Multiplicities, fluctuations and QCD: Interplay between soft and hard physics? *

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Multiplicity fluctuations are studied both globaly (in terms of high-order moments) and locally (in terms of small phase-space intervals). The ratio of cumulant factorial to factorial moments of the charged-particle multiplicity distribution shows a quasi-oscillatory behaviour similar to that predicted by the NNLLA of perturbative QCD. However, an analysis of the sub-jet multiplicity distribution at perturbative scales shows that these oscillations cannot be related to the NNLLA prediction. We investigate how it is possible to reproduce the oscillations within the framework of Monte-Carlo models. Furthermore, local multiplicity fluctuations in angular phase-space intervals are compared with Monte-Carlo models and with first-order QCD predictions. While JETSET reproduces the experimental data very well, the predictions of the Double Leading Log Approximations and estimates obtained in Modified Leading Log Approximations deviate significantly from the data.

1. INTRODUCTION

The shape of a distribution can be analyzed in terms of its moments [1]. In the case of the multiplicity of particles produced in particle collisions at high energies, such an approach is prefered both theoretically (generating function) and experimentally (stability).

The normalised factorial moment of rank q of the multiplicity distribution P_n is defined by:

$$F_q = \frac{\sum_{n=q}^{\infty} n(n-1)....(n-q+1)P_n}{\left(\sum_{n=1}^{\infty} nP_n\right)^q} . \qquad (1)$$

It expresses the normalized phase-space integral over the q-particle density function. If particles are produced independently, the multiplicity distribution is a Poissonian (for which the factorial moments of all ranks are equal to one). If the particles are positively correlated, the distribution is broader than Poisson and the factorial moments are greater than one.

The cumulant factorial moments are obtained

from the factorial moments by:

$$K_q = F_q - \sum_{m=0}^{q-1} \frac{(q-1)!}{m!(q-m-1)!} K_{q-m}F_m . \qquad (2)$$

K_q expresses the normalized phase-space integral over the q-particle correlation function, i.e., that function which describes the genuine correlations between q particles.

Here, we shall use these moments, for full phase space, as well as in ever smaller intervals of it, to study the interplay between hard and soft physics in particle production. In doing so, we use perturbative QCD (and its anomalous dimension) not as a model to be tested, but as a description of the perturbative background at the onset of unknown non-perturbative hadronization phenomena.

2. THE GLOBAL MULTIPLICITY DISTRIBUTION

2.1. High-order correlations

Since $|K_q|$ and F_q increase rapidly with increasing q, it is useful to define [2] the ratio $H_q = K_q/F_q$, which has the same order of magnitude over a large range of q. One can view this ratio as a cumulant factorial moment normalised
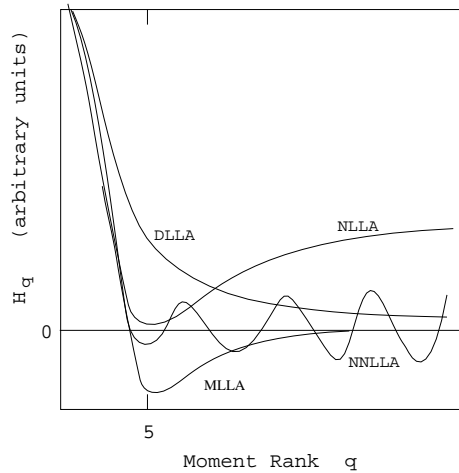
Figure 1. Predictions at different order of perturbative QCD for the H_q as a function of the rank q [2].

Figure 2. H_q versus q measured by the SLD collaboration [3].

to the factorial moment. It reflects the genuine q-particle correlation relative to the q-particle density.

The H_q have been calculated for the gluon multiplicity distribution at different orders of perturbative QCD [2] (see Fig. 1):

- For the Double Leading Logarithm Approximation (DLLA), H_q decreases to 0 as q^{-2}.
- For the Modified Leading Logarithm Approximation (MLLA), H_q decreases to a negative minimum at $q = 5$, and then rises to approach 0 asymptotically.
- For the Next to Leading Logarithm Approximation (NLLA), H_q decreases to a positive minimum at $q = 5$ and goes to a positive constant value for large q.
- For the Next to Next to Leading Logarithm Approximation (NNLLA), H_q decreases to a negative first minimum for $q = 5$ and shows quasi-oscillation around 0 for $q > 5$.

Assuming the validity of the Local Parton Hadron Duality hypothesis (LPHD), such behaviour is also expected for the charged-particle multiplicity distribution.

