Archives

Ref. TH. 803

A REGGE POLE MODEL FIT OF PION AND K MESON PHOTOPRODUCTION

J.P. Ader, M. Capdeville
Laboratoire de Physique Théorique, Bordeaux *)

Ph. Salin
CERN - Geneva

A B S T R A C T

Differential cross-sections in the forward direction for π^+, π^0 and K^+ photoproduction are evaluated with Regge trajectories exchange in the t channel. Kinematical singularities, constraints among helicity amplitudes, conspiracy, evasion, have been studied very carefully. A very good fit is obtained for $\gamma p \to \pi^0 p$, assuming ω and B exchange, and with only three parameters, for $2 \leq E_\gamma \leq 5$ GeV and transfer momentum t up to $-3(\mathrm{GeV}/c)^2$. Good agreement is also observed for $\gamma p \to \pi^+ n$ with π and B exchange, and for $\gamma p \to \Lambda^0 K^+$ and $\gamma p \to \Sigma^0 K^+$ with K and K^* exchange. Present experimental data seem to exhibit violations of the SU_3 symmetry.

*) Postal address: Laboratoire de Physique Théorique, Chemin du Solarium, 33 Gradignan, France.

67/922/5 - TH. 803
12 July 1967

1. INTRODUCTION

In elementary particle physics involving energies less than 1 GeV, the study of pion photoproduction has always been a complementary tool to elastic π - nucleon scattering understanding. The complications introduced by the spin 1 of the photon was a matter of algebra rather than a matter of dynamics. This is not the case in the high energy domain. Because photoproduction involves three kinds of different masses, and because the spin of particles is high enough, one encounters both the problem of daughter trajectories and of conspiracy relations. That is why this process and the associated one of vector meson production are really very interesting at the present time. The possibility that the scattering amplitude for photoproduction might be reggeized has been been cast in doubt by Diu and Le Bellac [1].

We have therefore tried to confront the usual Regge pole phenomenology with the present experimental information on $\gamma p \to p \pi^0$, $\gamma p \to \pi^+ n$, $\gamma p \to \Lambda K$ and $\gamma p \to \Sigma K$ at high energies.

We have assumed that the daughter mechanism works, in order to recover the usual s^α dependence of the scattering amplitudes, and paid much attention to the conspiracy relation and t dependence of helicity amplitudes in the forward direction. Our results seem to indicate that helicity amplitudes seem to choose evasion.

In all cases we found that a reasonable fit might be achieved taking into account the exchange of only two trajectories, $B(1^+)$ and ω for $\gamma p \to \pi^0 p$, B and π for $\gamma p \to \pi^+ n$, K and K^* for $\gamma p \to \Lambda K$ and $\gamma p \to \Sigma K$.

The plan of our paper is the following: in Section 2 we give the general structure of the helicity amplitudes and cross-section; then, the possible intermediate states in the t channel are examined in Section 3 and their contributions to each helicity amplitude discussed in Section 4. Section 5 deals with the reggeization of helicity amplitudes and the fit for pion photoproduction is discussed in Section 6. Finally, Section 7 is devoted to the K meson photoproduction. Implications of SU_3 invariance are also examined as well as connections with usual perturbation theory.

2. SCATTERING AMPLITUDES AND CROSS-SECTION

Using standard techniques the usual invariant functions A_i given by Ball [2] may be projected on the helicity amplitudes in the t channel. One obtains

$$
\begin{aligned}
\bar{f}^{+}_{\frac{1}{2}\frac{1}{2}10} &= \bar{f}^{+}_{01} = -\frac{k_t\sqrt{t}}{2}\,(A_1 - 2mA_4) \\
\bar{f}^{-}_{\frac{1}{2}\frac{1}{2}10} &= \bar{f}^{-}_{01} = p_t k_t\,(A_1 + tA_2) \\
\bar{f}^{+}_{\frac{1}{2}-\frac{1}{2}10} &= \bar{f}^{+}_{11} = \frac{k_t}{2}\,(2mA_1 - tA_4) \\
\bar{f}^{-}_{\frac{1}{2}-\frac{1}{2}10} &= \bar{f}^{-}_{11} = -p_t k_t\sqrt{t}\,A_3
\end{aligned}
\tag{1}
$$

where

$$\bar{f}_{\lambda\mu} = [\cos\theta_t/2]^{-|\lambda+\mu|}\,[\sin\theta_t]^{-|\lambda-\mu|}\,f_{\lambda\mu}$$

$$\bar{f}^{\pm}_{\lambda\mu} = \bar{f}_{\lambda\mu} \pm \bar{f}_{-\lambda\mu}$$

and k_t, p_t, θ_t are respectively the boson momentum, nucleon momentum and scattering angle in the c.m. system of t channel.

The isospin expansion is defined by

$$A_i = A_i^+ \delta_{3\alpha} + \frac{1}{2}[\tau_\alpha, \tau_3] A_i^- + \tau_\alpha A_i^0 \tag{2}$$

The A_i are free of kinematical singularities and we recall that A_2^- has a pole for $t = \mu^2$ due to gauge invariance. The kinematical singularities of the helicity amplitudes are readily obtained from (1) as well as the well-known conspiracy relation

$$\lim_{t \to 0} [i \bar{f}_{01}^- + \bar{f}_{11}^+] = O(\sqrt{t}) \tag{3}$$

In terms of the $\bar{f}$ the cross-section is then given by

$$\frac{d\sigma}{dt} = \frac{1}{\pi s k_s^2} \Big\{ (| \bar{f}_{01}^+ + \bar{f}_{01}^- |^2 + | \bar{f}_{01}^+ - \bar{f}_{01}^- |) \sin^2\theta_t + (1 - \cos\theta_t)^2 \times$$
$$\times | \bar{f}_{11}^+ - \bar{f}_{11}^- |^2 + (1 + \cos\theta_t)^2 \, | \bar{f}_{11}^+ + \bar{f}_{11}^- |^2 \Big\} \tag{4}$$

where k_s is the initial c.m. momentum in the s channel.

$$\cos\theta_t = (2s + t - 2m^2 - \mu^2) / 4 P_t k_t$$

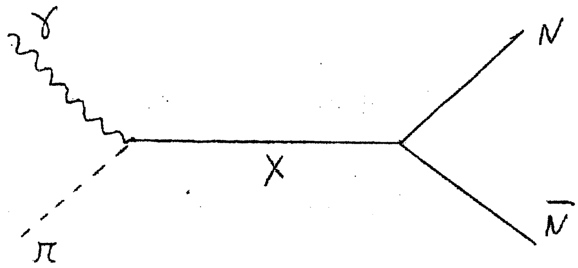
$$\sin\theta_t = [\Phi(s,t)]^{1/2} / 2 P_t k_t \sqrt{t}$$

3. POSSIBLE INTERMEDIATE STATES IN THE t CHANNEL

We first consider the two following sets of reactions

1) $\begin{cases} \gamma\pi^- \to n\bar{p} \\ \gamma\pi^+ \to p\bar{n} \end{cases}$

2) $\begin{cases} \gamma\pi^o \to p\bar{p} \\ \gamma\pi^o \to n\bar{n} \end{cases}$



As shown by Childers and Holladay [3], the possible intermediate state X which might be exchanged in the t channel must have quantum numbers such that

a. B, I, P, G, C, S and Q are conserved at the $XN\bar{N}$ vertex

b. B, P, C, S and Q are conserved at the $\gamma\pi X$ vertex

c. the eigenvalue of the charge conjugation operator C is a good quantum number only for neutral states.

The only known states with $B = S = 0$ and even parity, are the A_1, A_2 and $B^{*)}$ resonances and the two vacuum trajectories. A_1, A_2 and the two vacuum trajectories are forbidden in 2) by C conservation, and the two vacuum trajectories are also forbidden in 1) by charge conservation. The known particles with $B = S = 0$ and odd parity are ρ, ω, ϕ, π and η. The η exchange is forbidden in 1) and 2) for the same reasons as the vacuum trajectory was, ω and ϕ are forbidden in 1) by isospin and charge conservation and π is forbidden in 2) by C invariance.

The only possible states which remain are then in each case the following:

---

*) We assume that the B meson is a 1^+ resonance with $G = +1$ and $C = -1$ and a mass of 1210 MeV.

B, ω , ϕ, ρ for $\gamma N \to \pi^0 N$
B, ρ , π , A_1, A_2 for $\gamma p \to \pi^+ n$ and $\gamma n \to \pi^- p$.

4. CONTRIBUTIONS TO HELICITY AMPLITUDES

We now determine to which helicity amplitudes each state contributes.

