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CURRENT ALGEBRA AND $\eta \to 3\pi$

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A B S T R A C T

A critical review is given of developments in the theory subsequent to the demonstration that in first approximation $\eta \to 3\pi$ is forbidden. It remains difficult to understand the process with conventional ideas without reducing to an accident the success of current algebra, and a simple linear matrix element, in $K \to 3\pi$.

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1. INTRODUCTION

Hara and Nambu [1], and Elias and Taylor [2], obtained strikingly good results in applying current algebra and PDDAC to the decay $K \to 3\pi$. It was subsequently shown [3] that these results do not depend on the local non-leptonic weak interaction originally assumed, but are obtained equally when the weak interaction is mediated by an intermediate boson. The decay $\eta \to 3\pi$, mediated by a virtual photon, could then be expected to be rather analogous. However, Sutherland [4] showed that a parallel treatment required <u>zero</u> amplitude for $\eta \to 3\pi$. Of course by relaxing the assumptions, or improving the approximations, residual non-vanishing terms could be found; the problem was to use these terms in a credible way without reducing the significance of the kaon work, where such terms had been ignored: if the kaon results were dismissed as accidental, there would be no problem.

The question has subsequently been analyzed by a number of groups. Our object is to give a critical review of these developments. It will be seen that one line of attack, the improvement of the approximation PDDAC, has been especially enlightening. As a result of it we no longer wish to describe the η decay as forbidden, but rather to say that the k decay is enhanced. Theoretically η decay is <u>not</u> enhanced; unfortunately it seems to be very much enhanced experimentally. So the problem remains unresolved in conventional theory.

A number of papers have also appeared in which substantial modifications of conventional ideas are proposed. Some [5], either implicitly or explicitly, abandon electromagnetism as the main origin of the η decay. One proposes [6] a radical change from the usual notion of electromagnetic current. We will not examine these theories here, apart from some passing remarks. Discussion of the implications of a possible $\Delta I = 3$ interaction will be given elsewhere [7] by one of us and Veltman.

2. THE FIRST APPROXIMATION

Let us first restate the original argument [4], in a slightly different style. Denote the amplitudes for the charged $(\pi^+\pi^-\pi^0)$ and neutral $(\pi^0\pi^0\pi^0)$ modes by

$$A(E_1, E_2, E_3) \qquad B(E_1, E_2, E_3)$$

respectively, where the E's are the centre-of-mass energies of the pions. Of course $E_1+E_2+E_3 = m_\eta$, but it is more symmetric to retain the three variables. We continue these amplitudes off the mass shell of the third pion (neutral in both cases) by the conventional formula containing a T^* product [8] of the axial current divergence and a pair of electromagnetic currents. Because the various commutators are zero, we find that the continued amplitudes must vanish at $q^3_\mu = 0$, i.e., zero four-momentum for pion 3. Neglecting (PDDAC) the dependence of the continued amplitude on the extra variable $q^3_\mu q^3_\mu$, we have then

$$A\left(\tfrac{1}{2}m_\eta, \tfrac{1}{2}m_\eta, 0\right) = B\left(\tfrac{1}{2}m_\eta, \tfrac{1}{2}m_\eta, 0\right) = 0 \qquad (1)$$

Assuming the final state to be isospin 1, and Bose symmetry,

$$A(E_1, E_2, E_3) = A(E_2, E_1, E_3)$$

$$B(E_1, E_2, E_3) = A(E_1, E_2, E_3) + A(E_2, E_3, E_1) + A(E_3, E_1, E_2)$$

it follows that

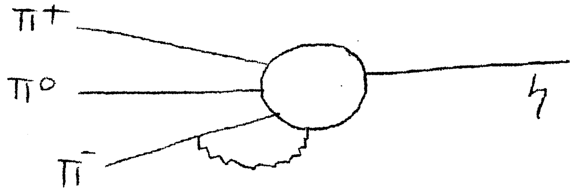
$$A\left(\frac{m_\eta}{2}, \frac{m_\eta}{2}, 0\right) = A\left(\frac{m_\eta}{2}, 0, \frac{m_\eta}{2}\right) = A\left(0, \frac{m_\eta}{2}, \frac{m_\eta}{2}\right) = 0 \qquad (2)$$

So if A is simply a linear function of the energy of the odd pion, as assumed [1),2)] in the successful treatment of kaon decay

$$A(E_1, E_2, E_3) = \alpha + \beta E_3 \qquad (3)$$

we must have $\alpha = \beta = 0$. The decay is forbidden.