The H_q of the charged-particle multiplicity distribution have been measured by the SLD collaboration [3] for $e^+e^- \longrightarrow$ hadrons at $\sqrt{s} = M_Z$. They indeed observe (Fig. 2) a negative minimum
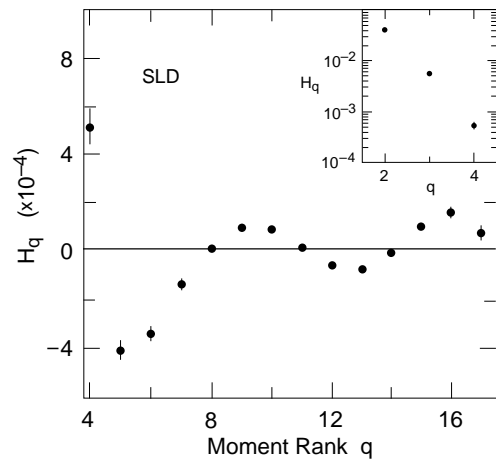
for $q = 5$ followed by quasi-oscillations about 0 for higher q. This result has been interpreted as confirmation of the NNLLA prediction.

Earlier, the H_q moments were calculated [4,5] from charged particle multiplicity distributions of e^+e^- experiments between 22 and 91 GeV. Similar behaviour with a negative first minimum followed by quasi-oscillation about 0 is obtained. Furthermore, the same behaviour has been observed in hadron collisions between 20 and 900 GeV [4,6] and in hA an AA collisions [7] .

While it is tempting to conclude that the observed behaviour is a confirmation of the NNLLA prediction, it must be realized that such a conclusion rests strongly on the validity of LPHD. Although the LPHD hypothesis is accepted for single-particle distributions and mean multiplicity, difficulties have been detected when resonances are involved [8,9]. Further, heavy quark decays will cause a modification of the multiplicity distribution which is unlikely to be accounted by LPHD [5]. For these reasons, we consider the assumption of LPHD as highly questionable.

Therefore, we present measurements of H_q not only of the charged-particle multiplicity distribution but also of the sub-jet multiplicity distributions. By considering sub-jets, i.e., jets obtained with small value of jet resolution parameter, we can, in some sense, follow the development of par-

ton to hadron by varying the jet resolution parameter (and hence the energy scale). The variation of the mean sub-jet multiplicity has been found [10] to agree well with the analytic perturbative QCD calculation in the perturbative region (energy scale greater than about $1-2$ GeV) and with Monte Carlo in both the perturbative and non-perturbative region. These sub-jet multiplicity distributions should therefore enable us to extend this analysis to the energy scales of partons and to that of perturbative QCD, avoiding (or at least lessening) dependence on the assumption of LPHD.

2.2. The data

The analysis is based on data collected by the L3 detector [11] at the energy of Z, corresponding to approximately 1.5M hadronic Z decays. Events are selected using a procedure based on the track momenta measured in the Central Tracking Detector including the Silicon Micro-vertex Detector. About 1M events survive this selection.

The charged-particle multiplicity distribution was corrected for the detector acceptance and efficiency using a matrix [12] determined from Monte-Carlo events generated by JETSET [13] and passed through detector simulation. Effects from selection procedure and initial state radiation are corrected from detector simulation. Charged particles originating from K_S^0 and Λ decay are left in the sample.

The same procedure was applied to correct the sub-jet multiplicity distributions.

To evaluate the errors of the H_q, we generated 10000 multiplicity distributions from the experimental one, by allowing a Gaussian fluctuation of each $P(n)$. The widths of the Gaussians were given by the statistical errors of the $P(n)$. The statistical error on the H_q is taken as the standard deviation of the 10000 values of H_q.

2.3. Charged-particle multiplicity analysis

The detector-corrected charged-particle multiplicity distribution is shown in Fig. 3 and compared to that of JETSET. Reasonably good agreement is found.

