Ref.TH.1413-CERN

SOME CONSEQUENCES OF RESONANCE PRODUCTION ACCORDING TO THE SCALING HYPOTHESIS

L. Brink, W.N. Cottingham *) and S. Nussinov **)

CERN, Geneva

ABSTRACT

The scaling behaviour of particles coming from decay of resonances is investigated. It is shown that resonance production can give some characteristic features in inclusive distributions.

---

*) A visitor from the University of Bristol

**) On leave from the Tel-Aviv University, Tel-Aviv. Now at SUNY at Stony Brook. - Supported in part by the U.S. National Bureau of Standards.

Ref.TH.1413-CERN
5 October 1971

There seems to be growing evidence that the distribution of pions observed in inclusive reactions scales according to the formula [1)]

$$E\frac{d^2\sigma}{dP_T^2\,dP_L} = \frac{d^2\sigma}{dP_T^2\,dy} = F(x,P_T^2) \qquad (1)$$

where

$$x = \frac{2P_L}{\sqrt{s}} = \frac{2m_T}{\sqrt{s}}\sinh y$$

and y is the rapidity, $y = \log\left(\frac{E+P_L}{m_T}\right)$. m_T is the transverse mass, $m_T = \sqrt{m_\pi^2 + P_T^2}$. L stands throughout the text for longitudinal and T for transverse.

The assumption of scaling is that for large s $F(x,P_T^2)$ is dependent on s and P_L only through x [2)].

It is the purpose of this letter to show that if meson resonance production obeys the scaling law Eq. (1), then the pion spectrum resulting from the resonance decay will also substantially obey the scaling hypothesis, with, however, a very different scaling function. Consider as an example a ρ meson produced longitudinally with momentum P_L^ρ, which decays into two pions at an angle θ to the longitudinal direction in the ρ rest frame. In the centre-of-mass system the pions have momenta

$$P_L^\pi = \frac{1}{2}P_L^\rho \pm \frac{1}{2}\sqrt{P_L^{\rho 2} + m_\rho^2}\;\gamma\cos\theta$$
$$P_T^\pi = \frac{1}{2}m_\rho\gamma\sin\theta \qquad (2)$$

where

$$\gamma = \left(1 - \frac{4m_\pi^2}{m_\rho^2}\right)^{1/2} = 0.93$$

It can be seen from Eq. (2) that a backward-going pion from the decay of a fast ρ will have substantially less momenta than the ρ itself, $P_L^\pi \approx 0.07\,P_L^\rho$. This effect will produce a proportionally larger number of pions with low momentum.

Consider first a model in which ρ mesons (the discussion goes through in essentially the same manner for any resonance decaying into two pions) of given helicity are produced according to Eq. (1) and with zero transverse momentum. We then get

$$\frac{d^2\sigma}{dy_\rho \, dp_T^{\rho 2}} = F\left(\frac{2m_\rho \sinh y_\rho}{\sqrt{s}}\right) \; \delta(p_T^2) \tag{3}$$

It can be seen from Eq. (2) that a ρ with rapidity y_ρ will decay into two pions with rapidity

$$y_\pi = y_\rho \pm \frac{1}{2} \log\left(\frac{1+\gamma\cos\theta}{1-\gamma\cos\theta}\right) \tag{4}$$

The distribution in $\cos\theta$ is simply related to the helicity state of the ρ meson and is some function

$$g(\cos\theta)\, d\cos\theta = g\left(\left(1 - \frac{4p_T^{\pi 2}}{m_\rho^2\gamma^2}\right)^{1/2}\right) \cdot \frac{2\, dp_T^{\pi 2}}{(m_\rho^2\gamma^2 - 4p_T^{\pi 2})^{1/2} \, m_\rho \gamma} \tag{5}$$

The pions will then be distributed according to

$$\frac{d^2\sigma}{dy_\pi \, dp_T^{\pi 2}} = G(p_T^\pi)\left[F\left(\frac{2m_\rho}{\sqrt{s}} \sinh\left(y_\pi + \frac{1}{2}\log\frac{1+\gamma\cos\theta}{1-\gamma\cos\theta}\right)\right) + F\left(\frac{2m_\rho}{\sqrt{s}}\sinh\left(y_\pi - \frac{1}{2}\log\frac{1+\gamma\cos\theta}{1-\gamma\cos\theta}\right)\right)\right] \tag{6}$$

where θ is the angle for one of the pions and G can be obtained from Eq. (5). This can now be rewritten

$$E\frac{d^2\sigma}{dp_L^\pi \, dp_T^{\pi 2}} = G(p_T^\pi)\left\{F\left[\frac{m_\rho}{m_T}\left(\frac{2p_L^\pi}{\sqrt{s}}\cosh\left(\frac{1}{2}\log\frac{1+\gamma\cos\theta}{1-\gamma\cos\theta}\right) + \frac{2\sqrt{p_L^{\pi 2}+m_T^2}}{\sqrt{s}}\sinh\left(\frac{1}{2}\log\frac{1+\gamma\cos\theta}{1-\gamma\cos\theta}\right)\right)\right] + F\left[\frac{m_\rho}{m_T}\left(\frac{2p_L^\pi}{\sqrt{s}}\cosh\left(\frac{1}{2}\log\frac{1+\gamma\cos\theta}{1-\gamma\cos\theta}\right) - \frac{2\sqrt{p_L^{\pi 2}+m_T^2}}{\sqrt{s}}\sinh\left(\frac{1}{2}\log\frac{1+\gamma\cos\theta}{1-\gamma\cos\theta}\right)\right)\right]\right\} \tag{7}$$

