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ON SIXTH ORDER RADIATIVE CORRECTIONS TO THE MUON g-FACTOR

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A B S T R A C T

The contribution to the anomalous magnetic moment of the muon from the Feynman diagrams shown in Fig. 1 is found to be

$$\left(\frac{\alpha}{\pi}\right)^3 \left\{ \left(\frac{119}{27} - \frac{4}{9}\pi^2 \right) \log\left(\frac{m_\mu}{m_e}\right) - \frac{61}{162} + \frac{1}{27}\pi^2 + O\left[\left(\frac{m_e}{m_\mu}\right)^2\right] \right\}.$$

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We have calculated the contribution to the anomalous magnetic moment of the muon, $\frac{1}{2}(g_\mu -2)$, from the Feynman diagrams shown in Fig. 1. If we denote by $\mu^{(2)}_{II\,e}$ the total contribution to $\frac{1}{2}(g_\mu -2)$ from these diagrams, we find that

$$\mu^{(2)}_{IIe} = \left(\frac{\alpha}{\pi}\right)^3 \left\{ \left(\frac{119}{27} - \frac{4}{9}\pi^2\right) \log\left(\frac{m_\mu}{m_e}\right) - \frac{61}{162} + \frac{1}{27}\pi^2 + O\left[\left(\frac{m_e}{m_\mu}\right)^2\right] \right\} \tag{1}$$

Numerically,

$$\mu^{(2)}_{IIe} = 0.10 \left(\frac{\alpha}{\pi}\right)^3 . \tag{2}$$

The Feynman diagrams shown in Fig. 1 are the electron-positron vacuum polarization correction to the Feynman diagram shown in Fig. 2. The contribution to $\frac{1}{2}(g_\mu -2)$ from this diagram was called $\mu_{II\,e}$ by Karplus and Kroll who first calculated it to be [1),2)]

$$\mu_{IIe} = \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{119}{36} - \frac{1}{3}\pi^2\right) . \tag{3}$$

It can be seen from the expressions given in Eqs. (1) and (3) that the coefficient of $\log\left(\frac{m_\mu}{m_e}\right)$ in Eq. (1) is precisely $\left(\frac{\alpha}{\pi}\right)\frac{4}{3}\mu_{II\,e}$. This agrees with a result already obtained by Kinoshita, Refs. 3), 4), using the techniques of the renormalization group.

To our knowledge, the term $\frac{-61}{162} + \frac{1}{27}\pi^2$ in Eq. (1) is the <u>only</u> constant term (i.e., which does not involve the ratio of electron to muon masses) of second order electron-positron vacuum polarization corrections to the fourth order contributions to $\frac{1}{2}(g_\mu -2)$ which so far has been calculated exactly [5)]. It has been suggested by Kinoshita [4)] that contributions from <u>these</u> constant terms to $\frac{1}{2}(g_\mu -2)$ are probably small compared to the corresponding logarithm terms.

The result obtained in Eq. (1) supports this conjecture, the contribution from the logarithm term is $0.1115(\alpha/\pi)^3$ while the contribution from the constant term is $-0.0110(\alpha/\pi)^3$.

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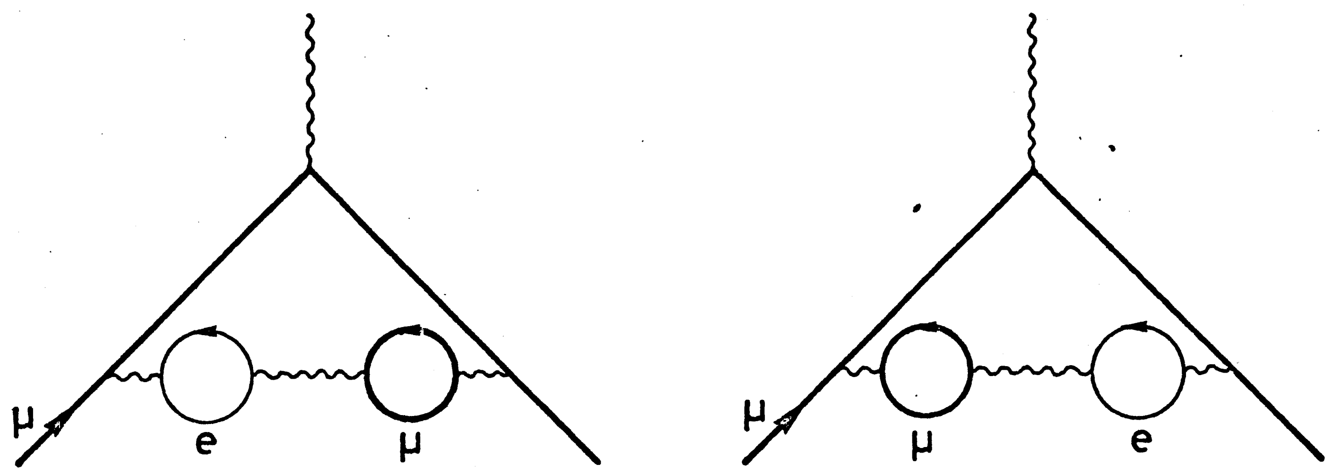

FIG. 1

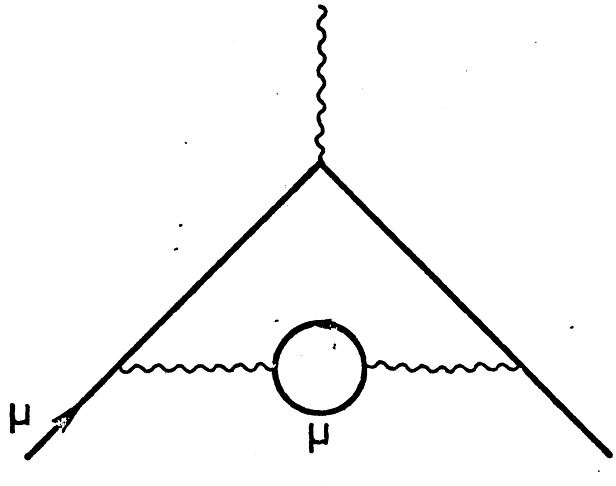

FIG. 2