Note that in this version only a neutral pion is taken off the mass shell. Amati observed that continuing off the mass shell for charged pions could be delicate because of thresholds [9] induced by diagrams of the type



The above argument is one way of avoiding this question.

We proceed to consider the consequences of relaxing the assumptions in various ways.

3. HIGHER ORDER ELECTROMAGNETISM

We can quickly dispose of the possibility that higher orders in the electromagnetic coupling constant [10] would by themselves be very helpful. This is because the equations (1) hold to all orders [11]. No matter how many electromagnetic currents appear in the T^* product, the zero order theorem depends on commutators all of which are zero. The assumption of $I = 1$ for the final state is no longer appropriate, so we do not obtain (2). However, from (1) alone it follows that in the linear form (3), $\alpha = 0$ and

$$A = \beta E_3$$

This is in complete disagreement with experiment. So even if the extra powers of the fine structure constant could be tolerated, this approach would not seem fruitful.

4. NON-LINEAR ENERGY DEPENDENCE

The main reason for considering a simple linear dependence on E_3, apart from simplicity, is just that it worked for kaons. The validity of such a dependence far outside the physical boundaries of the Dalitz plot, where it has some experimental support, has no real theoretical basis. If one permits dependence on E_3 only, but allows quadratic terms, (2) requires

$$A \propto E_3\left(E_3 - \tfrac{1}{2}m_\eta\right)$$

Now the factor $(E_3-\frac{1}{2}m_\eta)$ by itself would represent the data rather well. But the fit is spoiled by the factor E_3, which would have to be replaced by something more nearly constant over the physical region while still vanishing at $E_3 = 0$. So quite an elaborate dependence is required. This viewpoint is taken, for example, by Itzykson, Mahoux and Jacob [12]. They feel that the possibility of extrapolating the Dalitz plot to zero near $E_3 = \frac{1}{2}m_\eta$, which is not far outside the physical region [13], is the significant thing; they appeal to strong non-linearities to produce a zero at the more remote point $E_3 = 0$. Such a view would be more palatable if some reason were advanced for the success of the simple linear form in kaon decay.

Instead of imposing the zeros, one could also attempt to fit the physical region independently and see how it extrapolates. The difficulties of such an approach can be illustrated by a recent analysis of Crawford and Price [14], who retained up to cubic terms in the matrix element

$$A = \alpha\left(1 + \beta E_3 + \gamma E_3^2 + \delta E_3^3\right)$$

They were interested to see how much the prediction of the branching ratio R, of $\pi^0\pi^0\pi^0$ relative to $\pi^+\pi^-\pi^0$, varied from the value ≈ 1.7 associated with a linear fit. They found indeed that with good cubic fits they could obtain the very low values of R suggested by some, but not all, experiments [15]. For these cubic fits we quote the extrapolated values [16], normalizing A to unity at the middle of the Dalitz plot :

	Re A	Im A
$E_3 = \frac{1}{2}m_\eta$	-4.9 ± 6.4	-6.0 ± 8.0
$E_3 = 0$	-25.6 ∓ 51.2	50.4 ∓ 64.0

In each column the signs are associated as indicated. The magnitude of the errors illustrates the well-known difficulty of extrapolating empirical data without theoretical prejudices.

From a theoretical point of view, it has sometimes been felt that Weinberg current algebra prediction [17] of very small $\pi - \pi$ scattering lengths, implying weak final state interactions, removed a possible source of non-linearity in the η and K decay amplitudes. However as Weinberg himself remarked his results were only consistent with, rather than implied by, the theory. Subsequent authors [18),19] have shown how other values can be obtained. They have also shown [19] that Weinberg's small scattering lengths can coexist with strong pion-pion s wave phase shifts at a few hundred MeV. At $E_3 = 0$ in η or K decay, the relevant dipion mass is that of the η or K. A substantial $I = 0$ s wave phase shift ($\approx 50^\circ$) in this region is indicated by recent analyses of pion pair production in peripheral pion-nucleon collisions [20], of pion-nucleon scattering [21], and of $K \to 2\pi$ decays [22]. On the other hand, a large phase shift does not necessarily imply a strongly non-linear η decay amplitude. The correct incorporation of pion-pion interaction into the three-pion final state is quite complicated [23]. Moreover even if we try a simple Watson factor [24]

$$k^{-1} e^{i\delta} \sin\delta$$

in the amplitude, at least some of Donnachie et al.'s [21] many fits vary in such a way as to leave a linear form credible.

