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The Octonionic Membrane

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ABSTRACT

We generalize the supermembrane solution of $D = 11$ supergravity by permitting the 4-form G to be either self-dual or anti-self-dual in the eight dimensions transverse to the membrane. After analyzing the supergravity field equations directly, and also discussing necessary conditions for unbroken supersymmetry, we focus on two specific, related solutions. The self-dual solution is not asymptotically flat. The anti-self-dual solution is asymptotically flat, has finite mass per unit area and saturates the same mass=charge Bogomolnyi bound as the usual supermembrane. Nevertheless, neither solution preserves any supersymmetry. Both solutions involve the octonionic structure constants but, perhaps surprisingly, they are unrelated to the octonionic instanton 2-form F, for which $TrF \wedge F$ is neither self-dual nor anti-self-dual.

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1 Introduction

The eleven-dimensional supermembrane [1] is one of the cornerstones of M-theory [2, 3, 4]. The usual supermembrane solution of $D = 11$ supergravity [5] has symmetry $P_3 \times SO(8)$ and preserves half of the spacetime supersymmetry. The only non-vanishing components of the 4-form G_{MNPQ} $(M, N, \ldots = 0, 1, \ldots, 10)$ are the G_{012m} $(m, n, \ldots = 3, 4, \ldots, 10)$. In this paper we introduce generalizations of this solution which are obtained by permitting the 4-form G_{mnpq} to be non-vanishing and either self-dual or anti-self-dual in the eight dimensions transverse to the membrane.

With G_{mnpq} non-zero, the $SO(8)$ transverse symmetry of the usual supermembrane solution is necessarily broken to some subgroup (since no $SO(8)$-invariant fourth-rank antisymmetric tensors exist). There are many possible choices for G_{mnpq}, each with their own symmetry properties. Here we investigate in detail two particularly natural choices involving a certain constant $SO(7)$-invariant tensor which is related to the algebra of octonions. As a result, both solutions have symmetry $P_3 \times SO(7)$. The self-dual solution is not asymptotically flat. The anti-self-dual solution is asymptotically flat, has finite mass per unit area and saturates the same mass=charge Bogomolnyi bound as the usual supermembrane. Nevertheless, neither solution preserves any supersymmetry and neither is free of curvature singularities. Although both solutions implicitly involve the octonionic structure constants, perhaps surprisingly, neither is related to the octonionic $SO(7)$ instanton 2-form F [7, 8, 9], for which $TrF \wedge F$ is neither self-dual nor anti-self-dual.

2 The eleven-dimensional supermembrane

The bosonic sector of eleven-dimensional supergravity is described by the action

$$I_{11} = \frac{1}{2\kappa^2} \int d^{11}x \sqrt{-g} \left(R - \frac{1}{2 \cdot 4!} G_{MNPQ} G^{MNPQ} \right) - \frac{1}{12\kappa^2} \int C \wedge G \wedge G, \qquad (2.1)$$

where g_{MN} $(M,N = 0,1,\ldots,10)$ is the metric, C_3 is a three-form gauge field with four-form field-strength $G_4 = dC_3$, and $*$ denotes Hodge duality. The equations of motion are

$$R_{MN} - \frac{1}{2}g_{MN}R = \frac{1}{12}\left(G_{MPQR}G_N{}^{PQR} - \frac{1}{8}g_{MN}G_{PQRS}G^{PQRS}\right), \tag{2.2}$$

and

$$d * G = -\frac{1}{2}G \wedge G. \tag{2.3}$$

It was shown in [5] that the supergravity action I_{11} admits a fundamental membrane solution preserving half the spacetime supersymmetries. The solution for a single membrane invariant under $P_3 \times SO(8)$, where P_3 is the $d = 3$ Poincaré group, is given by

$$ds^2 = e^{2C/3}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-C/3}\delta_{mn}dy^m dy^n, \tag{2.4}$$

and

$$C_{012} = \mp e^C, \tag{2.5}$$

with

$$e^{-C} = 1 + K/y^6, \tag{2.6}$$

where $\mu,\nu = 0,1,2$ are indices in the "membrane" directions, $m,n = 1,2,\ldots,8$ are indices in the eight-dimensional space transverse to the membrane, y is the radial coordinate in this transverse space, and K is a constant.

