B Decays Involving η and η' in Light of the $B \to K\eta'$ Process

Amol S. Dighe
Enrico Fermi Institute and Department of Physics, University of Chicago, 5640 S. Ellis Avenue, Chicago, Illinois 60615

Michael Gronau*
Theoretical Physics Division, CERN, CH-1211 Geneva 23, Switzerland

Jonathan L. Rosner
Enrico Fermi Institute and Department of Physics, University of Chicago, 5640 S. Ellis Avenue, Chicago, Illinois 60615

(Received 4 August 1997)

The observation by the CLEO Collaboration of the decays $B^{(+,0)} \to K^{(+,0)}\eta'$ is shown to imply a significant but still uncertain contribution from the flavor-SU(3)-singlet component of the η'. By comparing the rate for these decays with others for decays of B mesons to two pseudoscalar mesons, it is shown that the prospects for observing CP-violating asymmetries in certain modes such as $B^+ \to \pi^+\eta$ and $B^+ \to \pi^+\eta'$ are quite good. [S0031-9007(97)04696-6]

PACS numbers: 13.25.Hw, 11.30.Er, 12.15.Ji, 14.40.Nd

The CLEO Collaboration [1] has recently reported the decays $B^+ \to K^+\eta'$ and $B^0 \to K^0\eta'$ with branching ratios of $(7.1^{+2.5}_{-2.1} \pm 0.9) \times 10^{-5}$ and $(5.3^{+2.8}_{-2.2} \pm 1.2) \times 10^{-5}$, respectively. In the present paper we show that these results, when combined with other information on decays of B mesons to pairs of light pseudoscalar mesons, indicate that the $K\eta'$ decays receive a significant contribution from the flavor-SU(3)-singlet component of the η'. We use the present information to predict the rates for charged and neutral B's to decay to (π or K) + (η or η'). By searching for processes in which contributions of different weak decay amplitudes are comparable to one another, we show that there is a high likelihood for observable CP-violating asymmetries in the decays $B^+ \to \pi^+\eta$ and $B^+ \to \pi^+\eta'$. A similar conclusion was reached earlier by Barshay, Rein, and Sehgal [2] on the basis of a different analysis. Others have emphasized previously the potential for CP-violating rate asymmetries to be exhibited in decays of B mesons to pairs of charmless mesons [3].

The contribution to $B^+ \to K^+\eta'$ from a new penguin amplitude, occurring only in decays involving a flavor-SU(3)-singlet component in the final pseudoscalar meson state, was noted in Refs. [4–6]. While one possibility for this contribution [2,7,8] is an intrinsic $c\bar{c}$ component in the η', more conventional mechanisms [8,9] (e.g., involving gluons) also seem adequate to explain the observed rate. Some enhancement of conventional mechanisms may be needed to explain the large rate for the *inclusive* process $B^+ \to \eta' + X$. We shall not be concerned here with the inclusive process.

We list the relevant decay amplitudes associated with a flavor-SU(3) decomposition [5,6,10–15] in Tables I and II. Unprimed amplitudes denote $\Delta S = 0$ decays; primed amplitudes denote $|\Delta S| = 1$ decays. An amplitude t (t') describes a tree-graph contribution, c (c') describes a color-suppressed process, p (p') describes a penguin graph contribution coupling to a pair of quark-antiquark mesons, and s (s') describes an additional penguin contribution coupling specifically to the flavor-SU(3)-singlet component of the η or η'. All of these amplitudes are defined in such a way [15] as to include contributions from electroweak penguin terms [16].

We assume the η and η' are mixed so that $\eta = (u\bar{u} + d\bar{d} - s\bar{s})/\sqrt{3}$ and $\eta' = (u\bar{u} + d\bar{d} + 2s\bar{s})/\sqrt{6}$, corresponding to an octet-singlet mixing angle of $\theta = -19.5°$. The p' contribution to $B \to K\eta$ vanishes for this mixing [14,15,17]. More details justifying this assumption are discussed, for example, in Refs. [4,5,12]. Other phase conventions for pseudoscalar mesons may be found in Ref. [13]. We have neglected all annihilation- and exchange-type amplitudes, which are expected to be highly suppressed in comparison with those shown.

