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Fermion Mass Textures in an M-Inspired Flipped $SU(5)$ Model Derived from String

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Abstract

We are inspired by the facts that M theory may reconcile the supersymmetric GUT scale with that of quantum gravity, and that it provides new avenues for low-energy supersymmetry breaking, to re-examine a flipped $SU(5)$ model that has been derived from string and may possess an elevation to a fully-fledged M-phenomenological model. Using a complete analysis of all superpotential terms through the sixth order, we explore in this model a new flat potential direction that provides a pair of light Higgs doublets, yields realistic textures for the fermion mass matrices, and is free of R-violating interactions and dimension-five proton decay operators.

1. Introduction

String model building is currently in a state of flux. Although quasi-realistic string models have been constructed [1], particularly in the free-fermion formulation of the weakly-coupled heterotic string [2], further progress was held back by poor understanding of the non-perturbative string effects that should determine the correct string vacuum. Moreover, a major problem for most weakly-coupled string models was the apparent discrepancy between the string unification scale calculated from first principles and the supersymmetric grand unification scale inferred from measurements of the Standard Model gauge couplings at LEP and elsewhere [3]. In fact, this discrepancy and a better understanding of non-perturbative string effects contributed to the motivation for study of the strong-coupling limit of string theory, and it is now known that the unification-scale discrepancy may be removed if the Theory of Everything turns out to be a strong-coupling limit of M theory [4], corresponding to an eleventh dimension which is much larger than the Planck length [5]. This may enable string models constructed directly in four dimensions [2] to reconcile the measured gauge couplings and four-dimensional Planck mass. Another issue which is cast in new light by advances in our understanding of non-perturbative effects in string theory is that of supersymmetry breaking. Previous upper limits on the rank of the effective four-dimensional gauge group have been abolished [6], which may open new horizons for gaugino condensation, and the Scherk-Schwarz mechanism has been revived [7]. Since it seems that many of the previous weakly-coupled string models may be elevated to consistent vacua of M theory, it is a good moment to re-investigate them, and explore whether their remaining phenomenological deficiencies can be overcome.

Among the phenomenological issues to be addressed by effective low-energy models derived from strings before they can be considered serious candidates for describing the elementary particle world are the flat directions of the effective potential and the associated choice of vacuum expectation values, the absence of light Higgs triplets and the presence of light electroweak Higgs doublets, the texture of the fermion mass matrices, R violation and proton decay. In particular, the stability of the proton had become particularly troublesome with the advent of supersymmetric GUTs, when it was realised that dimension-five operators of the form $QQQL$ and/or $u^c u^c d^c e^c$ could be generated by the exchange of coloured GUT states, inducing rapid proton decay [8]. In the string context, such operators could also be induced by exchanges of heavy string modes. However, it has recently been shown [9] that there is a class of free-fermion string models in which an enhanced custodial symmetry forbids the appearance of such dangerous operators to all orders in perturbation theory, and arguments have been given [9] that these models may be elevated to M theory.

Since the problems of the unification scale and proton stability may be on the way to resolution with such elevations of 'traditional' string models, we now explore some of their other phenomenological aspects in more detail, focussing in particular on predictions for fermion masses and mixing angles. There has been considerable work on the fermion mass problem during the last few years. In order to understand the observed hierarchies and mixings in the most predictive way, model builders have proposed specific textures of mass matrices at the unification scale with a minimal number of parameters. Inspired by string model building, several groups have attempted to use simple family U(1) or discrete symmetries to obtain the required textures. However, despite valiant efforts [10, 11, 12], a fully realistic pattern of fermion masses and mixing has never been derived from a string model, which is the objective of this paper.

We work in the context of three-generation superstring models derived in the free-fermionic formulation [2], which have the advantages of working directly in four dimensions and yielding easily unified models that reduce to the minimal supersymmetric extension of the Standard Model. The class of models that we favour is based on string-derived versions of flipped SU(5) [13] [1]. Using a certain three-generation string basis in the free-fermion formulation, it is easy to produce variant models with similar group structures but different phenomenological characteristics, by making slight modifications of the string basis. This gives us hope that minor 'defects' of an otherwise successful model can be cured.

Indeed, we exhibit in this paper an explicit string-derived model with a variant pattern of scalar vacuum expectation values that ensures heavy Higgs triplets, provides two light Higgs doublets, yields a qualitatively successful pattern of fermion masses and mixing, and has neither R-violating interactions nor proton decay operators to the order studied, which includes all sixth-order terms in the effective superpotential.

