(\mathrm{stat}) \pm 0.9(\mathrm{syst})) \times 10^{-3}$.

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