In Tables I and II we have also calculated expected squares of contributions of individual amplitudes to decays. We ignore, for the present purposes, any interference between tree (t or t') and other amplitudes. We consider two possibilities for the relative phase of the two predominant amplitudes, p' and s', in the decay $B^+ \to K^+\eta'$. The cases (a) and (b) listed in the Tables correspond to constructive interference and no interference between these amplitudes. [Destructive interference would imply a singlet amplitude s' so large that the predicted value of $\mathcal{B}(B^+ \to K^+\eta)$ would exceed the current 90% confidence level (C.L.) bound [1] $\mathcal{B}(B^+ \to K^+\eta) < 8 \times 10^{-6}$.]

Interference between amplitudes becomes important when they are not too different in magnitude, which occurs in several cases which we shall presently identify. We do not quote contributions of color-suppressed amplitudes, neglecting them in the ensuing discussion. We determine amplitudes in the following manner.

0031-9007/97/79(22)/4333(4)$10.00

TABLE I. Summary of predicted contributions to selected $\Delta S = 0$ decays of B mesons. Rates are quoted in branching ratio units of 10^{-6}. Rates in italics are assumed inputs.

Decay	Amplitudes	Denom. factor	$\lvert t\rvert^2$ rate	$\lvert p\rvert^2$ rate	$\lvert s\rvert^2$ rate (a)[a]	(b)[b]
$B^+ \to \pi^+\pi^0$	$t + c$	$-\sqrt{2}$	*4.1*	0	0	0
$\to K^+\overline{K}^0$	p	1	0	0.8	0	0
$\to \pi^+\eta$	$t + c + 2p + s$	$-\sqrt{3}$	2.8	1.0	0.06	0.24
$\to \pi^+\eta'$	$t + c + 2p + 4s$	$\sqrt{6}$	1.4	0.5	0.4	1.9
$B^0 \to \pi^+\pi^-$	$t + p$	-1	*8.3*	0.8	0	0
$\to \pi^0\pi^0$	$p - c$	$\sqrt{2}$	0	0.4	0	0
$\to K^0\overline{K}^0$	p	1	0	0.8	0	0
$\to \pi^0\eta$	$2p + s$	$-\sqrt{6}$	0	0.5	0.03	0.12
$\to \pi^0\eta'$	$p + 2s$	$\sqrt{3}$	0	0.26	0.2	0.9

[a] Constructive interference between p' and s' amplitudes assumed in $B^+ \to K^+\eta'$.
[b] No interference between p' and s' amplitudes assumed in $B^+ \to K^+\eta'$.

(A) The magnitude of the p' amplitude is estimated by averaging the observed branching ratios [18]

$$\mathcal{B}(B^0 \to K^+\pi^-) = (15^{+5+1}_{-4-1} \pm 1) \times 10^{-6} \qquad (1)$$

and

$$\mathcal{B}(B^+ \to K^0\pi^+) = (23^{+11+2}_{-10-2} \pm 2) \times 10^{-6} \qquad (2)$$

to obtain the estimate $\lvert p'\rvert^2 = 16.3 \pm 4.3$, where all squares of amplitudes in the Tables are quoted in branching ratio units of 10^{-6}. In $B^0 \to K^+\pi^-$ we have neglected the small t' contribution, an assumption which will be seen to be justified. If the rates for Eqs. (1) and (2) are found to be unequal, the neglect of the t' amplitude (or of some other contribution) may not be valid. In that case, the possibility of a *CP* asymmetry in, say, $B^0 \to K^+\pi^-$ may be significantly enhanced.

(B) The magnitude of the p amplitude is estimated to be $\lvert p\rvert = \lvert V_{td}/V_{ts}\rvert\,\lvert p'\rvert$, where V_{td} and V_{ts} are elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. With an uncertainty of about a factor of 2, $\lvert p\rvert^2 \simeq \lvert p'\rvert^2/20 \simeq 0.8$.