2. A Free-Fermion Model and its Spectrum

From among the relatively rich variaty of free-fermion models [13, 14, 15, 16, 17, 18] we choose to work in the context of the $SU(5) \times U(1)$ model [13]. This model is defined by a set of basis vectors defining boundary conditions on the world-sheet fermions [2] that span a finite additive group, and the physical states in the Hilbert space of a given sector are obtained by acting on the vacuum with bosonic and fermionic operators and then applying generalized GSO projections that ensure consistency with the string constraints. The construction of the flipped $SU(5)$ model can be seen in two stages. First, a set of five vectors $(1, S, b_{1,2,3})$ is introduced which define an $SO(10) \times SO(6) \times E_8$ gauge group with N=1 supersymmetry. Next, adding the vectors $b_{4,5}, \alpha$[13] [2], the number of generations is reduced to three and the observable-sector gauge group obtained is $SU(5) \times U(1)$ accompanied by additional four $U(1)$ factors and a hidden-sector $SU(4) \times SO(10)$ gauge symmetry.

The massless spectrum generated by the above basis, consists of the supergravity and gauge multiplets, the latter arising from the Neveu-Schwarz sector, and the seventy chiral superfields listed below with their non-Abelian group representation contents and their $U(1)$ charges. A generic feature shared with all such $k = 1$ constructions is that there are no adjoint or higher-dimensional representations [19, 20]. In this model, all states in the observable sector belong to the 1,5,10 of $SU(5)$ and their conjugates. This is why higher-level constructions are needed to obtain traditional GUT theories, such as $SU(5)$ or $SO(10)$,

[1] This class of models corresponds geometrically to $Z_2 \times Z_2$ orbifold compactification at the maximally-symmetric point in the Narain moduli space, and their three-generation nature is directly related to the $Z_2 \times Z_2$ orbifold structure.

[2] These correspond to Wilson lines in the orbifold formulation.

which need adjoint Higgs representations to break down to the Standard Model [3]. On the other hand, flipped $SU(5)$ is one of the few GUTs which only require Higgs representations smaller than the adjoint, since 10 and $\overline{10}$ representations suffice to break the symmetry down to $SU(3) \times SU(2) \times U(1)$. We recall also that the observable quarks and leptons are in the $10, \bar{5}, 1$, but with assignments and electric charges 'flipped' relative to conventional $SU(5)$.

Field Content of the Flipped $SU(5)$ String Model

$F_1(10, \frac{1}{2}, -\frac{1}{2}, 0, 0, 0)$	$\bar{f}_1(\bar{5}, -\frac{3}{2}, -\frac{1}{2}, 0, 0, 0)$	$\ell_1^c(1, \frac{5}{2}, -\frac{1}{2}, 0, 0, 0)$
$F_2(10, \frac{1}{2}, 0, -\frac{1}{2}, 0, 0)$	$\bar{f}_2(\bar{5}, -\frac{3}{2}, 0, -\frac{1}{2}, 0, 0)$	$\ell_2^c(1, \frac{5}{2}, 0, -\frac{1}{2}, 0, 0)$
$F_3(10, \frac{1}{2}, 0, 0, \frac{1}{2}, -\frac{1}{2})$	$\bar{f}_3(\bar{5}, -\frac{3}{2}, 0, 0, \frac{1}{2}, \frac{1}{2})$	$\ell_3^c(1, \frac{5}{2}, 0, 0, \frac{1}{2}, \frac{1}{2})$
$F_4(10, \frac{1}{2}, -\frac{1}{2}, 0, 0, 0)$	$f_4(5, \frac{3}{2}, \frac{1}{2}, 0, 0, 0)$	$\bar{\ell}_4^c(1, -\frac{5}{2}, \frac{1}{2}, 0, 0, 0)$
$\bar{F}_5(\overline{10}, -\frac{1}{2}, 0, \frac{1}{2}, 0, 0)$	$\bar{f}_5(\bar{5}, -\frac{3}{2}, 0, -\frac{1}{2}, 0, 0)$	$\ell_5^c(1, \frac{5}{2}, 0, -\frac{1}{2}, 0, 0)$

$h_1(5, -1, 1, 0, 0, 0)$	$h_2(5, -1, 0, 1, 0, 0)$	$h_3(5, -1, 0, 0, 1, 0)$
$h_{45}(5, -1, -\frac{1}{2}, -\frac{1}{2}, 0, 0)$		

$\phi_{45}(1, 0, \frac{1}{2}, \frac{1}{2}, 1, 0)$	$\phi_+(1, 0, \frac{1}{2}, -\frac{1}{2}, 0, 1)$	$\phi_-(1, 0, \frac{1}{2}, -\frac{1}{2}, 0, -1)$
$\Phi_{23}(1, 0, 0, -1, 1, 0)$	$\Phi_{31}(1, 0, 1, 0, -1, 0)$	$\Phi_{12}(1, 0, -1, 1, 0, 0)$
$\phi_i(1, 0, \frac{1}{2}, -\frac{1}{2}, 0, 0)$	$\Phi_i(1, 0, 0, 0, 0, 0)$	

