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IMPLICATIONS OF THE EXISTENCE OF A HEAVY MUON-PROTON RESONANCE

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A B S T R A C T

An enhancement that might be due to a resonant effect has been experimentally found in the $\nu n \to \mu p$ process. The implications of such an effect are reviewed in some detail.

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1. - INTRODUCTION

A rather unexpected enhancement in the reaction

$$\text{muon neutrino } (\nu_\mu) + \text{neutron (n)} \rightarrow \text{muon } (\mu) + \text{proton (p)} \qquad (1)$$

has been found in the data of the CERN Heavy Liquid Bubble Chamber Group [1),2)]. Its statistical significance is not yet established, this paper will analyze it on the hypothesis that it might survive further neutrino experiment statistics. Section 2 is a résumé of the experimental situation. Section 3 deals with the connection of this effect with some theories of weak interactions. Section 4 analyzes the possibilities of detecting it elsewhere. Our conclusions are summarized in Section 5.

2. - RESUME OF THE EXPERIMENTAL SITUATION

The search for narrow resonances in the cross-section for process (1) is handicapped by the poor energy resolution associated with the spread of the neutrino momentum distribution and the Fermi motion of the neutron within the nucleus. A way to avoid this difficulty is to plot number of events against the invariant mass of the (μ,p) system, which can be measured with a much better resolution (typically ± 50 MeV). This has been done by Yoshiki et al. [1),2)] who found an enhancement at about $\sqrt{s} = 1.9$ GeV. This peak is ~ 3 standard deviations from the background attributed to the usual Fermi weak interaction. Furthermore, it appears in the same position in both freon and propane runs.

The extra number of events is of the same order $(\sim\frac{2}{3})$ as the background weak process, which has a cross-section $\sim 10^{-38}$ cm^2. The centre-of-mass muon distribution is strongly peaked forward (all events in the resonant bin lie in the forward hemisphere). If not just a statistical effect, such an enhancement would correspond to a "lepto-baryonic" resonance, with non-zero spin and the rather odd quantum numbers $L_\mu = B = 1$.

On denoting by g_μ and g_ν the coupling constants at the muon-proton and neutrino-neutron vertices (see Fig. 1), experiment also tells us that g_μ is not much smaller than g_ν. Otherwise a large amount of neutrons would be produced and detected at the resonant energy. The same holds for the possible strangeness non-conserving coupling of the leptobaryon, since only very few "strange" events have been observed in the whole experiment. The three-body final states do not present a clear resonant character. As a consequence, only the μp and νn decay channels of the possible resonance will be considered hereafter.

3. - THE YOSHIKI EFFECT AND THE TANIKAWA-WATANAKE-KINOSHITA-ITAMI-NAKAMURA-MUGIBAYASHI BOSON

The appearance of resonances in weak interaction processes like (1) has been theoretically conjectured by Kinoshita [3], in the framework of the weak interaction theory of Tanikawa and Watanabe (TW) [4]. In their theory, the weak interactions would be mediated by a scalar boson B, coupling to lepton-baryon pairs. The relevant Lagrangian for the process under study would be

$$\mathcal{L} = \mathcal{L}_\mu + \mathcal{L}_\nu \tag{1a}$$

$$\mathcal{L}_\mu = g_\mu \bar{\mu} (1-\gamma_5) p^c B \tag{1b}$$

$$\mathcal{L}_\nu = g_\nu \bar{n^c} (1+\gamma_5) \nu B^* \tag{1c}$$

where μ and ν are the muon and neutrino fields, p^c and n^c the charge-conjugate proton and neutron fields, respectively. The usual "V-A" four-field form of the weak interaction is reproduced from the second order transition $O(g_\mu, g_\nu)$, by the use of the Fierz transformation identity

$$\bar{\mu}\,(1-\gamma_5)\,p^c\;\bar{n^c}\,(1+\gamma_5)\,\nu \;=$$

$$= \frac{1}{2}\;\bar{n^c}\,\gamma_\sigma\,(1-\gamma_5)\,p^c\;\bar{\mu}\,\gamma_\sigma\,(1+\gamma_5)\,\nu \;= \qquad (2)$$

$$= \frac{1}{2}\;\bar{p}\,\gamma_\sigma\,(1+\gamma_5)\,n\;\bar{\mu}\,\gamma_\sigma\,(1+\gamma_5)\,\nu$$

From processes (such as all decays) in which the four-momentum squared of the boson is small compared with its squared mass, we may demand that

$$\frac{g_\mu g_\nu}{2M^2} \approx \frac{G}{\sqrt{2}} \approx \frac{1}{\sqrt{2}}\,10^{-5}\,m^{-2} \qquad (3)$$

where G = Fermi coupling constant, M = mass of the boson B, m = mass of the proton.

If we further assume $g_\mu = g_\nu = g$ and $M \sim 2m$, then (3) requires

$$g \sim 7.\times 10^{-3} \qquad (4)$$

The renormalization of the hadronic axial charge in the TW theory was attributed to strong interaction effects. The advantage of the TW scheme is its renormalizability. Its main draw back is that the V-A type of interaction is only obtained after a Fierz transformation, and well established properties of the usual approach, like CVC, would then seem rather artificial [3].