(C) The $\lvert p'\rvert^2$ contribution to the decay $B^+ \to K^+\pi^0$ is estimated to be about 8, whereas [19]

$$\mathcal{B}(B^+ \to K^+\pi^0) + \mathcal{B}(B^+ \to \pi^+\pi^0) = (16^{+6+3}_{-5-2} \pm 1)10^{-6}. \qquad (3)$$

Thus there is room for a significant $B^+ \to \pi^+\pi^0$ signal. While 90% C.L. upper limits of $\mathcal{B}(B^+ \to \pi^+\pi^0) < 20 \times 10^{-6}$ and $\mathcal{B}(B^0 \to \pi^+\pi^-) < 15 \times 10^{-6}$ are quoted in Ref. [1], Ref. [19] also quotes a 2.8σ signal of

$$\mathcal{B}(B^+ \to \pi^+\pi^0) = (9^{+6}_{-5}) \times 10^{-6} \qquad (4)$$

and a 2.2σ signal of

$$\mathcal{B}(B^0 \to \pi^+\pi^-) = (7 \pm 4) \times 10^{-6}. \qquad (5)$$

Taking (4) as an estimate of $\lvert t\rvert^2/2 = 9 \pm 5.5$ (neglecting the color-suppressed amplitude c in $B^+ \to \pi^+\pi^0$) and (5) as an estimate of $\lvert t\rvert^2 = 7 \pm 4$ (neglecting the penguin amplitude p in $B^0 \to \pi^+\pi^-$), we find $\lvert t\rvert^2 = 8.3 \pm 3.8$.

(D) The value of $\lvert t'\rvert = \lvert V_{us}/V_{ud}\rvert\,\lvert t\rvert$ is estimated without accounting for SU(3) breaking to lead to $\lvert t'\rvert^2 \simeq \lvert t\rvert^2/20 \simeq 0.4$. It could be slightly higher if one applied a correction [14] of a factor of $\lvert f_K/f_\pi\rvert^2$.

(E) The $\lvert p'\rvert^2$ contribution to the $B \to K\eta'$ branching ratio (in units of 10^{-6}) is $(3/2)\lvert p'\rvert^2 = 24 \pm 6$; it cannot

TABLE II. Summary of predicted contributions to selected $\lvert\Delta S\rvert = 1$ decays of B mesons. Rates are quoted in branching ratio units of 10^{-6}. Rates in italics are assumed inputs.

Decay	Amplitudes	Denom. factor	$\lvert t'\rvert^2$ rate	$\lvert p'\rvert^2$ rate	$\lvert s'\rvert^2$ rate (a)[a]	(b)[b]
$B^+ \to K^0\pi^+$	p'	1	0	*16*	0	0
$\to K^+\pi^0$	$t' + c' + p'$	$-\sqrt{2}$	0.20	8	0	0
$\to K^+\eta$	$t' + c' + s'$	$-\sqrt{3}$	0.13	$\simeq 0$	1.2	4.9
$\to K^+\eta'$	$t' + c' + 3p' + 4s'$	$\sqrt{6}$	0.07	24	9	39
$B^0 \to K^+\pi^-$	$t' + p'$	-1	0.4	*16*	0	0
$\to K^0\pi^0$	$p' - c'$	$\sqrt{2}$	0	8	0	0
$\to K^0\eta$	$c' + s'$	$-\sqrt{3}$	0	$\simeq 0$	1.2	4.9
$\to K^0\eta'$	$c' + 3p' + 4s'$	$\sqrt{6}$	0	24	9	39

[a] Constructive interference between p' and s' amplitudes assumed in $B^+ \to K^+\eta'$.
[b] No interference between p' and s' amplitudes assumed in $B^+ \to K^+\eta'$.