In order that the above membrane provide a solution at $y = 0$, it was necessary to introduce an explicit source term. The source is a supermembrane sigma-model action

$$S_3 = T_3 \int d^3\xi \left(-\frac{1}{2}\sqrt{-\gamma}\gamma^{ij}\partial_i X^M \partial_j X^N g_{MN} + \frac{1}{2}\sqrt{-\gamma} \mp \frac{1}{3!}\epsilon^{ijk}\partial_i X^M \partial_j X^N \partial_k X^P C_{MNP}\right) \tag{2.7}$$

where T_3 is the membrane tension, ξ^i are the coordinates on the membrane worldvolume and γ_{ij} is the worldvolume metric. The net effect of the sigma-model source term is to add a delta-function to the right-hand side of the pure supergravity equations of motion and for the ansatz given above (and with $X^\mu = \xi^\mu, X^m = constant$) they then reduce to the single equation

$$\delta^{mn}\partial_m\partial_n e^{-C} = -2\kappa^2 T_3 \delta^8(y). \tag{2.8}$$

This fixes the constant $K = \kappa^2 T_3/3\Omega_7$, where Ω_7 is the volume of the unit seven-sphere. Although this takes care of the delta function at $y = 0$, the solution can be analytically continued to the region $r = 0$ where r is a Schwarzschild-like coordinate defined by $r^6 = y^6 + K$ [10, 11] and the solution still exhibits a curvature singularity at $r = 0$. This singularity contrasts with the fivebrane soliton solution [12] which is everywhere nonsingular. The mass per unit area of the membrane $\mathcal{M}_3$ is equal to its tension:

$$\mathcal{M}_3 = T_3. \tag{2.9}$$

This elementary solution of the supergravity equations coupled to a supermembrane source carries a Noether "electric" charge

$$Q = \frac{1}{\sqrt{2}\kappa}\int_{S^7}(*G + \frac{1}{2}C \wedge G) = \sqrt{2}\kappa T_3. \tag{2.10}$$

Hence the solution saturates the Bogomol'nyi bound

$$\sqrt{2}\kappa\mathcal{M}_3 \geq Q. \tag{2.11}$$

This follows from the preservation of half the supersymmetries, although the converse is not true [13] (as we shall rediscover in section 6). It is also intimately linked with the worldvolume kappa symmetry of the fundamental supermembrane [1].

3 Membrane solutions with (anti-)self-dual 4-forms

We now wish to generalize the membrane solution of section 2 by allowing for a non-vanishing G_{mnpq}. We make the same ansatz (2.4) for the metric and (2.5) for the components C_{012} but we allow the function C and the transverse components C_{mnp} to be as yet unspecified functions of the transverse coordinates y^m. (The "mixed" components of C_{MNP} are zero.) The $012mnpqr$ components of (2.3) are then equivalent to

$$\partial_m[e^C(G^{mnpq} \mp \frac{1}{4!}\epsilon^{mnpqrstu}G_{rstu})] = 0, \tag{3.1}$$

while the remaining components give

$$\delta^{mn}\partial_m\partial_n e^{-C} = \mp\frac{1}{2.4!}G_{mnpq} * G_{mnpq}. \tag{3.2}$$

Transverse indices are now understood to be raised and lowered with the flat metric δ_{mn} and $*$ denotes the Hodge dual in eight-dimensional Euclidean space. Turning to the Einstein equations (2.2), we find that the $\mu\nu$ components are satisfied if

$$\delta^{mn}\partial_m\partial_n e^{-C} = -\frac{1}{2.4!}G_{mnpq}G^{mnpq} \tag{3.3}$$

while the only additional content of the mn components is the vanishing of the eight-dimensional stress tensor:

$$G_{mpqr}G_n{}^{pqr} - \frac{1}{8}\delta_{mn}G_{pqrs}G^{pqrs} = 0. \tag{3.4}$$

We now observe that equations (3.1) and (3.4) are satisfied automatically when the 4-form G is self-dual or anti-self-dual in the eight dimensions transverse to the membrane. Furthermore, the remaining equations (3.2) and (3.3) then coincide. So we conclude that the complete set of supergravity field equations is satisfied for any 3-form C_{mnp} whose field strength obeys

$$G_{mnpq} = \pm\frac{1}{4!}\epsilon_{mnpqrstu}G_{rstu}, \tag{3.5}$$

provided the function C, which determines the remaining fields through (2.4) and (2.5), is a solution of the single equation (3.3). Concrete examples of such solutions are given in section 6.