Kinoshita [3] made the obvious but important remark that a Lagrangian of type (1) would produce resonances in neutrino-neutron reactions. At resonance, the cross-section $\nu\,n \to \mu\,p$ would reach the value

$$\sigma = \frac{g_\mu^2 g_\nu^2}{(g_\mu^2 + g_\nu^2)^2} \frac{4\pi M^2}{(M^2 - m^2)^2} \sim 5.8\ 10^{-28}\ \text{cm}^2. \qquad (5)$$

where the muon mass has been neglected. The cross-section (5) is many orders of magnitude larger than the usual Fermi cross-section, which is $\sim 10^{-38}$ cm^2. However, a resonance with a coupling satisfying (3) (and with $g_\mu = g_\nu$) would be very narrow ($\Gamma \sim 10$ keV). Averaged over the experimental resolution Δ (say, 100 MeV), the observed cross-section is considerably lowered $\sigma_{obs} \sim \sigma \Gamma/\Delta \sim$ $\sim 5.10^{-31}$ cm^2. Since the possibly resonant cross-section observed is still many orders of magnitude smaller, it does not correspond to the TW boson.

Itami, Nakamura, Mugibayashi and Takinawa [5] have proposed a mechanism through which the cross-section [5] could be much smaller, the condition (3) still holding. Their proposal was to lift the restriction $g_\mu = g_\nu$. Taking [6] $g_\mu / g_\nu \sim 10^3$, condition (3) implies $g_\mu^2/4\pi \sim 4.10^{-3}$, which is smaller than the electromagnetic coupling and could have escaped detection [5]. The cross-section (5) diminishes by a factor $(g_\mu/g_\nu)^{-2} \sim 10^{-5}$ and there is hope to reach the magnitude of Yoshiki's effect. Unfortunately, the observed integrated cross-section is proportional to the total width of the resonance, $\Gamma (\sim g_\mu^2 + g_\nu^2)$, and σ_{obs} is reduced by only a factor $\sim 3.10^{-3}$. In fact, no value of g_μ/g_ν can be made consistent with both the experimentally observed cross-section and relation (3) [7]. A further criticism could be that very different values of the coupling of the resonance to the μ p and ν n channels seem unnatural. But, on the other hand, since such a resonance would anyhow be a completely new kind of effect, one has to recognize that such a reasoning is at best based on (presumably sound) prejudice.

The TWK boson, with spin zero, was in fact already excluded by the angular distribution. We have seen in this Section that the magnitude of the cross-section would also be grossly incorrect. Thus, we are forced to conclude that the Yoshiki effect is a totally new and "weaker-than-weak" phenomenon, outside the realm of present known theories.

4. - POSSIBLE EXPERIMENTAL CONSEQUENCES OF THE YOSHIKI BOSON

The purpose of the present Section is to take seriously the "leptobaryonic resonance" B found in $\nu n \to \mu p$ scattering, and study whether there is hope to detect this very weak effect elsewhere. Only better statistics in neutrino experiments will definitely tell us whether this is a sensible question. To begin with, let us study the information provided by the neutrino experiment itself.

The angular distribution of events excludes a spin zero effect, and we shall assume $J = 1$ [8] as the next simple possibility. The strongest forward peaking (experimentally favoured) is produced by a μp coupling of the form $\bar{\mu} \gamma_\sigma (1+\gamma_5) p^c$ [9], the νn coupling being restricted by the neutrino definite helicity to be $\overline{n^c} \gamma_\sigma (1+\gamma_5) \nu$. Using the Fierz identity :

$$\bar{\mu} \gamma_\alpha (1+\gamma_5) p^c \, \overline{n^c} \gamma_\beta (1+\gamma_5) \nu =$$

$$= -\frac{1}{2} (g_{\alpha\sigma} g_{\beta\tau} + g_{\alpha\tau} g_{\beta\sigma} - g_{\alpha\beta} g_{\sigma\tau}) \bar{p} \gamma_\sigma (1-\gamma_5) n \, \bar{\mu} \gamma_\tau (1+\gamma_5) \nu \qquad (6)$$

and remembering that such an expression will be contracted with the symmetric tensor $g_{\alpha\beta} + k_\alpha k_\beta / M^2$, one [5] deduces that the extra weak interaction induced by the new effect couples via a V+A current $\bar{p} \gamma_\sigma (1-\gamma_5) n$.

The ν experiment does not measure either the coupling constant g_μ or g_ν independently, but rather a relation between them. For a $J = 1$ resonance, at the resonance energy, we have

$$\sigma_{eff} \approx \sigma_{res} \frac{\Gamma}{\Delta} = \frac{g_\mu^2 g_\nu^2 (2M^2 + m^2)}{(g_\mu^2 + g_\nu^2) M^2 m \Delta} \tag{7}$$

which, for $M \approx 1.9$ GeV, $\sigma_{obs} \sim \frac{2}{3} 10^{-38}$ cm^2 and $\Delta \sim 100$ MeV means

$$\frac{g_\mu^2 g_\nu^2}{g_\mu^2 + g_\nu^2} \sim 7.5 \ 10^{-13} \tag{8}$$

Let us define G' as the coupling strength of the induced V+A interactions, in analogy with (3) :

$$G' = \frac{g_\mu g_\nu}{M^2} \sqrt{2} \tag{9}$$

From Eq. (8) we see that the minimum absolute magnitude of G' (g_μ and g_ν are taken to be real throughout) occurs when $|g_\mu / g_\nu| = 1$, with the value

$$\min |G'| \sim 5. \ 10^{-13} \ m^{-2} \sim \frac{G}{2.10^7} \tag{10}$$

indeed, a rather weak interaction.