For large $\sqrt{s}$ this becomes a scaling function

$$E \frac{d^2\sigma}{dP_L^\pi dP_T^{\pi 2}} = G(P_T^\pi)\left[F\left(\frac{2x}{1-\gamma\cos\theta}\right) + F\left(\frac{2x}{1+\gamma\cos\theta}\right)\right] \tag{8}$$

It is interesting to note from Eq. (7) that the scaling limit is approached most slowly at small P_L^π. More interesting, at small P_T^π this scaling function for the secondary pions will be the sum of two functions with quite different scales, coming from the mesons, which decay forward or backward. For example, for $P_T^\pi = 0$

$$E \frac{d^2\sigma}{dP_L^\pi dP_T^{\pi 2}} \approx G(P_T^\pi)\left[F(29x) + F(x)\right] \tag{9}$$

It is important to consider to what extent such substructure as given by Eq. (9) will be washed out by the ρ mesons, which are themselves produced with non-negligible transverse momenta.

If ρ meson production follows the scaling law, then at very high energies most of the ρ mesons will be produced with relativistic velocities. Consider such a relativistic ρ meson with longitudinal and transverse momentum P_L^ρ and $\vec{P}_T^\rho$. The resulting pions will have momenta

$$P_L^\pi = \frac{P_L^\rho}{2}(1+\gamma\cos\theta) \tag{10}$$

$$\vec{P}_T^\pi = \frac{\vec{P}_T^\rho}{2}(1+\gamma\cos\theta) + \vec{\pi}_T \tag{11}$$

where $\vec{\pi}_T$ is the transverse momentum of the pion in the rest frame of the ρ, again $|\vec{\pi}_T| = (m_\rho/2)\,\gamma\sin\theta$.

We can anticipate from experiment that the significant values of P_T are confined to the region $|P_T| < 0.3$ GeV/c. From the preceding discussion we know that it is the backward-going pions that produce the sharp substructure for small x and P_T^π. From Eq. (11) it can be seen that for P_T^ρ within the limits above and for small P_T^π ($P_T^\pi \lesssim 0.2$ GeV/c) the backward going pions are confined to

$$1+\gamma\cos\theta \lesssim 0.07 + \frac{2}{\gamma}\left[\frac{P_T^\pi + 0.04 P_T^\rho}{m_\rho}\right]^2 \tag{12}$$

This factor is almost as small as in the case of $P_T^{\rho} \equiv 0$ already considered and from Eq. (10) it can be seen that these pions will have a correspondingly small longitudinal momentum.

It would seem, therefore, that the structure at small P_T^{π} in the scaling function of the pions will be a genuine feature of primary ρ production. It will, of course, also be a feature of the scaling production of any resonances, but in particular the ρ , f and g mesons, which have two-pion decay modes. The effect could, however, be obscured somewhat at finite energies at small longitudinal pion momenta ($P_L^{\pi} \lesssim 0.5$ GeV/c) by the low momentum resonances for which Eqs. (10) and (11) are not correct. It is very likely that the scaling limit is approached more quickly for primary pion production than for resonance production because of the much larger masses of the resonances. If the direct pion production is the dominating feature of the process, one should be able to see the secondary pion production as a fine structure in the distribution function that eventually scales.

It should also be mentioned that it is possible to build up a rather steeply decreasing distribution with resonance distributions that are uniform [which is the result from the multiperipheral model [3)]], if resonance production is dominating.

We would like to thank D. Horn and M. Bishari for helpful discussions and we are very grateful to D. Horn for critically reading of the manuscript.

REFERENCES

1) L.G. Ratner et al., Phys. Rev. Letters 27, 68 (1971);
See also D. Horn, "Many-Particle Production", CERN preprint TH. 1387 (1971), to be published in Physics Reports.

2) R.P. Feynman, Phys. Rev. Letters 23, 1415 (1969);
C.N. Yang et al., Phys. Rev. 188, 2159 (1969) and Phys. Rev. Letters 25, 1072 (1970);
D. Amati, S. Fubini and A. Stanghellini, Nuovo Cimento 26, 896 (1962).

3) L. Bertocchi, S. Fubini and M. Tonin, Nuovo Cimento 25, 626 (1962);
D. Amati, S. Fubini and A. Stanghellini, Nuovo Cimento 26, 896 (1962).