To be less sensitive to the large statistical fluctuations in the high-multiplicity tail due to a fi-
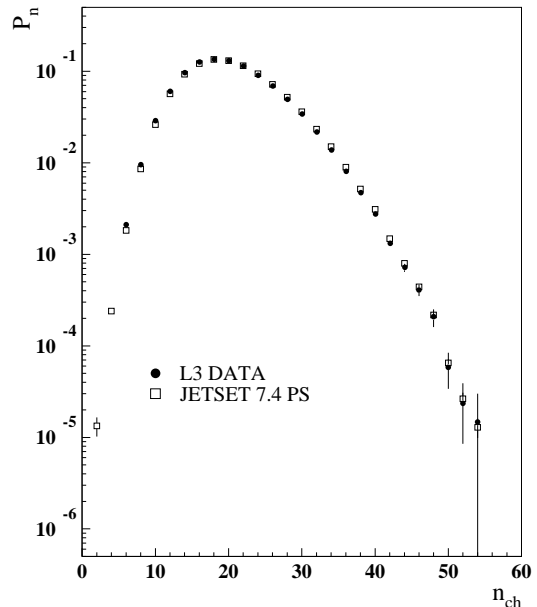

Figure 3. The fully corrected multiplicity distribution compared to JETSET 7.4 PS.

nite data sample, the largest multiplicities ($N_{ch} > 50$) are removed. The ratios H_q are shown in Fig. 4 for both the initial charged-particle multiplicity distribution and the truncated one. These results are in qualitatively good agreement with those of the SLD collaboration.

In both the truncated and the non-truncated cases, the H_q have a negative first minimum at $q = 5$ and quasi-oscillations around 0 for higher q, but the amplitudes are slightly larger in the truncated case and are found to increase for further truncation (not shown).

The H_q calculated from JETSET 7.4 PS agree well with the data both, for the entire and the truncated distributions, when the generated multiplicities are truncated in the same way as the data. The H_q calculated from HERWIG 5.9 [14] do not agree with the data. They also show an oscillatory behaviour, but with a negative first minimum at $q = 7$ and all minima and maxima shifted by 2 units of q.

2.4. Sub-jet multiplicity analysis

Since the behaviour of the H_q agrees qualitatively with that predicted in NNLLA, it is nat-
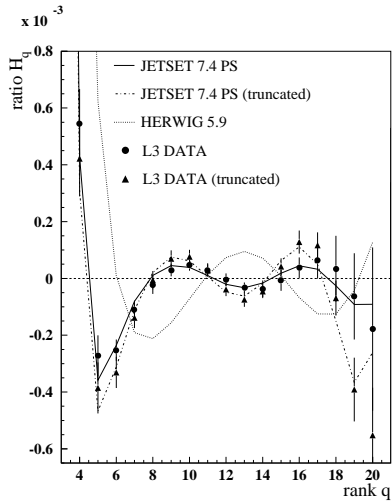
Figure 4. H_q obtained from the charged-particle multiplicity distribution.
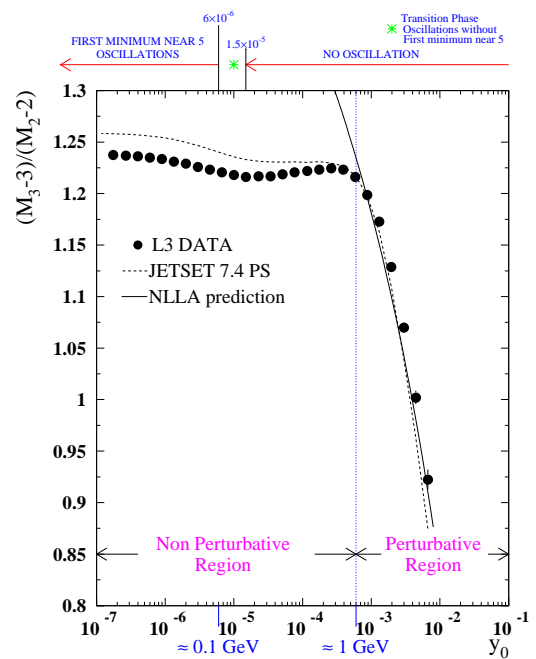

Figure 5. The soft gluon production rate as a function of y_0 is compared to the NLLA prediction. Furthermore at the top of the figure, is indicated the evolution of the behaviour of H_q with the energy scales.

ural to interpret this agreement as a confirmation of the NNLLA prediction, and SLD [3] indeed gave this interpretation. However, JETSET agrees with the data and HERWIG shows qualitatively similar behaviour, in spite of parton-shower modellings not being at the NNLLA order of pQCD and, therefore, not expected to oscillate. That they do, together with doubt on the LPHD assumption, suggests that we view the NNLLA interpretation with scepticism.

The sub-jet analysis assumes that the sub-jet multiplicity distribution is related to the parton multiplicity distribution at the energy scale corresponding to the jet resolution. By choosing a scale where perturbative QCD should be applicable (≥ 1 GeV), we should be able to test the pQCD predictions for the behaviour of the H_q without the LPHD assumption. We define sub-jets using the Durham algorithm [15], minimizing the hadronization effects by associating particles with large angles and small transverse energies.