Taking $g_\mu = g_\nu$ as an estimate of the minimum muon-proton resonance coupling (we know from experiment that $g_\mu \gtrsim g_\nu$) we find

$$\min \alpha_\mu \equiv \min \frac{g_\mu^2}{4\pi} \sim 10^{-13} \sim \frac{\alpha}{10^{11}} \qquad (11)$$

Here α ($\sim 1/137$), the electromagnetic fine structure constant is used for a comparison of coupling strengths. It seems clear at first sight that interactions as weak as (10) or (11) would not have been detected against stronger backgrounds. Nevertheless there exist experimental measurements that place very stringent limits on any new kind of interaction. We have in mind the anomalous magnetic moment of leptons. Also a weak but resonant effect can always be enhanced with a high resolution (in μ p scattering, say). We therefore proceed to study the upper limits placed on G' or α_μ by the former kind of experiments, and the accuracy to which one would have to push the latter experiment in order to independently detect a μ p resonance. Other processes are also considered at the end of the present Section.

A - <u>The muon anomalous magnetic moment</u>

The "g-2" factor of the muon has been measured to a great accuracy, [10]

$$\frac{1}{2}(g-2)\Big|_{exp} = (116616 \pm 31)\, 10^{-8} \qquad (12)$$

The theoretical calculation, in the present state of affairs, predicts [11]

$$\frac{1}{2}(g-2)_{th} = (116587 \pm 3)\, 10^{-8} \qquad (13)$$

in beautiful agreement with experiment. A μ p resonance would contribute to g-2 through the process shown in Fig. 2. In order not to spoil the agreement between (12) and (13) we want the contribution of Fig. 2 to be less than $\sim 3.10^{-7}$.

A vector coupling of the form $g_\mu \bar{\mu} \gamma_\sigma (1+\lambda\gamma_5) p^c$ is taken at the μ p B vertex. The calculation has to be made with minimal electromagnetic coupling at the photon vertex (forgetting the complications due to strong interactions), since no information exists on the off-mass-shell dependence of the proton electromagnetic form factors. A power counting argument on the graph of Fig. 2 would suggest a logarithmic divergence. The external muons being on their mass shell, the explicit calculation shows that this argument is misleading : the graph is in fact convergent. For the sake of simplicity, a power expansion in μ/m (muon over proton mass) is made. The results of a standard calculation are :

$$\frac{1}{2}(g-2)\Big|_{O(\mu/m)} = \alpha_\mu (1-\lambda^2)\frac{\mu}{m}\frac{1}{\pi}\int_0^1 dz \int_{-1}^{1} dx\, z^2 \frac{\left[xz + \frac{m^2}{4M^2}(1-xz)\right]}{\frac{M^2-m^2}{2m^2}(1-z^2)+1-xz} \tag{14a}$$

$$\frac{1}{2}(g-2)\Big|_{O(\frac{\mu}{m})^2} = \alpha_\mu (1+\lambda^2)\left(\frac{\mu}{m}\right)^2\frac{1}{\pi}\int_0^1 dz \int_{-1}^{1} dx\, z^2 \frac{\left[\frac{1}{8}\left(2+\frac{m^2}{M^2}\right)(z^2-2xz+x^2z^2)-1\right]}{\frac{M^2-m^2}{2m^2}(1-z^2)+1-xz} \tag{14b}$$

Note that in the experimentally favoured situation (i.e., $\lambda \sim 1$) the first order contribution (14a) vanishes [12]. Thus the upper bound on α_μ is less stringent than in the case where λ is very different from unity. Equation (14b), together with our desire not to spoil the quantum electrodynamic agreement between (12) and (13) provides the condition (for $\lambda \simeq +1$)

$$\max \alpha_\mu \approx \frac{\alpha}{100} \sim 10^{-4} \tag{15}$$

Combining (15) and the constraint (8) from the neutrino experiment, we get an upper bound to G' :

$$\max |G'| \approx 10^{-8} m^{-2} \approx \frac{G}{10^3} \qquad (16)$$

If we take these upper bounds to be the true value of the coupling constants, then the ratio g_μ / g_ν assumes a rather "radical" value of 4.10^4 !