4 Supersymmetry

The fact that a bosonic field configuration is annihilated by one or more supersymmetries is usually assumed to imply that it must also satisfy the equations of motion, although the converse is not true. Having discussed the structure of the equations of motion in the last section, we will now examine the complementary issue of supersymmetry. Once again, we consider a general field configuration given by (2.4) and (2.5) with C and C_{mnp} functions only of the coordinates y^m.

The full $D = 11$ supergravity theory involves the bosonic action I_{11} in (2.1) coupled to a gravitino field ψ_M. Under a supersymmetry transformation with local parameter ζ, the variation of the gravitino is

$$\delta\psi_M = \nabla_M\zeta - \frac{1}{288}G_{PQRS}(\Gamma_M{}^{PQRS} - 8\delta_M^P\Gamma^{QRS})\zeta. \tag{4.1}$$

In the standard fashion, it suffices to consider bosonic field configurations which are supersymmetric in the sense that the above variation vanishes for one or more choice of ζ.

Corresponding to the metric ansatz (2.4), we can decompose the supersymmetry parameter in the form $\zeta = \epsilon \otimes \xi$ and the $D = 11$ curved space gamma-matrices can be written

$$\Gamma_\mu = e^{C/3}(\gamma_\mu \otimes \gamma_9), \qquad \Gamma_m = e^{-C/6}(1 \otimes \gamma_m), \tag{4.2}$$

where γ_μ and γ_m are gamma-matrices for flat $D = 3$ Minkowski space and $D = 8$ Euclidean space respectively, and γ_9 is the eight-dimensional chirality operator. Using

this decomposition in conjunction with (2.4) and (2.5) we find

$$\delta\psi_\mu = \partial_\mu\zeta - \frac{1}{6}e^{C/2}\partial_m C(\gamma_\mu \otimes \gamma_m\gamma_9)\zeta \\ \pm\frac{1}{6}e^{C/2}\partial_m C(\gamma_\mu \otimes \gamma_m)\zeta - \frac{1}{288}e^{C}G_{mnpq}(\gamma_\mu \otimes \gamma_{mnpq}\gamma_9)\zeta \qquad (4.3)$$

and

$$\delta\psi_m = \partial_m\zeta - \frac{1}{12}\partial_n C(1 \otimes \gamma_{mn})\zeta \\ \mp\frac{1}{6}\partial_m C(1 \otimes \gamma_9)\zeta \pm \frac{1}{12}\partial_n C(1 \otimes \gamma_{mn}\gamma_9)\zeta \\ +\frac{1}{24}e^{C/2}G_{mnpq}(1 \otimes \gamma_{npq})\zeta - \frac{1}{288}e^{C/2}G_{pqrs}(1 \otimes \gamma_m\gamma_{pqrs})\zeta \qquad (4.4)$$

In each of these equations, the term in the first line on the right-hand side comes from the spin-connection, while the remaining terms come from the decomposition of the antisymmetric tensor fields.

In order to arrange that all components of the gravitino have zero variation, we cancel terms with the same gamma-matrix structure by setting

$$\zeta = e^{C/6}(\epsilon \otimes \xi^\pm), \qquad \gamma_9\xi^\pm = \pm\xi^\pm, \qquad (4.5)$$

with ϵ and $\xi^\pm$ constant. The condition for a supersymmetric field configuration then reduces to a single equation in the transverse space:

$$G_{mnpq}\gamma_{npq}\xi^\pm = 0. \qquad (4.6)$$

(Related equations made their appearance in the physically different context of finding Calabi-Yau fourfold compactifications of $D = 11$ supergravity down to $D = 3$ [22].)

We emphasize that we have assumed nothing so far about C_{mnp} or its field strength G_{mnpq}. Note that if $G_{mnpq} = 0$ we recover the fundamental membrane solution of section 2, and (4.6) then gives no additional restriction on $\xi^\pm$, implying that half of the spacetime supersymmetries of $D = 11$ supergravity are preserved. In the case with

G_{mnpq} non-zero, we would like to understand how the condition for supersymmetry (assuming (4.5)) fits together with our analysis of the equations of motion; or in other words, whether (4.6) can be satisfied when G_{mnpq} is self-dual or anti-self-dual.

