# R-charge Kills Monopoles

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# Abstract

Large charge density, unlike high temperature, may lead to nonrestoration of global and gauge symmetries. Supersymmetric GUTs with the appealing scenario of unification scale being generated dynamically naturally contain global continuous R symmetries. We point out that the presence of a large R charge in the early Universe can lead to GUT symmetry nonrestoration. This provides a simple way out of the monopole problem.

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#### I. INTRODUCTION

The existence of magnetic monopoles is one of the most beautiful aspects of the idea of Grand Unification. Unfortunately, due to their super large mass and overproduction in the early Universe, this at the same time represents a cosmological catastrophe known as the monopole problem [1]. The conventional solution to this problem could be divided into three categories: inflation [2], Langacker-Pi mechanism [3] and symmetry nonrestoration [4]. Recently, another interesting scenario has been suggested in which the monopoles are swept away by domain walls [5].

In this paper we focus on the symmetry nonrestoration scenario, which in itself is a fascinating phenomenon that defies common intuition [6,7]. Unfortunately, it may not work in gauge theories due to the large next to leading order correction [8]. On the other hand it has been known that a large background charge density provides a natural setting for the breakdown of gauge symmetries in the early Universe [9–11]. We showed recently that this may provide a simple solution of the monopole problem based on the simple extension of the standard model and a large lepton number [12].

In recent years supersymmetric Grand Unified Theories (GUTs) have become increasingly more popular for two fundamental reasons. Low energy supersymmetry naturally protects large mass hierarchies and, equally important, it leads to the unification of gauge couplings [13]. It is thus a particular challenge to solve the monopole problem in supersymmetric GUTs. In this context a large background charge may be important for it has been shown recently that it provides high temperature symmetry nonrestoration in supersymmetry too [14]. The point is that without any external charge, in supersymmetry the internal symmetries are necessarily restored [15,16]. This is true even when one includes the higher dimensional operators [17], in spite of some interesting attempts on the contrary [18].

In this letter we point out that the role of the background charge of the Universe may be naturally played by global R charges. Our motivation and inspiration lies in the simple well known fact that the Minimal Supersymmetric Standard Model (MSSM) possesses a global  $U(1)_R$  symmetry. Actually, this is true even in the general case when all the R-parity breaking terms are allowed [19]. Of course, the soft supersymmetry breaking terms in the potential also break this  $U(1)_R$  symmetry, so that today the background R charge of the Universe would have necessarily been washed out. However, at very high temperature their effects get suppressed [14] so that it is perfectly sensible to speak of possibly large and conserved R charge in the early Universe. This is not the whole story though, since this R charge must also be compatible with Grand Unification. Our work is devoted precisely to this issue. In the following sections we argue that  $U(1)_R$  symmetries are naturally present in supersymmetric GUTs, which generate large mass hierarchies dynamically. In such theories there may be no monopole problem whatsoever.

In what follows we first give a simple example of a gauge model with an automatic global  $U(1)_R$  symmetry and discuss the connection between the large R charge and symmetry nonrestoration in the early Universe. We then turn to the minimal supersymmetric SU(5) theory and its simple extensions which incorporate our scenario.

### II. A PROTOTYPE TOY EXAMPLE

In order to illustrate our mechanism we discuss the simplest supersymmetric gauge model, that is the supersymmetric QED with coupling constant g [14]. The minimal spectrum consists of the chiral superfields  $\Phi_+$  and  $\Phi_-$  with gauge charges +1 and -1 respectively and with the most general renormalizable superpotential

$$W = m\Phi_{+}\Phi_{-} . \tag{1}$$

This model possesses an automatic R symmetry,

$$\Phi_{\pm} \to e^{i\alpha} \Phi_{\pm} , \ \theta \to e^{i\alpha} \theta .$$
(2)

Following the reference [14] we assume that there is a non vanishing background density  $n_R$  of the  $U(1)_R$  charge. The effective potential at high temperature and high density can be computed using the usual techniques [20,21]:

$$V_{eff}(n_R, T) = g^2 T^2 \phi^2 + \frac{3 n_R^2}{5T^2 + 24\phi^2} , \qquad (3)$$

where  $\phi = \phi_+ = \phi_-$  is easily seen to be the minimum (this explains the vanishing of the D-term in the potential). It is easy to see that for

$$n_R > n_R^{crit} = \frac{5g}{6\sqrt{2}}T^3 \tag{4}$$

 $\phi$  gets a nonzero vacuum expectation value (vev) and breaks the U(1) gauge symmetry.