The Table summarizes upper and lower limits, and their origin

$G/2.10^7$	(estimate from ν experiment)	$\lesssim$	$G' \lesssim G/10^3$	estimate from (g-2) plus ν experiment
$\alpha/10^{11}$	(estimate from ν experiment)	$\lesssim$	$\alpha_\mu \leq \alpha/100$	estimate from (g-2)
~ 1			$\leq g_\mu / g_\nu \leq 4.10^4$	

B - Muon-proton scattering

Quite an obvious place where to look for a μ p resonance is μ p $\rightarrow \mu$ p scattering [13]. Since the μ p $\rightarrow$ B $\rightarrow \mu$p process would have to be observed against an electromagnetic background, a much better resolution than in ν n $\rightarrow \mu$ p scattering is necessary. Let us study the possibility of detecting B in muon-proton scattering.

The observed [14] differential cross-section, at the resonance energy, for the process μ p $\rightarrow$ B $\rightarrow \mu$p is :

$$\left.\frac{d\sigma_{obs}}{d\Omega}\right|_{c.m.} = \frac{\alpha_\mu}{1+g_\nu^2/g_\mu^2} \frac{3}{\Delta 4 M^2 m (2M^2+m^2)} \left\{ (1+\lambda^2)^2 \left(M^2 [M^2+2m^2] + [M^2-m^2]^2 x^2 \right) + 8\lambda^2 M^4 x \right\} \qquad (17)$$

where Δ is the experimental resolution, x is the cosine of the scattering angle in the centre of momentum system and the μpB coupling is taken to be $\sim g_\mu B^\alpha \bar{\mu} \gamma_\alpha (1+\lambda\gamma_5) p^c$. The cross-section (17) is to be compared with the purely electromagnetic background [15]. Both are peaked forward, (17) being most peaked for $\lambda = \pm 1$ ($\lambda = +1$ is the situation favoured by ν n $\to \mu$ p data). The shape of (17) and the electromagnetic differential cross-section are such that there is an optimal angle at which their ratio is at its maximum. Using the experimental knowledge of the electromagnetic form factors of the proton [16] we can calculate that angle. It turns out to be (for $\lambda = +1$) $\theta_{lab} \sim 90^\circ$. At this angle, with $s = M^2$, the electromagnetic cross-section is $d\sigma(\text{lab}) \sim 1.2 \cdot 10^{-34}$ cm^2 $d\Omega$. On the other hand, the resonant cross-section ranges (for a 1 MeV energy resolution, say) from $1.2 \cdot 10^{-38}$ to $2.4 \cdot 10^{-29}$ cm^2 $d\Omega$, corresponding to the range of values of α_μ quoted in the Table. If g_μ / g_ν were about 40 ($\alpha_\mu \sim 1.6 \cdot 10^{-8} \alpha$), the deviation from the electromagnetic cross-section would be 20 % - a considerable "anomaly". If, however, $g_\mu \sim g_\nu$, a resolution of ~1 keV would be needed to observe a 10 % effect. Such an energy resolution would be of the order of 1 p.p.m. and does not seem easy to obtain experimentally. Of the μ^-p experiments done at present, only one [17] might have included the B particle in its energy range (muon momentum spread from 1.5 GeV/c to 6 GeV/c ; M = 1.94 GeV corresponds to 1.53 GeV/c), but the energy spread was rather large (~4.5 GeV) and the scattering angles were not the optimal. That no "anomaly" had been reported places some upper bound on α_μ. This bound is not better than the g-2 bound, however.

In conclusion, a typical μ^-p experiment (1 % energy resolution and 2 % precision in measuring σ) would not detect the Yoshiki boson unless g_μ / g_ν were larger than 20. This is a marked improvement, of course, over the bound obtained from g-2. Nevertheless, to be sure of detecting the effect (i.e., down to $g_\mu / g_\nu \sim 1$) in μp experiments, the energy resolution must be fantastically good.

C - Other processes

The elastic scattering of muons on protons described above essentially compares the magnitude of $\alpha_\mu M/\Delta$ with α^2. This would also be the case for photoproduction or strong production of muon pairs ($\gamma p \to p \mu^+ \mu^-$ or $\pi^+ n \to p \mu^+ \mu^-$, respectively). Experiments of the first kind have been reported [18], but the Yoshiki resonance does not lie in their kinematical region. The experiments are much more difficult than μp scattering. Similarly, their theoretical analysis in terms of a B resonance would be complicated due to strong interaction effects. Consequently we feel that both experimental and theoretical analysis are worthwhile only if the μp scattering data confirmed the existence of the resonance.

A μp resonance would also modify the energy levels in muonic atoms. The resonant amplitude is, at non-relativistic energies, once more disregarding strong interaction complications :

$$\mathcal{M} \cong \frac{i g_\mu^2}{(m+\mu)^2 - M^2} \bar{\mu}_2 \gamma_\mu (1+\gamma_5) p_2^c \bar{p}_1^c \gamma_\nu (1+\gamma_5) \mu_1 \left[g_{\mu\nu} - g_\mu g_{\nu_0} \frac{(m+\mu)^2}{M^2} \right]$$

$$\simeq \frac{i g_\mu^2}{(m+\mu)^2 - M^2} \chi_2^+(\mu) \chi_2^+(p) \left[1 + \vec{\sigma}_\mu \vec{\sigma}_p \right] \chi_1(\mu) \chi_1(p) \qquad (18)$$

where we have used the Fierz transformation, Eq. (6), and where we omitted some multiplicative mass-dependent factors of the order of unity. In Eq. (18), $\chi_{1,2}(\mu, p)$ represent Pauli spinors for initial and final muon and proton, and $\vec{\sigma}_{\mu,p}$ are the spin operators on the corresponding particles. At low energies, we can say that the amplitude (18) results from a repulsive potential

$$V(\vec{x}) \simeq \frac{g_\mu^2}{M^2 - (m+\mu)^2} \delta^3(\vec{x}) \left[1 + \vec{\sigma}_p \vec{\sigma}_\mu \right] \qquad (19)$$