$\Delta_1(0, 1, 6, 0, -\frac{1}{2}, \frac{1}{2}, 0)$	$\Delta_2(0, 1, 6, -\frac{1}{2}, 0, \frac{1}{2}, 0)$	$\Delta_3(0, 1, 6, -\frac{1}{2}, -\frac{1}{2}, 0, \frac{1}{2})$
$\Delta_4(0, 1, 6, 0, -\frac{1}{2}, \frac{1}{2}, 0)$	$\Delta_5(0, 1, 6, \frac{1}{2}, 0, -\frac{1}{2}, 0)$	

$T_1(0, 10, 1, 0, -\frac{1}{2}, \frac{1}{2}, 0)$	$T_2(0, 10, 1, -\frac{1}{2}, 0, \frac{1}{2}, 0)$	$T_3(0, 10, 1, -\frac{1}{2}, -\frac{1}{2}, 0, \frac{1}{2})$
$T_4(0, 10, 1, 0, \frac{1}{2}, -\frac{1}{2}, 0)$	$T_5(0, 10, 1, -\frac{1}{2}, 0, \frac{1}{2}, 0)$	

Table 1: *The chiral superfields are listed with their quantum numbers [13]. The F_i, $\bar{f}_i$, ℓ_i^c, as well as the h_i, h_{ij} fields and the singlets are given in terms of their $SU(5) \times U(1)' \times U(1)^4$ quantum numbers. Conjugate fields have opposite $U(1)' \times U(1)^4$ quantum numbers. The fields Δ_i and T_i are tabulated in terms of their $U(1)' \times SO(10) \times SO(6) \times U(1)^4$ quantum numbers.*

As can be seen explicitly above, the matter and Higgs fields in this string model carry additional charges under surplus $U(1)$ symmetries [13], there are a number of neutral singlet fields, and a hidden-sector matter fields which transform non-trivially under the $SU(4) \times SO(10)$ gauge symmetry, as sextets under $SU(4)$, namely $\Delta_{1,2,3,4,5}$, and as decuplets under $SO(10)$, namely $T_{1,2,3,4,5}$. There are also fourplets of the $SU(4)$ hidden symmetry which possess fractional charges, however, as we discuss later, these are confined and will not concern us here.

[3]Important progress has recently been made in higher-level string constructions of $SU(5)$ and $SO(10)$ models [21], which may eventually lead to more realistic versions. Intrinsically M-theoretical compactifications might also be useful in this respect.

We recall that the flavour assignments of the light Standard Model particles in this model are as follows:

$$\bar{f}_1 : \bar{u}, \tau, \quad \bar{f}_2 : \bar{c}, e/\mu, \quad \bar{f}_5 : \bar{t}, \mu/e$$
$$F_2 : Q_2, \bar{s}, \quad F_3 : Q_1, \bar{d}, \quad F_4 : Q_3, \bar{b}$$
$$\ell_1^c : \bar{\tau}, \quad \ell_2^c : \bar{e}, \quad \ell_5^c : \bar{\mu} \tag{1}$$

up to mixing effects which we discuss later.

3. An Interesting Flat Direction of the Effective Potential

Any string model such as that reviewed above has a degenerate potential with many flat directions along which combinations of scalar fields may acquire large vacuum expectation values. The detailed phenomenological properties of the model depend on the choices of these moduli of the vacuum, which are subject to flatness conditions associated with both the D and the F terms in the effective potential, but cannot be fixed unambiguously with the string technology currently available. A new feature of this paper is a different choice of flat direction from those considered previously [10, 22]. We choose non-zero vacuum expectation values for the following singlet and hidden-sector fields:

$$\Phi_{31}, \bar{\Phi}_{31}, \Phi_{23}, \bar{\Phi}_{23}, \phi_2, \bar{\phi}_{3,4}, \phi^-, \bar{\phi}^+, \phi_{45}, \bar{\phi}_{45}, \Delta_{2,3,5}, T_{2,4,5} \tag{2}$$

The vacuum expectation values of the hidden-sector fields must satisfy additional constraints

$$T^2_{3,4,5} = T_i \cdot T_4 = 0, \;\; \Delta^2_{3,5} = 0, \;\; T_2^2 + \Delta_2^2 = 0 \tag{3}$$

which are imposed by the flatness conditions. As we discuss below, an acceptable scenario for supersymmetry breaking may still be possible, despite this breaking of the hidden-sector gauge group, at least within the context of M-theory.

