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HADRONIC CONTRIBUTIONS TO THE MUON g FACTOR

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A B S T R A C T

Using recently reported results from the Orsay colliding beam experiments, we have made a new estimate of the hadronic contributions to the anomalous magnetic moment of the muon due to vacuum polarization corrections.

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1. - INTRODUCTION

The importance of vacuum polarization corrections due to the electromagnetic interaction of hadrons became apparent soon after the discovery of the ρ meson resonance in the $\pi - \pi$ system and the subsequent determination of its quantum numbers. It was first pointed out by Bouchiat and Michel [1] that such a type of correction to the photon propagator could lead to an observable effect in precision measurements of the anomalous magnetic moment of the muon : $a_\mu \equiv \frac{1}{2}(g_\mu - 2)$. Detailed calculations of this effect were also made, independently, by Durand [2]. These authors estimate of the hadronic contributions to a_μ was of the order of 10^{-7}, i.e., far below the error in the measurements at that time, which was $\pm 5 \times 10^{-6}$ [3].

Since then, a new precision measurement of a_μ has been made at CERN [4], and, simultaneously, colliding beam experiments have been performed at Novosibirsk [5] and Orsay [6],[7],[8] which yield precious information on the electromagnetic interactions of hadrons. In the new g-2 experiment [4], the error in the measurement of a_μ is $\pm 3.1 \times 10^{-7}$; and it is likely to improve significantly in the next generation of g-2 experiments. It is clear that a more detailed evaluation of the hadronic contributions to a_μ is now required.

Indeed, new estimates of hadronic contributions to a_μ have been made [9],[10],[11] ; however, they all have been done prior to the recently reported results from the Orsay colliding beam experiments [6],[7],[8]. We believe that these results warrant a new detailed evaluation of the hadronic contributions to a_μ and devote this note to it and to the comparison with previous calculations.

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2. - METHOD OF CALCULATION

The general method to obtain the hadronic contributions to a_μ, due to vacuum polarization corrections, is well known *). The resulting expression, corresponding to the class of Feynman diagrams shown in Fig. 1, has the following structure

$$a_\mu(\text{hadrons}) = \frac{1}{\pi} \int_0^\infty \frac{dt}{t} \, \mathrm{Im}\, \Pi^{(H)}(t) \, K(t) \tag{1}$$

where

$$K(t) = \left(\frac{\alpha}{\pi}\right) \int_0^1 dz \, \frac{z^2(1-z)}{z^2 + \frac{t}{m_\mu^2}(1-z)} \tag{2}$$

and $\mathrm{Im}\,\Pi^{(H)}(t)$ is the hadronic spectral function, given by $(p^2=t)$

$$\theta(p)\, \mathrm{Im}\, \Pi^{(H)}(t) = \frac{-1}{6p^2} \sum_n (2\pi)^4 \delta^{(4)}(p-p_n) \langle 0|J^\mu(0)|n\rangle\langle n|J_\mu(0)|0\rangle \tag{3}$$

where $\sum_n$ also means summation over the phase space available to hadrons in each possible state n. The function $\mathrm{Im}\,\Pi^{(H)}(t)$ is related to the total electron-positron annihilation cross-section into hadrons at total centre-of-mass energy equal to $\sqrt{t}$:

$$\sigma_{tot}(e^+e^- \to \text{hadrons}) = \frac{4\pi^2\alpha}{t} \frac{1}{\pi} \, \mathrm{Im}\, \Pi^{(H)}(t) \tag{4}$$

with $\alpha = e^2/4\pi \simeq 1/137$. It is precisely this relation which shows the relevance of the colliding beam experiments to the evaluation of a_μ (hadrons) given by Eq. (1). We have

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*) See, e.g., Refs. 1) and 2).

$$a_\mu(\mathrm{hadrons}) = \frac{1}{4\pi^2\alpha}\int_{4m_\pi^2}^{\infty} dt\, \sigma_{tot}(e^+e^-\to \mathrm{hadrons})\, K(t) \qquad (5)$$

The explicit form of the function $K(t)$ defined in Eq. (2) is known *). For $t \geq 4m_\mu^2$, a convenient parametrization is the following [13], with $\beta_\mu = \sqrt{1-(4m_\mu^2/t)}$ and $x = (1-\beta_\mu)/(1+\beta_\mu)$,

$$K(t) = \left(\frac{\alpha}{\pi}\right)\left\{\frac{x^2}{2}(2-x^2) + (1+x)^2(1+x^2)\frac{\log(1+x) - x + \frac{x^2}{2}}{x^2} + \frac{1+x}{1-x}x^2\log x\right\} \qquad (6)$$