This local interaction would add up as a small perturbation to the Coulomb force in a muonic atom. To first order in g_μ^2 (a very good approximation, according to our anomalous magnetic moment bound), the interaction, Eq. (19), produces a shift and a hyperfine splitting of the energy levels :

$$\Delta E = \frac{g_\mu^2 Z}{M^2-(m+\mu)^2} |\psi_{nlm}(0)|^2 \left(1 + \langle \vec{\sigma}_p \rangle \vec{\sigma}_\mu\right) \qquad (20)$$

where Z is the atomic number, ψ_{nlm} the muon wave function and $\langle \vec{\sigma}_p \rangle$ the nuclear average of the proton spin operator.

In muonic hydrogen, and for S waves (the ones that most penetrate the nucleus) the energy shifts are :

$$\Delta E_{(S)}^{(T)} = \binom{3}{7} \frac{\alpha_\mu}{n^3} \mu \frac{\mu^2}{M^2-(m+\mu)^2} \alpha^3 \qquad (21)$$

where T and S refer to the triplet and singlet states of the muon and proton spins and n is the principal quantum number.

The strongest shift occurs in the singlet ground level and has a value

$$\Delta E_s^1 \sim .9\, \alpha_\mu \text{ eV} \lesssim 10^{-1} \text{eV} \qquad (22)$$

where the bound is taken from the Table. There are no experimental data on muon-hydrogen spectral lines so far. The 2P-1S transition being $\Delta E \sim 1.9$ keV, the energy resolution needed to improve the (g-2) limit on α_μ would have to be $\sim \frac{1}{2} 10^{-7}$.

It is interesting to speculate what the effect of an e-p resonance similar to the μ p one would be on the normal hydrogen energy levels. These are measured to an accuracy of ~ 0.1 MHz [19]. Unfortunately, the energy shifts (21) are roughly proportional to the cube of the electron mass, and in the most favourable case are

$$\Delta E_s^1 \text{ (hydrogen)} \approx 30\, \alpha_e \text{ MHz} \qquad (23)$$

If $\alpha_e \lesssim \alpha/100$ too, it follows that the shift cannot be detected in the present data. It might at first seem surprising that an interaction whose coupling constant is 1% of the electromagnetic one produces such a small effect. The reasons for it are the mass ratios involved in Eq. (21) and the related fact that the interaction being local, its effect is very small (proportional to the muon's probability density at the origin).

Very accurate data (2 °/oo) are at present available on the spectral lines of heavy muonic atoms [20]. For low lying levels the effect of the electron cloud is small, but the nuclear finite size effects are most important. The energy shift (20) for a 1S state would be of the order

$$\Delta E_s' \approx 4 \alpha_\mu Z^4 \alpha^3 \mu \frac{\mu^2}{M^2-(m+\mu)^2} \qquad (24)$$

since the operator $1 + \vec{\sigma}_\mu \vec{\sigma}_p$ has expectation values of the order of unity. For $\alpha_\mu \sim 1/100$ and $Z \sim 80$, $\Delta E = 1.7$ keV. The $2P \rightarrow 1S$ transition has an energy of ~6 MeV and is measured with an experimental accuracy of ~9 keV. Even if the nuclear effects, as independently measured in electron scattering, could be exactly evaluated, a higher resolution would be needed to reach the upper limit on α_μ that we estimated from (g-2). On the other hand, Eq. (24) is an overestimate, since the finite nuclear volume and the corresponding spread of the muon's wave function (which is maximum at the origin) both tend to diminish ΔE_{1S}. It suffices to relax one of these approximations (the pointlike nucleus) to get a much smaller upper bound on the energy shift. Indeed, taking a square well distribution of nuclear matter

$$|\psi_N|^2 = \frac{3Z}{4\pi r_0^3} \theta(r_0 - r) \qquad (25)$$

with $r_0 \sim 1.07\, A^{1/3}$ [20] and changing $\delta(\bar{x})$ into $\delta(\bar{x}-\bar{y})$ in Eq. (19), we have

$$\Delta E_S' \leq \int d^3x\, d^3y\, V(\vec{x},\vec{y})\, |\psi_N(x)|^2\, |\psi_\mu(y)|^2 =$$

$$= \frac{4\alpha_\mu Z (\mu Z\alpha)^3}{M^2-(m+\mu)^2}\, \frac{3}{r_0^3} \int_0^{r_0} r^2 e^{-2\mu Z\alpha r}\, dr \qquad (26)$$