We now discuss in more detail the F-flatness conditions. We have verified that a pattern of vacuum expectation values of the form (2) is compatible with flatness up to sixth order non-renormalizable terms in the superpotential. Here and later in the paper, we make a computerized search of all possible superpotential terms up to this order, discarding all terms that are disallowed by gauge symmetries and string selection rules [23, 24]. We do not calculate explicitly the remaining terms, but assume that they appear with generic coefficients of order unity. For the vacuum expectation values of interest, the relevant F-flatness conditions are:

$$\frac{\partial W}{\partial \Phi_{12}} \; : \; e^{i\gamma}\Phi_{23}\Phi_{31} + \frac{1}{2}\phi_2^2 \approx 0 \tag{4}$$

$$\frac{\partial W}{\partial T_4} \; : \; \frac{1}{\sqrt{2}}\phi_2 T_5 + T_4(\Phi_{23} + \frac{1}{2}e^{i\delta_1}\bar{\Phi}_{31}\phi_2^2) \approx 0 \tag{5}$$

$$
\begin{aligned}
\frac{\partial W}{\partial T_5} &: \quad \frac{1}{\sqrt{2}}\phi_2 T_4 + T_5(\Phi_{31} + \frac{1}{2}e^{i\delta_2}\bar{\Phi}_{23}\phi_2^2) \approx 0 && (6)\\
\frac{\partial W}{\partial \bar{\Phi}_{12}} &: \quad e^{iw}\bar{\Phi}_{23}\bar{\Phi}_{31} + \frac{1}{2}(\bar{\phi}_3^2 + \bar{\phi}_4^2) + (F_1\bar{F}_5)^2 \approx 0 && (7)\\
\frac{\partial W}{\partial \phi_2} &: \quad T_2 \cdot T_4 \Delta_2 \cdot \Delta_5 \approx 0 && (8)\\
\frac{\partial W}{\partial \Delta_4} &: \quad \frac{1}{\sqrt{2}}\Delta_5\bar{\phi}_3 + \Delta_4\bar{\Phi}_{23} + 2e^{ip_1}\Phi_{31}(\bar{\phi}_3^2 + \bar{\phi}_4^2) \approx 0 && (9)\\
\frac{\partial W}{\partial \Delta_5} &: \quad \frac{1}{\sqrt{2}}\Delta_4\bar{\phi}_3 + \Delta_5\bar{\phi}_{31} + 2e^{ip_2}\Phi_{23}(\bar{\phi}_3^2 + \bar{\phi}_4^2) \approx 0 && (10)\\
\frac{\partial W}{\partial T_2} &: \quad T_2(\Phi_{31} + 2e^{ip_3}\phi_2^2\bar{\Phi}_{23}) + \phi_2\Delta_2\Delta_5 T_4 \approx 0 && (11)\\
\frac{\partial W}{\partial \Delta_2} &: \quad \Delta_2(\Phi_{31} + 2e^{ip_4}\phi_2^2\bar{\Phi}_{23}) + \phi_2 T_2 T_4 \Delta_5 \approx 0 && (12)\\
\frac{\partial W}{\partial \Phi_3} &: \quad \phi_{45}\bar{\phi}_{45} + \phi_i\bar{\phi}_i \approx 0, \quad i = 4, \pm && (13)
\end{aligned}
$$

where δ_i, w, p_i are phases that are in principle calculable, that we leave indefinite. The approximate equality means that these conditions are valid up to a certain order. One should bear in mind that they will be further modified when higher-order corrections are taken into account.

We see from the above that if we demand $\Delta_2 \cdot \Delta_5 \neq 0$, which is needed in order to obtain non-zero (2,3) mixing in the V_{CKM} matrix, as we discuss later, we are forced to impose the condition $T_2 \cdot T_4 = 0$. Suppressing constants of order unity and various phases, the above equations may be satisfied if the following relations hold between the different non-zero vacuum expectation values:

$$
\begin{aligned}
\phi_2^2 &\sim \Phi_{31}\bar{\Phi}_{23} + \cdots && (14)\\
1 + \cdots &\sim |(\Phi_{31}\bar{\Phi}_{31} - 1)(\Phi_{23}\bar{\Phi}_{23} - 1)| && (15)\\
\bar{\phi}_3^2 + \bar{\phi}_4^2 &\sim \bar{\Phi}_{23}\bar{\Phi}_{31} + \cdots && (16)\\
\bar{\phi}_3^2 + \cdots &\sim \bar{\Phi}_{23}\bar{\Phi}_{31}(1 - \Phi_{31}\bar{\Phi}_{31})(1 - \Phi_{23}\bar{\Phi}_{23}) && (17)\\
\phi_{45}\bar{\phi}_{45} &\sim \phi_i\bar{\phi}_i + \cdots && (18)
\end{aligned}
$$

where the dots stand for higher-order corrections to the F-flatness conditions. Some such corrections may be crucial in ensuring that all the singlet fields may have non-zero vacuum expectation values in the perturbative regime, i.e., $\langle\phi_i\rangle \leq M_{String}/10$. In particular, we assume that (18) is valid only when such corrections are taken into account, which can be easily satisfied when at least one of the singlet fields $\phi_{45}, \bar{\phi}_{45}$ develops a relatively small vev. It can be easily checked that the above set (2) of vacuum expectation values (vevs) satisfies also the D-flatness conditions.