It is natural to expect that there are no spinors $\xi^{\pm}$ which satisfy (4.6) when $*G = \mp G$. This is because the sign specifying the chirality of the spinor in the condition for unbroken supersymmetry is correlated with the sign appearing in the ansatz (2.5). But from section 3 we know that the sign in (2.5) is in turn correlated with the sign in (3.5) if we are to have a solution of the field equations. A spinor $\xi^{\pm}$ which satisfied (4.6) with $*G = \mp G$ would therefore imply the existence of a supersymmetric field configuration which did not satisfy the equations of motion, which runs counter to conventional wisdom. Indeed, it can be shown directly from (4.6) that no such configurations are possible.

To establish this, we first note that for any commuting spinors $\xi^{\pm}$ with $\gamma_9\xi^{\pm} = \pm\xi^{\pm}$, the tensors

$$X^{\pm}_{mnpq} = \xi^{\pm T}\gamma_{mnpq}\xi^{\pm} \tag{4.7}$$

are self-dual and anti-self-dual:

$$X^{\pm}_{mnpq} = \pm\frac{1}{4!}\epsilon^{mnpqrstu}X^{\pm}_{rstu}. \tag{4.8}$$

Now on squaring (4.6) and performing some gamma-matrix algebra we find

$$2(\xi^{\pm})^2 G_{mnpq}G_{mnpq} = 3G_{mnpq}G_{mnrs}X^{\pm}_{pqrs}. \tag{4.9}$$

But it is also easy to show that $*G = \mp G$ and $*X^{\pm} = \pm X^{\pm}$ together imply that the expression on the right-hand side must vanish, and hence that $G_{mnpq} = 0$.

In conclusion, we have shown that for a non-zero field-strength G which is self-dual or anti-self-dual, the only unbroken supersymmetries allowed by (4.6) are given by spinors with positive or negative chirality respectively. Whether there actually are

any such unbroken supersymmetries of this type is then a question which depends on the detailed structure of G.

5 Octonions and related tensors

We now turn from the general considerations of the previous sections to discuss some specific solutions. Although no configuration with non-zero G can be $SO(8)$-invariant, we can find solutions with $SO(7)$ symmetry by making use of a certain tensor c_{mnpq} which is related to the algebra of octonions. We will begin by introducing this tensor from a slightly different point of view, however.

Choosing any commuting, positive-chirality spinor η and normalizing it so that $\eta^T\eta = 1$, we define

$$c_{mnpq} = \eta^T \gamma_{mnpq} \eta. \tag{5.1}$$

As we mentioned in the last section, this tensor is self-dual, obeying

$$c_{mnpq} = \frac{1}{4!}\epsilon^{mnpqrstu} c_{rstu}. \tag{5.2}$$

Some other useful identities are

$$c_{mnpr}c^{mnqs} = 12\delta_{[pr]}^{\ \ [qs]} - 4c_{pr}^{\ \ qs}, \quad c_{mnpr}c^{mnps} = 42\delta_r^{\ s}, \quad c_{mnpq}c^{mnpq} = 336, \tag{5.3}$$

which can also be deduced using standard properties of gamma-matrices.

It is immediate from its definition that the tensor c_{mnpq} is invariant under a maximal subgroup $SO(7) \subset SO(8)$ with respect to which the positive-chirality spinors decompose as $8 \to 7 \oplus 1$, with η belonging to the singlet. The eight-dimensional vector and negative-chirality spinor representations remain irreducible under this subgroup. Any two choices of the fixed spinor η are equivalent, in that they correspond to conjugate $SO(7)$ embeddings in $SO(8)$.

To explain the relationship to octonions, we introduce the totally antisymmetric octonionic structure constants c_{abc} by means of the multiplication rule

$$o_a o_b = -\delta_{ab} + c_{abc} o_c, \tag{5.4}$$

where o_a are unit imaginary octonions with $a, b, \ldots = 1, 2, \ldots, 7$. With suitable choices of bases, the connection between c_{mnpq} and c_{abc} is simply

$$c_{abc8} = c_{abc}, \qquad c_{abcd} = \frac{1}{3!}\epsilon_{abcdefg} c_{efg}. \tag{5.5}$$

The description in terms of octonions is attractive from many points of view. It has one disadvantage, however, in that it makes manifest only a G_2 subgroup (the octonion automorphism group) of the full $SO(7)$ symmetry of c_{mnpq}.