account for the observed value of 63^{+20}_{-18} (our average for charged and neutral modes, where $|t'|^2$ and $|c'|^2$ contributions are assumed to be negligible). Assuming constructive interference between p' and s' in $B \to K\eta'$, we find the $|s'|^2$ contribution to the rate to be about $(8/3)\,|s'|^2 = 9$, with an additional contribution of 30 from the s'–p' interference term. (The enhancement of the $B \to K\eta'$ rate by a modest s' amplitude interfering constructively with p' was noted by Lipkin, last part of Ref. [17].) If the interference term is absent (i.e., if the relative phase of the amplitudes is $\pi/2$) then one needs an $|s'|^2$ contribution of $(8/3)\,|s'|^2 = 39$ to the rate. Henceforth, we shall work only with central values of amplitudes for illustrative purposes; the uncertainty in $|s'|$ due to the uncertainty in its phase relative to $|p'|$ generally exceeds that due to experimental error. (If one allows the $B \to K\eta'$ branching ratio to be at its -1σ value, s' can even be considerably smaller, with $(8/3)\,|s'|^2 \simeq 4$, when s' and p' interfere constructively in this decay.)

(F) Since we expect $|s/s'| = |V_{td}/V_{ts}|$ if both s and s' are dominated by the top quark, we choose $|s|^2 = |s'|^2/20$. (If in fact $|s/s'| = |V_{cd}/V_{cs}|$ as a result of charmed quark dominance of this type of penguin contribution, the result is the same.)

The results in the Tables may be interpreted in the following manner.

(i) Any contribution of order 10 or greater (corresponding to a branching ratio of 10^{-5}) has been observed.

(ii) A contribution greater than or equal to 1 should be observable in the next generation of CLEO experiments, with improved sensitivity and particle identification. Thus the decays $B^0 \to \pi^+\pi^-$, $B^0 \to K^0\pi^0$, $B^+ \to K^+\eta$, $B^+ \to \pi^+\eta$, and $B^+ \to \pi^+\eta'$ should all make their appearance, while $B^+ \to K^+\pi^0$ and $B^+ \to \pi^+\pi^0$ should be resolved from one another. For example, one expects $\mathcal{B}(B^+ \to \pi^+\eta) \simeq 4 \times 10^{-6}$ and $\mathcal{B}(B^+ \to \pi^+\eta') = (2 \text{ to } 4) \times 10^{-6}$, where the current upper bounds [1] are 8×10^{-6} and 45×10^{-6}, respectively. The first limit is already quite close to our prediction. The above branching ratios are about a factor of 2 larger than those predicted in Ref. [2].

(iii) The amplitudes for $B^0 \to K^0\pi^0$, $B^0 \to K^0\eta$, and $B^0 \to K^0\eta'$ satisfy

$$3\sqrt{2}\,A(B^0 \to K^0\pi^0) - 4\sqrt{3}\,A(B^0 \to K^0\eta) = \sqrt{6}\,A(B^0 \to K^0\eta')\,. \qquad (6)$$

Aside from small c' contributions, the terms in this triangle relation are dominated by p' and s' contributions. Since p' and s' are expected to have the same weak phase, the shape of the triangle will tell us about the relative strong phase of these amplitudes. Neglecting t' contributions as well, one can write

$$3A(B^+ \to K^0\pi^+) - 4\sqrt{3}\,A(B^+ \to K^+\eta) = \sqrt{6}\,A(B^+ \to K^+\eta')\,, \qquad (7)$$

which is easier to measure. The main uncertainty lies in the value of the branching ratio for $B^+ \to K^+\eta$. If s' involves strong rescattering from charm-anticharm states [2,3,8,9], its strong phase could differ from that of p' (and, hence, also possibly t').

(iv) Processes with two contributions, both of which exceed one, are prime candidates for observable direct CP violation if both strong and weak phases of the two amplitudes differ from one another. The weak phases of the t (tree) and p (penguin) contributions in $B^+ \to \pi^+\eta$ are expected to be γ and $-\beta$ [20], respectively, while the relative strong phases are unknown. In the case of $B^+ \to \pi^+\eta'$, if its s contribution is dominated by the charmed quark penguin, a significant strong phase shift could arise from the real $c\bar{c}$ intermediate state. One could thus have a large strong phase shift difference between the s and t amplitudes in $B^+ \to \pi^+\eta'$. The weak phases of these two amplitudes are also different: The charm penguin is approximately real, while the t amplitude has a weak phase γ.