An additional constraint on the vacuum expectation values of the hidden fields above is that they should be consistent with the confinement of fractionally-charged states [25]. We have verified that there is a pattern of values for the Δ_i vevs which preserves unbroken an $SO(4)$ subgroup of the hidden-sector $SU(4)$. Moreover, the residual unbroken gauge group is asymptotically free, so that all fractionally-charged states are confined, though at a lower energy scale than in previous versions of flipped $SU(5)$.

We also note that the hidden-sector $SO(10)$ breaks down to $SO(7)$, which may still be sufficient to seed supersymmetry breaking by gaugino condensation. We note, however, that this may not be necessary when the model is elevated to M theory. In this case, the rank of the gauge group may be enhanced, providing other gauge subgroups that might lead to gaugino condensation. Moreover, it has been suggested that the Scherk-Schwarz mechanism may operate in M theory [7], either supplementing or replacing gaugino condensation in a realistic scenario for supersymmetry breaking.

4. Light Higgs Doublets

We now discuss the appearance of light Higgs doublets and the large masses required for dangerous colored particles that share common $SU(5)$ representations with the standard model matter. Taking the latter issue first, and working to sixth order in the superpotential, as above, we find the following mass terms

$$\begin{aligned}\mathcal{W} &\rightarrow \Phi_{31}h_3\bar{h}_1 + \bar{\Phi}_{31}h_1\bar{h}_3 + \Phi_{23}h_3\bar{h}_2 + \bar{\Phi}_{23}h_2\bar{h}_3 + \phi_{45}\bar{h}_3h_{45} + \bar{\phi}_{45}h_3\bar{h}_{45} \\ &+ F_1F_1(h_1 + h_2\phi_i^2 + h_{45}\Phi_{31}\phi_{45}) \\ &+ \bar{F}_5\bar{F}_5(\bar{h}_2 + \bar{h}_{45}\bar{\phi}_{45}\Phi_{23} + \bar{h}_1\phi_i^2 + \Delta_{1,4}^2\bar{h}_3 + T_1^2\bar{h}_1) \end{aligned} \tag{19}$$

With our choice of vacuum expectation values, it can be easily checked that all extra colour-triplet pairs become massive.

Turning now to the more delicate issue of the survival of light Higgs doubles, we note that the mass matrix for the Higgs fiveplets $h_{1,2,3}$, h_{45} which may include the doublets required in the Standard Model takes the following form to fifth order:

$$m_h = \begin{pmatrix} 0 & \Phi_{12} & \bar{\Phi}_{31} & T_5^2\bar{\phi}_{45} \\ \bar{\Phi}_{12} & 0 & \Phi_{23} & \Delta_4^2\bar{\phi}_{45} \\ \Phi_{31} & \bar{\Phi}_{23} & 0 & \bar{\phi}_{45} \\ \Delta_5^2 & T_4^2\phi_{45} & \phi_{45} & 0 \end{pmatrix}, \tag{20}$$

With our choice (2) of singlet vacuum expectation values: $\langle\Phi_{12}, \bar{\Phi}_{12}\rangle = 0$ and with the supplementary conditions $\langle\Delta_i^2\rangle = 0$ and $\langle T_i^2\rangle = 0$, there are two massless combinations [26, 27] $h \sim \cos\theta h_1 + \cdots$, and $\bar{h} \sim \bar{h}_{45} + \cdots$.

Going on to sixth order, we find the following superpotential terms that might *a priori* mix Higgs doublets and leptons:

$$\begin{aligned}\mathcal{W} &\rightarrow \bar{F}_5F_2^2\bar{f}_2\bar{h}_{45} + F_2\Delta_2\Delta_5\bar{f}_5\bar{h}_{45} + F_1T_1T_4\Phi_{2,4}\bar{f}_5\bar{h}_{45} \\ &+ F_1\bar{\phi}_1\phi_{45}\Phi_{31}\bar{f}_1\bar{h}_2 + F_1\phi_{45}(\bar{\Phi}_{12}\bar{\phi}_1 + \phi_2\Phi_4)\bar{f}_1\bar{h}_3 + F_1\bar{\phi}_2\Phi_4\bar{\Phi}_{12}\bar{f}_1\bar{h}_{45} \end{aligned} \tag{21}$$

However, these are not in fact dangerous. The first two terms do not contribute to the Higgs mass matrix because we choose $\langle F_2\rangle = 0$. Moreover, the choice (2) also implies that $\langle \Phi_i\rangle = \langle \bar{\phi}_{1,2}\rangle = 0$, so the Higgs mass matrix (20) remains intact through sixth order. A complete dicussion of the light Higgs multiplets would need an enumeration of many higher-order terms in the superpotential, which would take us beyond the scope of this paper.