It has been shown [10,11,16] in previous papers (see for review [17]) that sub-jet mean multiplicities agree well with the NLLA prediction in the perturbative region, i.e., for values of y_{cut} greater than 10^{-3} which corresponds to transverse energies greater than ~ 1 GeV.

Since this study starts with the charged-particle multiplicity distribution, we have also constructed sub-jets using only charged tracks. The charged-particle and sub-jets multiplicities are, of course, identical for sufficiently small values of y_{cut} ($y_{cut} < 10^{-8}$). We have verified that using both charged and neutral particles gives similar results.

The average sub-jet multiplicities in 2-jet and 3-jet events (M_2 and M_3) are obtained for a set of values of cut-off parameter y_0: $10^{-7} \leq y_0 < y_1$, where $y_1 = 0.01$ is the cut-off parameter which resolves 2-jet and 3-jet events. $M_2(y_0)$ and $M_3(y_0)$ correspond to the number of gluons radiated at different energy scales (at different places in the parton shower evolution) for 2-jet and 3-jet events, respectively.

We have calculated the ratio $\frac{M_3-3}{M_2-2}(y_0)$, which corresponds to the ratio of the amount of soft gluon production, when the primary quarks and the hard gluon have been taken out. This ratio, which has the advantage of being independent of α_s, depends only on the resolution pa-
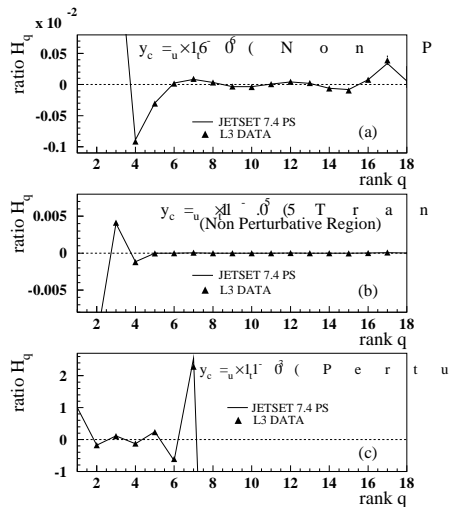
Figure 6. H_q for sub-jet multiplicities, (a) for $y_{cut} = 6 \times 10^{-6}$, (b) for $y_{cut} = 1.5 \times 10^{-5}$, (c) for $y_{cut} = 1 \times 10^{-3}$.
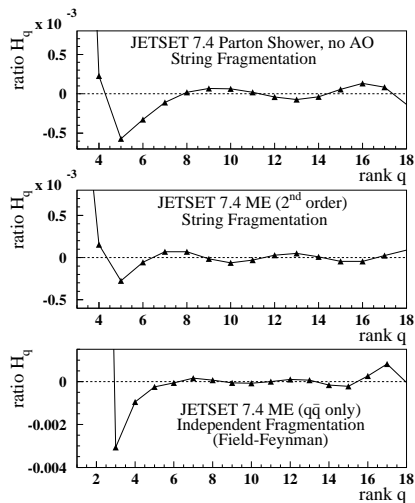

Figure 7. H_q for the charged particle multiplicity distribution obtained from Monte Carlo generators using different methods of parton generation and different models of fragmentation.

rameters y_0 and y_1. The NLLA prediction [18] with $N_f = 3$ [19] is seen (Fig. 5) to be in good agreement with the data in the perturbative region. This result supports the assumption that we can perform the sub-jet analysis with sub-jets obtained from charged particles.

The sub-jet multiplicity distributions used for the calculation of the H_q have been corrected using the same procedure as used for the charged-particle multiplicity distribution (i.e., correction matrix and correction for ISR, K_S^0 and Λ decays). The resulting H_q are shown in Fig. 6 for three different values of y_{cut}.

A first minimum at $q = 4$ followed by quasi-oscillation about 0, is seen for $y_{cut} = 6 \times 10^{-6}$ (energy scale ~ 100 MeV) (Fig. 6a). This behaviour is qualitatively similar to that for the charged-particle multiplicity distribution, as well as for sub-jet multiplicity distributions with smaller y_{cut}, but both the position of the first minimum and of the subsequent maxima and minima are shifted to lower values of q.

As y_{cut} is increased, the negative first minimum near 5 disappears. For $q \leq 5$ each H_q alternates between positive and negative values. For higher q, the oscillations begin to disappear. For $y_{cut} = 1.5 \times 10^{-5}$ (energy scale ~ 160 MeV) these

oscillations have disappeared (Fig. 6b).