This simple example illustrates perfectly the general situation: if the field in question carries an R charge, for sufficiently large values of this charge, the gauge symmetry will be spontaneously broken even at high temperature. This phenomenon takes place because the charge cannot entirely reside in the thermal excited modes if the conserved charge stored in the system is larger than some critical value: the charge must flow into the vacuum and this is an indication that the vev of the charged field is non-zero. A natural candidate for our considerations is represented by the superheavy Higgs field in the adjoint representation of a GUT theory to which we now turn our attention.

## III. GRAND UNIFICATION AND GAUGE CHARGES

In what follows we shall discuss theories made on SU(N) groups. Of course, the minimal supersymmetric SU(5) model is of our primary interest, but it will turn out that we must go beyond it. To set the discussion and to facilitate the computations we now discuss SU(N) models in general.

With the superheavy Higgs superfield  $\Phi$  being in the adjoint representation, the superpotential takes the form

$$W = m \operatorname{Tr} \Phi^2 + \lambda \operatorname{Tr} \Phi^3. \tag{5}$$

For m=0 the theory has a  $U(1)_R$  global symmetry

$$\Phi \to e^{i\alpha} \Phi \quad , \quad \theta \to e^{i3\alpha/2} \theta.$$
 (6)

The trouble is that  $m \neq 0$  is necessary in order to achieve the possibility of non vanishing vev  $\langle \Phi \rangle \neq 0$  at zero temperature. However, for temperatures T larger than m one could still hope that the rate  $\Gamma_R$  at which R-violating processes take place is slower than the expansion rate of the Universe, H.

Now, for  $m \neq 0$ , just as in the toy example, we could achieve a non vanishing vev  $\langle \Phi \rangle$  for  $T \gg m$  if the background R charge of the Universe is large enough. Then  $\langle \Phi \rangle \sim n_R^{1/3} \sim T$ . By looking at the equation of motion of the R charge density is it easy to convince oneself that the rate of the R-violating processes is as fast as  $\Gamma_R \simeq \Lambda^2/T$  where

$$\Lambda^4 \simeq m \langle \Phi(T) \rangle^3 \simeq mT^3, \tag{7}$$

and thus

$$\Gamma_R \simeq \sqrt{mT}$$
. (8)

For  $m \sim 10^{16}$  GeV and obviously for T larger than  $10^{16}$  GeV there is an epoch when  $\Gamma_R > H \sim \sqrt{g_*}T^2/M_P$ , and thus any previous R charge could have been washed out. However, we cannot guarantee this without the precise computation of the wash-out rate. More important, at temperatures of the order of the GUT scale thermal equilibrium is not easy to attain and all the phenomena, including the GUT phase transition leading to the possible formation of monopoles, may have taken place out-of-equilibrium.

The above theory may not work. On the other hand, it suffers from a serious drawback: the large GUT scale m is put by hand.

A more complete theory should try to compute the above ratio, in which case m should be determined dynamically. This philosophy fortunately cries out for a global R-symmetry.

# A. SUSY GUTs with a dynamical determination of the unification scale

Here the philosophy is very simple. One eliminates the mass term from the superpotential and attempts to compute the ratio of the GUT and the electroweak mass scales dynamically. Here the results depend dramatically on whether N of SU(N) (where N > 4 in realistic theories) is even or odd, as we describe now. For m = 0 and for a diagonal  $\Phi_{ij} = \phi_i \delta_{ij}$  which makes the D-potential vanish,  $F_i = 0$  imply  $\phi_i^2 = \phi_0^2$ . Since the trace of  $\Phi$  is zero, it is easy to see that for odd N there is only a trivial solution  $\phi_0 = 0$ , and this case will be treated separately in detail for the physically relevant case of SU(5).

On the other hand, for N even (N=2n) the solution has the form

$$\langle \Phi \rangle = \phi_0 \operatorname{diag}(I, -I) , \qquad (9)$$

where  $\phi_0$  denotes the flat direction and I is the  $n \times n$  unity matrix. Thus for  $\phi_0 \neq 0$  the original SU(2n) symmetry is broken down to SU(n)×SU(n)×U(1). Obviously, the minimal such theory which contains the standard model is based on SU(6) gauge group. This flat direction is a characteristic of the R-symmetry above and it is lifted with the soft supersymmetry breaking terms, the same terms that break the R symmetry. As is well known,

along the lines of ref. [22], these soft terms then induce a large vev  $\phi_0 = M_X \simeq 10^{16} GeV$  through radiative corrections along the flat direction. Thus, this is a perfectly consistent and realistic scenario with a dynamical generation of the GUT scale [23].