To the order studied, $\bar{h}$ (which contains a component of $\bar{h}_{45}$) remains light and hence is available to give a mass to the top quark through the coupling $F_4\bar{f}_5\bar{h}_{45}$. Similarly, h is available to give a mass to the bottom quark, also via a trilinear coupling, whose magnitude depends on a Higgs mixing angle θ, which is as yet undetermined. This model therefore predicts a heavy top quark, but allows the bottom to be significantly lighter, without requiring a large ratio of Higgs vacuum expectation values.

5. Fermion Mass Matrices and Mixing

A deeper treatment of fermion mass matrices requires the consideration of higher-order non-renormalizable terms which fill in entries that vanish at lower order [13]. Due to the additional $U(1)$ symmetries in this type of model, and string selection rules [23], only a few Yukawa couplings are available at any given order. A complete discussion would require going to very high order, and would need, for consistency, to discuss flat directions and the choice of vacuum expectation values to comparable order. As before, we restrict our attention to terms of at most sixth order, which are sufficient to discuss interesting qualitative features of the fermion mass matrices.

To this order, for $\langle h_2\rangle \ll \langle h_1\rangle$, the **down-quark** mass matrix takes the form

$$M_D = \begin{pmatrix} 0 & \Delta_2\Delta_3\bar{\Phi}_{23} & \Delta_5\Delta_3\bar{\phi}_3 \\ \Delta_2\Delta_3\bar{\Phi}_{23} & (\bar{\phi}_3^2+\bar{\phi}_4^2) & \Delta_2\Delta_5\bar{\phi}_4 \\ \Delta_5\Delta_3\bar{\phi}_3 & \Delta_2\Delta_5\bar{\phi}_4 & 1 \end{pmatrix} \lambda_b(M_{GUT})\langle h_1\rangle \tag{22}$$

where at the unification scale $\lambda_b(M_{GUT}) = \sqrt{2}g$. The zero entry should be understood as being of higher than sixth order, and each of the non-zero higher-order entries should be understood as having numerical factor of order unity, with possible phases. We notice that these entries have the right orders of magnitude to reproduce the correct hierarchy of the down quark masses. In the approximation $\Delta_2\Delta_5\bar{\phi}_4 < \Delta_2\Delta_3\bar{\Phi}_{23} \sim \Delta_2\Delta_3$, we find a bottom-quark Yukawa coupling of magnitude $g\sqrt{2}$ to the h_1 component of the h field, so that $m_b \sim g\sqrt{2}\sin\theta\langle h\rangle$, which may be much smaller than m_t, and the (1,2) mixing element is given by

$$V_{12}^d = \sqrt{\frac{m_d}{m_s}} = \frac{\Delta_2\Delta_3\bar{\Phi}_{23}}{\bar{\phi}_3^2+\bar{\phi}_4^2} \tag{23}$$

The **up-quark** mass matrix in the particular flat direction we study takes the form

$$M_U = \begin{pmatrix} 0 & 0 & \Delta_3\Delta_5\bar{\phi}_3 \\ 0 & \bar{\phi}_4 & \Delta_2\Delta_5\bar{\phi}_4 \\ 0 & \Delta_2\Delta_5 & 1 \end{pmatrix} \lambda_t(M_{GUT})\langle \bar{h}_{45}\rangle, \tag{24}$$

with the same remarks as above concerning the meaning of the zero entries and the presence of unspecified numerical coefficients of order unity in the non-zero entries. Since $m_u \neq 0$ only to higher order, we see that $m_u < m_d$ is a natural possibility. It is striking that the above string-derived Ansatz for the quark mass matrices belongs to one [28, 29] of the few cases [30] based on the appearance of texture zeros which describe correctly the low-energy quark masses and mixings using only a minimal set of parameters at the GUT scale. We should point out that the above matrices are defined at the unification scale. To compare with the experimentally measured fermion masses, one has to take into account the renormalisation group effects.