Notice that for large t, $K(t)$ goes to zero as t^{-1}

$$K(t) = \left(\frac{\alpha}{\pi}\right)\left\{\frac{1}{3}\frac{m_\mu^2}{t} + O\left[\left(\frac{m_\mu^2}{t}\right)\log\left(\frac{t}{m_\mu^2}\right)\right]\right\} \qquad (7)$$

It appears thus that the high energy (t large) contributions to a_μ (hadrons) are depressed by the factor $K(t)$ and the integral in Eq. (5) is dominated by the low values of t ; in particular, by those values of t corresponding to the mass squared of the low lying resonances which have the quantum numbers of the photon, i.e., ρ, ω and φ. It is in these regions that we have now data of $e^+ e^-$ annihilation cross-sections which we shall use to evaluate a_μ (hadrons) as given by Eq. (5).

We shall, of course, work at the one photon exchange approximation for the $e^+ e^-$ annihilation into hadrons. The final state has then isospin $I = 0$ and $I = 1$, and it is convenient to split a_μ (hadrons) correspondingly :

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*) See, e.g., Refs. [12] and [13]. Notice that $K(t=0) = \alpha/2\pi$, which is the well-known second order contribution to the anomalous magnetic moment of the electron and of the muon.

$$a_\mu(\text{hadrons}) = a_\mu(I=0) + a_\mu(I=1) \qquad (8)$$

In the next Section, we compute the isoscalar part a_μ (I=0) using vector meson dominance and taking into account the contribution from the ω and φ mesons. Section IV is dedicated to the calculation of the isovector part a_μ (I=1). Here, we assume the $I = 1$ hadronic system to be dominated, in the region of interest, by the $\pi - \pi$ P wave neglecting other non-resonating states.

3. - ISOSCALAR CONTRIBUTIONS

The isoscalar contributions to the anomalous magnetic moment of the muon can be written in the general form

$$a_\mu(I=0) = \frac{1}{4\pi^2\alpha} \int_{9m_\pi^2}^{\infty} \sigma_{tot}(e^+e^- \to I=0)\, K(t)\, dt \qquad (9)$$

The total cross-section $\sigma_{tot}(e^+e^- \to I=0)$ is dominated by the ω and φ meson contributions and we assume the t dependence of the cross-section to be given by the denominators of the Breit-Wigner formulae associated to the ω and φ mesons

$$\sigma(e^+e^- \to V \to I=0) \simeq 12\pi \frac{\Gamma(V \to e^+e^-)\,\Gamma_V}{(t-M_V^2)^2 + M_V^2\Gamma_V^2}, \quad V = \omega, \varphi$$

We then obtain for the isoscalar contribution an approximate expression

$$a_\mu(I=0) \simeq \sum_{V=\omega,\varphi} \frac{3}{\alpha\pi} \Gamma(V \to e^+e^-)\,\Gamma_V\, K(M_V^2) \int_{9m_\pi^2}^{\infty} \frac{dt}{(t-M_V^2)^2 + M_V^2\Gamma_V^2}$$

In the narrow width approximation, the last integral takes the value $\pi/M_V\Gamma_V$ and the final result is simply

$$a_\mu(I=0) \simeq \sum_{V=\omega,\varphi} \frac{3}{\alpha} K(M_V^2) \frac{\Gamma(V\to e^+e^-)}{M_V} \qquad (10)$$

Using now the Orsay results [7),8)]

$$\Gamma(\omega\to e^+e^-) = (0.94 \pm 0.18)\ \mathrm{KeV}$$

$$\Gamma(\varphi\to e^+e^-) = (1.64 \pm 0.26)\ \mathrm{KeV}$$

and evaluating the function $K(t)$ at the vector meson mass [*)]

$$K(M_\omega^2) = \left(\frac{\alpha}{\pi}\right)\times 5.34\times 10^{-3}$$

$$K(M_\varphi^2) = \left(\frac{\alpha}{\pi}\right)\times 3.28\times 10^{-3}$$

we obtain for the ω meson contribution

$$a_\mu(\omega) = (0.61 \pm 0.12)\times 10^{-8} \qquad (11)$$

and, for the φ meson contribution

$$a_\mu(\varphi) = (0.50 \pm 0.08)\times 10^{-8} \qquad (12)$$

Adding both results, the isoscalar contribution is thus estimated to be

$$a_\mu(I=0) = (1.11 \pm 0.20)\times 10^{-8} \qquad (13)$$

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*) Let us note that, to evaluate $K(M_\omega^2)$, the asymptotic form for $K(t)$ [see Eq. (7)], is not sufficiently accurate and the complete expression given in Eq. (6) must be used even at these energies.

