INFLUENCE OF ABSORPTION DUE TO COMPETING PROCESSES ON PERIPHERAL REACTIONS

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ABSTRACT

The peripheral or one-meson exchange model is modified to include absorptive effects due to competition from other inelastic channels. These effects are most important for small impact parameters (low partial waves) and produce an appreciable reduction in cross-section, and a pronounced collimation of the angular distribution. The absorption thus provides a natural explanation of the highly peaked angular distributions previously ascribed to ad hoc form factors. The necessary information concerning the absorption is inferred from the elastic scattering in the initial and final state in a manner first proposed by Sopkovich. Because of the importance of the decay correlations of resonances produced in these reactions, the spins of the particles are treated in a consistent way. It is found that a proper treatment of the spins is also essential in evaluating the angular distribution of production. The reaction $\pi^- p \to \rho^- p$ at 4 GeV/c is discussed in detail ; the theoretical cross-section is in good agreement with experiment, both as to angular dependence and absolute value. The ρ meson's spin density matrix is found to be altered somewhat from the unmodified one-pion exchange result, especially at larger production angles. The calculated density matrix is in agreement with the available data on the decay correlations.

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1. INTRODUCTION

It is now common knowledge [1] that the unadorned peripheral model cannot account for the data on high energy quasi two-body collisions. Over an appreciable range of energy, and for a large variety of reactions, the empirical angular distributions are much more peaked in the forward direction than the theoretical distributions predicted by the one particle exchange diagram. The data therefore appear to imply the existence of an interaction mechanism responsible for these processes which is considerably more peripheral than that employed by the peripheral model, even though the decay correlations indicate a specific type of exchange. There already exist several somewhat unconvincing escapes from this paradoxical situation. For example, one can assume that the difference between the observed angular distribution and that arising from the one-meson exchange diagram is due to the momentum transfer (Δ) dependence of the vertex functions [1),2)]. The dependence required for this is far too rapid for comfort, however ; in the case of the $\bar{N}N\rho$ vertex, for example, it appears to be incompatible with the electromagnetic form factor of the nucleon. Another "explanation" involves the notion of Regge-pole exchange [3] ; although this device has the virtue of relating quasi-two-body reactions to a description of the elastic scattering, it is of a rather ad hoc nature, inasmuch as it introduces several unknown parameters (characteristics of the Regge trajectory). In short, it is not clear whether one is doing much more than curve fitting by introducing the rapidly varying form factors or the Regge-pole exchange.

There is, however, one very important effect, which undoubtedly exists at the energies in question, and which has been neglected in the peripheral model and in the modification thereof just referred to. This effect is the existence of many competing open channels. A single quasi-two-body production channel usually constitutes only a small fraction of the total inelastic cross-section. Furthermore, numerous reactions with more complex final state configurations also exist with appreciable cross-sections. On the crudest of intuitive grounds, one would expect these complex reactions to be initiated by collisions with small impact parameters. If this were so, the small impact parameter encounters (i.e., those of low relative angular momentum) are less likely to participate in the quasi-two-body reactions. Loosely speaking, one therefore expects the existence of competing open channels to reduce the low partial wave reaction amplitudes below the values given by the simple peripheral model, while leaving the higher partial waves essentially unchanged. This means that the main consequences of the coupling to other channels are (a) a reduction of the reaction cross-section, and (b) important modifications of the angular distribution. The specific change in the angular dependence must be determined by calculation ; as we shall see here, a drastic collimation in the forward direction results.

The influence of absorptive effects on production amplitudes at high energies was first considered by Feld [4]. Baker and Blankenbecler [5] treated the coupling of many open channels by dispersion theory and found the qualitative features described above. Unfortunately it appears that their model cannot lead to strong absorption. A simpler approach, based on the distorted wave Born approximation, and which we shall adopt, was developed by Sopkovich [6] who used it to treat $p\bar{p}$ reactions. Recently Dar, Kugler, Dothan and Nussinov [7] proposed an even simpler model wherein the two-body production process (such as $p\bar{p} \longrightarrow \Lambda\bar{\Lambda}$) is fed by encounters having a definite impact parameter, under the assumption that all smaller impact parameter collisions are completely taken up by complex, highly inelastic, reactions. This model gives rise to angular distributions which are quite well collimated in the forward direction. But, these distributions possess strong diffraction maxima and minima which do not appear to be observed, except perhaps in reactions involving antibaryons. There is, in fact, no evidence for a sharply defined absorptive region in the interactions between elementary particles. On the contrary, all the data on elastic scattering at high energies indicate a diffuse structure which can be characterized by an absorptivity having a Gaussian dependence on impact parameter. Furthermore, the exchange interaction responsible for the production process is surely not concentrated on a well-defined impact parameter. This last shortcoming of the model of Dar <u>et al</u>. has been removed by Dar and Tobocman [8], who eliminate all partial waves below a certain value of j from the one-meson exchange amplitude. In this way they retain a more realistic asymptotic form for the interaction.

During the preparation of this manuscript, two papers [9],[10] employing the distorted wave Born approximation appeared. These authors reach conclusions qualitatively similar to those reached here, but there are significant quantitative differences since our treatment, in contrast to theirs, incorporates spin effects completely.

None of the papers cited take the spins of the various particles properly into account [*]. This is a serious drawback for several reasons. First of all, in the peripheral model the spins of the various participants in the collision have a dramatic influence on the energy and momentum transfer dependences of the cross-section. In fact, for many of the reactions of interest, the peripheral model actually gives an angular distribution which fails completely to show collimation in the forward direction. By neglecting the spin one therefore reduces the discrepancy between the model and the data in a spurious fashion. But, more important, one must treat the spins consistently if one wishes to understand the decay correlations [11] in those reactions where unstable particles are produced. The significance of the decay correlations is that they appear to be far more sensitive indicators of the nature of the interaction than the angular distribution of production [**]. In view of this importance of the decay correlations we have formulated the theory with spin effects fully taken into account. In fact, we have also found that a correct treatment of spins is of importance in evaluating the angular distribution of production.

In the present paper we apply our methods to the reaction $\pi^- p \rightarrow \rho^- p$ at 4 GeV/c, assuming pion exchange as the basic mechanism. Other reactions, in particular those resulting from KN interactions, will be considered in a subsequent publication.

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*) See, however, Feld [4].