1) Reactions $\gamma N \to \pi^\pm N$

In terms of the parity eigenstates

$$|JM\lambda\lambda'\rangle_\pm = |JM\lambda\lambda'\rangle \pm |JM -\lambda -\lambda'\rangle$$

where λ and λ' refer to the nucleon helicity, one obtains

$$G|JM\lambda\lambda\rangle_\pm = (-1)^{J+1}|JM\lambda\lambda\rangle_\pm$$

$$G|JM\lambda\lambda'\rangle_\pm = \pm(-1)^{J+1}|JM\lambda\lambda'\rangle_\pm \quad (\lambda \neq \lambda')$$

because $p\bar{n}$ or $n\bar{p}$ are pure isovectors.

a. ρ contribution. The parity and signature of the ρ trajectory are odd, and it couples only to states $|JM\lambda\lambda'\rangle_+$. So, its contribution arises in $\bar{f}^{J+}_{\frac{1}{2}\frac{1}{2}10}$ and $\bar{f}^{J+}_{\frac{1}{2}-\frac{1}{2}10}$.

b. π contribution. The π trajectory has even signature and odd parity. States with different nucleon helicity being forbidden by G parity, the π will contribute only to $\bar{f}^{J-}_{\frac{1}{2}\frac{1}{2}10}$.

c. B contribution. For the B trajectory, states with different nucleon helicity are also forbidden. Because it has odd signature and even parity it will contribute to $\bar{f}^{J-}_{\frac{1}{2}\frac{1}{2}10}$ only.

d. A_2 contribution. The A_2 meson has even signature and even parity. Both helicity states are allowed for the nucleons. One gets a contribution to $\bar{f}^{J+}_{\frac{1}{2}\frac{1}{2}10}$ and $\bar{f}^{J+}_{\frac{1}{2}-\frac{1}{2}10}$.

e. A_1 contribution. A_1 has odd signature and even parity. Only states with opposite nucleon helicities are allowed by G. Then it contributes to $\bar{f}^{J-}_{\frac{1}{2}-\frac{1}{2}10}$.

2) Reactions $\gamma N \rightarrow \pi^0 N$

The G parity is always conserved because the photon may have both isoscalar and isovector character. However, C has to be conserved at both vertices, and one obtains

$$C\,|JM\lambda\lambda'\rangle_{\pm} = \pm(-1)^J\,|JM\lambda\lambda'\rangle_{\pm} \quad (\lambda \neq \lambda')$$

$$C\,|JM\lambda\lambda\rangle_{\pm} = (-1)^J\,|JM\lambda\lambda\rangle_{\pm}$$

a. Vector meson contributions. The ρ, ω and ϕ have odd signature and odd parity. Both helicity states of the nucleons are allowed so that they contribute to $\bar{f}^{J+}_{\frac{1}{2}\frac{1}{2}10}$ and $\bar{f}^{J+}_{\frac{1}{2}-\frac{1}{2}10}$.

b. B contribution. States with different nucleon helicity are this time forbidden by C, and B contributes to $\bar{f}^{J-}_{\frac{1}{2}\frac{1}{2}10}$.

5. REGGEIZATION OF HELICITY AMPLITUDES

1) Daughter trajectories

Because in our problem we have three kinds of different masses, $\cos\theta_t$ is no longer growing like s in the forward direction. We assume that this s dependence is recovered by a daughter trajectory mechanism [4].

Then, using the standard method developed by Wang [5] and keeping only the first poles of the Gamma functions in the asymptotic expansion of Legendre polynomials, we obtain the following contributions of a single Regge pole to helicity amplitudes

$$\begin{aligned}
\bar{f}^{\,+}_{\frac{1}{2}\frac{1}{2}10} &= -\frac{k_t\sqrt{t}}{2}\,(a_1 - 2m a_4)\,\xi_\alpha \\
\bar{f}^{\,-}_{\frac{1}{2}\frac{1}{2}10} &= p_t\, k_t\,(a_1 + t\, a_2)\,\xi_\alpha \\
\bar{f}^{\,+}_{\frac{1}{2}-\frac{1}{2}10} &= \frac{k_t}{2}\,(2m a_1 - t\, a_4)\,\xi_\alpha \\
\bar{f}^{\,-}_{\frac{1}{2}-\frac{1}{2}10} &= -p_t\, k_t\sqrt{t}\, a_3\,\xi_\alpha
\end{aligned} \qquad (5)$$

with

$$\xi_\alpha = \alpha(\alpha+1)\,\frac{1 \pm e^{-i\pi\alpha}}{\sin\pi\alpha}\left(\frac{s}{s_0}\right)^{\alpha-1}\Pi_\alpha$$

where

$$\Pi_\alpha \;\text{ is }\; (\alpha+2) \;\text{ or }\; (\alpha+2)(\alpha+3)$$

and, as usual, we take $s_0 = 1\ \mathrm{GeV}^2$.

2) Constraints for $t = 0$

In the preceding expressions a_1, a_2, a_3 and a_4 are assumed to be regular functions, without kinematical singularities. They are in fact related to the invariant functions for which the Mandelstam representation is usually assumed to hold and, therefore, may have some poles due to gauge invariance, from which we shall only retain the pion pole in A_2^-. As we have already mentioned, for t going to zero, the helicity amplitudes are no longer independent and are related through the constraint equation, (3).

Three mechanisms have been proposed to satisfy this kind of relations [1]: conspiracy, evasion, daughter trajectories interferences. From (5) it appears that any Regge pole will lead to helicity amplitudes of order $s^{\alpha-1}$, and that, therefore, any daughter trajectory will give a contribution of order $s^{\alpha-2}$ or less. If the main trajectory contributes to $\bar{f}^{-}_{01}$ or to $\bar{f}^{+}_{11}$ with terms of order $s^{\alpha-1}$, then, it is clear that the daughter contributions will never satisfy (3). They will be of some help only when the main trajectory gives a $s^{\alpha-1}$ contribution to f^{-}_{11}, to cancel the $s^{\alpha-2}$ contribution to f^{+}_{11}. (This situation is met only with A_1 exchange.) In fact, we shall always neglect the $s^{\alpha-2}$ terms arising from a main trajectory.

Therefore, we conclude that the daughter mechanism (alternative 3 of Ref. [1]) is of very little interest in photoproduction to make the constraint equation at $t=0$ satisfied, and, at least, that there are very few chances that it might be checked by experiment. In the following we shall examine for each particular case which of the two mechanisms, conspiracy, or evasion, may be considered. We shall see that it is very unlikely that known particles are conspirators, and that perturbation theory always supports the evasion mechanism.

6. COMPARISON WITH EXPERIMENT

We begin first by the easiest case, namely, the reaction $\gamma p \to \pi^0 p$.

1) Neutral pion photoproduction

As we have already mentioned, only B and vector mesons could contribute to this process. Now it is well known from low energy photoproduction analysis, from vector meson photoproduction, from SU_3 and from many other analyses, that the $\gamma\pi\rho$ and $\gamma\pi\phi$ couplings are strongly depressed with regard to the $\gamma\pi\omega$ coupling.

We shall then disregard the ρ and ϕ contributions and assume that only ω and B contribute to this reaction. Of course, our ω contribution may be considered as an effective vector meson contribution.

Now, in terms of the usual isospin expansion (2) of the scattering amplitude, one sees that B contributes only to T^0 and ω only to T^+. Then, any conspiracy between these two trajectories is impossible in $\gamma\, p \rightarrow \pi^0 n$.

We then take the perturbation theory as a guide to see if ω chooses evasion or not. Elementary ω exchange leads to the following contribution

$$T_\omega^+ = [C_1 M_4 - C_2(t M_1 - M_2)] \frac{\lambda_{\gamma\pi\omega}}{t - m_\omega^2}$$

where C_1 and C_2 are the usual ω-$N\bar{N}$ coupling constants.

In this calculation the contribution to helicity amplitudes is then given by

$$\bar{f}^+_{\frac{1}{2}\frac{1}{2}10} = \frac{k_t\sqrt{t}}{2}\,[C_2 t + 2m C_1] \tag{7}$$

$$\bar{f}^+_{\frac{1}{2}-\frac{1}{2}10} = -\frac{k_t t}{2}\,[2m C_2 + C_1]$$

From Eq. (7) one deduces that the conspiring amplitude f^+_{11} is going to zero like $\sqrt{t}$. This supports an evasion mechanism for the ω exchange.

For the B exchange only A_2 in Eq. (1) is non-zero, so that the B contribution is always vanishing like $\sqrt{t}$. It should be noticed that even if ρ and ϕ contributions were taken into account, conspiracy would be excluded by the preceding reasoning. By different arguments it has been concluded by Halpern [6] that ω, ϕ, ρ and B are not conspirators.

We have assumed that the ω and B trajectories were parallel to the ρ trajectory with a mass difference displacement, such that

$$\alpha_\omega(t) = 0.56 + t$$

$$\alpha_B(t) = -0.30 + t$$

The parametrization of helicity amplitudes for $\gamma p \rightarrow \pi^0 n$ is then given by

$$\bar{f}^{+}_{01} = -\frac{k_t\sqrt{t}}{2}\left[t\, a_1^\omega - 2m\, a_4^\omega\right]\xi_{\alpha_\omega}$$

$$\bar{f}^{+}_{11} = \frac{k_t}{2}\, t\, \left[2m\, a_1^\omega - a_4^\omega\right]\xi_{\alpha_\omega}$$

$$\bar{f}^{-}_{01} = P_t\, k_t\, t\, a_2^B\, \xi_{\alpha_B}$$

with

$$\xi_{\alpha_\omega} = \alpha(\alpha+1)(\alpha+2)\left[\frac{1-e^{-i\pi\alpha}}{\sin\pi\alpha}\right]\left(\frac{s}{s_0}\right)^{\alpha-1}$$

$$\xi_{\alpha_B} = \alpha(\alpha+1)(\alpha+2)(\alpha+3)\left[\frac{1-e^{-i\pi\alpha}}{\sin\pi\alpha}\right]\left(\frac{s}{s_0}\right)^{\alpha-1}$$

The extra factors $(\alpha+2)$ and $(\alpha+3)$ have been introduced in order to fit the experimental data up to $t = -3(\mathrm{GeV}/c)^2$, because $\alpha_\omega = -2$ for $t = -2.5(\mathrm{GeV}/c)^2$ and $\alpha_B = -3$ for $t = -2.7\ (\mathrm{GeV}/c)^2$, and a_1^ω, a_4^ω and a_2^B are assumed to be constants.