4. - ISOVECTOR CONTRIBUTIONS

The isovector contributions to the anomalous magnetic moment of the muon can be written in the general form

$$a_\mu(I=1) = \frac{1}{4\pi^2\alpha}\int_{4m_\pi^2}^{\infty} \sigma_{tot}(e^+e^-\to I=1)\,K(t)\,dt \tag{14}$$

We assume the total cross-section $\sigma_{tot}(e^+e^- \to I=1)$ to be dominated by the 2π meson contributions, and from the well known expression

$$\sigma_{tot}(e^+e^-\to \pi^+\pi^-) = \frac{\pi\alpha^2}{3}\frac{1}{t}\left(1-\frac{4m_\pi^2}{t}\right)^{3/2}|F_\pi(t)|^2$$

where $F_\pi(t)$ is the pion electromagnetic form factor, we deduce

$$a_\mu(I=1) = \frac{\alpha}{12\pi}\int_{4m_\pi^2}^{\infty}\frac{dt}{t}\left(1-\frac{4m_\pi^2}{t}\right)^{3/2}|F_\pi(t)|^2 K(t) \tag{15}$$

As a first approximation, we can estimate the integral in Eq. (14) by the method which we have used in the last Section to evaluate the isoscalar contributions. Using again the Orsay results [6], we obtain

$$a_\mu(\rho) = (5.0 \pm 0.3)\times 10^{-8} \tag{16}$$

One outstanding feature which comes from this estimate is the relative smallness of the ω and φ contributions [see Eqs. (11) and (12)] as compared to the ρ contribution. This is due, in part, to the difference in the couplings of ρ, ω and φ to the photon (γ_ω and γ_φ are larger than γ_ρ by a factor ~ 3), and also to the larger mass value for the φ. We believe that the estimate of a_μ (I=0) obtained in the last Section [see Eq. (13)] is already sufficiently precise for a comparison of theory with experiment, given the accuracy in the latter. However, this is not the case for a_μ (I=1). The peak approximation to evaluate $\sigma(e^+e^- \to \rho \to I=1)$ is certainly not justified in the case of a large width for the ρ resonance. Other factors than just the usual Breit-Wigner denominator must be taken into account.

We have evaluated the integral in Eq. (15) using for $F_\pi(t)$ the expression proposed by Gounaris and Sakurai [14]. This expression fits extremely well the Orsay data with the following values for the mass and the width of the ρ meson [6]

$$M_\rho = (770 \pm 4)\,\text{MeV}\,, \quad \Gamma_\rho = (111 \pm 6)\,\text{MeV} \tag{17}$$

The expression of the form factor is [14]

$$F_\pi(t) = \frac{M_\rho^2\left(1 + d\,\Gamma_\rho/M_\rho\right)}{M_\rho^2 - t + H(t) - i\,M_\rho\,\Gamma_\rho\left(\frac{k}{k_\rho}\right)^3 M_\rho/\sqrt{t}} \tag{18}$$

with

$$k = \sqrt{\frac{t}{4} - m_\pi^2} \qquad\qquad k_\rho = \sqrt{\frac{M_\rho^2}{4} - m_\pi^2}$$

$$d = \frac{3}{\pi}\,\frac{m_\pi^2}{k_\rho^2}\,\log\left(\frac{M_\rho + 2k_\rho}{2m_\pi}\right) + \frac{M_\rho}{2\pi k_\rho} - \frac{m_\pi^2 M_\rho}{\pi k_\rho^3} \simeq 0.48$$

$$H(t) = \frac{\Gamma_\rho M_\rho^2}{k_\rho^3}\left\{k^2\left[h(t) - h(M_\rho^2)\right] + k_\rho^2\, h'(M_\rho^2)(M_\rho^2 - t)\right\}$$

and

$$h(t) = \frac{2}{\pi}\,\frac{k}{\sqrt{t}}\,\log\left(\frac{\sqrt{t} + 2k}{2m_\pi}\right)$$

This expression for $F_\pi(t)$ has been used in the complete range of integration in Eq. (15), whereas the experimental data have been fitted at a centre-of-mass energy ranging from 644 MeV to 885.8 MeV, i.e., around the region corresponding to the ρ resonance. Looking at the χ^2 fit results,[6], the formula given by Eq. (18) which takes into account the P wave character of the 2π meson final state is strongly favoured with respect to a single S wave Breit-Wigner expression without momentum dependence.

The numerical integration of Eq. (15) with $F_\pi(t)$ given by Eq. (18) and M_ρ and Γ_ρ as given in Eq. (17) finally leads to the result

$$a_\mu(I=1) = (5.4 \pm 0.3) \times 10^{-8} \tag{19}$$

somewhat larger than the narrow width estimate given in Eq. (16).