For $Z \sim 80$, $A \sim 200$ and $\alpha_\mu \sim \frac{\alpha}{100}$ the upper bound becomes 0.2 keV. This figure is even smaller than the present experimental precision in the induced nuclear gamma ray transitions that occur in presence of a 1S muon [21]. (Their energy is of the order of a few hundred keV and they have been measured to a ~1 keV accuracy.) Moreover, the extra isomer shifts are only proportional to the change of nuclear charge distributions. The extra hyperfine structure is damped relative to the estimate, Eq. (26), by a factor 1/Z, which is the order of the average proton spin operator. The muon-proton potential, Eq. (19), would also produce a fine muon spin-orbit splitting. The form of the spin orbit interaction is

$$V' = \frac{1}{2\mu^2}\, \frac{1}{r}\, \frac{dV}{dr}\, \vec{L}\vec{S} \qquad (27)$$

The strongest splitting is the $2P_{\frac{1}{2}} - 2P_{\frac{3}{2}}$ one. The energy difference is :

$$\Delta E \simeq \alpha_\mu \frac{3}{96} Z (Z\alpha)^5 \frac{\mu^2}{M^2-(m+\mu)^2}\, \mu \qquad (28)$$

which is two orders of magnitude smaller than the shift of the 1S line for the highest Z atoms, Eq. (24), six orders of magnitude smaller in hydrogen. There is, therefore, no chance to improve the limits of α_μ on grounds of an observable fine structure.

The hypothetical μ p resonance could also intervene in several decay processes. If it were present in the μ p system, but not in e-p, for instance, it would spoil the μ e universality prediction for the ratio $(\pi \to \mu\nu)/(\pi \to e\nu)$. The deviation would be of the order G'/G. Recalling that max G' $\sim$ G/1000, we see that lowering the present experimental errors [22] of 3% would not be a worth while method towards measuring G'. The same holds for the ratio $(\rho \to \mu^+\mu^-)/(\rho \to e^+e^-)$. The deviation from the $\rho \to \gamma \to$ lepton pair phase space ratio is of the order of α_μ / α_{em} ($<10^{-2}$ from g-2) also inside the experimental [22] errors (~20%).

The situation in $K_L \to \mu^+\mu^-$ decay is more interesting. The usual understanding of this decay proceeds via a two photon intermediate state, providing a decay rate $\Gamma \propto G^2 \alpha^4$. An "estimated rigorous lower limit" [23] exists :

$$R_{th} = \left.\frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K_L \to \text{all})}\right|_{th} \gtrsim 6 \cdot 10^{-9}$$

A recent experimental lower limit [24] is $R_{exp} < 2.1 \cdot 10^{-7}$. A possible diagram to which B contributes is shown in Fig. 3a. Figure 3b shows a similar diagram via a strangeness non-conserving coupling of B, if it exists. If the contribution, Fig. 3a, is not to add to the electromagnetic transition in a way that spoils the agreement with the experimental lower limit, a very rough order of magnitude calculation imposes

$$G^2 \alpha_\mu^2 \left(\frac{m_\mu}{m_K}\right)^2 + G^2 \alpha^2 \alpha_\mu \frac{m_\mu}{m_K} \lesssim G^2 \alpha^4 \frac{R_{exp}}{R_{th}} \qquad (29)$$

or $\alpha_\mu < \alpha/10$, worse than our g-2 estimate, that could moreover be made convergent [25]. For the contribution of Fig. 3b, a similar argument places a limit on the product $g_\mu g_s$, where g_s is the strangeness changing coupling of the resonance. Explicitly,

$$\alpha_\mu \alpha_s \left(\frac{m_\mu}{m_K}\right)^2 + \alpha^2 \sqrt{\alpha_\mu \alpha_s}\, G\, m_K^2 \frac{m_\mu}{m_K} \lesssim \frac{R_{exp}}{R_{th}} G^2 \alpha^4 \qquad (30)$$

The relation (30) tells us that, if g_μ is big (close to the upper limit of the Table), the new strangeness changing coupling would have to be extremely small ($\alpha_s \equiv g_s^2/4\pi < \alpha/10^{11}$). If g_μ is minimal, the restriction on α_s is weaker than the one obtained from the ν experiment itself (few "strange" events, $g_\mu \sim g_\nu$ implies $\alpha_s \lesssim \alpha/10^{12}$).

Finally, we should mention that, if indeed there was a μp resonance not existing in the e-p system, it would contribute to the μ e mass difference with the right sign. Unfortunately the leading contribution would be quadratically divergent : $m_\mu - m_e = m_e \frac{3}{2} \alpha_\mu \left(\frac{\Lambda}{M}\right)^2$, where Λ is a cut-off.

5. - CONCLUSIONS

The hypothetical μp resonance is an extremely weak effect. It is not the one predicted by Kinoshita in the framework of the Tanikawa-Watanabe theory, even in its modified versions. It would be extremely difficult to detect elsewhere, unless an unconventionally high ratio of its decay rates into the μp and νn channels is assumed. In this case, its strangeness changing couplings would have to be relatively very small.

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6) Having $g_\mu/g_\nu \ll 1$ would imply a flood of ν n final pairs. This is not observed (see comments in Section 2).