**) Thus, in the simple peripheral model the decay correlation reveals the spin of the exchanged particle [1].

2. THE DISTORTED WAVE BORN APPROXIMATION AT HIGH ENERGIES

We first treat a familiar non-relativistic situation, and shall then adapt the final result to our own (highly relativistic) problem. Consider an inelastic process which, in lowest non-vanishing order, is caused by the interaction V. Let $U^{(+)}$ and $U^{(-)}$ be the <u>remaining</u> interaction between the colliding systems in the initial and final state, respectively. ($U^{(+)} = U^{(-)}$ only if there is no rearrangement during the collision.) The scattering amplitude to first order in V, but to all orders in $U^{(\pm)}$, is then given by

$$M_{fi} = \langle \psi_f^{(-)} | V | \psi_i^{(+)} \rangle , \qquad (2.1)$$

where $\psi_f^{(+)}$ and $\psi_i^{(-)}$ satisfy

$$\psi_i^{(+)} = \phi_i + \frac{1}{E-(H-V)+i\eta} U^{(+)} \phi_i \;,$$

$$\psi_f^{(-)} = \phi_f + \frac{1}{E-(H-V)-i\eta} U^{(-)} \phi_f \;.$$

At high energies and small momentum transfers one can use the Glauber [12] approximation for the wave functions. For simplicity, we assume that the system is described by one (3-dimensional) degree of freedom. We also allow the interactions $U^{(\pm)}$ to be complex (optical) potentials; in this connection it should be noted that if the interactions are the same in the entrance and exit channels, $U^{(-)} = U^{(+)*}$. Under these circumstances the approximate wave functions are

$$\psi_{\underset{\sim}{q}}^{(\pm)}(\underset{\sim}{b}, z) \simeq e^{i \underset{\sim}{q} \cdot \underset{\sim}{r}} \exp\left\{ \frac{-i}{v} \int_{\mp\infty}^{z} U^{(\pm)}(\underset{\sim}{b} + \hat{\underset{\sim}{k}} z')\, dz' \right\} , \qquad (2.2)$$

where v is the relative velocity, the z axis is chosen along $\underset{\sim}{k} = \underset{\sim}{q}+\underset{\sim}{q}'$, $\underset{\sim}{q}$ and $\underset{\sim}{q}'$ being the initial and final momenta, and $\underset{\sim}{b}$, the impact parameter vector, is perpendicular to $\underset{\sim}{k}$. In the same approximation, the scattering amplitude becomes

$$M_{fi} \simeq \int d^2b \int_{-\infty}^{\infty} dz \, e^{i\underset{\sim}{\Delta}\cdot\underset{\sim}{b}} \, V(\underset{\sim}{b}+\hat{\underset{\sim}{k}}z)$$

$$\times \exp\left\{ \frac{-i}{v_-} \int_z^{\infty} U^{(-)}(\underset{\sim}{b}+\hat{\underset{\sim}{k}}z')^* dz' \right\} \tag{2.3}$$

$$\times \exp\left\{ \frac{-i}{v_+} \int_{-\infty}^{z} U^{(+)}(\underset{\sim}{b}+\hat{\underset{\sim}{k}}z'') dz'' \right\},$$

where $\underset{\sim}{\Delta}$ is the 3-momentum transfer.

It does not appear possible to go further without some assumptions or approximations. The simplest situation is the one where the interactions in the initial and final channels are the same, i.e., $U^{(-)*} = U^{(+)} \equiv U$, and $v_+ \simeq v_- \equiv v$. In this case (2.3) is easily reduced to

$$M_{fi} = 2\pi \int_0^{\infty} J_0(\Delta b) \, e^{i\chi(b)} \, B(b) \, b\,db , \tag{2.4}$$

where $J_0(x)$ is the Bessel function of zeroth order,

$$\chi(b) = -\frac{1}{v} \int_{-\infty}^{\infty} U(\underset{\sim}{b}+\hat{\underset{\sim}{k}}z)\,dz \tag{2.5}$$

is the phase shift suffered by a wave packet travelling through the potential U at an impact parameter b, and

$$B(b) = \int_{-\infty}^{\infty} V(\underset{\sim}{b}+\hat{\underset{\sim}{k}}z)\,dz \tag{2.6}$$

is, aside from a factor of $-v^{-1}$, the same quantity for the potential V. Thus (2.4) is the impact parameter representation of the distorted wave Born approximation. The ordinary Born approximation is obtained by setting $e^{i\chi}$ equal to unity.

When the difference between $U^{(+)}$ and $U^{(-)*}$ is appreciable, one can only simplify (2.3) if the range of V is much smaller than that of $U^{(+)}$ or $U^{(-)}$. If this limiting situation holds, (2.3) becomes

$$M_{fi} = 2\pi \int_0^\infty b\,db\; J_0(\Delta b)\; B(b) \exp\{\tfrac{i}{2}[\chi^{(+)}(b) + \chi^{(-)}(b)]\}, \tag{2.7}$$

where

$$\chi^{(+)}(b) = -\frac{1}{v_+}\int_{-\infty}^{\infty} U^{(+)}(\underset{\sim}{b} + \hat{\underset{\sim}{k}} z)\,dz,$$
$$\chi^{(-)}(b) = -\frac{1}{v_-}\int_{-\infty}^{\infty} U^{(-)}(\underset{\sim}{b} + \hat{\underset{\sim}{k}} z)\,dz. \tag{2.8}$$

The partial wave expansion equivalent to Eq. (2.7) was first given by Sopkovich [6].

It is an easy matter to relate $\chi^{(\pm)}$ to the elastic scattering amplitude in the initial and final channels. For example, the elastic amplitude due to $U^{(+)}$ is

$$f_+(\Delta, q) = -iq\int_0^\infty b\,db\; J_0(\Delta b)\left(e^{i\chi^{(+)}(b)} - 1\right), \tag{2.9}$$

or

$$e^{i\chi^{(+)}(b)} = 1 + \frac{i}{q}\int_0^\infty \Delta d\Delta \; f_+(\Delta,q)\, J_0(\Delta b) \; . \tag{2.10}$$

The amplitude f_+ (and the similarly defined elastic amplitude f_- for the final configuration) is not the actual elastic amplitude observed when the constituents of the initial (and final) state are allowed to collide. The true elastic amplitude also includes terms where V acts an even number of times. If, however, the inelastic channel in question has a cross-section which is very small compared to the total inelastic cross-section, the true elastic amplitude will not differ appreciably from f_+ of Eq. (2.9). As this is the case in the reactions of interest to us, we shall henceforth identity $f_+(f_-)$ with the amplitude which describes the elastic scattering of the incident (outgoing) systems.

At high energies the elastic cross-sections generally extrapolate to the optical theorem point at zero momentum transfer. Hence at least at small angles the elastic amplitudes are essentially imaginary, implying that the elastic scattering is almost entirely the shadow of the inelastic processes. For the momentum transfers of interest to us, the experimental angular distribution is well fitted by a Gaussian :

$$f(\Delta,q) = i\,\frac{\sigma_T q}{4\pi}\, e^{-\frac{1}{2}A\Delta^2} , \tag{2.11}$$

where σ_T is the total cross-section, and A is a slowly varying function of energy. Hence

$$e^{i\chi(b)} = 1 - \frac{\sigma_T}{4\pi A}\, e^{-b^2/2A} , \tag{2.12}$$

where for consistency we require $(\sigma_T/4\pi A) \leqslant 1$.

When these expressions are substituted into the inelastic amplitude (2.7), one sees explicitly how the coupling to competing channels depletes the inelastic processes which would otherwise originate from small impact parameter collisions.

3. INCLUSION OF SPIN

As stressed in the Introduction, spins have a profound effect on the undistorted Born approximation. Furthermore, we wish to compute the decay angular distributions of the reaction products. For both of these reasons we must generalize the foregoing treatment to include spin.