5. - DISCUSSION OF THE RESULTS

Combining now isoscalar and isovector contributions, we finally obtain

$$a_\mu(\text{hadrons}) = (6.5 \pm 0.5) \times 10^{-8} \tag{20}$$

Previous estimates of a_μ (hadrons) give

$$a_\mu(\text{hadrons}) = 7.5 \times 10^{-8}$$ (Kinoshita and Oakes [9])

$$a_\mu(\text{hadrons}) = (3.4 {}^{+1.9}_{-0.9}) \times 10^{-8}$$ (Bowcock [10])

In both these calculations, the contributions from the ω and φ mesons were obtained from the corresponding ρ meson contribution using SU(3) invariance and a ω - φ mixing angle of 40°. The estimate of Kinoshita and Oakes was made prior to the colliding beam experiments. The estimate of Bowcock uses the results from the Novosibirsk colliding beam experiment on $e^{+} e^{-}$ annihilation into two π mesons. Our result (20) is obviously compatible with these previous calculations.

The error quoted in Eq. (20) reflects only the uncertainty from the experimental data used in our estimate of a_{μ} (hadrons). It should be taken into account that theoretical uncertainties from other hadronic contributions to a_{μ} than those discussed in this note could enlarge this error by a factor of two. In particular, we have in mind all those diagrams where hadrons can give contributions to the vacuum polarization <u>tensor</u> and thus to the anomalous magnetic moment of the muon (see Fig. 2) *). Although order of magnitude estimates indicate that they are $< (\alpha/\pi)^3$, one should nevertheless remember that very little is known about light by light scattering contributions to the anomalous magnetic moment of the muon, even in pure quantum electrodynamics.

The result in Eq. (20), when added together with the second [15] and fourth [16] order calculations from quantum electrodynamics, leads to the prediction

$$\tfrac{1}{2}(g_\mu - 2)_{Th.} = (116560.7 \pm 0.8) \times 10^{-8} \tag{21}$$

to be compared with the experimental result [4]

$$\tfrac{1}{2}(g_\mu - 2)_{Exp.} = (116616 \pm 31) \times 10^{-8} \tag{22}$$

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*) Several of these diagrams were pointed out to us by Professor J. Prentki.

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FIGURE CAPTION

Figure 1 Vacuum polarization diagrams contributing to $\frac{1}{2}(g-2)$.

Figure 2 Light by light scattering diagrams contributing to $\frac{1}{2}(g-2)$.

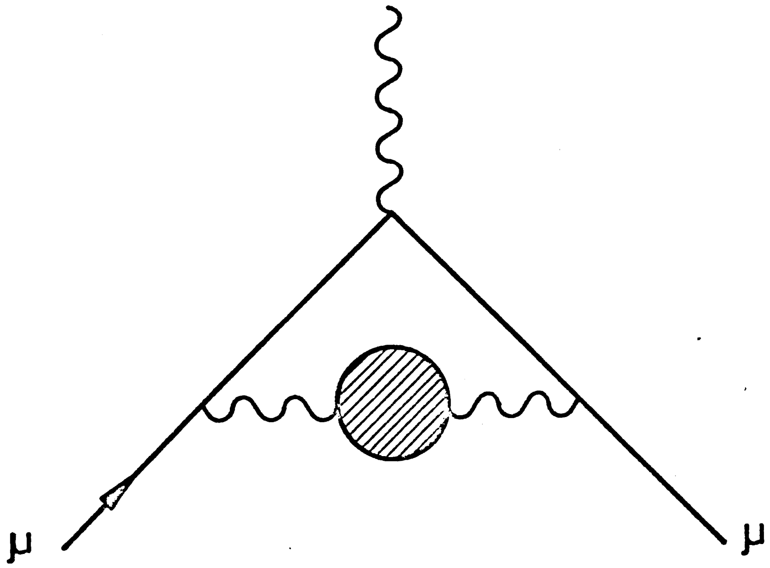

FIG.1

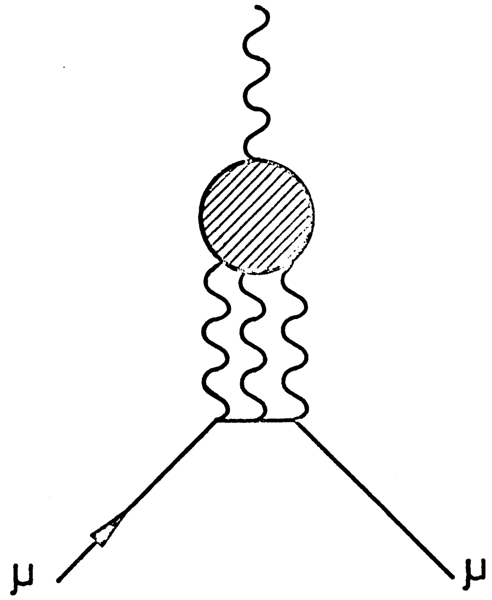

FIG.2