We finally discuss the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. From the particular form of our mass matrices, we deduce that the $(1-2)$ Cabibbo angle is essentially obtained from the down quark mass matrix. Ignoring for simplicity the $(1-3)$ mixing, we find that $V_{12}^{CKM} \sim V_{12}^d$ approximately [10]. Specifically, after diagonalization of the quark mass matrices, one finds the following form of the CKM matrix to first order in perturbation theory and up to order-unity coefficients

$$V^{CKM} \approx \begin{pmatrix} 1 & \Delta_2\Delta_3\bar{\Phi}_{23}/(\bar{\phi}_3^2+\bar{\phi}_4^2) & \Delta_3\Delta_5\bar{\phi}_3 \\ -\Delta_2\Delta_3\bar{\Phi}_{23}/(\bar{\phi}_3^2+\bar{\phi}_4^2) & 1 & \Delta_2\Delta_5\bar{\phi}_4 \\ -\Delta_3\Delta_5\bar{\phi}_3 & -\Delta_2\Delta_5\bar{\phi}_4 & 1 \end{pmatrix}, \tag{25}$$

where the vacuum expectation values of the fields are normalised with respect to the heavy mass scale of the theory, which should presumably be identified with some M-theory scale $\sim 10^{16}$ GeV [4].

Charged Leptons in this model are accommodated in the $\bar{f}_1, \bar{f}_2,\ \bar{f}_5$ and $\ell_1^c, \ell_2^c, \ell_5^c$ representations in the light spectrum shown above. The remaining representations $f_4, \bar{f}_3$, $\bar{\ell}_4^c, \ell_3^c$ are expected to gain masses from terms

$$(f_4\bar{f}_3 + \bar{\ell}_4^c\ell_3^c)\left(T_3\cdot T_5(\phi_2\bar{\Phi}_{23} + \bar{\phi}_2\bar{\Phi}_{31})\right) \tag{26}$$

Trilinear superpotential couplings yield

$$\bar{f}_1\ell_1^c h_1 + (\bar{f}_2\ell_2^c + \bar{f}_5\ell_5^c)h_2 \tag{27}$$

where one should expect as before higher-order corrections involving products of the singlet and hidden-sector fields $\phi_i, \phi^{\pm}$ and $\bar{\phi}_i, \bar{\phi}^{\pm}$, etc.. In order to obtain the known hierarchy of lepton masses, we see that a sensible choice is to accommodate the τ as $\ell_1^c, \bar{f}_1$, while choosing $\langle h_1\rangle \gg \langle h_2\rangle$ for the Higgs vacuum expectation values. Indeed, as can be seen directly from the superpotential, now bottom and τ couplings are exactly the same at the unification scale [31], which agrees with the measured values after renormalisation effects are taken into account.

It is not clear yet what combinations of $\bar{f}_{2,5}$ and $\ell_{2,5}^c$ should be interpreted as first- and second-generation leptons, because there are higher-order terms mixing these latter fields with some heavy states. We plan to return to these in a later publication, together with the complicated issue of neutrino masses.

6. R Violation

The types of terms discussed may not exhaust the pattern of superpotential couplings in a string-derived model, since there may, in particular, be terms that violate R parity. These might *a priori* cause serious phenomenological problems, because there are stringent upper bounds on several individual couplings, and especially on the simultaneous presence of some flavour-changing products of couplings [32].

In our case, no renormalizable R-violating contributions are allowed, because they are not invariant under the GUT group, *in contrast* to conventional $SU(5)$. The flipped $SU(5)$ transformation properties of the R-violating couplings are:

$$\begin{aligned} LL\bar{E} &\rightarrow \bar{5}\times\bar{5}\times 1 \\ LQ\bar{D} &\rightarrow \bar{5}\times 10\times 10 \\ \bar{U}\bar{D}\bar{D} &\rightarrow \bar{5}\times 10\times 10 \end{aligned} \tag{28}$$

This could be regarded as another potential phenomenological asset of flipped $SU(5)$, in the absence of any confirmed R-violation in Nature.

However, effective terms that break R-parity may be generated by non-renormalizable higher-order terms [33, 34], and in all the cases (28) we need simply to add a field that transforms as a 10 of $SU(5)$ in order to obtain an $SU(5)$-invariant combination. If and when this field gets a vacuum expectation value, effective operators may be generated. The only candidate for such a field is F_1, but we must check in each case whether the transformations of the fields match under the $U(1)$ of the flipped $SU(5)$, as well as the other $U(1)$ factors, and also check the string selection rules.

Working as before to sixth order in the superpotential, we find in our flat direction the following candidate field combinations that are invariant under the gauge symmetries of the theory:

<u>$\bar{U}\bar{D}\bar{D}$ operators :</u>

$$\bar{U}_1\bar{D}_2\bar{D}_3F_1\phi_{45}\Phi_{31}$$

<u>$LQ\bar{D}$ operators :</u>

$$\begin{aligned} &L_1Q_2\bar{D}_2F_1\phi_{45}\bar{\Phi}_{23}, \quad L_2Q_2\bar{D}_2F_1\phi_{45}\bar{\Phi}_{23} \\ &L_1Q_3\bar{D}_3F_1\phi_{45}\Phi_{31}, \quad L_2Q_3\bar{D}_3F_1\phi_{45}\Phi_{31} \\ &L_3Q_2\bar{D}_3F_1\phi_{45}\Phi_{31}, \quad L_3Q_3\bar{D}_2F_1\phi_{45}\Phi_{31} \end{aligned}$$