(v) Our focus has been on the observability of direct CP violation in decays such as $B^+ \to \pi^+\eta$ and $B^+ \to \pi^+\eta'$. These processes may not be the first to exhibit CP asymmetries; asymmetric B factories will search for mixing-induced asymmetries, in which the time dependence of the decays must be studied. A time-dependent asymmetry measurement in $B^0 \to K_S\eta'$ would provide a clean determination of the weak phase β. Our result, $|p/t| = 0.3$, implies a rather large "penguin pollution" in the analysis of the time-dependent decay asymmetry in $B^0 \to \pi^+\pi^-$, compatible with previous estimates [14,21]. In order to resolve such effects using isospin symmetry [22], one would have to measure $B^0 \to \pi^0\pi^0$, for which the $|p|^2$ contribution to the branching ratio is only 0.4×10^{-6}. (The contribution of the color-suppressed amplitude c is highly uncertain but unlikely to be much larger.) An alternative way to resolve the penguin pollution question in $B^0 \to \pi^+\pi^-$ is to rely on flavor SU(3) to link this decay with various $B \to K\pi$ modes [21,23], all of which have large rates.

In summary, we have used existing data on $B \to K\eta'$ and other two-body modes involving pairs of light pseudoscalar mesons to anticipate observable CP-violating effects in the decays $B^+ \to \pi^+\eta$ and $B^+ \to \pi^+\eta'$. Since experimental errors are still quite large, the same procedure, based only on flavor SU(3), can and should be applied to better data when they become available. In that case, one will be able to test for effects of interference among various amplitudes which have been ignored here (applying, for example, amplitude relations noted in Refs. [4–6]). One welcome improvement in data will be a better estimate of $|p/p'|^2$ and $|s/s'|^2$, which, under the assumption of top quark dominance, we have taken to be $|V_{td}/V_{ts}|^2 \simeq 1/20$, with an uncertainty of a factor of 2.

A rule of thumb for observable CP-violating effects is that one must at least be able to observe the *square* of the lesser of two interfering amplitudes at the $n\sigma$ level in order to observe an asymmetry at this level [24]. Our results indicate that this sensitivity threshold is passed for decays of the form $B^+ \to \pi^+ \eta$ and $B^+ \to \pi^+ \eta'$ when branching ratios of order 10^{-6} become detectable in experiments sensitive to both charged and neutral final-state particles.

We are grateful to J. Alexander, D. Atwood, K. Berkelman, P. Drell, H. Fritzsch, and L. Sehgal for helpful discussions. This work was performed in part at the Aspen Center for Physics, and supported in part by the United States Department of Energy under Grant No. DE FG02 90ER40560 and by the United States–Israel Binational Science Foundation under Research Grant Agreement 94-00253/2.

---

*Permanent address: Physics Department, Technion-Israel Institute of Technology, 32000 Haifa, Israel.

[1] CLEO Collaboration, Cornell University Report No. CLEO CONF 97-22, presented at the Lepton-Photon Symposium, Hamburg, 1997; see also J. Smith, Aspen Winter Physics Conference on Particle Physics, Aspen, 1997 (to be published); B. Behrens and J. Alexander, The Second International Conference on *B* Physics and *CP* Violation, Honolulu, 1997 (to be published); F. Würthwein, Les Rencontres du Moriond: QCD and High Energy Hadronic Interactions, Les Arcs, 1997, Report No. hep-ex/9706010. In quoting rates we do not distinguish between a process and its charge conjugate, though such a difference is precisely what we search for when looking for direct *CP* violation.

[2] S. Barshay, D. Rein, and L. M. Sehgal, Phys. Lett. B **259**, 475 (1991).

[3] M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. **43**, 242 (1979); J. M. Gérard and W. S. Hou, Phys. Rev. Lett. **62**, 855 (1989); Phys. Rev. D **43**, 2909 (1991); G. Kramer, W. F. Palmer, and H. Simma, Nucl. Phys. **B428**, 77 (1994); Z. Phys. C **66**, 429 1995).

[4] M. Gronau and J. L. Rosner, Phys. Rev. D **53**, 2516 (1996).

[5] A. S. Dighe, M. Gronau, and J. L. Rosner, Phys. Lett. B **367**, 357 (1996); **377**, 325(E) (1996).