Now, as we said in the introduction, we must make sure that the original SU(2n) symmetry is not restored at the temperatures above the GUT scale. Namely, this is the scale which corresponds to the usual monopole production, since it is at this scale that the U(1) symmetry appears first. The effective potential at high temperature and high density in this case is

$$V = \frac{\lambda^2}{2} \left[ \text{Tr} \left( \Phi^2 \Phi^{\dagger 2} \right) - \frac{1}{N} \text{Tr} \left( \Phi^2 \right) \text{Tr} \left( \Phi^{\dagger 2} \right) \right] + g^2 \text{Tr} \left( \left[ \Phi, \Phi^{\dagger} \right]^2 \right)$$

$$+ \left[ \left( \frac{N^2 - 4}{16N} \right) \lambda^2 + Ng^2 \right] T^2 \text{Tr} \left( \Phi \Phi^{\dagger} \right) + \frac{n_R^2 / 2}{3(N^2 - 1)T^2 / 4 + 4 \text{Tr} \left( \Phi \Phi^{\dagger} \right)}$$

$$(10)$$

It is easy to see that for R-charge density  $n_R$  bigger than the critical

$$n_R^{crit} = \frac{3}{4}(N^2 - 1) \left[ \left( \frac{N^2 - 4}{32N} \right) \lambda^2 + \frac{Ng^2}{2} \right]^{1/2} T^3$$
 (11)

the symmetry breaking is in the same direction as at T=0 (9) with  $\phi_0$  now given by

$$\phi_0^2 = \frac{3}{16} \left( \frac{N^2 - 1}{N} \right) \left( \frac{n_R - n_R^{crit}}{n_R^{crit}} \right) T^2 . \tag{12}$$

Notice that the direction of the vev of  $\Phi$  is fixed by the supersymmetric terms  $V_F$  and  $V_D$  (the first line in (10)) and thus obviously has the same form  $\langle \Phi \rangle = \phi_0 \operatorname{diag}(I, -I)$  as at zero temperature. In this case  $V_F$  and  $V_D$  play no role in determining the critical density and the magnitude  $\phi_0$ .

The fact that the vacuum has the same form at all temperatures is a remarkable fact and it provides a solution to another serious problem of supersymmetric GUTs. Namely, in the usual minimal GUTs with degenerate minima at zero temperature and symmetry restoration at high temperature the preferred high T vacuum is for  $\langle \Phi \rangle = 0$ . Obviously, the system prefers to remain in this vacuum even at T = 0. In our case no such problem exists.

In other words, for  $n_R > n_R^{crit}$  there is no phase transition whatsoever: as the Universe cools down below  $T \simeq M_X$  the Higgs field remains in the same broken phase. Notice that for the monopole problem it is not really essential that the direction of symmetry breaking is the same at high and low temperature. Even if these directions were different the rank of the broken group would be the same since the adjoint representation cannot change the rank of the original group. Thus, there would be in any case explicit U(1) factor below and above the GUT scale. This is sufficient for the solution of the monopole problem, as we discuss at the end of this section. The crucial point here is that unlike in the minimal SU(5) theory at T=0 there is a flat direction but the direction is unique. It is enough to eliminate the  $\Phi=0$  minimum at high T (as in the case of large charge density) and the T=0 minimum is necessarily in the right direction.

#### B. Realistic models

**SU(5) model.** We look first for a situation in the SU(5) theory in which m, instead of being put in by hand, is the vev of a singlet field S. An obvious attempt,  $W = S \operatorname{Tr} \Phi^2 + \operatorname{Tr} \Phi^3$ , does not work, for it implies  $\langle \Phi \rangle = 0$ . We must go beyond the minimal model and the simplest extension is to postulate another adjoint superfield  $\tilde{\Phi}$  with a superpotential

$$W = \lambda_1 S \operatorname{Tr} \tilde{\Phi} \Phi + \lambda_2 \operatorname{Tr} \tilde{\Phi} \Phi^2 . \tag{13}$$