5. IMPROVEMENT OF PDDAC

Suppose now that dependence of the amplitude on the invariant squares of the pion masses is no longer neglected. That is to say that the strict form of PDDAC is relaxed [25]. Let the off shell amplitudes, defined by T^* products with three axial current divergences, be

$$A(E_1, q_1^2, E_2, q_2^2, E_3, q_3^2) \quad , \quad B(E_1, q_1^2, E_2, q_2^2, E_3, q_3^2)$$

where B is again obtained from A by symmetrization, and A is again already symmetric in 1 and 2. As before

$$\begin{aligned} A\left(\tfrac{1}{2}m_\eta, -m_\pi^2, -\tfrac{1}{2}m_\eta, -m_\pi^2, 0, 0\right) &= 0 \\ A\left(\tfrac{1}{2}m_\eta, -m_\pi^2, 0, 0, \tfrac{1}{2}m_\eta, -m_\pi^2\right) &= 0 \\ A\left(0, 0, \tfrac{1}{2}m_\eta, -m_\pi^2, \tfrac{1}{2}m_\eta, -m_\pi^2\right) &= 0 \end{aligned} \tag{4}$$

We now allow terms linear [9] in the q^2's :

$$A = \alpha + \beta E_3 + \gamma q_3^2 + \delta(q_1^2 + q_2^2) \tag{5}$$

From (4) we have immediately

$$\alpha = 2m_\pi^2 \delta \qquad \beta = \frac{2m_\pi^2}{m_\eta}(\gamma + \delta) \tag{6}$$

but this leaves two undetermined parameters.

It was shown by Dolgov et al. [26] and by Bardeen et al. [27] that additional information can be gained by considering the neutral pion at zero momentum with the charged pions off the mass shell. Values have then to be attributed to the commutators

$$[A_0^3 , \partial_\mu A_\mu^\pm]$$

and they are set equal to zero as suggested by, for example, a form of the quark model [23]. Then A must vanish for $q_3^2 = E_3 = 0$ whatever the values of q_1^2 and q_2^2. This requires $\alpha = \delta = 0$. Thus

$$A = \gamma\left(q_3^2 + 2\frac{m_\pi^2}{m_4}E_3\right) \tag{7}$$

$$B = \gamma\left(q_1^2 + q_2^2 + q_3^2 + 2m_\pi^2\right) \tag{8}$$

On the mass shell

$$A = -\gamma m_\pi^2\left(1 - \frac{2}{m_4}E_3\right) \tag{9}$$

Leaving aside the over-all normalization, this fits the data very well.

Now the small quantity m_π^2 appears in (9) precisely because we are here appealing to residual terms to make the process go. The quantity γ must contain as a factor the fine structure constant, so we might guess

$$\gamma \approx 10^{-2}\, m^{-2} \tag{10}$$

where m is some characteristic mass. If we take for m the rho mass for example, then the decay amplitude, essentially γm_π^2, is very small indeed. However a current algebra estimate to be described immediately actually gives

$$\gamma \approx 5\; 10^{-2}\, F_\pi^{-2} \tag{11}$$

where F_π is the pion decay coupling constant :

$$F_\pi \approx 1.31\; m_\pi \tag{12}$$

whence

$$\gamma m_\pi^2 \approx 3\ 10^{-2} \tag{13}$$

The influence of the small "forbiddeness" factor, m_π^2, has been cancelled. We wish to comment on this result before becoming involved in the detailed calculation.

The appearance of the factor F_π^{-1} is characteristic in current algebra PDDAC arguments relating many-pion processes to fewer pion processes. These enhancement factors multiply equally the leading terms, so that when these do not vanish they can still be supposed dominant. That is to say that the multipion process as a whole can be said to be enhanced [28]. Such a factor F_π^{-1} occurs for example in the empirically successful relation [1),2)] of $K \to 3\pi$ to $K \to 2\pi$. Another such factor would occur in relating $K \to 2\pi$ to $K \to \pi$. The latter vertex is not empirically accessible without making unreliable models. However it has been noted already that $K \to 2\pi$ is somehow larger [29] than simple dimensional estimates, some enhancement process being indicated to cancel the inhibiting effect of the Cabibbo angle.

The point of view just outlined leads us to refer to the η decay, because the leading terms vanish, as "not enhanced" rather than "forbidden". A matrix element of magnitude (13) could reasonably be regarded as allowed.

6. η-π MIXING

A possibility which has been considered repeatedly [30] is that of relating the amplitude for $\eta \to \pi^0\pi^0\pi^0$, by taking two pions to zero momentum, to the mixing amplitude $\eta \to \pi^0$ and then by U spin symmetry to kaon and pion electromagnetic mass differences. The treatment of Brown et al. [27] seems to us the most straightforward. We follow it closely below, obtaining however a rather different answer. Note from (8) that the off mass shell amplitude for $\eta \to \pi^0\pi^0\pi^0$ is parametrized as

$$B = \gamma(q_1^2 + q_2^2 + q_3^2 + 2m_\pi^2)$$

The on shell amplitude is just the constant, $-\gamma m_\pi^2$, which we now try to determine.