For $t \to 0$ only the ω contribution to $\bar{f}^{+}_{01}$ survives, so that one may expect the differential cross-section to be very small in the forward direction, and then to rise quickly. This is in good agreement with experimental data. Also the dip for $t \sim -0.6 (\mathrm{GeV}/c)^2$, is well accounted for by the vanishing of the ω trajectory at this energy.

The best fit obtained is shown on Figs. 1, 2 and 3, together with the experimental data of Refs. [7),8),9)], for the following values of the parameters

$$a_1^{\omega} = -22.8 \sqrt{\mu b}\ (\mathrm{GeV}/c)^{-3}$$

$$a_4^{\omega} = 4.56 \sqrt{\mu b}\ (\mathrm{GeV}/c)^{-2}$$

$$a_2^{B} = 34 \sqrt{\mu b}\ (\mathrm{GeV}/c)^{-3}$$

The general feature of the differential cross-section as a function of t and E_γ is nicely reproduced in this three parameter fit, for $E_\gamma \geqslant 3$ GeV. It is really not astonishing that the fit is not so good for $E_\gamma = 2$ and 2.5 GeV, because there is certainly some resonant contributions at these low energies. Nevertheless we think that our fit is better than the one obtained by Locher and Rollnik [10)] using only ω exchange + background, and helicity amplitudes which do not satisfy the analytic properties we have imposed on ours. The contribution of ω is also shown on Fig. 2. It should be noticed that the two contributions are purely additive in the cross-section, so that one may write:

$$\frac{d\sigma}{dt}(\gamma p \to \pi^0 p) = \frac{d\sigma^{\omega}}{dt} + \frac{d\sigma^{B}}{dt}$$

2) Charged pion photoproduction

The situation with regard to the π^+ photoproduction case is much more complicated both from a theoretical point of view because the number of possible exchanged states is much larger, and from an experimental point of view because the general behaviour of the angular distribution is rather different from the π^o case and seems to exhibit different qualitative features above 3 GeV. It has even been suggested [1] that the π^+ photoproduction might be a definite example of failure of the Regge pole model.

To begin with, we have tried to describe the process $\gamma\, p \rightarrow \pi^+ n$ in the simplest way, namely, with a two trajectory fit. The two trajectories were chosen to be the B whose importance has been shown in π^o photoproduction and, of course, the π which is known to be very important. The ρ contribution was excluded for the reasons explained in the preceding paragraph and the A_1 and A_2 contributions neglected.

The B contribution to $\gamma\, p \rightarrow \pi^+ n$ is simply deduced by isospin invariance from $\gamma\, p \rightarrow \pi^o n$. One obtains

$$\bar{f}_{01}^{-}(\pi^+) = -\sqrt{2}\, \bar{f}_{01}^{-}(\pi^o)$$

For the pion it seems reasonable to assume that its contribution to $\bar{f}_{01}^{-}$ vanishes also for $t \rightarrow 0$. From perturbation theory one knows that the one-pion exchange term contributes only to A_2^-, which leads to a $\sqrt{t}A_2$ contribution to $\bar{f}_{01}^{-}$, and it has also been shown by Halpern that probably the pion was not conspiring. We therefore get the following expression for the π exchange

$$\bar{f}_{01}^{-} = P_t k_t \frac{t}{t-\mu^2}\, a_2^{\pi}\, \xi_{\alpha_\pi}$$

with

$$\xi_{\alpha_\pi} = \alpha(\alpha+1)(\alpha+2)(\alpha+3)\left[\frac{1+e^{-i\pi\alpha}}{\sin\pi\alpha}\right]\left(\frac{s}{s_0}\right)^{\alpha-1}$$

If we assume that for $t=\mu^2$ we should recover the usual perturbation calculation, then for $s\to\infty$:

$$s_0 a_2^{\pi} = \sqrt{2}\ e g \pi/48 = 7.8\sqrt{\mu b} \times (GeV/c)$$

and the π^+ photoproduction is completely determined, without any free parameter. But, whatever the parameters are, we then get in trouble, because with the assumed contributions for π and B, the differential cross-section should vanish in the forward direction. Experimentally, this does not seem to be the case, because a diffraction peak is observed in the forward direction [11], at least up to 3 GeV.

Nevertheless, the recent experimental results of Dowd et al. [12] for $E_\gamma = 3.25$ and 4.17 GeV suggest that this forward peak may disappear at high energy. It is in fact well known that this peak is already present at low energy, even at 600 MeV, and that at these energies it is due to an interference between a resonant background and the pion term. More precise experimental information in the forward direction at high energy would be of much help. Within the present experimental status, we consider the two following possibilities:

a. the forward peak disappears at high energy; then, the experimental cross-section is well described by π and B exchange only; the results are shown on Figs. 4, 5, 6 and 7 *); it should be noticed that this is an absolute prediction without any free parameters;

---

*) Experimental results for $3.4 \leq E_\gamma \leq 4.1$ GeV from Ref. [13] have been excluded because of uncertainties in the energy determination and in the absolute normalization of the cross-section.

b. the forward peak does not disappear at high energy; then, only in the forward direction B and π are not sufficient to explain the data, and one has to search for a different mechanism; the more natural explanation would probably be in terms of a resonant background, but at the present time the more popular would certainly be a possible conspiracy mechanism.

7. K MESON PHOTOPRODUCTION

With the K meson photoproduction we are faced with the kinematics of a four-different-masses process, and we shall adopt the same notations as in the preceding sections, the main difference being that now p_t, the baryon momentum in the t channel, is given by

$$p_t = [t-(m+Y)^2]^{1/2}\,[t-(m-Y)^2]^{1/2}/2\sqrt{t}$$

where Y is the hyperon mass.

We choose the same system of invariant functions, except that M_4 is now

$$M_4 = 2\gamma_5(\gamma.\varepsilon\ \ P.k - \gamma.k\ \ P.\varepsilon) - i\gamma_5(m+Y)\,\gamma.\varepsilon\ \gamma.k$$

and the helicity amplitudes are given in terms of the corresponding A_i by the following equations:

$$\begin{aligned}
f^{+}_{01} &= -\frac{k_t}{2}[t-(m-Y)^2]^{1/2}\,[A_1-(m+Y)A_4] \\
f^{-}_{01} &= p_t k_t\left[\frac{t}{t-(m-Y)^2}\right]^{1/2}\left[(t-(m-Y)^2)A_2 + A_1 + (m-Y)A_3\right] \\
f^{+}_{11} &= \frac{k_t}{2}\left[\frac{t-(m-Y)^2}{t}\right]^{1/2}\left[(m+Y)A_1 - t A_4\right] \\
f^{-}_{11} &= -p_t k_t\frac{1}{[t-(m-Y)^2]^{1/2}}\left[t A_3 + (m-Y)A_1\right]
\end{aligned} \qquad (9)$$

If we invert now these relations and express the A_i in terms of the helicity amplitudes, we find as expected [14] that the A_i are non-singular for $t=0$, or, in other words, that there are no constraints among helicity amplitudes for $t=0$. Nevertheless, there are constraints for $t=(m-Y)^2$, which is not far from the physical domain.

If we put

$$x = \left[t-(m-Y)^2\right]^{1/2}$$

we find that the following relations are to be satisfied for $x \rightarrow 0$

$$\bar{f}_{01}^{+} + \frac{Y+m}{Y-m}\,\bar{f}_{11}^{+} = O(x) \qquad (10a)$$

$$\bar{f}_{01}^{-} - \bar{f}_{11}^{-} = O(x^2) \qquad (10b)$$

The first relation is always satisfied in a trivial manner, because from (9) we see that both $\bar{f}_{01}^{+}$ and $\bar{f}_{11}^{+}$ have to vanish as x when $x \rightarrow 0$, otherwise A_1 and A_4 should have a pole in $1/x$. The second relation is not trivial and is interesting by itself because it is completely different from the one obtain for $t=0$ when the two baryons have equal masses, and implies a relation between $\bar{f}_{01}^{-}$ and $\bar{f}_{11}^{-}$ instead of $\bar{f}_{01}^{-}$ and $\bar{f}_{11}^{+}$. This demonstrates once more, if necessary, that there is really no continuity relation between equal mass case and unequal mass case. Moreover, it should be noticed, that if one takes into account the neutron-proton mass difference (the fact that this is an electromagnetic mass difference has nothing to do here, because it is a pure problem of kinematics), then it is relation (10b) which has to be used for $\gamma p \rightarrow \pi^+ n$ instead of (3). The discussion usually developed about conspirators for $\gamma p \rightarrow \pi^+ n$ is, in fact, incorrect. Nevertheless, the evasion procedure we have assumed in Section 6 will show up to be correct in K meson photoproduction.