<u>$LL\bar{E}$ operators :</u>

$$\begin{aligned} &L_1L_2\bar{E}_1F_1\phi_{45}\bar{\Phi}_{23}, \quad L_1L_2\bar{E}_2F_1\phi_{45}\bar{\Phi}_{23} \\ &L_1L_3\bar{E}_3F_1\phi_{45}\Phi_{31}, \quad L_2L_3\bar{E}_3F_1\phi_{45}\Phi_{31} \end{aligned}$$

However, it turns out, by inspection, that none of these satisfies all of the string selection rules!

At the present level of understanding, we cannot exclude the possibility that R-violating couplings may appear at higher orders, and the upper limits on some couplings and combinations are so severe that it would be necessary for many higher-order terms to vanish before any model could be considered safe. Nevertheless, we consider the vanishing to this order a very positive point for this particular string model.

7. Dimensional-Five Proton-Decay Operators

In this model, fourth-order superpotential terms provide no dimension-five proton-decay operators. The following are the potentially-dangerous dimension-five operators generated by fifth-order non-renormalizable terms [35, 36]:

$$F_4F_4F_3\bar{f}_3\Phi_{31}, \quad F_2F_2F_3\bar{f}_3\bar{\Phi}_{23}, \quad F_1F_1F_3\bar{f}_3\Phi_{31} \tag{29}$$

$$F_3\bar{f}_3\bar{f}_1\ell_1^c\Phi_{31}, \quad F_3\bar{f}_3\bar{f}_5\ell_5^c\bar{\Phi}_{23}, \quad F_3\bar{f}_3\bar{f}_2\ell_2^c\bar{\Phi}_{23} \tag{30}$$

$$F_3\bar{f}_2\bar{f}_2\ell_3^c\Phi_{31}, \quad F_3\bar{f}_1\bar{f}_1\ell_3^c\Phi_{31}, \quad F_3f_5\bar{f}_5\ell_3^c\bar{\Phi}_{23} \tag{31}$$

However, with our choice of vacuum expectation values (2), none of these terms are dangerous, because they do not involve particles in the light Standard Model part of the spectrum. At sixth order, the following potentially-dangerous operators appear:

$$F_4F_3F_3\phi_+\bar{f}_5\bar{\Phi}_{23}, \quad F_4F_3F_3\bar{f}_5\bar{\phi}_-\Phi_{31} \tag{32}$$

However, the singlet fields ϕ_+ and $\bar{\phi}_-$ do not acquire vacuum expectation values (2), so these also pose no problems.

In contrast to the string model described in [9], currently we do not see any symmetry reason why dimension-five proton-decay operators should be absent in this model to all orders. If they do appear at some higher order, this may signal that the current model requires some adjustment, but this should be regarded as an open question for the time being.

8. Conclusions

We have discussed in this paper a new variant of a flipped $SU(5)$ model derived from string [13] which has many positive features. It exploits an appealing flat direction that accommodates a pair of light Higgs doublets. These provide up- and down-quark mass matrices that can reproduce the observed hierarchy of quark masses and V_{CKM} matrix elements, since they have one of the patterns of flavour texture zeros that had been proposed previously on purely phenomenological grounds. The mass matrices of charged leptons and neutrinos remain open issues, to which we hope to return in a forthcoming paper. One very positive feature of this model is the absence of R-violating interactions to the order studied. This is in part due to the representation content of flipped $SU(5)$, but is also due in part to string selection rules that forbid certain couplings apparently permitted by the gauge symmetries. Another positive feature of the model is the absence of dimension-five proton-decay operators, which is essentially due to details of the vacuum we have chosen and the corresponding light-particle spectrum.

Outstanding problems that should be addressed in future work include the discussion of lepton and neutrino masses mentioned above, and the extension of all the analysis of superpotential terms to higher orders, using systematically all the string selection rules as well as the gauge symmetries of the model. In the longer run, we should also like to explore the elevation of this model to an intrinsic M-theory compactification. Here we have assumed that this is possible, and only used the inspiration of M-theory to motivate the absence of intermediate-scale degrees of freedom, which are no longer needed to reconcile the bottom-up and top-down calculations of the unification scale, and to motivate tolerance of a model with symmetry breaking in the hidden sector, on the grounds that M theory may provide other mechanisms for supersymmetry breaking. Knowledge of M-theory compactifications is accumulating, but none of those exhibited explicitly so far has as many attractive features as the model discussed here. Perhaps this or a related model may play a useful rôle in focussing M-phenomenology?

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