In the perturbative region (energy scale $\geq$ 1 GeV), the oscillations and the negative first minimum near 5 have disappeared (Fig. 6c). Instead, H_q alternates between positive and negative values for each q (mind the scale!).

Although good agreement with NLLA is found for the production rates of soft gluons seen in the perturbative region, none of the predictions made at different orders of pQCD for the H_q is observed at LEP I energy. Even though pQCD describes collective processes (like the average sub-jet multiplicity) it seems to be unable to describe the shape of these distributions. Therefore, the minimum and oscillatory behaviour of the H_q seen for the charged-particle multiplicity distribution and for very low values of y_{cut} appears unrelated to the behaviour of the H_q calculated in NNLLA.

However JETSET PS reproduces the observed behaviour at all values of y_{cut}, and HERWIG shows a qualitatively similar behaviour.

2.5. Monte Carlo study of the oscillatory behaviour of the H_q

Given the success of JETSET and HERWIG and the failure of the NNLLA prediction seen in

the previous sub-section, we attempted to find that aspect of the Monte Carlo generators which is responsible for the agreement.

We varied several options in JETSET and studied their influence on the behaviour of the H_q. First we used different models of parton generation using in all cases the Lund string fragmentation:

- no angular ordering in the parton shower;
- $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$ matrix elements;
- matrix element production of only q$\bar{\text{q}}$.

Next we considered the possibility that the observed behaviour could come from the fragmentation model. We used JETSET 7.4 with the options of parton generation given above but with independent instead of string fragmentation. We also used HERWIG 5.9 which uses cluster fragmentation.

In all cases the H_q have a negative first minimum near $q = 5$ and quasi-oscillations for higher q. Some very different examples are shown in Fig. 7. This Monte-Carlo study shows that one can reproduce the oscillatory behaviour of H_q without the need for NNLLA of pQCD.

We further found that the oscillations do not exist on the parton level of JETSET at 91 GeV, even when reducing the cut-off Q_0 to 0.7 GeV. They do, however, set in at $\sqrt{s} = 190$ GeV.

2.6. Conclusions from the global distribution

The ratio H_q of cumulant factorial to factorial moments has been measured for both the charged-particle and sub-jet multiplicity distributions. A perturbative QCD calculation at the next-to-next-to-leading-logarithm order predicts H_q to have a negative first minimum at $q \sim 5$ and quasi-oscillation around 0 for higher q, thus confirming previous results. This behaviour is indeed observed for the charged-particle multiplicity distribution. However, our analysis reveals that this behaviour appears only for very small y_{cut}, corresponding to energy scales < 100 MeV, far away from the perturbative region. Furthermore, an investigation performed on very different models of parton generation and fragmentation shows similar oscillatory behaviour for all cases.

Contrary to [3], we conclude that the oscilla-

tions of H_q in the charged-particle multiplicity distribution are unrelated to the behaviour predicted by the NNLLA perturbative QCD calculations at the given energy.

3. LOCAL FLUCTUATIONS

3.1. Introduction

Local multiplicity fluctuations have been studied for many years in terms of a variety of phase-space variables [1], but only recently has substantial progress been made to derive analytical QCD predictions for these observables [20–22]. Assuming LPHD once again, these predictions for the parton level can be compared to experimental data. DELPHI [23] has shown that the analytical predictions tend to underestimate the correlations between particles in small angular windows for a QCD dimensional scale of $\Lambda \sim 0.1-0.2$ GeV. However, a reasonable agreement is achieved for an effective $\Lambda \sim 0.04$ GeV, surprisingly smaller than theoretical QCD estimates [24].

In an investigation [25] of local multiplicity fluctuations in terms of bunching parameters [26], L3 concluded that the local fluctuations inside jets are of multifractal type. This is qualitatively consistent with the QCD predictions. Here, we extend this study and present a quantitative comparison of the analytical first-order QCD predictions [20–22] with the L3 data using normalized factorial moments of ranks $q = 2$ to 5 in angular phase-space intervals.