1) Reggeization of helicity amplitudes

The three possible trajectories are K, K^* and K^{**}. For simplicity, due to the lack of experimental information, we have neglected the K^{**} contribution.

The contribution of the K trajectory is given by

$$\bar{f}_{0\bar{1}} = p_t k_t \sqrt{t} \; \frac{[t-(m-Y)^2]}{t-m_K^2} \; a_2^K \, \xi_{\alpha_K} \tag{11}$$

with

$$\xi_{\alpha_K} = \alpha(\alpha+1)(\alpha+2)\cancel{(\alpha+3)} \frac{1+e^{-i\pi\alpha}}{\sin\pi\alpha} \left(\frac{s}{s_0}\right)^{\alpha-1}$$

and

$$\alpha_K = t - m_K^2$$

In (11) we have factorized the pole in A_2 due to gauge invariance, and it is also manifest that (10b) is satisfied.

For the K^* contribution we obtain

$$\bar{f}_{01}^{\,+} = -\frac{k_t}{2}[t-(m-Y)^2]^{1/2}[a_1^{K^*} - (m+Y)\, a_4^{K^*}]\, \xi_{\alpha_{K^*}}$$

$$\bar{f}_{11}^{\,+} = \frac{k_t}{2}\left[\frac{t-(m-Y)^2}{t}\right]^{1/2}\left[(m+Y)a_1^{K^*} - t\, a_4^{K^*}\right] \xi_{\alpha_{K^*}}$$

with

$$\xi_{\alpha_{K^*}} = \alpha(\alpha+1)(\alpha+2)\; \frac{1-e^{-i\pi\alpha}}{\sin\pi\alpha}\left(\frac{s}{s_0}\right)^{\alpha-1}$$

and

$$\alpha_{K^*} = 0.37 + t$$

[We assume once more that the K^* trajectory is parallel to the ρ trajectory with $\alpha_{K^*}(o) - \alpha_\rho(o) = m_\rho^2 - m_{K^*}^2$.]

The cross-section is then also given by (4) with appropriate expressions for $\cos\theta_t$ and $\sin\theta_t$.

2) Comparison with experiment

The experimental information about K meson photoproduction at high energy is very poor up to now. There are only differential cross-sections at 3.7 GeV from Elings et al. [13] for $\gamma p \to K^+\Lambda$ and $\gamma p \to K^+\Sigma^o$. Any definite conclusion is, of course, impossible at the present time. Nevertheless, we found that a one-trajectory fit was impossible because this would lead in any case to a vanishing cross-section for $0.4 \leqslant -t \leqslant 0.8 (\mathrm{GeV/c})^2$, in contradiction with experiment.

A tentative fit is presented on Fig. 8 for $\gamma p \to K^+\Lambda^o$ with the following values for the parameters

$$a_1^{K^*} = 4.5 \sqrt{\mu b}\ (\mathrm{GeV/c})^{-3}$$
$$a_4^{K^*} = 22.3 \sqrt{\mu b}\ (\mathrm{GeV/c})^{-2}$$
$$a_2^{K^*} = 81.5 \sqrt{\mu b}\ (\mathrm{GeV/c})^{-1}$$

The differential cross-section for $\gamma p \to K^+\Sigma^o$ is shown on Fig. 9, the theoretical cross-section was deduced from the $\gamma p \to K^+\Lambda^o$ cross-section by multiplying it by a factor 2.22. We found that the $\Lambda^o - \Sigma^o$ mass difference gives only 5% difference in the absolute normalization of the cross-section.

3) SU_3 symmetry

It has already been noticed that the SU_3 symmetry encounters some difficulties in photoproduction at high energy. The two reactions $\gamma p \to \Lambda K^+$ and $\gamma p \to \Sigma^0 K^+$ differ only at the baryon vertex. From SU_3 we predict at once that for any isospinor exchange in the t channel

$$\frac{d\sigma}{dt}(\Sigma^0) \Big/ \frac{d\sigma}{dt}(\Lambda^0) = 3\left(\frac{d-f}{3f+d}\right)^2$$

where f and d are the usual symmetric and antisymmetric couplings. For K exchange, if we use $f/d = \frac{1}{2}$, this leads to

$$\sigma(\Sigma^0)/\sigma(\Lambda^0) = \frac{3}{25}$$

For K^* exchange, if we assume $d = 0$, then

$$\sigma(\Sigma^0)/\sigma(\Lambda^0) = \frac{1}{3}$$

Because there is no interference in the differential cross-section between K and K^* contributions, we find that in any case $\sigma(\Sigma^0) \ll \sigma(\Lambda^0)$, which is in contradiction with the experimental result $\sigma(\Sigma^0) = 2.22\,\sigma(\Lambda^0)$.

From our results one gets $f/d = 0.03$ for the 0^- octet and 1^- octet. This is completely unreasonable, when compared with previous analysis.

CONCLUSION

At the present time one can conclude that the Regge pole model works for photoproduction. We think that it is interesting to notice that our best results have been obtained for π^0 photoproduction, which is the most restrictive case, because as we have shown, only two states may be exchanged in the t channel. This is to some extent analogous to elastic π - nucleon scattering for which analysis of $\pi^- p \rightarrow \pi^0 n$, with only one possible trajectory exchanged, was a great success of the Regge pole model (except for polarization!).

But, of course, some problems remain unsolved. For π^+ photo-production it is clear that the absence or presence of a forward peak will be decisive to prove or disprove that a simple picture may work, and at last we think that the K meson photoproduction will be a very important check of the validity of SU_3 symmetry for strong and electromagnetic interactions.

R E F E R E N C E S

1) B. Diu and M. Le Bellac, Orsay preprint, TH/198 (April 1967).

2) J.S. Ball, Phys.Rev. 124, 2014 (1961).

3) R.W. Childers and W.G. Holladay, Phys.Rev. 132, 1809 (1963).

4) D. Freedman and J. Wang, Phys.Rev. 153, 1596 (1967).

5) L.L. Wang, Phys.Rev. 142, 1187 (1966).

6) M.B. Halpern, Princeton preprint (1967).

7) R. Alvarez et al., Phys.Rev.Letters 12, 707 (1964).

8) M. Braunschweig, D. Husmann, K. Lübelsmeyer and D. Schmitz, Phys. Letters 22, 705 (1966).

9) G.C. Bolon et al., Phys.Rev.Letters 18, 926 (1967).

10) M.P. Locher and H. Rollnik, Phys.Letters 22, 696 (1966).

11) G. Buschhorn et al., Phys.Rev.Letters 17, 1027 (1966) and 18, 571 (1967);
P. Schmüser, DESY Interner Bericht (March 1967).

12) J.P. Dowd, D.O. Caldwell, K. Heinloth and T.R. Sherwood, CEA preprint (1967).

13) V.B. Elings et al., Phys.Rev.Letters 16, 474 (1966).

14) H. Högaasen and Ph. Salin, CERN preprint, TH. 788, (1967).

FIGURE CAPTIONS

Figure 1 : Differential cross-section for $\gamma p \to \pi^{o} p$, experimental data from Refs. [7),8),9)].

Figure 2 : Differential cross-section for $\gamma p \to \pi^{o} p$ at 3 GeV, experimental data from Ref. [8)]; dash line: ω contribution.

Figure 3 : Differential cross-section for $\gamma p \to \pi^{o} p$ at 2 GeV, experimental data from Refs. [7),8)].

Figure 4 : Differential cross-section for $\gamma p \to \pi^{+} n$ at 2.6 GeV, experimental data from Ref. [11)].

Figure 5 : Differential cross-section for $\gamma p \to \pi^{+} n$ at 2.88 GeV, experimental data from Ref. [11)].

Figure 6 : Differential cross-section for $\gamma p \to \pi^{+} n$ at 3.25 GeV, experimental data from Ref. [12)].

Figure 7 : Differential cross-section for $\gamma p \to \pi^{+} n$ at 4.17 GeV, experimental data from Ref. [12)].

Figure 8 : Differential cross-section for $\gamma p \to K^{+} \Lambda^{o}$ at 3.7 GeV, experimental data from Ref. [13)].

Figure 9 : Differential cross-section for $\gamma p \to K^{+} \Sigma^{o}$ at 3.7 GeV, experimental data from Ref. [13)].