The way to do this will be clear once we rederive (2.7) from the partial wave expansion. For spinless particles we have

$$M_{fi} = \sum_{\ell=0}^{\infty} (\ell+\tfrac{1}{2})\, T_\ell \, P_\ell(\cos\vartheta) . \tag{3.1}$$

At high energies many partial waves contribute, and one may replace the sum by an integration. This is facilitated for small scattering angles ϑ by the asymptotic formula $P_\ell(\cos\vartheta) \simeq J_0((2\ell+1)\sin\frac{1}{2}\vartheta)$, which allows one to write (3.1) as

$$M_{fi} \simeq \int_0^\infty x\,dx \;\; T(x)\, J_0(\omega x) , \tag{3.2}$$

where

$$\omega = 2\sin\tfrac{1}{2}\vartheta . \tag{3.3}$$

Upon applying the WKB approximation to the distorted wave Born approximation for $T(x)$, one retrieves (2.7),

$$T(x) \simeq e^{\frac{i}{2}\chi^{(+)}(x)}\; B(x)\, e^{\frac{i}{2}\chi^{(-)}(x)} , \tag{3.4}$$

where, needless to say, $x = qb$.

The inclusion of spin is now seen to be a simple matter. Our starting point is the generalization of (3.1), namely the Jacob-Wick partial wave expansion :

$$\langle \lambda_c \lambda_d / M / \lambda_a \lambda_b \rangle = \sum_j (j+\tfrac{1}{2}) \langle \lambda_c \lambda_d / T_j / \lambda_a \lambda_b \rangle \, d^j_{\lambda\mu}(\vartheta), \tag{3.5}$$

where λ_a, λ_b are the helicities of the incident particles, λ_c, λ_d those of the outgoing particles, and $\mu = \lambda_c - \lambda_d$, $\lambda = \lambda_a - \lambda_b$. For values of the angular momentum j large compared to $|\lambda|$, $|\mu|$, and small scattering angles (i.e., $\sin^2 \frac{1}{2} \vartheta \ll 1$), we can use the asymptotic formula

$$d^j_{\lambda\mu}(\vartheta) \simeq J_n((2j+1) \sin \tfrac{1}{2}\vartheta), \tag{3.6}$$

with

$$n = \mu - \lambda = \lambda_c - \lambda_d - \lambda_a + \lambda_b , \tag{3.7}$$

to convert (3.5) into an integral :

$$\langle \lambda_c \lambda_d / M / \lambda_a \lambda_b \rangle \simeq \int_0^\infty x\,dx \, \langle \lambda_c \lambda_d / T(x) / \lambda_a \lambda_b \rangle \, J_n(\omega x) . \tag{3.8}$$

Since the angular distribution of elastic scattering extrapolates to the optical theorem point, the helicity changing parts of the elastic amplitudes are presumably quite small, and will always be ignored by us. Therefore, in the generalization of (3.4) to include spin effects, only the Born amplitude will carry the dependence on the various helicities, while the "penetration factors" continue to be given by (2.12). Thus we assume that the distorted wave Born approximation to the reaction amplitude including spins is

$$\langle\lambda_c\lambda_d|T(x)|\lambda_a\lambda_b\rangle = e^{\frac{i}{2}\chi^{(+)}(x)}\langle\lambda_c\lambda_d|B(x)|\lambda_a\lambda_b\rangle e^{\frac{i}{2}\chi^{(-)}(x)}. \tag{3.9}$$

Before taking up a specific reaction, we should record several other relationships of physical significance involving the expression (3.8). First of all, we must give the normalization of the states entering into the matrix element M ; it is such that the differential cross-section for unpolarized incident beams is

$$\frac{d\sigma_{ab\to cd}}{d\Omega} = \frac{1}{64\pi^2 s}\ \frac{q'}{q}\ \frac{1}{(2s_a+1)(2s_b+1)}\sum_{\lambda}|\langle\lambda_c\lambda_d\,|\,M\,|\,\lambda_a\lambda_b\rangle|^2, \tag{3.10}$$

where s_a is the spin of particle a, etc., q, q', are the initial and final centre-of-mass momenta, respectively, and s is the square of the centre-of-mass energy. From (3.8) and the orthogonality properties of the Bessel functions it then follows that the total cross-section for the particular channel in question is

$$\sigma_{ab\to cd} = \frac{1}{32\pi s}\ \frac{q'}{q}\ \frac{1}{(2s_a+1)(2s_b+1)}\sum_{\lambda}\int_0^{\infty} x\,dx\,|\langle\lambda_c\lambda_d|T(x)|\lambda_a\lambda_b\rangle|^2. \tag{3.11}$$

Unitarity imposes a bound on the partial wave amplitudes for the individual reactions. To see how this comes about, we note that the total inelastic cross-section follows from (2.9) in the form

$$\sigma_{in} = \frac{2\pi}{q^2} \int_0^\infty x\,dx \left(1 - e^{-2\,Im\,\chi(x)}\right) \quad . \tag{3.12}$$

The integrand of (3.11), which describes only <u>one</u> inelastic channel, cannot exceed the upper limit of (3.12), obtained by setting $\exp(-2\mathrm{Im}\,\chi(x)) = 0$. Therefore

$$1 \geqslant \frac{q q'}{64\pi^2 s} \; \frac{1}{(2S_a+1)(2S_b+1)} \sum_\lambda |\langle \lambda_c \lambda_d | T(x) | \lambda_a \lambda_b \rangle|^2 \tag{3.13}$$

is the bound set by unitarity.

4. THE REACTION $\pi^- p \rightarrow \rho^- p$ AT 4 GEV/C

We shall now analyze the reaction

$$\pi^- p \rightarrow \rho^- p \tag{4.1}$$

in detail at 4 GeV/c, and compare to the recently published data [13].

On the assumption that (4.1) is a one-pion exchange process, the six independent Born helicity amplitudes are readily found to be

$$B_0^{(1)} \equiv \langle 0+|B|+\rangle = -ia\,\frac{c_0}{m_c}\,\xi_-\,\frac{1}{\epsilon^2+\omega^2}\left(1-\alpha-\tfrac{1}{2}\omega^2\right)\sqrt{1-\tfrac{1}{4}\omega^2}, \tag{4.2}$$

$$B_0^{(2)} \equiv \langle 1+|B|-\rangle = \frac{-ia}{2\sqrt{2}}\,\xi_+\,\frac{\omega^2}{\epsilon^2+\omega^2}\sqrt{1-\tfrac{1}{4}\omega^2}, \tag{4.3}$$

$$B_1^{(1)} \equiv \langle 0-|B|+\rangle = -\frac{i}{2}\,a\,\frac{c_0}{m_c}\,\xi_+\,\frac{\omega}{\epsilon^2+\omega^2}\left(1-\alpha-\tfrac{1}{2}\omega^2\right), \tag{4.4}$$

$$B_1^{(2)} \equiv \langle 1+|B|+\rangle = \frac{-ia}{\sqrt{2}}\,\xi_-\,\frac{\omega}{\epsilon^2+\omega^2}\left(1-\tfrac{1}{4}\omega^2\right), \tag{4.5}$$

$$B_1^{(3)} \equiv \langle 1-|B|-\rangle = -B_1^{(2)}, \tag{4.6}$$

$$B_2 \equiv \langle 1-|B|+\rangle = B_0^{(2)}. \tag{4.7}$$

The subscript on $B_n^{(k)}$ is simply the quantum number n defined in (3.7) ; the helicities of the ρ , the outgoing, and the incident nucleon are specified from left to right, respectively. The other six amplitudes are related to these by space reflection through the general formula *)

$$\langle -\lambda_\rho - \lambda' | M | -\lambda \rangle = (-1)^{\lambda_\rho + \lambda + \lambda'} \langle \lambda_\rho \lambda' | M | \lambda \rangle . \tag{4.8}$$

The other quantities not yet defined (recall $\omega = 2 \sin \frac{1}{2} \vartheta$) are

$$qq'\epsilon^2 = m_\pi^2 + (q-q')^2 - \frac{1}{4s}\left[m_a^2 - m_b^2 - m_c^2 + m_d^2\right]^2, \tag{4.9}$$

$$\xi_\pm = \sqrt{(b_0+m_b)(d_0+m_d)}\left\{\frac{q}{m_b+b_0} \pm \frac{q'}{m_d+d_0}\right\}, \tag{4.10}$$

and

$$\alpha = \frac{q' a_0}{q\, c_0},$$

where b_0 is the energy and m_b the mass of particle b, etc. The coupling constants enter through

$$a = 2\, g_{\pi\pi\rho}\, g_{\pi NN} / q',$$

where $g_{\pi\pi\rho}^2/4\pi \simeq 2.0$ for $\Gamma_\rho \simeq 100$ MeV, and $g_{\pi NN}^2/4\pi \simeq 14.5$. The definitions of coupling constants, sign conventions, etc., are those of Ref. [1].