6 An octonionic membrane

Using the tensor c_{mnpq} introduced in the last section, we can construct both self-dual and anti-self-dual solutions of the supergravity field equations which are invariant under $P_3 \times SO(7)$.

To find a self-dual G we write

$$C_{mnp} = \frac{1}{a} c_{mnpq} y^q, \tag{6.1}$$

where a is a constant, and hence

$$G_{mnpq} = \frac{1}{a} c_{mnpq} \tag{6.2}$$

is manifestly self-dual. To find an anti-self-dual G we write

$$C_{mnp} = \frac{a^7}{y^8} c_{mnpq} y^q \tag{6.3}$$

and hence

$$G_{mnpq} = -\frac{a^7}{y^{10}} (y^2 c_{mnpq} + 8 y_{[m} c_{npq]r} y^r), \tag{6.4}$$

whose anti-self-duality follows from the self-duality of c_{mnpq}. (In general, c_{mnpq} self-dual and h_{mn} symmetric and traceless implies $c_{r[mnp}h_{q]r}$ is anti-self-dual.)

In the self-dual case, substituting into (3.3) we find

$$\frac{1}{y^7}\frac{\partial}{\partial y}(y^7\frac{\partial}{\partial y}e^{-C}) = -\frac{7}{a^2} \tag{6.5}$$

and hence

$$e^{-C} = 1 + \frac{K}{y^6} - \frac{7y^2}{16a^2}, \tag{6.6}$$

which yields a metric which is not asymptotically flat. In the anti-self-dual case, we find

$$\frac{1}{y^7}\frac{\partial}{\partial y}(y^7\frac{\partial}{\partial y}e^{-C}) = -\frac{7a^{14}}{y^{16}} \tag{6.7}$$

and hence

$$e^{-C} = 1 + \frac{K}{y^6} - \frac{a^{14}}{16y^{14}}, \tag{6.8}$$

which yields a metric which is asymptotically flat. Note the minus signs in both (6.6) and (6.8) which imply that the metric (2.4) is singular at the zeros of e^{-C}. In each case there is exactly one such zero at $y = y_0 > 0$. These zeros actually give rise to curvature singularities.

It is not difficult to read off the mass and charge of the new asymptotically flat, anti-self-dual solution and we find the same results as for the usual supermembrane, in spite of the non-vanishing of G_{mnpq}, essentially because the coefficient of the y^{-6} term in (6.8) remains unchanged. The bound (2.11) is therefore still saturated by this new solution.

From section 4, we know that the only possible unbroken supersymmetries for $*G = \pm G$ involve spinors $\xi^{\pm}$. For the specific G_{mnpq} appearing in both (6.6) and (6.8), we find that none of the supersymmetry survives. This may seem surprising in view of the fact that the anti-self-dual solution saturates the same Bogomolnyi bound between the mass and charge (2.11) as the usual supermembrane, however the

relation between supersymmetry and the Bogomolnyi bound is a subtle one [13] and can break down in the presence of singularities.

By replacing c_{mnpq} with other constant tensors it is clear that we can obtain a variety of similar solutions with invariance groups such as $SU(4)\times U(1)$ or $SO(4)\times SO(4)$ [7]. We do not expect any improvement in the singularity structure of these solutions, however. Furthermore, the $SO(7)$-invariant choice c_{mnpq} is particularly natural in conjunction with the isotropic metric ansatz (2.4), since this is the unique maximal subgroup under which the vector representation of $SO(8)$ remains irreducible. For any other subgroup, the condition that the metric should depend only on the radial transverse coordinate would be much less compelling. Allowing more general transverse behaviour also opens up many other possibilities of course, and we note in particular that G_{mnpq} is anti-self-dual whenever

$$C_{mnp} = c_{mnpk}\partial_k f(y^i), \qquad \delta^{mn}\partial_m\partial_n f = 0. \tag{6.9}$$

Our octonionic anti-self-dual solution utilizes the maximally symmetric harmonic function. Once again, we would not expect any less symmetric solution to have a more desirable singularity structure.