[6] A. S. Dighe, Phys. Rev. D **54**, 2067 (1996).

[7] K. Berkelman, CLEO note CBX 96-79 and supplement (unpublished).

[8] I. Halperin and A. Zhitnitsky, Reports No. hep-ph/9704412 and No. hep-ph/9705251; F. Yuan and K.-T. Chao, Phys. Rev. D **56**, R2495 (1997); A. Ali and C. Greub, DESY 97-126, Report No. hep-ph/9707251.

[9] D. Atwood and A. Soni, Phys. Lett. B **405**, 150 (1997); Report No. hep-ph/9706512; W.-S. Hou and B. Tseng, Report No. hep-ph/9705304; H.-Y. Cheng and B. Tseng, IP-ASTP-03-97/NTU-TH-97-08, Report No. hep-ph/9707316; A. Datta, X.-G. He, and S. Pakvasa, Report No. hep-ph/9707259; A. L. Kagan and A. A. Petrov, UCHEP-27/UMHEP-443, Report No. hep-ph/9707354; H. Fritzsch, CERN-TH/97-200, Report No. hep-ph/9708348.

[10] D. Zeppenfeld, Z. Phys. C **8**, 77 (1981).

[11] M. Savage and M. Wise, Phys. Rev. D **39**, 3346 (1989); **40**, 3127(E) (1989).

[12] L. L. Chau *et al.*, Phys. Rev. D **43**, 2176 (1991).

[13] M. Gronau, O. Hernández, D. London, and J. L. Rosner, Phys. Rev. D **50**, 4529 (1994).

[14] M. Gronau, O. Hernández, D. London, and J. L. Rosner, Phys. Rev. D **52**, 6356 (1995).

[15] M. Gronau, O. Hernández, D. London, and J. L. Rosner, Phys. Rev. D **52**, 6374 (1995).

[16] R. Fleischer, Z. Phys. C **62**, 81 (1994); Phys. Lett. B **321**, 259 (1994); **332**, 419 (1994); N. G. Deshpande and X.-G. He, Phys. Lett. B **336**, 471 (1994); Phys. Rev. Lett. **74**, 26 (1995); N. G. Deshpande, X.-G. He, and J. Trampetic, Phys. Lett. B **345**, 547 (1995).

[17] The suppression of the p' contribution to the $K\eta$ mode and its enhancement for the $K\eta'$ mode is a special case of a more general mechanism noted by H. J. Lipkin [Phys. Rev. Lett. **46**, 1307 (1981); Phys. Lett. B **254**, 247 (1991), and references therein; Argonne National Laboratory Report No. ANL-HEP-CP-97-45, presented at The Second International Conference on *B* Physics and *CP* Violation, Honolulu, 1997]. [See also Ref. [12], where the relatively large value $\mathcal{B}(B^+ \to K^+ \eta') = 3.6 \times 10^{-5}$ is predicted.] Lipkin points out that the same mechanism works in the opposite direction for $K^* + (\eta, \eta')$ final states, enhancing $K^*\eta$ and suppressing $K^*\eta'$ decays.

[18] J. Alexander, Ref. [1].

[19] F. Würthwein, Ref. [1].

[20] The phase $-\beta$ [$= \mathrm{Arg}(V_{td})$] in the $b \to d$ penguin term may receive corrections from u and c penguins; see A. J. Buras and R. Fleischer, Phys. Lett. B **341**, 379 (1995).

[21] J. Silva and L. Wolfenstein, Phys. Rev. D **49**, R1151 (1994).

[22] M. Gronau and D. London, Phys. Rev. Lett. **65**, 3381 (1990).

[23] M. Gronau and J. L. Rosner, Phys. Rev. Lett. **76**, 1200 (1996); A. S. Dighe, M. Gronau, and J. L. Rosner, Phys. Rev. D **54**, 3309 (1996); A. S. Dighe and J. L. Rosner, Phys. Rev. D **54**, 4677 1996.

[24] I. Dunietz and J. L. Rosner, Phys. Rev. D **34**, 1404 (1986).