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*) The labelling of the participants in the present reaction in the general notation of the preceding Section is $a = \pi$, $c = \rho$, etc.

The quantity α is actually the ratio of the velocities of the ρ and π in the centre-of-mass system.

Before carrying out the partial wave expansion of $B_n^{(k)}$, we make the small angle approximation, $\omega^2 \ll 1$. Since α is close to unity the term $\frac{1}{2}\omega^2$ in $(1-\alpha-\frac{1}{2}\omega^2)$ cannot be neglected, however. The essential tool in the partial wave analysis is the identity

$$\frac{\omega^\nu}{\epsilon^2+\omega^2} = \epsilon^\nu \int_0^\infty x\,dx\; J_\nu(\omega x)\, K_\nu(\epsilon x), \tag{4.11}$$

where K_ν is the modified Hankel function. Consider, for example, the amplitude $B_1^{(1)}$. According to (3.8) this must be expanded in terms of $J_1(\omega x)$. We therefore rewrite (4.4) as

$$B_1^{(1)} = -\frac{ia}{2}\frac{c_0}{m_c}\xi_+^{\varphi}\left\{\frac{\omega}{\epsilon^2+\omega^2}\left(1-\alpha+\frac{1}{2}\epsilon^2\right) - \frac{1}{2}\omega\right\}$$

$$= -\frac{ia}{2}\frac{c_0}{m_c}\xi_+^{\varphi}\left\{\left(1-\alpha+\frac{1}{2}\epsilon^2\right)\epsilon\int_0^\infty J_1(\omega x)\,K_1(\epsilon x)\,x\,dx - \frac{1}{2}\omega\right\}. \tag{4.12}$$

The term $\frac{1}{2}\omega$ represents an "anomalous" $j = \frac{1}{2}$ partial wave amplitude ; as is well known, the lowest member of a partial wave series frequently fails to follow the general pattern.

The influence of the competing channels on the one-particle exchange amplitude is now taken into account by the recipe of the preceding Section. Since the one-particle exchange potential is due to π exchange, the ranges of $U^{(\pm)}$ certainly do not exceed that of V, and therefore one is really not permitted to employ formula (3.9). This is not an important difficulty since we only know $\chi^{(+)}$ from $\pi^- p$ -elastic scattering in any case, and know nothing about ρp -elastic scattering, i.e., have no way of competing $\chi^{(-)}$. We shall therefore resort to Occam's razor, and assume that the πp -elastic scattering amplitude is approximately equal to that for ρp -elastic scattering. The only unambiguous way of investigating the validity of this assumption appears to be through photo-production of ρ mesons. Having set $\chi^{(+)} = \chi^{(-)} \equiv \chi$, the modification of (4.12) due to competing channels is

$$M_1^{(1)} = -\frac{ia}{2}\frac{c_0}{m_c}\xi_+\Big\{-\frac{1}{2}e^{i\chi(0)}\omega + (1-\alpha+\tfrac{1}{2}\epsilon^2)\epsilon\int_0^\infty J_1(\omega x)K_1(\epsilon x)e^{i\chi(x)}x\,dx\Big\}, \tag{4.13}$$

where we have designated the modified reaction amplitude by the same labels as $B_n^{(k)}$. It is now clear that for any reaction $M_n^{(k)}$ will have the form

$$M_n^{(k)} = A_n^{(k)}\int_0^\infty J_n(\omega x)K_n(\epsilon x)e^{i\chi(x)}x\,dx + \bar{A}_n^{(k)}e^{i\chi(0)}d^{j_0}_{\lambda\mu}(\vartheta), \tag{4.14}$$

where j_0 is the smallest eigenvalue of the angular momentum which is compatible with n. The quantities $A_n^{(k)}$ and $\bar{A}_n^{(k)}$ depend only on energy ; in fact the latter frequently vanishes identically. The modifications analogous to (4.13) for the remaining five amplitudes $B_n^{(k)}$ can now be easily derived, but we shall not exhibit them.

The absorption factor $e^{i\chi}$ is, according to (2.12), well represented by

$$e^{i\chi(x)} = 1 - C e^{-\gamma x^2} , \tag{4.15}$$

For $\pi^- p$ -elastic scattering at 4 GeV/c [13],

$$C = 0.765 \quad , \; \gamma = 0.038 , \tag{4.16}$$

where we used $b^2 \rightarrow x^2/qq'$ to determine γ from the experimental value of $A = 8.53 \pm 0.49 \; (\mathrm{GeV})^{-2}$. We have also carried out a complete set of calculations with $C = 1$, hoping in this way to simulate a possibly stronger ρp interaction in the final state.

In order to illustrate the effects of absorption on the partial wave amplitudes we have, in Fig. 1, plotted the right-hand side of (3.13) (note that $(2j+1)\pi/q^2$ times this quantity is the partial wave cross-section) for the unmodified and distorted wave Born approximations. As one observes, the absorptive effects keep the partial wave cross-sections well below the unitarity bound. One notes, moreover, that the influence of absorption extends to partial waves of rather large angular momentum.

The differential cross-section (3.10) for the reaction (4.1) is shown in Fig. 2 for extremely small scattering angles, and in Fig. 3 over the entire range of ϑ for which the small angle approximation is expected to be sensibly accurate. The normalization of these curves is not arbitrary, since the coupling constants

$g_{\pi\pi\rho}$ and $g_{\pi NN}$ are both known. Qualitatively speaking, both $C = 0.765$ and $C = 1$ give good fits to the data. But $C = 1$ gives a rather better fit at larger angles, and is also in better agreement with the total cross-section. In fact the theoretical cross-sections, integrated over the interval $1 > \cos\vartheta > 0.6$, are 0.85 mb for $C = 0.765$ and 0.58 mb for $C = 1$, while the experimental cross-section is 0.45 ± 0.08 mb. In Fig. 4 we compare the angular distribution predicted by the unadulterated peripheral model (curve a) with those already shown in Fig. 3 (curves b and c) ; the significance of the absorptive effects needs no further advocacy.