Obviously, at T=0 one of the degenerate minima is

$$\langle \Phi \rangle = \frac{\lambda_1}{\lambda_2} \langle S \rangle \operatorname{diag}(2, 2, 2, -3, -3).$$
 (14)

with  $\langle \tilde{\Phi} \rangle = 0$  and  $\langle S \rangle$  undetermined. Of course, among other minima there is also  $\langle \Phi \rangle$  in the diagonal direction (1,1,1,1,-4). From the toy model example of the previous section the reader can easily deduce what happens at high temperature. Needless to say, we assume again a large R charge background density of the Universe. As our fields  $\Phi$  and S carry non vanishing R charges just as in the previous case for  $n_R$  sufficiently large, they will have nonvanishing vevs even for temperatures much above  $10^{16}$  GeV. The critical value  $n_R^c$  can be easily computed following the previous calculation.

The above superpotential has two continuous U(1) R-symmetries:

$$i) \Phi \to e^{i\alpha} \Phi , S \to e^{i\alpha} S , \tilde{\Phi} \to \tilde{\Phi} , \theta \to e^{i\alpha} \theta ;$$
 (15)

$$ii) \Phi \to \Phi , S \to S , \tilde{\Phi} \to e^{i\alpha}\tilde{\Phi} , \theta \to e^{i\alpha/2}\theta .$$
 (16)

with corresponding charge densities  $n_R^{(1)}$  and  $n_R^{(2)}$ . In what follows we shall take  $n_R^{(1)} \equiv n_R \neq 0$  and  $n_R^{(2)} = 0$ . Now for us it is crucial to establish the nonrestoration at high temperature, but the precise value of the vevs is not so important. It has a generic form as in the example SU(2n); however since in this case it gets to be very complicated, we will not present it. Instead, we shall establish the fact of symmetry breaking and give the critical charge density.

The effective potential is

$$V = V_F + V_D + \Delta V_T + V_n , \qquad (17)$$

with

$$V_F = \frac{\lambda_1^2}{2} |S|^2 \operatorname{Tr} \left( \Phi \Phi^{\dagger} \right) + \frac{\lambda_1 \lambda_2}{2} \left[ S \operatorname{Tr} \left( \Phi \Phi^{\dagger 2} \right) + S^* \operatorname{Tr} \left( \Phi^{\dagger} \Phi^2 \right) \right] + \frac{\lambda_2^2}{2} \left[ \operatorname{Tr} \left( \Phi^2 \Phi^{\dagger 2} \right) - \frac{1}{5} \left| \operatorname{Tr} \Phi^2 \right|^2 \right] ,$$
(18)

$$V_D = g^2 \text{Tr} \left( \left[ \Phi, \Phi^{\dagger} \right]^2 \right) , \qquad (19)$$

$$\Delta V_T = \frac{T^2}{8} \left[ (\lambda_1^2 + \frac{21}{5}\lambda_2^2 + 40g^2) \text{Tr} \left( \Phi \Phi^{\dagger} \right) + 12\lambda_1^2 |S|^2 \right] , \qquad (20)$$

$$V_n = \frac{n^2/2}{49T^2/3 + 2|S|^2 + 4\text{Tr}\left(\Phi\Phi^{\dagger}\right)},$$
(21)

where we already used  $\langle \tilde{\Phi} \rangle = 0$ . We assume that  $\Phi$  can be diagonalized, which minimizes  $V_D$ . The critical charge density above which the adjoint  $\Phi$  gets a nonzero vev and so the symmetry gets broken can be calculated straightforwardly. If  $115\lambda_1^2 - 21\lambda_2^2 - 200g^2 > 0$  we get

$$n_R^{crit} = \sqrt{\frac{3}{2}} \frac{\lambda_1 T^3}{30} \left[ 835 - 63 \left( \frac{\lambda_2}{\lambda_1} \right)^2 - 600 \left( \frac{g}{\lambda_1} \right)^2 \right] , \qquad (22)$$

while for  $115\lambda_1^2 - 21\lambda_2^2 - 200g^2 < 0$  the solution is

$$n_R^{crit} = \frac{49}{12} T^3 \left( \lambda_1^2 + \frac{21}{5} \lambda_2^2 + 40g^2 \right)^{1/2} . \tag{23}$$

We have establish thus that the symmetry remains broken at temperature above the GUT scale. The question is in which of the two possible directions (2, 2, 2, -3, -3) or (1, 1, 1, 1, -4) the symmetry breaking takes place. As far as the monopole problem is concerned this is of no importance, for in any case we will have a U(1) factor even above the critical temperature. However, since the tunneling from the one minimum to the other is too slow, one must simply assume that we start with the correct vacuum (this does not have to be a global minimum).