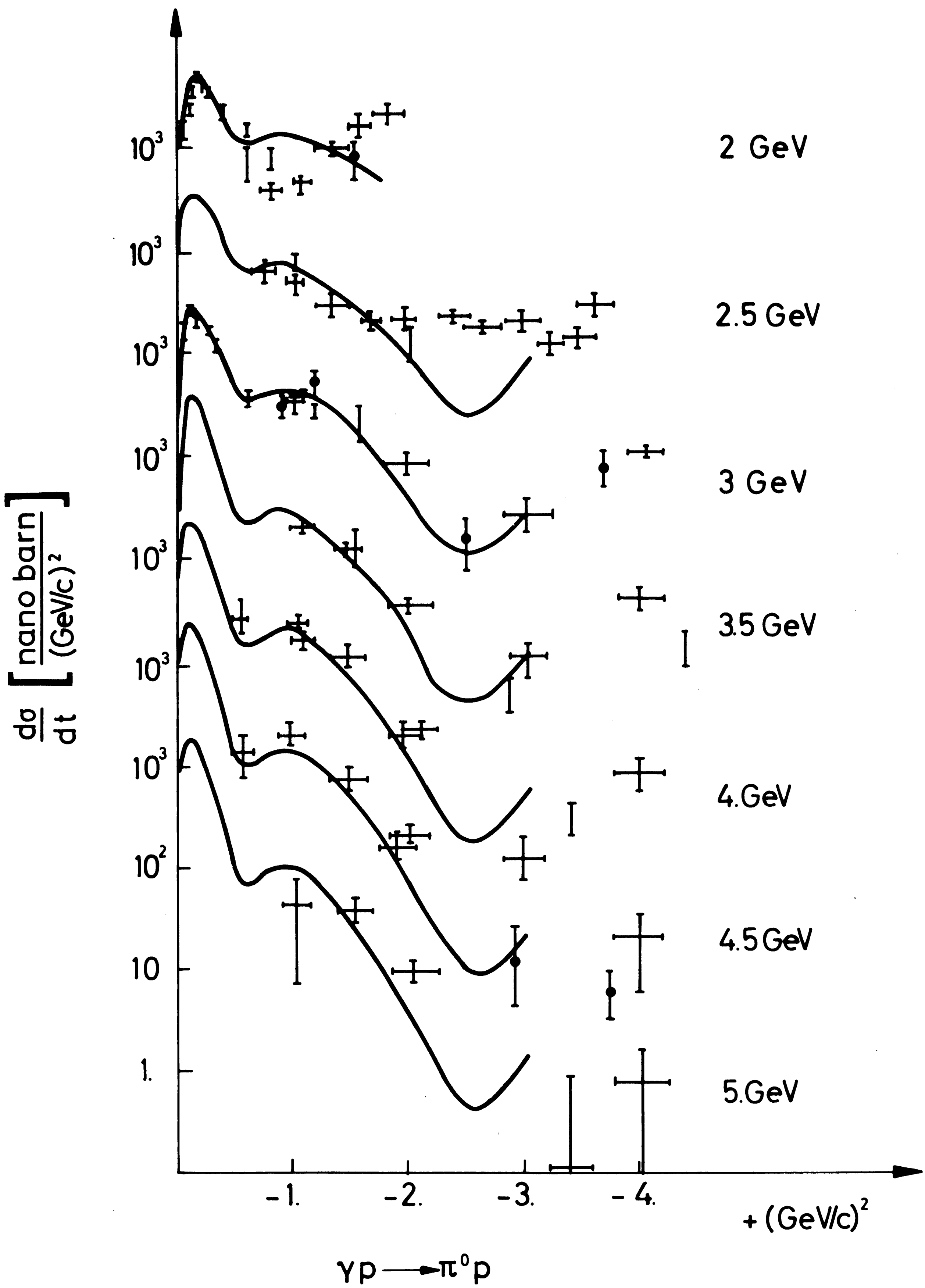
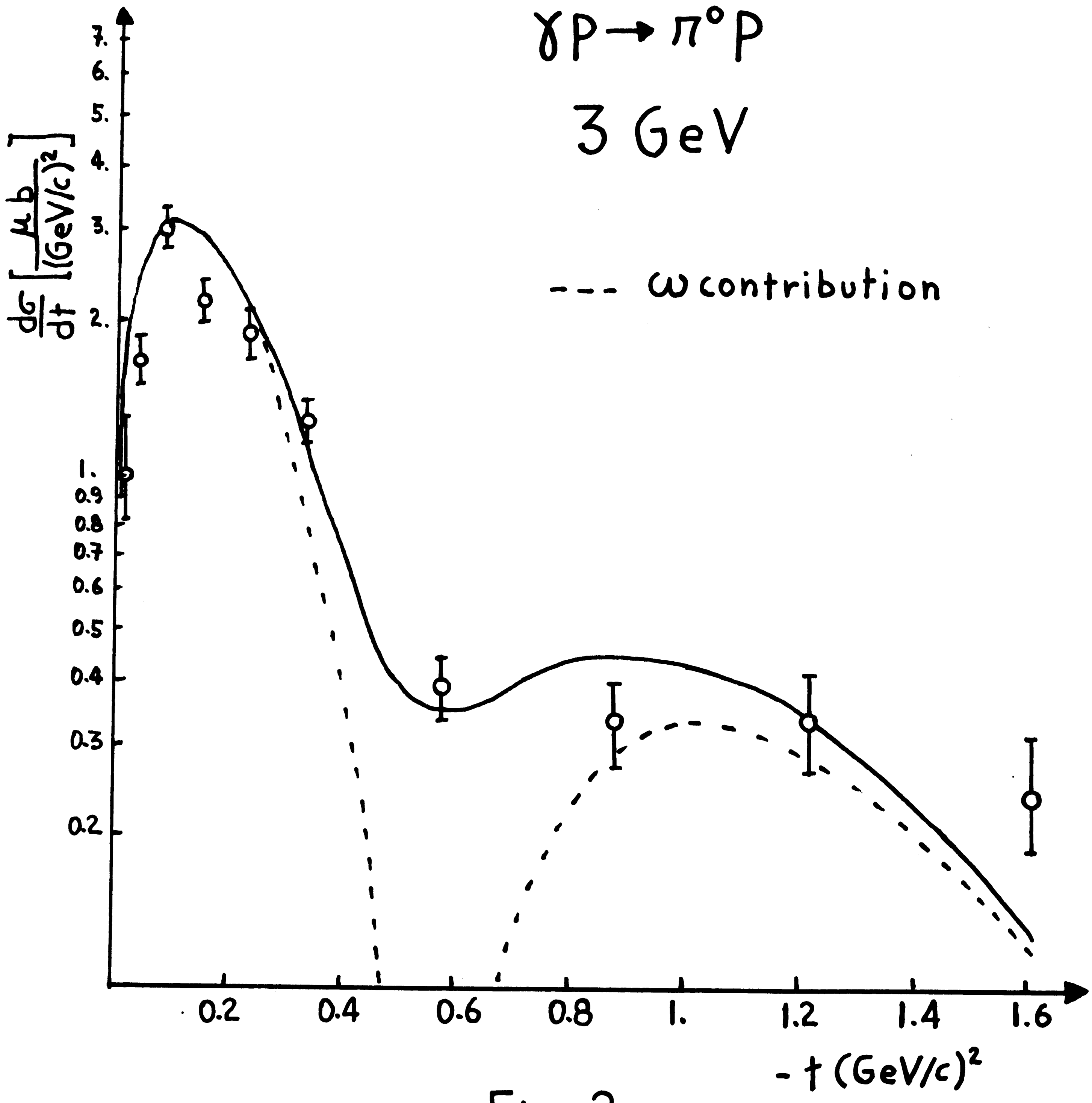