As we have stressed repeatedly, the proper treatment of the spin is of great importance, and not merely a pedantic detail. This point is borne out by a comparison of the angular distribution of Fig. 3 with simple recipes for treating the spin effects. The one-pion exchange cross-section (i.e., the sum of all twelve $|B_n^{(k)}|^2$) can be written as

$$\frac{d\sigma}{d\Omega} = S(s,t) \left| \frac{1}{\epsilon^2 + \omega^2} \right|^2 , \tag{4.17}$$

where the function $S(s,t)$ arises from the vertices and the traces over the nucleon spinors. One simple recipe is to modify only the propagator $(\epsilon^2 + \omega^2)^{-1}$ by setting

$$\frac{1}{\epsilon^2 + \omega^2} \longrightarrow \int_0^\infty J_0(\omega x)\, K_0(\epsilon x)\, e^{i\chi(x)}\, x\, dx \; .$$

The resulting angular distribution with $C = 1.0$ in (4.15) (curve d) is compared with the correct one (curve c) in Fig. 4. One notes that the simple recipe leads to diffraction zeroes which are spurious, since partial waves with $n \neq 0$ fill these in when the computation is done properly. In fact, as one goes to larger angles, waves with larger values of n become relatively more important. This is exemplified by Table I, where we list $d\sigma_n/d\Omega$, the partial cross-sections for different values of the quantum number n, for $C = 1$.

Finally, we must determine the decay angular distribution of the produced ρ meson. In the rest frame of the ρ, and in a co-ordinate system where the z axis is chosen parallel to the pion momentum as seen in that frame and the x axis is in the production plane, the decay distribution is [11])

$$W(\theta,\phi) = \frac{3}{4\pi}\left\{\rho_{11}\sin^2\theta + (1-2\rho_{11})\cos^2\theta - \rho_{1,-1}\sin^2\theta\cos 2\phi - \sqrt{2}\,\mathrm{Re}\,\rho_{10}\sin 2\theta\cos\phi\right\}. \tag{4.18}$$

The quantities $\rho_{mm'}$ are elements of the vector meson's spin space density matrix, with the above defined z axis being the quantization direction. We shall call this the canonical density matrix. The Treiman-Yang distribution $W_{TY}(\phi)$ is obtained from (4.18) by averaging over the polar angle, i.e., :

$$W_{TY}(\phi) = \frac{1}{2\pi}\left(1-2\rho_{1,-1}\cos 2\phi\right). \tag{4.19}$$

For the unmodified one-pion exchange model the canonical density matrix has an exceedingly simple structure :

$$\rho^{0}_{mm'} = \delta_{m0}\,\delta_{m'0}. \tag{4.20}$$

When absorption is taken into account, $\rho_{mm'}$ will have a more complicated structure. In order to determine the canonical matrix, we must first determine the density matrix in the helicity representation,

$$\hat{\rho}_{\mu\mu'} = N \sum_{\lambda\lambda'} \langle \mu\lambda' | M | \lambda \rangle \langle \mu' \lambda' | M | \lambda \rangle^* , \tag{4.21}$$

where these amplitudes are given by (4.14). The canonical matrix $\rho_{mm'}$ is then obtained from $\hat{\rho}_{\mu\mu'}$ by the unitary transformation *)

$$\rho_{mm'} = \sum_{\mu\mu'} d^1_{m\mu}(\Psi)\, \hat{\rho}_{\mu\mu'}\, d^1_{\mu' m'}(-\Psi) , \tag{4.22}$$

where Ψ is the angle between the helicity direction of the ρ and the z axis defined above. The explicit expression for this angle is

$$\sin \Psi = \frac{q}{a_c} \sin \vartheta ,$$

where $4m_c^2\, a_c^2 = [t - (m_a - m_c)^2][t - (m_a + m_c)^2]$; a_c is the pion's momentum in the rest frame of the ρ .

The various elements of $\rho_{mm'}$ computed in this fashion are shown in Fig. 5. For $C = 0.765$ the off-diagonal elements ρ_{10} and $\rho_{1,-1}$ are small, and ρ_{00} is fairly close to unity, in rough accord with the unmodified form (4.20).

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*) From (4.20) and (4.22), it follows that the Born amplitudes $B_n^{(k)}$ lead to the helicity representation density matrix

$$\hat{\rho}^{\,o}_{\mu\mu'} = d^1_{o\mu}(\Psi)\, d^1_{o\mu'}(\Psi)$$

For $C = 1$ the deviations from (4.20) are much more marked. But even in this case the deviations are not so striking once $\rho_{mm'}$ is averaged over the production angular distribution. For example, the average of $\rho_{1,-1}$ over the interval $1 \geqslant \cos\vartheta \geqslant 0.6$ is 0.064 for $C = 1$. The Treiman-Yang distribution is therefore still rather isotropic, in agreement with the data. From Fig. 5 it is in fact clear that the quantity which is most sensitive to deviations from the unmodified one-pion exchange is the interference element ρ_{10}, and not $\rho_{1,-1}$. Thus the determination of ρ_{10} from the decay data is of greater interest than the Treiman-Yang test.

5. CONCLUSIONS AND DISCUSSIONS

Several rather clear conclusions have emerged which we shall summarize here.

1) Competition with other channels results in a drastic collimation and over-all reduction of reaction cross-sections as computed from the peripheral model.

2) In the reaction $\pi p \longrightarrow \rho p$ at 4 GeV/c both the angular variation and absolute value of the theoretical cross-section are in good agreement with the data.

3) Proper treatment of spins is of considerable importance, even in the evaluation of the angular distribution. Cavalier handling of the spins can lead to spurious diffraction zeroes. (See Fig. 4 and Table I.)

4) In the present reaction, at least, the spin state of the unstable reaction product (the ρ meson) is not drastically different from that predicted by the peripheral model. The decay correlations are in agreement with the available data.

A number of remarks concerning these conclusions are in order here.

1) If the peripheral model enjoys any measure of validity, absorptive corrections due to competing channels are almost a direct consequence of general principles. The analogy with nuclear reactions which underlies our treatment may prove to be unrealistic, but corrections of this general type surely exist. The present work shows that the form factors (which are presumably always present) cannot be nearly as rapidly varying as previously assumed [1),2)].

2) Coupling constants cannot be determined if absorptive effects are not taken into account. Since the absorptive corrections are appreciable for many partial waves (see Fig. 1), the extraction of coupling constants - which is essentially an extrapolation to the pole - may be difficult for all but pion exchange.

3) In the absence of information on elastic scattering in the final state, we have permitted ourselves modest variations of the parameter C in (4.15). A value of $C = 1$ (i.e., total absorption of the s wave) gives a better fit than the value inferred from $\pi^- p$ - elastic scattering.

4) The most important factor in determining the angular distribution is the shape of the elastic diffraction pattern, the specific exchange mechanism playing a somewhat secondary role. What was supposedly a correction (i.e., the absorptive effect) has qualitatively changed the original result - a circumstance which is somewhat disquieting.

5) Even though the momentum transfer dependence of the one-meson exchange model has been altered beyond recognition by the absorptive effects, the decay correlations, in the present reaction at least, still reflect the spin character of the exchanged system. If this feature is found to occur generally, the model presented here may be considered as a reasonably realistic description of these processes, despite the reservations indicated above.

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TABLE I

Partial cross-sections $\frac{d\sigma_n}{d\Omega}$ (in mb/str) for different values of the quantum number n (3.7) as functions of $\cos\vartheta$.