3.2. Analytical predictions

QCD is inherently intermittent and QCD predictions [20–22] for normalized factorial moments (NFMs) have the following scaling behavior

$$F_q(\Theta) \propto \left(\frac{\Theta_0}{\Theta} \right)^{(D-D_q)(q-1)}, \tag{3}$$

where Θ_0 is the half opening angle of a cone around the jet-axis, Θ is the angular half-width of a ring around the jet-axis centered at Θ_0 (see Fig. 8), D is the underlying topological dimension ($D = 1$ for single angle Θ), and D_q are the so-called Rényi dimensions. The analytical QCD expectations for D_q are as follows [20–22]:
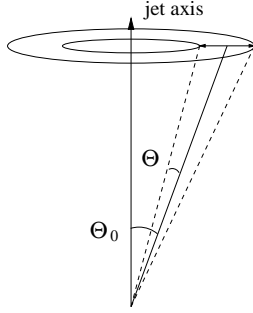
Figure 8. A schematic representation of the interval in the polar angle around the jet axis.

1) In the fixed coupling regime, for moderately small angular bins,

$$D_q = \gamma_0(Q)\frac{q+1}{q}, \qquad (4)$$

where $\gamma_0(Q) = \sqrt{2\,C_A\alpha_s(Q)/\pi}$ is the anomalous QCD dimension calculated at $Q \simeq E\Theta_0$, $E = \sqrt{s}/2$, and gluon color factor $C_A = N_c = 3$.

2) In the running-coupling regime, for small bins, the Rényi dimensions become a function of the size of the angular ring ($\alpha_s(Q)$ increases with decreasing Θ). It is useful to introduce a new scaling variable [22],

$$z = \frac{\ln(\Theta_0/\Theta)}{\ln(E\Theta_0/\Lambda)}, \qquad (5)$$

where the maximum possible region ($\Theta = \Theta_0$) corresponds to $z = 0$.

Three approximations derived in DLLA will be compared:

a) According to [21], the D_q have the form

$$D_q \simeq \gamma_0(Q)\frac{q+1}{q}\left(1 + \frac{q^2+1}{4q^2}z\right). \qquad (6)$$

b) Another approximation has been suggested in [22]:

$$D_q \simeq 2\,\gamma_0(Q)\frac{q+1}{q}\left(\frac{1-\sqrt{1-z}}{z}\right). \qquad (7)$$

c) In [20], a result has been obtained for the cumulant moments converging to factorial moments

for high energies,

$$D_q \simeq 2\,\gamma_0(Q)\frac{q-w(q,z)}{z(q-1)}, \qquad (8)$$

$$w(q,z) = q\sqrt{1-z}\left(1 - \frac{\ln(1-z)}{2q^2}\right). \qquad (9)$$

In [21], an estimate for D_q has, furthermore, been obtained in the Modified Leading Log Approximation (MLLA). In this case, (6) remains valid, but $\gamma_0(Q)$ is replaced by an effective $\gamma_0^{\text{eff}}(Q)$ depending on q:

$$\gamma_0^{\text{eff}}(Q) = \gamma_0(Q) + \gamma_0^2(Q)\frac{b}{4C_A} \cdot$$
$$\cdot \left[-B\frac{q-1}{2(q+1)} + \frac{q-1}{2(q+1)(q^2+1)} + \frac{1}{4}\right], \quad (10)$$
$$b = \frac{11C_A}{3} - \frac{2n_f}{3}, \qquad B = \frac{1}{b}\left[\frac{11C_A}{3} + \frac{2n_f}{3C_A^2}\right],$$

where n_f is the number of flavors.

For our comparison of data and theoretical predictions quoted above, we will use $n_f = 3$ and $\Lambda = 0.16$ GeV.

The low value of n_f is chosen since even at high energies the production of higher flavors will rarely happen in the jet and its evolution is still dominated by the light flavors [19]. (The contribution of heavy quarks will be discussed below.) The value of Λ chosen is that found in tuning the JETSET 7.4 ME program [13] on L3 data [27] and in our most recent determination of $\alpha_s(m_Z)$ [16].

For the angle Θ_0, we consider two possibilities: $\Theta_0 = 25^0$ and 35^0. The first value is chosen in order to compare our results with the DELPHI analysis [23], while the larger value of Θ_0 allows a larger range of Θ to be studied.

The effective coupling constant is evaluated at $Q \simeq E\Theta_0$. For $\Theta_0 = 25^0$, one obtains $\alpha_s(E\Theta_0) \simeq 0.144$ according to the first-order QCD expression for $\alpha_s(Q)$. This value leads to $\gamma_0(E\Theta_0) \simeq 0.525$. For $\Theta_0 = 35^0$, $\alpha_s(E\Theta_0) \simeq 0.135$ and $\gamma_0(E\Theta_0) \simeq 0.508$.