7) If g_μ/g_ν were higher than 10^4 and constraint (3) is still assumed to be valid, then the resonant shape of $\Gamma(B \to \mu p)$ would have been resolved.

8) The forward peaking, with the present statistics, seems to be even more intense than what can be reached with a spin 1 resonance at this mass (maximum Forward/Backward $\simeq 4$).

9) The maximal forward and backward peakings respectively correspond to $1 \pm \gamma_5$ type of couplings, i.e., maximum parity violation.

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12) A lower limit of the first order contribution would be $\sim \alpha\ (1-\lambda^2)\ 10^{-2}$.

13) Electron-proton scattering would be the place to look for its possible partner, since high energy electron-neutrino beams are not available. Experiments looking for a TW boson in electron-proton scattering have been performed :

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14) By "observed" we mean, again, the physical cross-section averaged over the resolution,

15) When the width of the resonance is much smaller than the experimental energy resolution, as will always be the case in our analysis the interference term might be neglected. Its contribution, when averaged over the width of typical energy resolution errors, is small compared to the contributions from the incoherent terms.

16) Dipole formulae $G(q^2) = \left[\frac{1}{1+q^2/0.71\text{GeV}^2}\right]^2$ are used.

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25) A Fierz transformation of the type of Eq. (6) shows that the decay would be forbidden by angular momentum conservation if $m_\mu = 0$. Thus, one might expect a suppression factor m_μ/m_K in the amplitudes. There is of course no model independent way of knowing its exact value. Its presence makes the bound worthless. We have also taken a constructive interference with the electromagnetic amplitude. Destructive interference would not affect the rough bound, unless something enhances it.