$\cos\vartheta$	$\frac{d\sigma_0}{d\Omega}$	$\frac{d\sigma_1}{d\Omega}$	$\frac{d\sigma_2}{d\Omega}$
1.000	1.42	0	0
0.993	0.248	1.52	0.114
0.990	0.143	1.43	0.168
0.970	0.004	0.632	0.265
0.950	0	0.294	0.253
0.900	0.002	0.058	0.173
0.800	0.001	0.004	0.078
0.700	0	0	0.041
0.600	0	0	0.024

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FIGURE CAPTIONS

Figure 1. Reaction cross-section for the process $\pi^- p \to \rho^- p$ at 4 GeV/c as a function of impact parameter. The variable x is the angular momentum and is proportional to the impact parameter $(x = qb)$. The ordinate is the partial wave cross-section divided by $(2j+1)\pi/q^2$ (i.e., the right-hand side of Eq. (3.13)), the normalization having been chosen so that the unitarity bound is unity. The dashed curve is the unmodified one-pion exchange result ; the solid curve includes the absorptive effects.

Figure 2. Comparison of the theoretical and experimental differential cross-section near the forward direction for the reaction $\pi^- p \to \rho^- p$ at 4 GeV/c. The theoretical curve is for the case of total absorption of the s wave, i.e., $C = 1$. The data are taken from Fig. 3 (a) of Ref. [13].

Figure 3. Same as Fig. 2, but over a wider angular interval.. Two theoretical curves are shown : the solid curve is for $C = 1$, whereas the broken curve is for $C = 0.765$, the value inferred from $\pi^- p$-elastic scattering. The data are taken from Fig. 14 (a) of Ref. [13].

Figure 4. Comparison of several theoretical cross-sections for the reaction $\pi^- p \to \rho^- p$ at 4 GeV/c. (a) is the cross-section predicted by the unmodified one-pion exchange model ; (b) and (c) are the theoretical cross-sections computed in this paper incorporating absorption and spin effects, and are also shown in Fig. 3 ; (b) is for $C = 0.765$ and (c) for $C = 1$. The dashed curve (d) is computed from a simple recipe that does not incorporate the spin effects consistently (see text for details).

Figure 5. Elements of the ρ meson's canonical density matrix as a function of production angle. In the unmodified one-pion exchange model $\rho_{00} = 1$, and all other elements vanish. The solid curves are for $C = 1$, and the broken curves for $C = 0.765$. The sign of ρ_{10} depends on the definition of the normal to the production plane, which we take as $\underset{\sim}{n} = \underset{\sim}{q} \times \underset{\sim}{q}'$.