3.3. The data
The data as well as the selection and correction procedures are the same as in Sect. 2.2, except

that a more severe cut is applied on the track quality and K_s^0, Λ^0 decay products are excluded from the sample. Furthermore, Bose-Einstein correlations and Dalitz decay are removed from the data using correction from MC.

The resolution of the L3 detector for a number of relevant variables was estimated in [28]. The resolution of polar angle defined with respect to the thrust axis is found to be approximately 0.01 radians. For higher-order NFMs, the minimum angle Θ is chosen according to the many-particle resolution studied in [28].

The error bars on the results include contributions from statistical errors on the raw quantities and both statistical and systematic errors on the correction factors.

As a systematic error, we take half of the difference between the correction factors determined using JETSET [13] and those using HERWIG 5.9 [14]. In addition, we checked how a variation of the cuts affects the NFMs. The influence of such variations on the observed signal was found to be negligible.

The sphericity axis is used to define the jet axis.

3.4. Analysis
3.4.1. Comparison with JETSET on hadronic and partonic levels

It has been suggested in [22] to consider the ratio $F_q(z)/F_q(0)$ in order to reduce hadronization effects on the actual behavior of the NFMs. In addition, this reduces a theoretical ambiguity for the evaluation of NFMs in full phase space ($z = 0$). In terms of $F_q(z)/F_q(0)$, the power law (3) reads as

$$\ln \frac{F_q(z)}{F_q(0)} = z(1 - D_q)(q - 1)\ln \frac{E\Theta_0}{\Lambda}. \qquad (11)$$

The behavior of the $\ln(F_q(z)/F_q(0))$ as a function of z is shown in Fig. 9.

To increase statistics, we evaluated the NFMs in each sphericity hemisphere of an event and averaged the results, thus assuming that the local fluctuations in each hemisphere are independent.

The open symbols show the predictions of the JETSET 7.4 PS model for hadronic (open circles) and partonic (open triangles) levels with the default cut-off $Q_0 = 1.0$ GeV. The Monte-Carlo
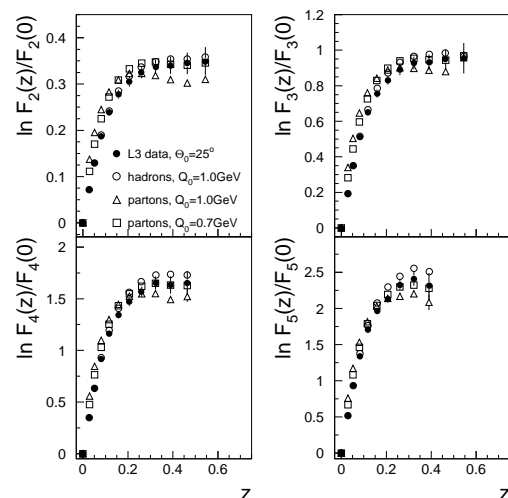

Figure 9. The behavior of $F_q(z)/F_q(0)$ ($q = 2, \ldots, 5$) and JETSET 7.4 PS predictions, on the partonic and hadronic levels. Data are corrected for detector effects, initial-state photon radiation, Bose-Einstein correlations and Dalitz decays.

prediction for hadrons gives a reasonable description of the fluctuations.

The partonic level of JETSET is close to the data, both for $Q_0 = 1.0$ GeV and $Q_0 = 0.7$ GeV, though a noticeable difference between the slopes for the data and the partonic level of JETSET is present. The similarity of partonic and hadronic level Monte-Carlo predictions is used as a measure of the degree of validity of LPHD.

The contributions of heavy flavors (c and b quarks) were estimated by rejection of these flavors on generator-level of JETSET. The effect was found to be negligible.

3.4.2. Comparison of the analytical predictions

The comparison of the analytical QCD predictions (4)-(10) to the corrected data is shown in Fig. 10 for $\Lambda = 0.16$ GeV and $\Theta_0 = 25^0$. For second order, the running-α_s predictions lead to the saturation effects observed in the data, but significantly underestimate the observed signal. Predictions for the higher moments are too low for low values of z, but tend to overestimate the data at larger z. The fixed coupling regime (thin solid lines) approximates the running coupling regime
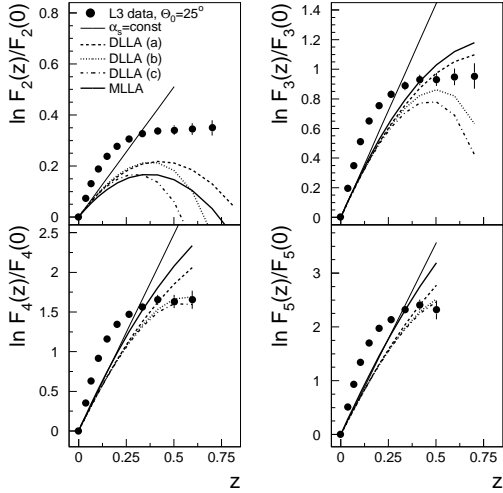
Figure 10. The analytical QCD predictions for $\Lambda = 0.16$ GeV: $\alpha_s = $ const (4); DLLA (a) (6); DLLA (b) (7); DLLA (c) (8); MLLA (10).