Our model serves to illustrate the essential role that R symmetries play, but need not be taken as a final theory. The crucial outcome lies in the fact that the GUT symmetry would not be restored. Notice that, in all the above we have assumed unbroken supersymmetry. When supersymmetry is softly broken,  $U(1)_R$  gets also explicitly broken because of the presence of soft trilinear scalar couplings in the Lagrangian. Therefore, the associated net charge vanishes [14]. However, this takes place at temperatures much below the GUT scale so that no phase transition and subsequent monopole production may occur.

Notice further that the demand of R symmetry on a full theory implies that the light Higgs and matter superfields transform non trivially under it. Thus at high temperature their vevs should also be non vanishing leading to an upside-down scenario of more symmetry breaking in the early Universe.

SU(6) model. As we said above, we can as well enlarge the gauge group. This is not just model building; in the minimal SU(6) GUT with the single adjoint representation the idea of the dynamical generation of the mass scale works automatically. The point is that its superpotential without the mass term

$$W = \lambda \operatorname{Tr} \Phi^3 \tag{24}$$

in this case has a nontrivial solution. Namely, this is the situation described above for SU(2n) with n=3. Thus we can immediately write down the critical value for the R-charge density,

$$n_R^{crit} = \frac{35}{8} \left( 18g^2 + \lambda^2 \right)^{1/2} T^3 ,$$
 (25)

and the vev at high density and temperature  $(T \gg M_X)$ 

$$\phi_0^2 = \frac{35}{32} \frac{n_R - n_R^{crit}}{n_R^{crit}} T^2 \ . \tag{26}$$

Once we have established the phenomenon of nonrestoration the solution to the monopole problem is almost automatic. The discussion proceeds along the lines of [4]. First, the nonrestoration of symmetry eliminates the essential cause of the problem, which is the overproduction of monopoles [1] during the phase transition via the Kibble mechanism [24]. This of course is not sufficient to claim the solution for one must worry about the thermal production at the temperature above the GUT scale [25]. Fortunately this can be easily shown to be under control as in the case of [4].

One more important comment. The reader may worry about the creation of monopoles even without the phase transition, since in any case the Higgs field is expected to take random values for correlations bigger that the horizon. Here one must resort to the idea of primordial inflation which presumably took place before, say at the Planckian scales. In such a case the whole Universe should have started from a causally connected region with the uniformly oriented Higgs field  $\Phi$ .

### IV. DISCUSSION AND OUTLOOK

It has been known for a long time that a large background charge density in the Universe can induce symmetry breaking at temperatures much above the physical mass scale of the system. This, among other implications may have important impact on the monopole problem. Now, many supersymmetric models are characterized by global R symmetries, and in fact the supersymmetric standard model possesses an automatic such U(1) symmetry. In this letter, we have shown that a corresponding sufficiently large charge asymmetry will cause GUT symmetry breaking even above the unification scale. This then leads to the solution of the monopole problem.

As in the minimal supersymmetric SU(5) model there is not any global R symmetry, at first glance this mechanism cannot work in this theory. However, a simple extension which provides a dynamical mechanism for the generation of the GUT scale naturally incorporates an R symmetry and therefore a dynamical generation of the GUT scale may imply no monopole problem whatsoever. The price one has to pay is the doubling of the adjoint representation and an additional singlet field.

The situation is far more appealing and natural in the SU(6) extension of the minimal grand unified theory. Remarkably enough, as long as the GUT scale is generated dynamically through radiative corrections there is automatically an R symmetry. Its large nonvanishing background charge in the Universe guarantees the GUT symmetry to be broken at high temperature and furthermore in the same direction as at zero temperature. The absence of a phase transition not only provides the solution to the monopole problem but also solves the problem of the high-T wrong vacuum in the usual GUTs. The point there is that symmetry restoration chooses the vanishing vev which at T=0 is one of the degenerate minima and the system simply prefers to remain in that state. No such problem is encountered in the SU(6) theory with a large enough R charge. In a sense this theory is tailor fit for the ideas described here.

The remarkable feature of this scenario is that the Universe today is left with no trace of the background charge, since at temperatures below the GUT scale the presence of the soft supersymmetry breaking terms will necessarily imply its washout.

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