We should also mention that having obtained a new membrane solution in $D = 11$, it follows by simultaneous dimensional reduction that we can obtain a new type IIA string solution in $D = 10$ [6, 5].

7 Not the octonionic instanton

The octonionic structure constants have appeared before in many different physical contexts [14, 7, 15, 8, 9, 16, 18, 17, 19, 20, 21, 25]. In particular, octonionic string soliton [18] and octonionic membrane soliton [20, 21] solutions of the heterotic string have been found which make use of the $SO(7)$ octonionic instanton [7, 8, 9] in eight

dimensions and the G_2 octonionic instanton [20, 21] in seven dimensions respectively. In the Horava-Witten [23] approach to deriving the $D = 10$ $E_8 \times E_8$ heterotic string by compactifying M-theory on S^1/Z_2, the equation

$$G \sim TrF^2 - \frac{1}{2}trR^2 \tag{7.1}$$

appears on one of the boundaries, where F is the Yang-Mills field strength of one of the E_8's. One might therefore have expected that our solution would be related to the $SO(7)$-invariant octonionic instanton where the tensor c_{mnpq} also makes its appearance in the equation

$$F_{mn} = \frac{1}{2}c_{mnpq}F^{pq}. \tag{7.2}$$

Indeed, for instanton size ρ one finds [8, 9]

$$TrF_{[mn}F_{pq]} = \frac{3\rho^2 + y^2}{(\rho^2 + y^2)^3}c_{mnpq} + \frac{4(4\rho^2 + y^2)}{(\rho^2 + y^2)^4}y_{[m}c_{npq]r}y^r \tag{7.3}$$

which has a similar tensor structure as (6.4). However, it is not difficult to verify that this expression is neither self-dual nor anti-self-dual and does not provide a solution for G_{mnpq}. In this respect we differ from the authors of [25] who claim that $TrF \wedge F$ is self-dual.

Another way in which our supermembrane solution differs from the octonionic string and membrane solutions of [18, 20, 21] is that the string has infinite mass per unit length and the membrane has infinite mass per unit area, whereas our anti-self-dual solution has the same finite mass per unit area as the usual supermembrane [5].

8 Conclusions

Although the octonionic instanton 4-form TrF^2 is neither self-dual nor anti-self-dual, we have not entirely given up on the possibility that Yang-Mills instantons may

provide a solution to the $D = 11$ supergravity equations. There are M-theoretic corrections to the $D = 11$ supergravity Lagrangian arising from a sigma-model Lorentz anomaly on the worldvolume of the fivebrane [27]. The 3-form field equation gets modified to

$$d * G = -\frac{1}{2} G \wedge G + (2\pi)^4 \beta X_8, \tag{8.1}$$

where β is related to the fivebrane tension T_6 by $T_6 = 1/(2\pi)^3\beta$ and where

$$X_8 = \frac{1}{(2\pi)^4}[-\frac{1}{768}(trR^2)^2 + \frac{1}{192}trR^4]. \tag{8.2}$$

Indeed, our original motivation for generalizing the supermembrane solution was to search for solutions involving eight-dimensional Yang-Mills instantons in the dimensions transverse to the membrane whose finite instanton size would smear out the singularity at $r = 0$ of the usual supermembrane solution [5]. We also expected that the $(R^2)^2$ and R^4 M-theoretic corrections (8.1), together with corresponding corrections to the Einstein equations demanded by supersymmetry [28], would also play a role. One's first inclination might be to look for solutions of this kind which preserve the $SO(8)$ symmetry of the usual supermembrane solution. Indeed, $SO(8)$ Yang-Mills instantons do exist [24] for which F^2 is in fact self-dual. However TrF^2 necessarily vanishes since there are no $SO(8)$-invariant antisymmetric tensors of rank 4. TrF^4 is non-zero, however, and indeed these instantons played a role in smoothing out [26] the singularity of the $SO(32)$ heterotic string soliton solution [29] by incorporating the one-loop TrF^4 corrections to the Lagrangian.

We intend to return elsewhere to this original goal of finding non-singular supersymmetric solutions involving the M-theoretic corrections. In this paper, however, we were diverted into finding singular, non-supersymmetric solutions for which the X_8 corrections vanish, but which may nevertheless prove to be of interest in their own right.

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