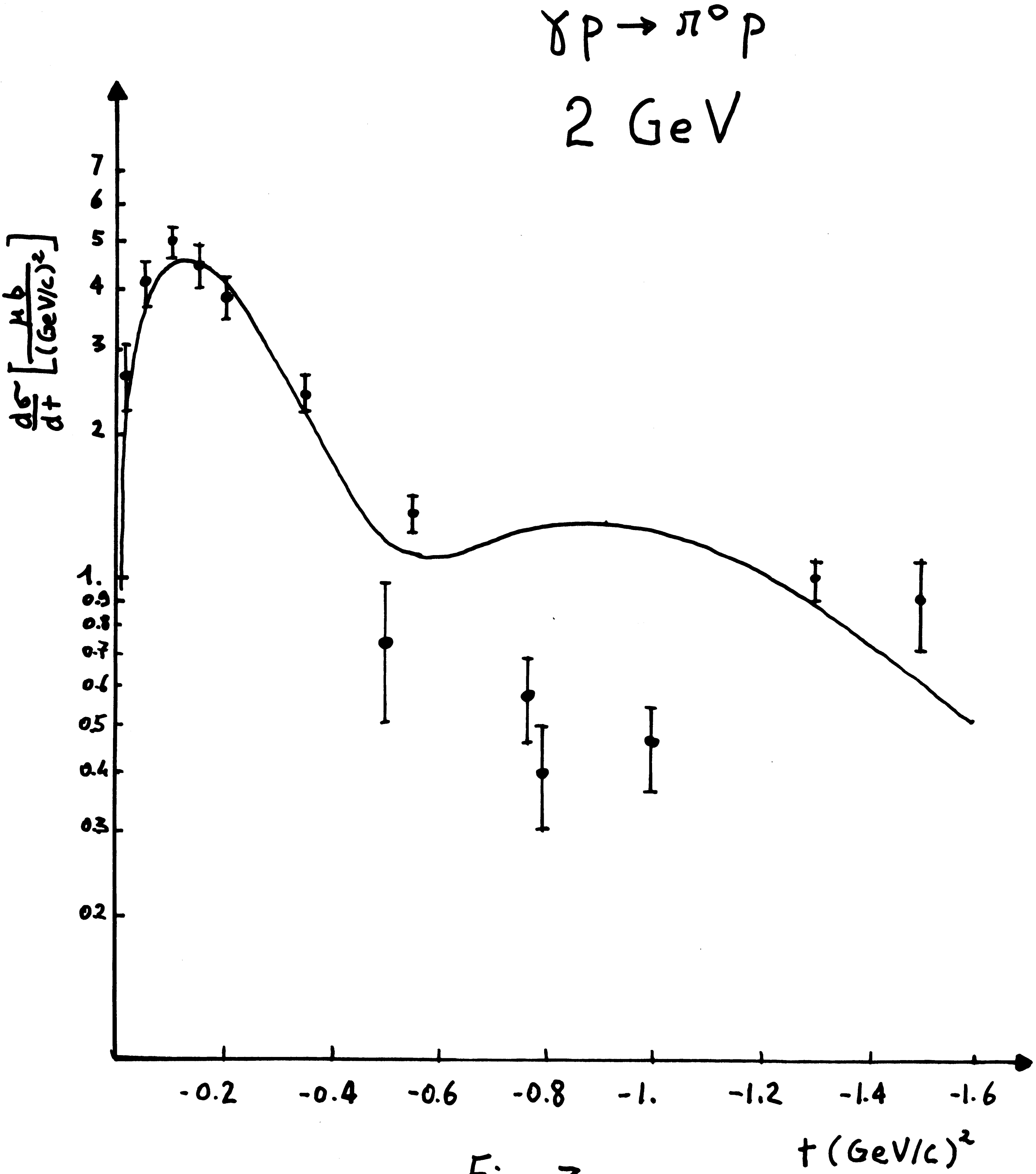
FIG.1

SIS/K 0 203



γP → π°P
3 GeV
--- ω contribution
dσ/dt [μb/(GeV/c)²]
7.
6.
5.
4.
3.
2.
1.
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.2
0.4
0.6
0.8
1.
1.2
1.4
1.6
-t (GeV/c)²

Fig.2



$\gamma p \to \pi^0 p$
2 GeV
$\frac{d\sigma}{dt}\left[\frac{\mu b}{(GeV/c)^2}\right]$
7
6
5
4
3
2
1.
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
-0.2
-0.4
-0.6
-0.8
-1.
-1.2
-1.4
-1.6
$t\ (GeV/c)^2$

Fig. 3

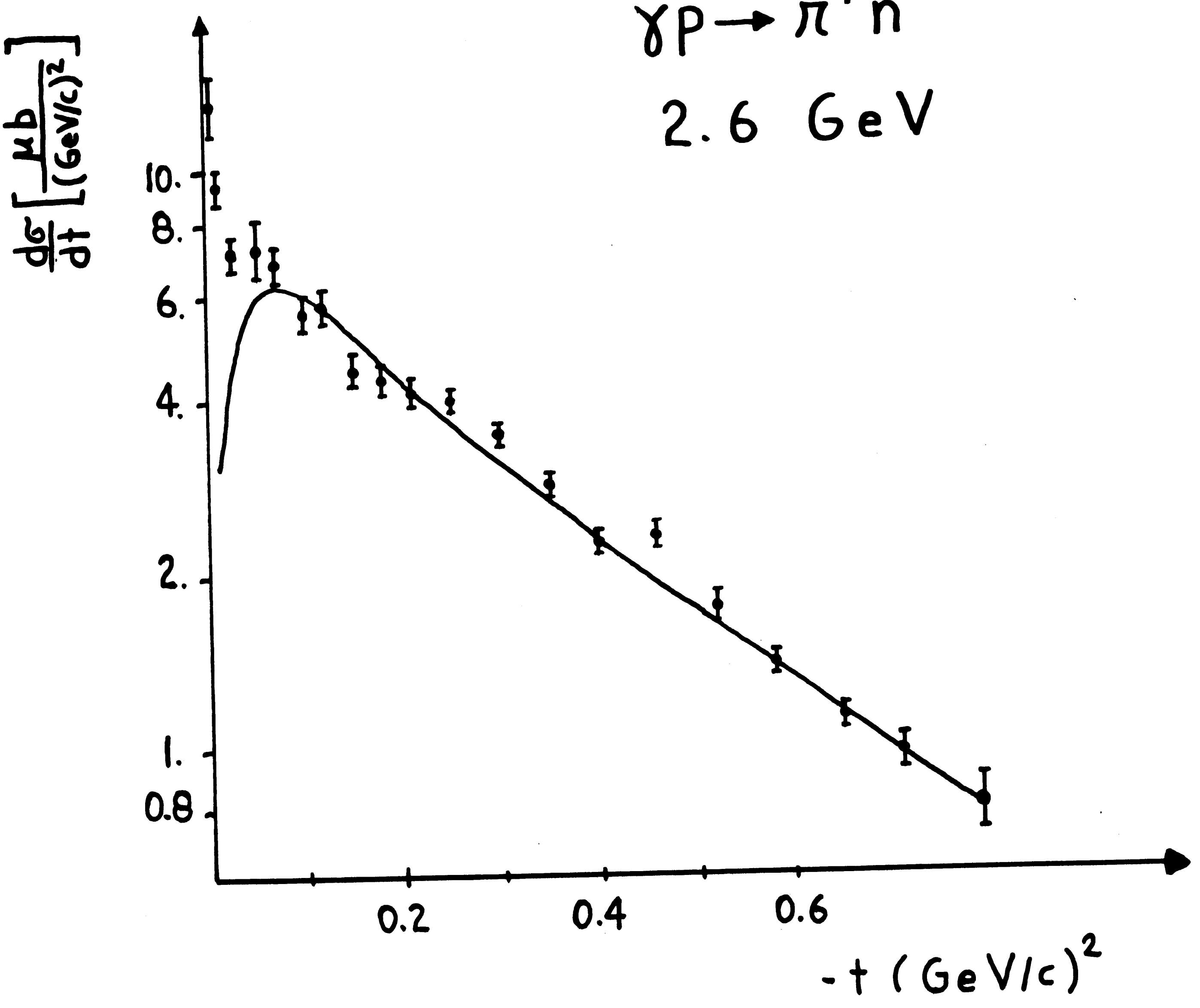
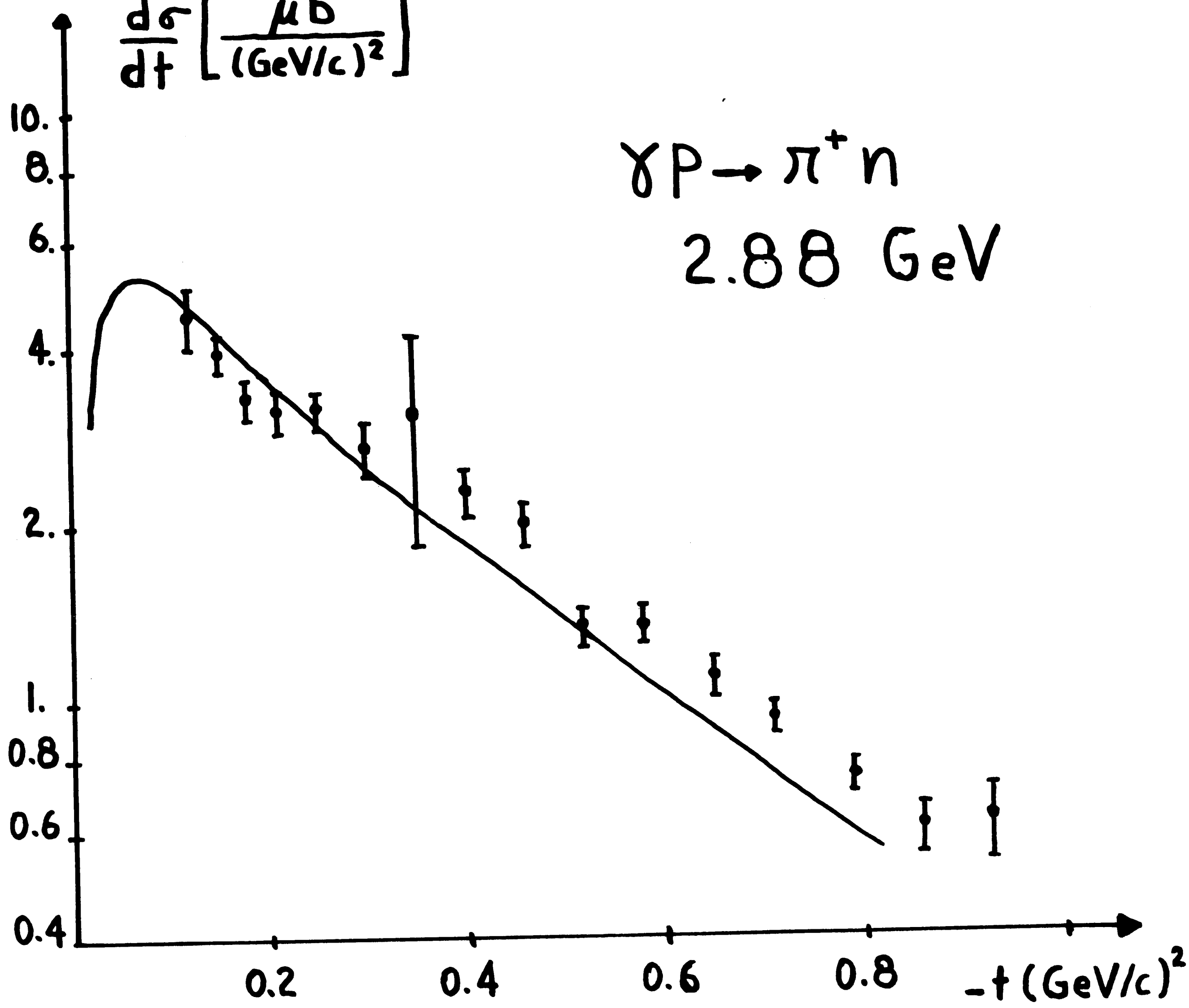