The notation to be used is indicated by the formula for the general off shell amplitude

$$B = \frac{m_\pi^2+q_1^2}{F_\pi m_\pi^2}\frac{m_\pi^2+q_2^2}{F_\pi m_\pi^2}\frac{m_\pi^2+q_3^2}{F_\pi m_\pi^2}\langle 0|T^*(\partial_\mu A_\mu(q_1), \partial_\mu A_\mu(q_2), \partial_\mu A_\mu(q_3), H(0))|\eta\rangle \tag{14}$$

in which

$$H(0) = \int dy\, d_{\mu\nu}(-y)\, J_\mu(0)\, J_\nu(y) \tag{15}$$

(where $d_{\mu\nu}$ is the photon propagator) is the effective electromagnetic transition operator, and

$$\partial_\mu A_\mu(q) = \int dx\, e^{-iqx}\, \partial_\mu A_\mu(x) \tag{16}$$

where A_μ is the neutral axial current. The T^* symbol is supposed to operate <u>inside</u> the various integrations. With our normalizations the renormalized pion field π is determined by

$$\partial_\mu A_\mu = F_\pi m_\pi^2 \pi \tag{17}$$

where F_π is given by (12) [31].

Denoting the η four-momentum by p, we respect the restriction

$$q_1 + q_2 + q_3 = p \tag{18}$$

We assume the commutators

$$\delta(x_0-y_0)\left[A_0(x), \partial_\mu A_\mu(y)\right] = \sigma(x)\,\delta(x-y) \qquad (19)$$

$$\delta(x_0-y_0)\left[A_0(x), \sigma(y)\right] = 4\,\partial_\mu A_\mu(x)\,\delta(x-y) \qquad (20)$$

as suggested, for example, by the quark model.

Setting q_3 on the mass shell in (14) and $q_1 = 0$, we obtain in the usual way

$$B = \frac{m_\pi^2 + q_2^2}{F_\pi^2 m_\pi^2}\langle\pi(q_3)|\,T^*(\sigma(q_2), H(0))\,|\eta\rangle \qquad (21)$$

Comparing this with (8)

$$\langle\pi(q_3)|\,T^*(\sigma(q_2), H(0))\,|\eta\rangle = F_\pi^2 m_\pi^2 Y \qquad (22)$$

Then taking pion 3 off the mass shell to zero, using the commutator (20) and PDDAC,

$$\frac{4}{F_\pi}\langle 0|\,T^*(\partial_\mu A_\mu(p), H(0))\,|\eta\rangle = F_\pi^2 m_\pi^2 Y \qquad (23)$$

in which we have used $q_2 = p$, from (18) with $q_1 = q_2 = 0$.

We have in (22) a matrix elcment suggestive of $\eta\pi$ mixing. However we have to be careful with momentum dependence. Consider the more general matrix element

$$\frac{m_\pi^2 + q^2}{F_\pi m_\pi^2}\;\frac{m_\eta^2 + p^2}{F_\eta m_\eta^2}\langle 0|\,T^*(\partial_\mu A_\mu(q), \partial_\mu A_\mu^{\prime}(p), H(0))\,|0\rangle \qquad (24)$$

where A_μ^η and F_η are the analogues for η of A_μ and F_π. The expression (24), because of vanishing commutators, must go to zero when either $q=0$, $p^2=-m_\eta^2$ or $p=0$, $q^2=-m_\pi^2$. So it would be unreasonable to approximate it by a constant. The next simplest assumption is a linear form in the invariants :

$$A + Bp^2 + C(p^2-q^2) + D(p^2 - pq) \tag{25}$$

The requirement that this vanish for $p=0$, $q^2=-m_\pi^2$ gives

$$A + Cm_\pi^2 = 0 \tag{26}$$

suggesting that A is a small quantity, if C is not unexpectedly large [32]. Returning to the case $q=p$, we note from (22)

$$F_\pi^2 m_\pi^2 \gamma = \frac{4m_\pi^2}{m_\pi^2 - m_\eta^2}(A - Bm_\eta^2) \tag{27}$$