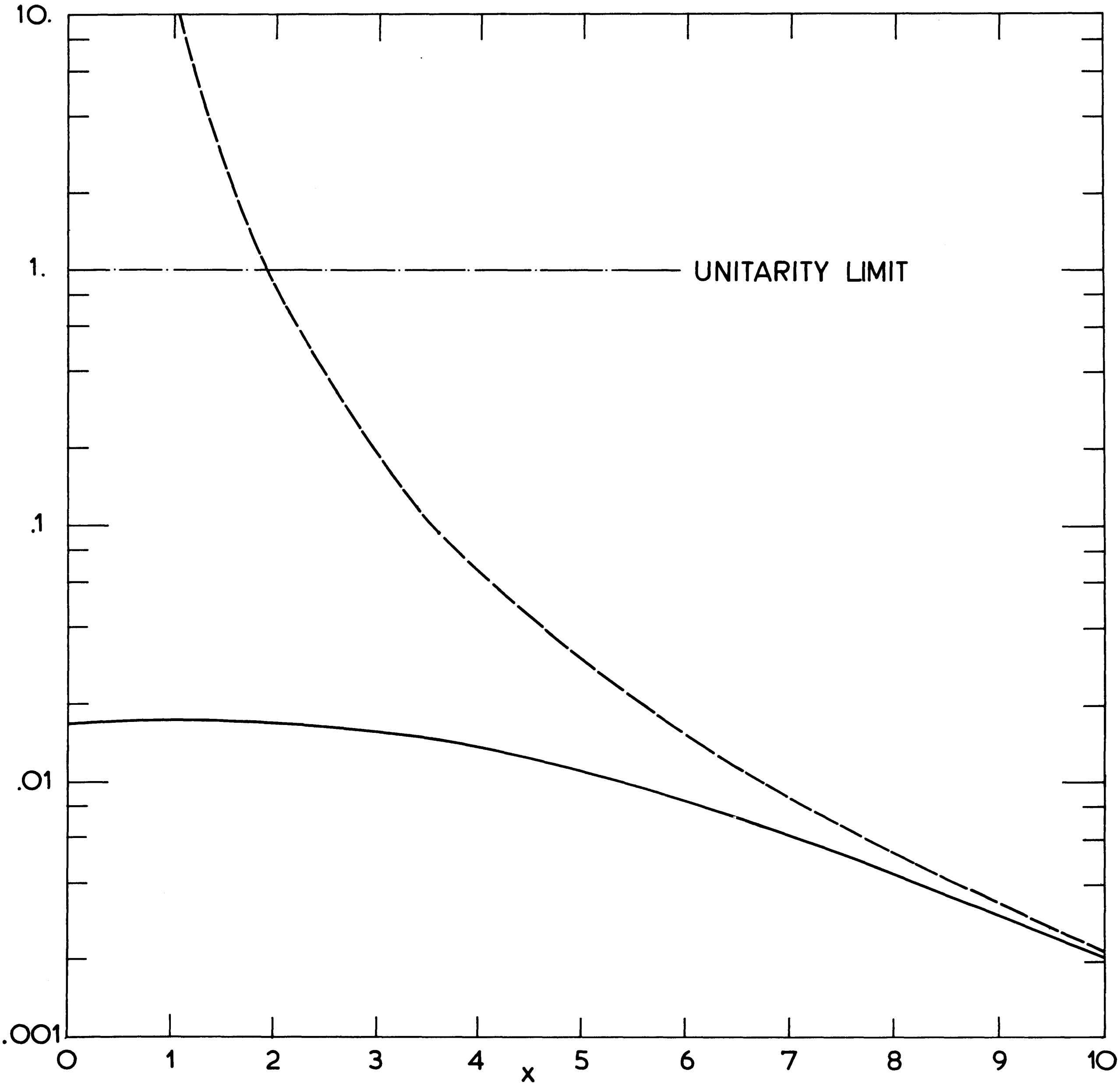

FIG. 1

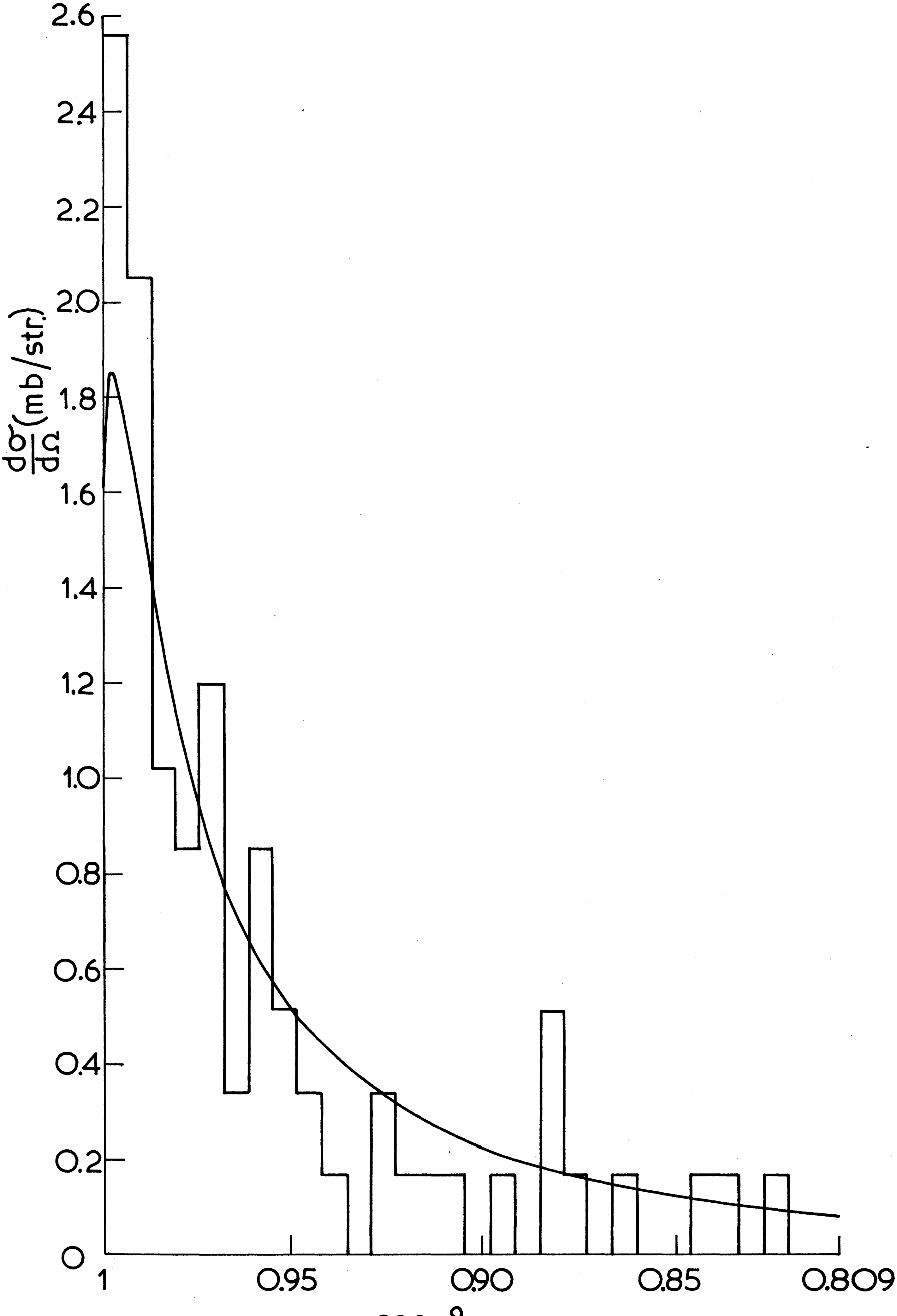

FIG. 2

SIS/R/8585

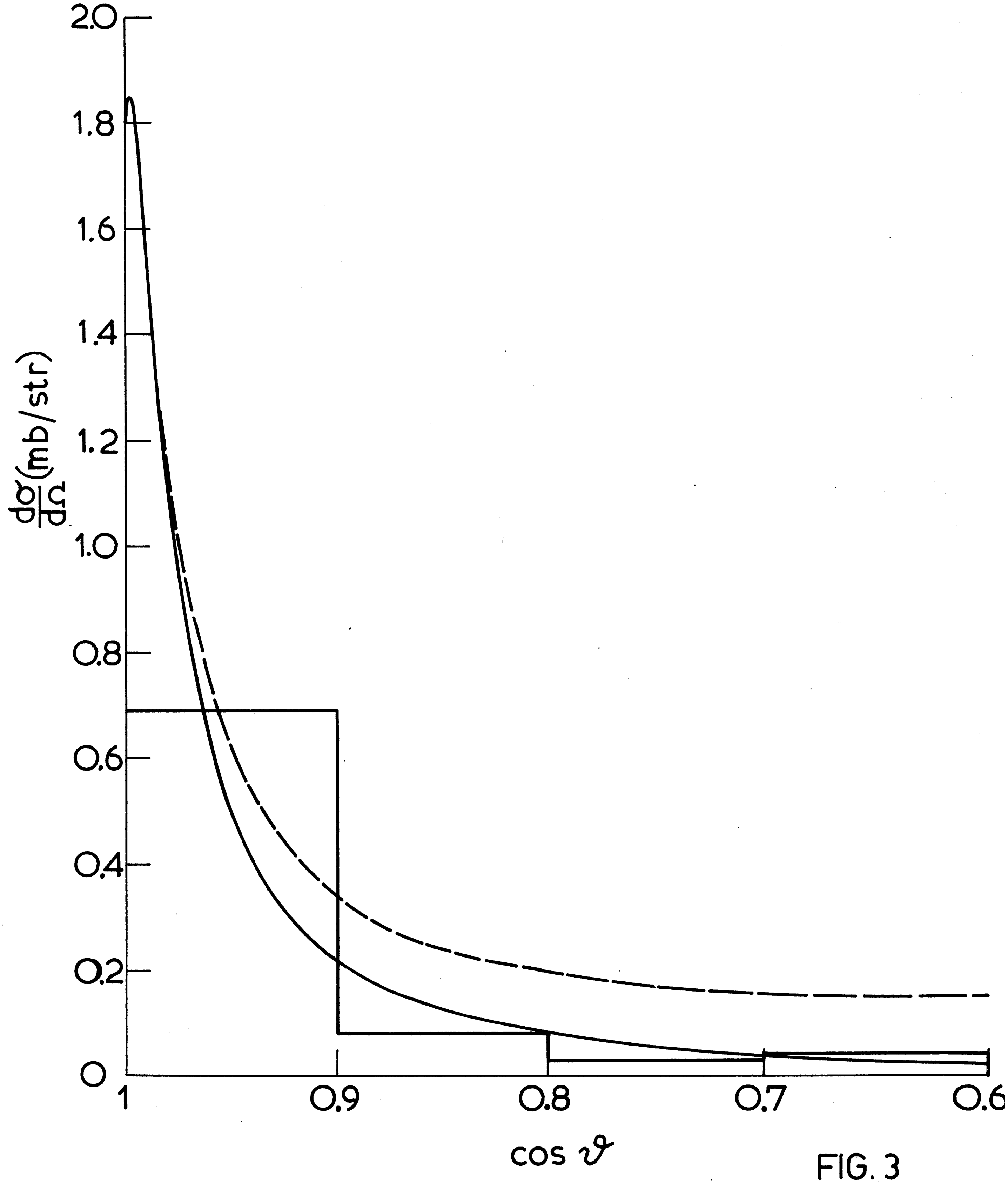

FIG. 3

SIS/R/8584

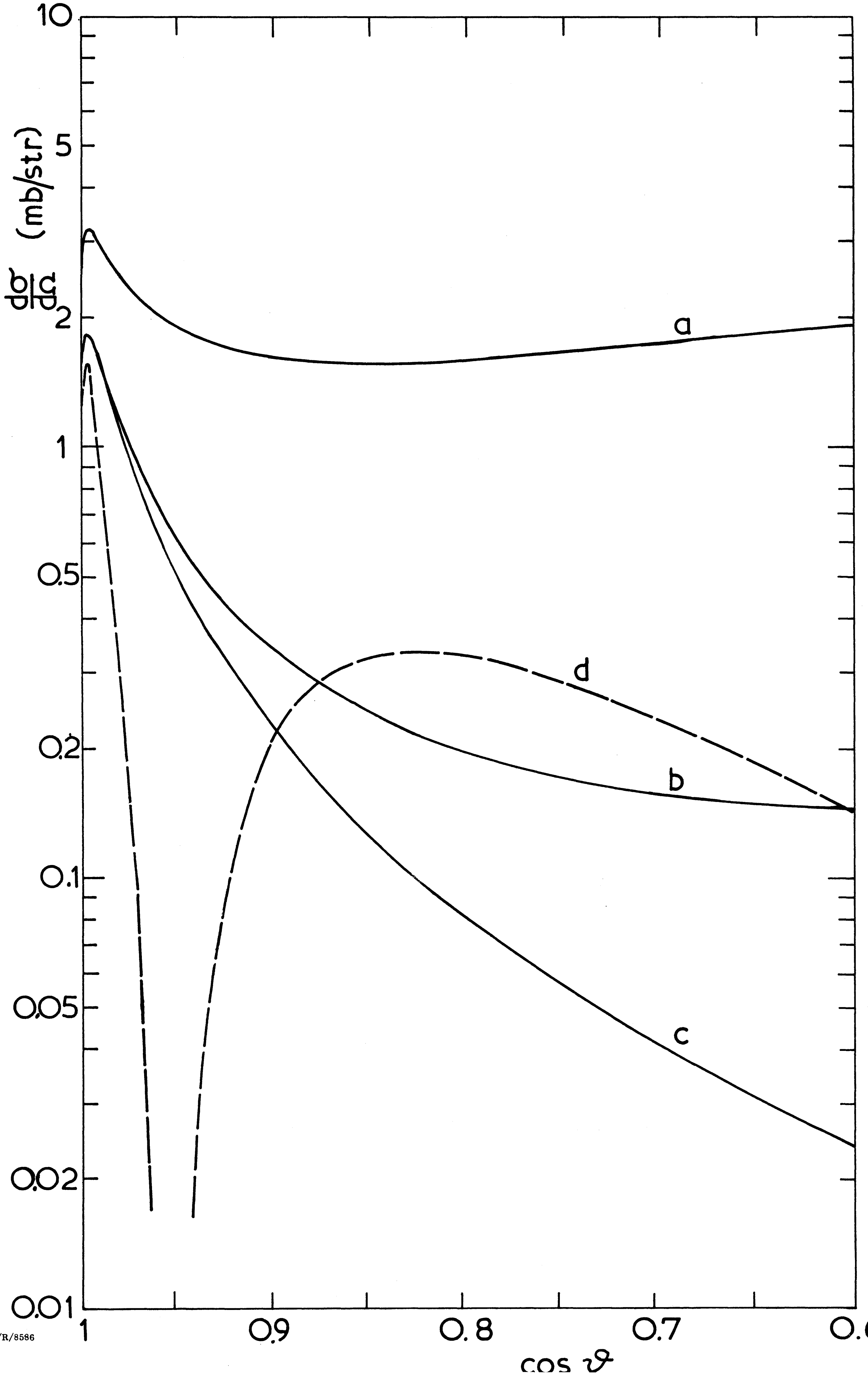

FIG.4