Fig. 4



$\frac{d\sigma}{dt}\left[\frac{\mu b}{(GeV/c)^2}\right]$
10.
8.
6.
4.
2.
1.
0.8
0.6
0.4
$\gamma P \to \pi^+ n$
2.88 GeV
0.2
0.4
0.6
0.8
$-t\ (GeV/c)^2$

Fig. 5

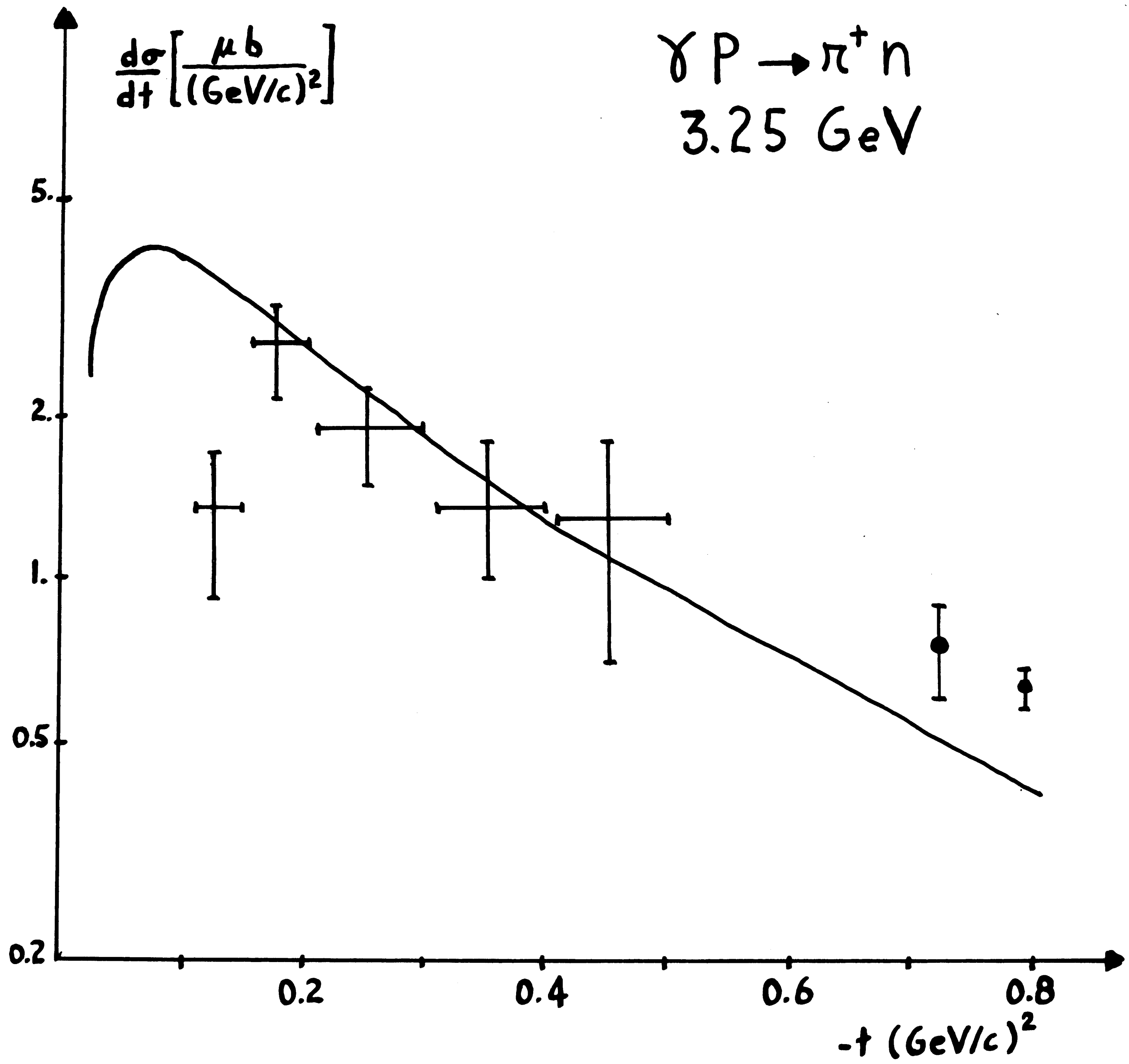
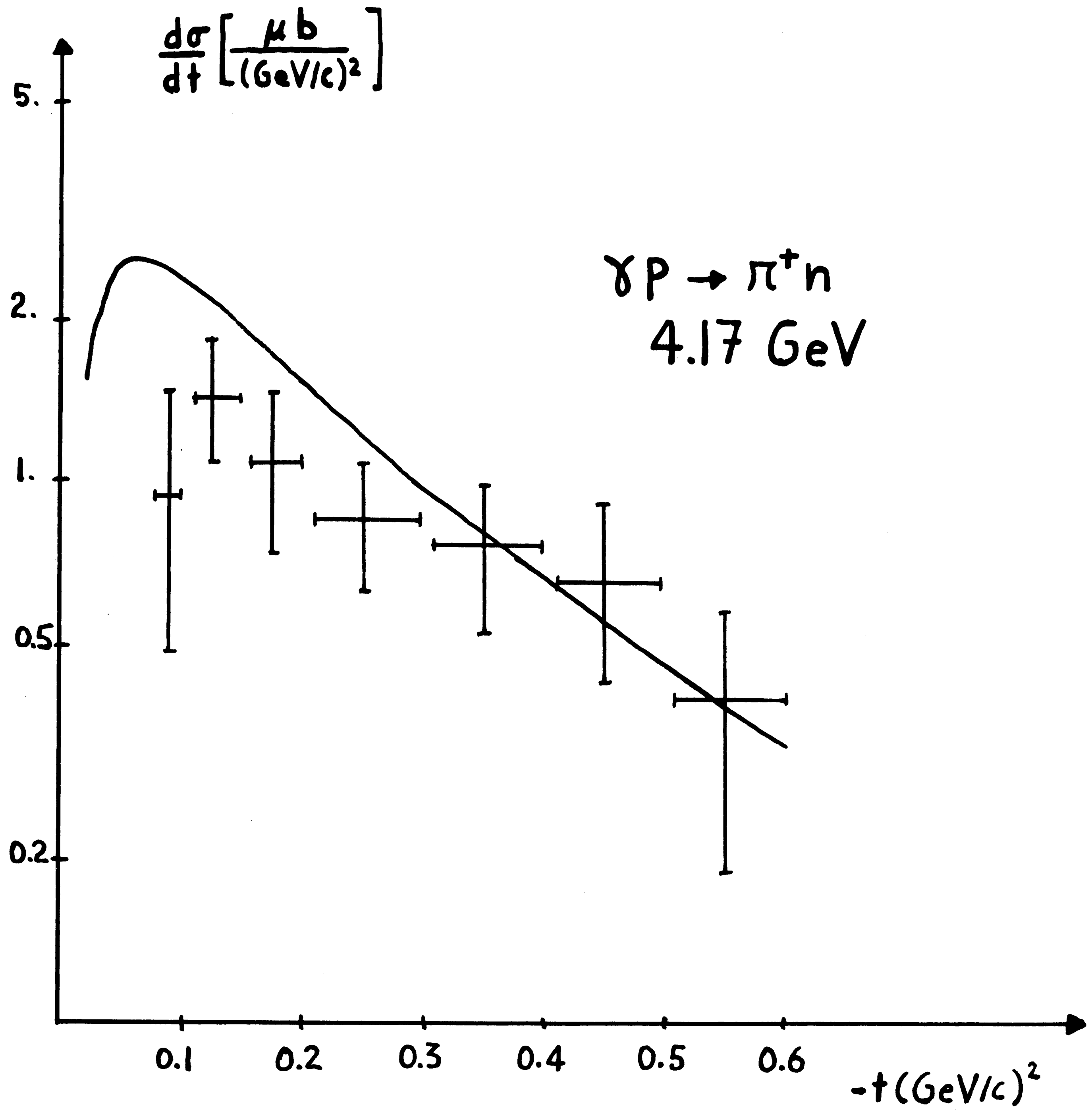
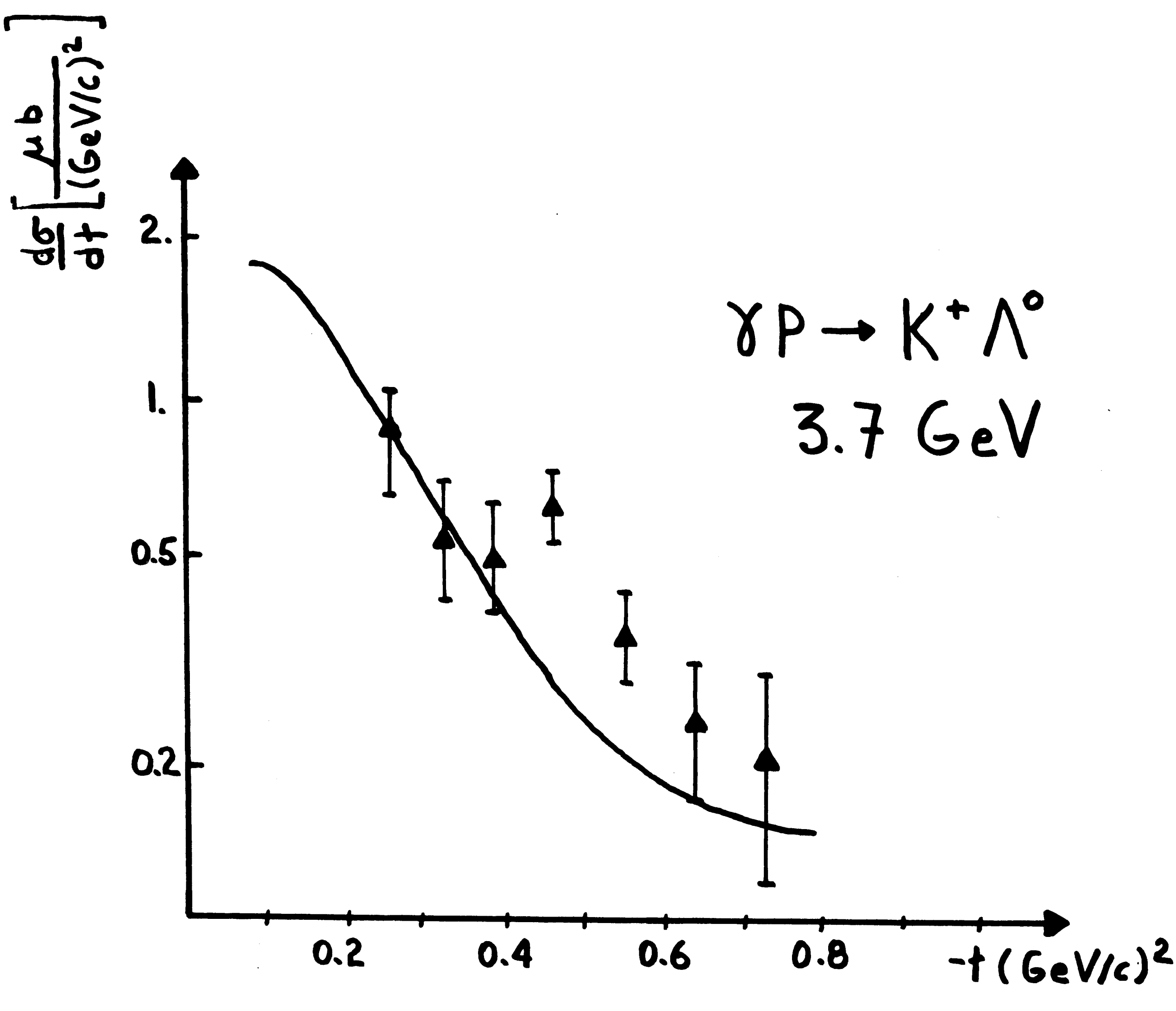