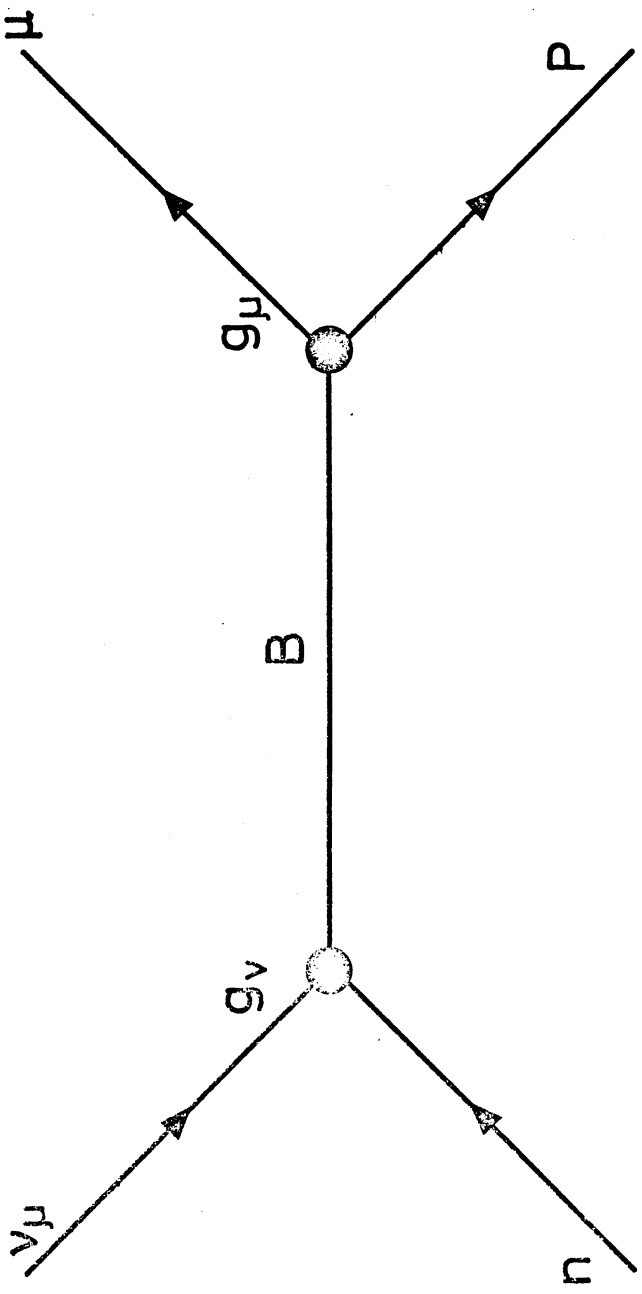


FIG.1

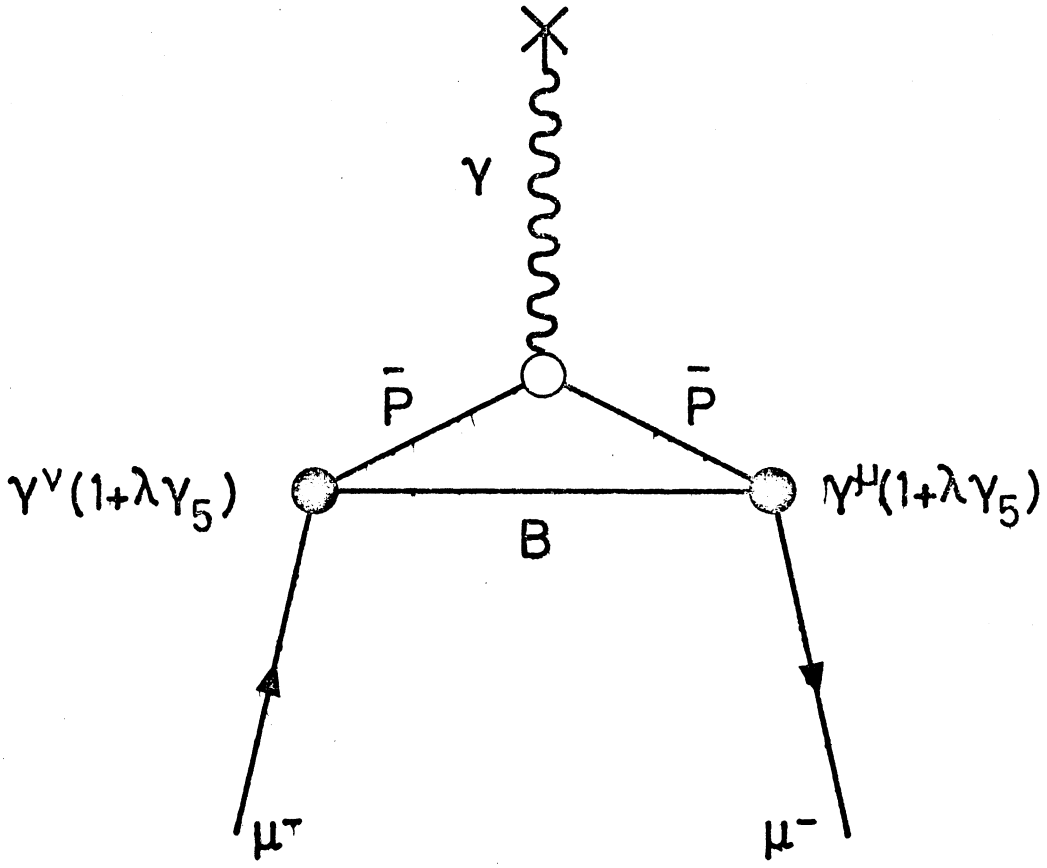
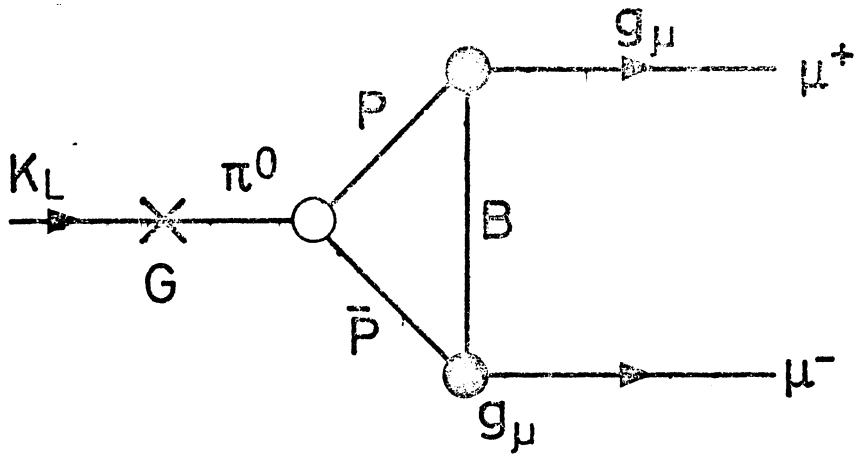
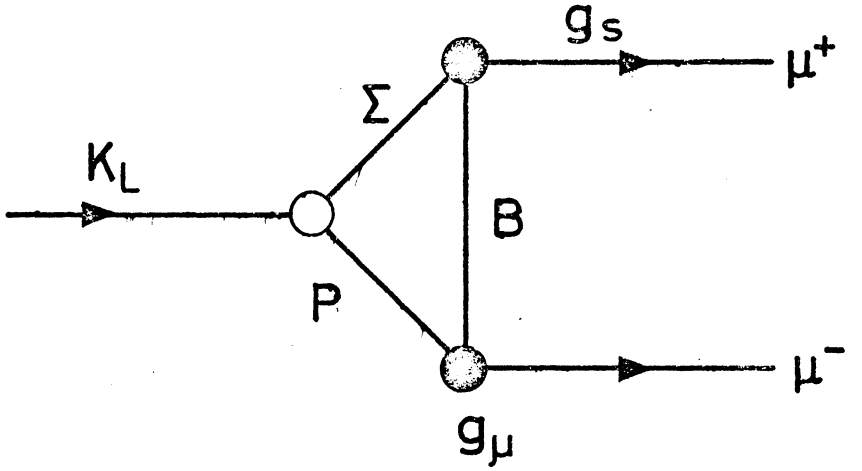

γ
$\bar{P}$
$\bar{P}$
$\gamma^\nu(1+\lambda\gamma_5)$
$\gamma^\mu(1+\lambda\gamma_5)$
B
μ^+
μ^-

FIG. 2



3a



3b

FIG. 3