Consider now approximate U spin symmetry. The application of a broken symmetry leaves much room for ambiguity, but we believe the following procedure to be in the best traditions [33]. The mass differences are determined by expressions of type (24), with $q=p$. Parametrizing them in a similar way, we obtain with obvious notation

$$m^2(K_0) - m^2(K_+) = A_K - B m_K^2 \tag{28}$$

$$m^2(\pi_+) - m^2(\pi_0) = A_\pi - B m_\pi^2 \tag{29}$$

We use U spin symmetry [34] for the A's and B's, obtaining for the $\eta\pi$ mixing amplitudes

$$-\sqrt{3}\, A = A_K + A_\pi$$

$$-\sqrt{3}\, B = B_K + B_\pi$$

Then with $A \approx 0$ as suggested by (26)

$$\gamma F_\pi^2 \approx \frac{4}{\sqrt{3}} \left\{ \frac{m^2(K_0) - m^2(K_+) - A_\pi}{m_K^2} + \frac{m^2(\pi_+) - m^2(\pi_0) - A_\pi}{m_\pi^2} \right\} \qquad (30)$$

Now Das et al. [35] have accounted very well for the pion mass difference in a calculation which ignores the m_π^2 term in (29). If we believe them

$$A_\pi \approx m^2(\pi_+) - m^2(\pi_0) \qquad (31)$$

Using this one obtains from (30) and the empirical masses

$$\gamma m_\pi^2 \approx 3\ 10^{-2} \qquad (32)$$

An uncertainty of about 10% in (31) would induce an uncertainty of about 30% in (32).

We are well aware of the uncertainties in such calculations, in connection with the use of simple linear forms over large ranges and especially with the application of broken U spin symmetry. Nevertheless if an answer is to be forced out at the present stage, we believe the argument given is reasonable. We were unable to find any equally reasonable basis for much larger values of γm_π^2, larger by $(m_K/m_\pi)^2$, quoted by other authors [30].

As far as we know the phenomenon of $\eta\pi$ mixing is not subject to empirical test. The $\eta\pi$ vertex appears in some models of isospin violation in nucleon-nucleon and nucleon-hypernucleon interaction [36], and of η' decay [37]. It may be that our revised treatment requires some revision there, but what that might be, like the basis of those models, is obscure to us.

As has been remarked, the result (32) is less small than could have been feared: the inhibition factor m_π^2 has been cancelled by the enhancement factor F_π^{-2}. However it still seems much too small empirically. The width for $\eta \to \pi^+\pi^-\pi^0$ is given by

$$\Gamma(\pi^+\pi^-\pi^0) = |\gamma m_\pi^2|^2 \frac{(m_\eta - 3m_\pi)^2}{3456\sqrt{3}\,\pi^2 m_\eta}$$

ignoring relativistic corrections in the phase space. Empirically

$$\Gamma(\pi^+\pi^-\pi^0) \simeq \frac{22}{31}\Gamma(\gamma\gamma)$$

Recently $\Gamma(\gamma\gamma)$ has been measured [38] to be 0.95 ± 0.25 keV. On the other hand, U spin symmetry together with the observed π^0 width suggests [39] $\Gamma(\gamma\gamma) \approx 160$ eV. According to whichever of these we use

$$|\gamma m_\pi^2| \simeq 1.13 \quad \text{or} \quad 0.45$$

These are to be compared with the theoretical result (32)

$$|\gamma m_\pi^2| \simeq 0.03$$

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$$R = \Gamma(3\pi^\circ)/\Gamma(\pi^+\pi^-\pi^\circ) = 0.94 \pm 0.16$$

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25) That this should be tried was urged on us some time ago by P.C. Csonka; an attempt in this direction was made also by L. Clavelli [Chicago preprint (1967), to appear in Nuovo Cimento]. They did not attempt to give the absolute rate, which, as we see later, presents difficulty. Clavelli appears also to neglect η pole terms and "σ terms". We learn from S.L. Glashow that S.S. Shai has given a treatment very close to that of Ref. 27).

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This result also appears in several of the non-electromagnetic theories of Ref. 5), but our subsequent discussion does not apply to them.

31) Our normalizations of currents are those of Bell [3] and Weinberg [17]. We take F_π from the pion decay rate. The Goldberger-Treiman formula, $F_\pi = 2m_N f_A(0) g^{-1}_{\pi NN}$ gives a value about 12% lower. In the argument of Section 6 the width $\Gamma(\eta \to 3\pi)$ is eventually proportional to F_π^{-4}. An increase of F_π by 12% corresponds to an increase of Γ by about 60%. In general authors do not say which value they take for F_π, and this may account for varying answers from similar formulae.

32) By taking both momenta to zero in (24), A could be related to the matrix element of a commutator of σ type, (19). The situation where such a σ term must be small by a consistency condition is typical.

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39) For discussion and references, see the second paper of Ref. 4).