$\frac{d\sigma}{dt}\left[\frac{\mu b}{(GeV/c)^2}\right]$
$\gamma P \rightarrow \pi^+ n$
3.25 GeV
5.
2.
1.
0.5
0.2
0.2
0.4
0.6
0.8
$-t\ (GeV/c)^2$

Fig. 6



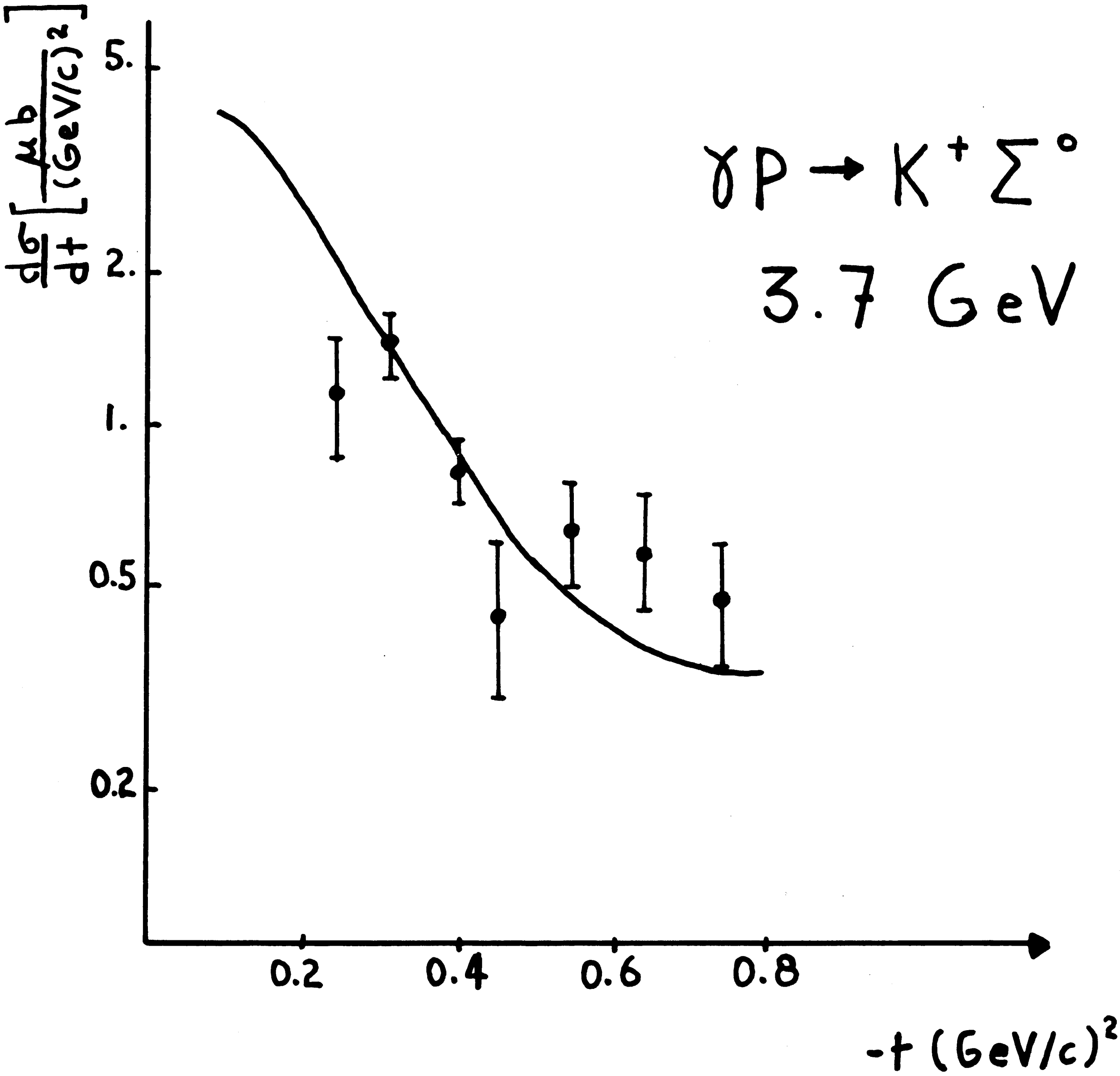
$\frac{d\sigma}{dt}\left[\frac{\mu b}{(GeV/c)^2}\right]$
5.
2.
1.
0.5
0.2
$\gamma p \to \pi^+ n$
4.17 GeV
0.1
0.2
0.3
0.4
0.5
0.6
$-t\,(GeV/c)^2$

Fig. 7



$\frac{d\sigma}{dt}\left[\frac{\mu b}{(GeV/c)^2}\right]$
2.
1.
0.5
0.2
0.2
0.4
0.6
0.8
$-t\ (GeV/c)^2$
$\gamma P \rightarrow K^+ \Lambda^0$
3.7 GeV

Fig. 8



$\frac{d\sigma}{dt}\left[\frac{\mu b}{(GeV/c)^2}\right]$
5.
2.
1.
0.5
0.2
0.2
0.4
0.6
0.8
$-t\ (GeV/c)^2$
$\gamma P \rightarrow K^+ \Sigma^\circ$
3.7 GeV

Fig. 9