Figure 11. Same as Fig. 10, but for $\Lambda = 0.04$ GeV.

for small z, but does not exhibit the saturation effect seen in the data. The DLLA approximation (6) and (7) differ significantly at large z. The predictions for cumulants (8) show a stronger saturation effect. The MLLA predictions do not differ significantly from the DLLA result (6).

At $\Theta_0 = 35^0$ (not shown) both data and predictions show a stronger increase than for $\Theta_0 = 25^0$. This indicates that fluctuations are larger for phase-space regions containing a larger contribution from hard-gluon radiation.

A better agreement of the QCD predictions with the data at low z-values can be achieved by decreasing the value of Λ. A similar observation has been made by DELPHI [23]. As an example, Fig. 11 shows the case of $\Lambda = 0.04$ GeV. Such an effective value makes the coupling constant smaller and this can expand the range of reliability of the perturbative QCD calculations (for $\Lambda = 0.04$ GeV, $\alpha_s(E\Theta_0) \simeq 0.112$, $\gamma_0(E\Theta_0) \simeq 0.46$). However, this leads to a large disagreement between the QCD predictions and the data for $z > 0.3$, where contributions from high-order perturbative QCD and hadronization are expected to be stronger. We have varied Λ in the range of $0.04 - 0.25$ GeV and found that there is no value of Λ in this range which provides agreement for
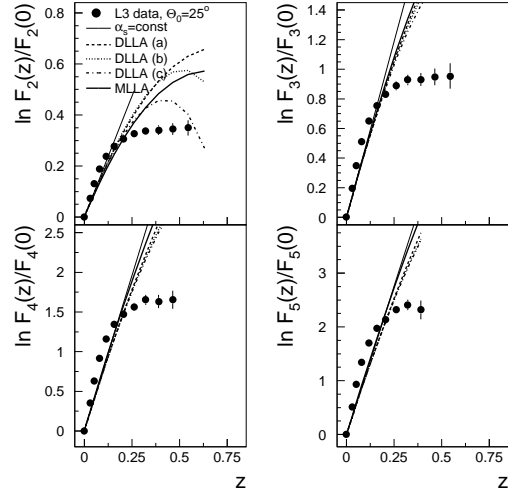
all orders of NFMs. Furthermore, increasing n_{f} to 4 and 5 reduces the second moment to reproduce the data at large z, but high-order moments still continue to overshoot.

3.5. Conclusions from local fluctuations

The first-order predictions of the DLLA of perturbative QCD are shown to be in disagreement with the local fluctuations as observed for hadronic Z decay. This conclusion is valid for standard values of Λ ($\Lambda = 0.16$ GeV) as well as for small values ($\Lambda = 0.04$ GeV). In the latter case, a reasonable estimate for $z < 0.3$ can be reached, consistent with the DELPHI conclusion [23]. However, our analysis shows that, in this case, the theoretical NFMs strongly overestimate the data for relatively large z (small Θ), where contribution from high-order perturbative QCD is stronger.

The prediction of MLLA shows very similar a result as DLLA. Note, however, that a full MLLA correction for the z-dependence of D_q has not been obtained, so far. The MLLA prediction quoted above only modifies γ_0, but the z-dependence of NFMs is parameterized by DLLA, which is only asymptotically correct. One large effect in the MLLA approximation is the angular recoil effect, which is important for small z. This effect can change the value of NFMs at $z = 0$ and, hence, the absolute normalization of the NFMs

[29]. Note that a recent study [30] of energy-conservation in triple-parton vertices shows that the energy-conservation constraint is indeed sizeable and leads to a stronger saturation effect.

Another likely reason for their disagreement with the experimental data is the asymptotic character of the QCD predictions, corresponding to an infinite number of partons in an event. Furthermore, the failure of the predictions can lie with the LPHD hypothesis, which is used to justify comparison of the predictions of perturbative QCD to hadronic data.

Acknowledgments

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