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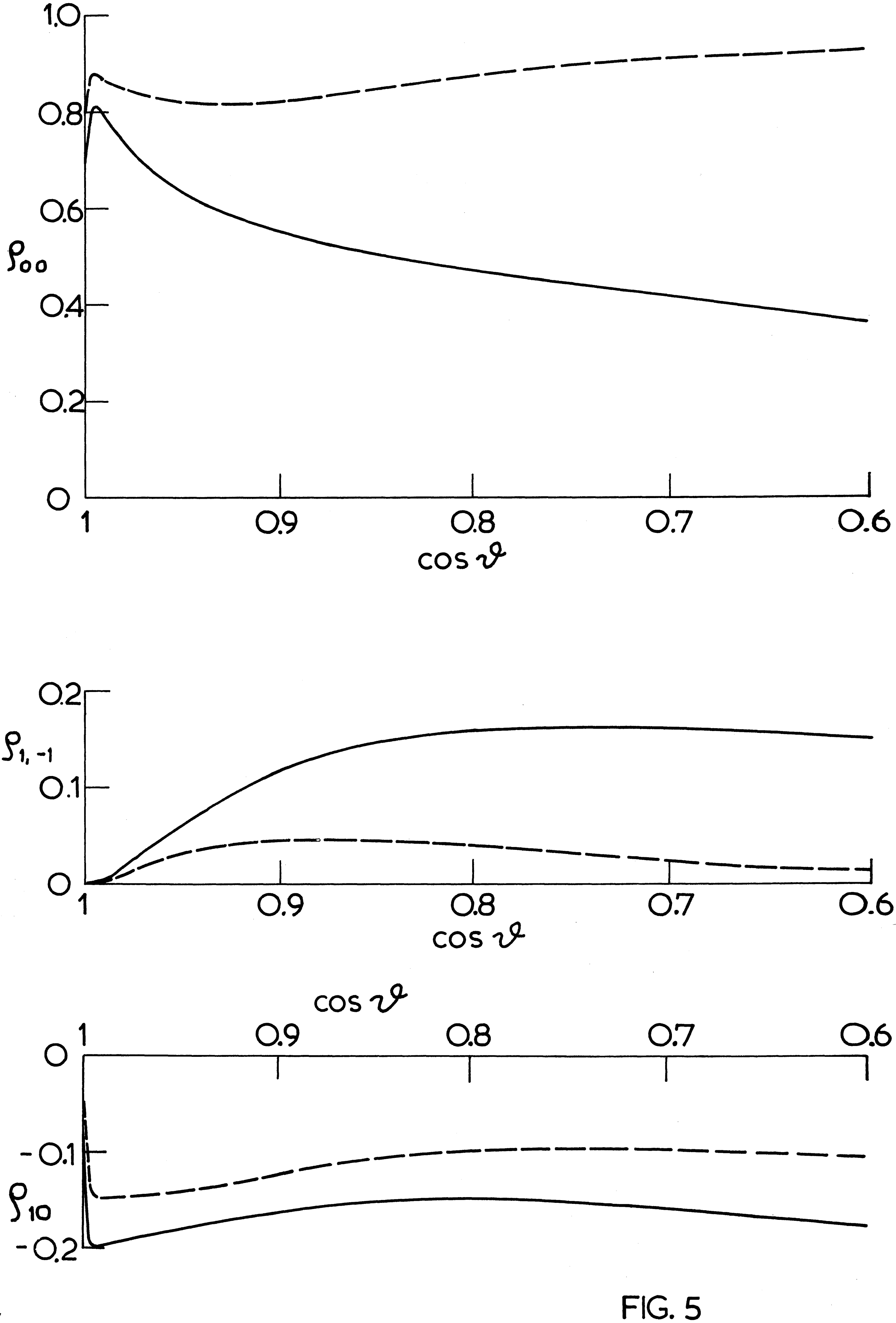
1.0
0.8
0.6
0.4
0.2
0
ρ_{00}
1
0.9
0.8
0.7
0.6
cos ϑ
0.2
0.1
0
$\rho_{1,-1}$
1
0.9
0.8
0.7
0.6
cos ϑ
cos ϑ
1
0.9
0.8
0.7
0.6
0
-0.1
ρ_{10}
-0.2

FIG. 5