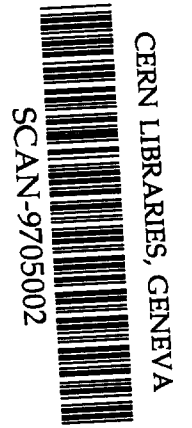


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The Short-Range Wakefields in the SBLC Linac

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Abstract.

The short-range longitudinal and transverse wakefields of a point charge in the SBLC linac are obtained using a modal summation technique. Simple functional fits to these wakes are given, which can be used as Green functions in beam dynamics simulations of bunches. These results, however, are valid only after the beam has traversed a critical number of cells N_{crit} . Using time domain computations with Gaussian bunches we have obtained results that are consistent with N_{crit} varying as $\alpha a^2 / (L\sigma_z)$, with a the iris radius and L the period length of the structure, σ_z the bunch length and α a constant on the order of 1. For the loss per cell to reach to within a few percent of the asymptotic value $\alpha \sim 0.5 - 1.0$.

Introduction

In the S-Band Linear Collider (SBLC) project [1] long trains of short, intense bunches of electrons and positrons are each accelerated through 16 km of linac to 250 GeV before colliding at the interaction point. Good luminosity requires that both the longitudinal and transverse emittance growth in the linacs be kept small. Important among the effects that can cause emittance growth are the interactions of the bunches with the short range and the long range wakefields of the linac structure. The long range wakefields, which can cause the multi-bunch projected emittance to grow, will be suppressed in the SBLC by having the iris tips coated with lossy material [2]. The short range wakefields, however, can still increase the single bunch energy spread and single bunch projected emittance of the beam.

The SBLC linac structure is a cylindrically symmetric, disk-loaded, constant gradient structure consisting of 180 cells, whose geometry varies gradually from beginning to end of the structure. The SBLC longitudinal bunch distribution is Gaussian with an rms length σ_z that is very short compared to the iris radius a : $\sigma_z = 300 \mu\text{m}$, a is typically 1.2 cm and $\sigma_z/a \sim .025$. Therefore, the beam samples primarily the high frequency impedance of the structure. It is known that for a single cavity with infinitely long beam tubes the longitudinal high frequency impedance varies with frequency as $\omega^{-1/2}$, implying that the longitudinal wakefield of a very short Gaussian bunch varies as $\sigma_z^{-1/2}$ [3]. For an infinitely periodic structure the longitudinal high frequency impedance (real part) varies as $\omega^{-3/2}$, implying that the wakefield per cell of a very short bunch approaches a constant value as $\sigma_z \rightarrow 0$ [4, 5, 6]. For a structure consisting of a finite number of repeating cells with infinitely long beam tubes the wake of the first cell is given by the single cell wakefield and the wake per cell asymptotically approaches that of the periodic case as the beam moves toward the end of the structure. For a Gaussian bunch the number of cells needed for the periodic solution to be valid, N_{crit} , is given by [4, 5]

$$N_{crit} = \frac{\alpha a^2}{L\sigma_z} \quad , \quad (1)$$

with L the structure period length and α a constant of proportionality on the order of one.

In the first part of this report we use a frequency domain approach to obtain the short-range longitudinal (monopole) and transverse (dipole) wakefields generated by an ultra-relativistic point charge in the SBLC linac structure. These functions can then, in turn, be used as Green functions for studying the single bunch, wakefield induced effects in the SBLC linac. They are the asymptotic wakefields, in that, for a given bunch length, they are only valid after the beam has traversed N_{crit} cells of structure. Note that it is difficult to obtain the wakefields appropriate for the short SBLC bunches through either a frequency or a time domain approach, since they must be accurate to very high frequencies. The wakefields in the SBLC linac have already been obtained using a time domain approach, for Gaussian bunches down to the nominal bunch length $\sigma_z = 300 \mu\text{m}$ [7] using a dedicated window technique [8]. Nevertheless, we believe that it is still useful to perform the calculations of the present report, since for beam dynamics studies the wakefields must be known to a fraction, say at least 1/4, of the nominal bunch length σ_z . Note that similar calculations have been performed before for the SLAC linac structure [9], which has a similar geometry to the SBLC linac, and for the NLC linac structure, [10] which is an X-band structure. In the second part of this report we use the MAFIA time domain module T2 [11] to test the validity of Eq. 1, and to find α , in order that we can estimate the number of cells that are needed for our asymptotic solutions to be valid.

The Calculation Method

In the SBLC linac cavity the iris radii and the cavity radii change gradually as one moves from the beginning to the end of the structure. With the change being so gradual—0.2% change in iris radius per cell—we believe we can locally approximate the structure by a periodic one. To obtain the wakefields for an entire SBLC cavity we first find the wakes for 5 purely periodic models, with cell dimensions that approximate 5 representative cells along the SBLC cavity. (See Fig. 1. The shape of a real SBLC cell is the same, except that the iris and one upper corner are rounded.) The wakes of the 5 models are then averaged to give us the wakefields representing the entire SBLC structure. These functions can, in turn, be used as Green functions to find the induced voltage or transverse kick over a distribution of particles. However, since the wake functions are derived using periodic models the results are valid only if the bunch length is not too short, such that the transient effects at the beginning of the structure are small and can be ignored, a subject to be discussed in a later section of this report.

To obtain the longitudinal wakefield of each of the 5 periodic models we proceed as follows [9]: We use the computer program KN7C [12] to obtain the synchronous frequencies and loss factors of the first few hundred monopole modes, which give us the low-frequency impedance. We approximate the high frequency contribution using the so-called Sessler-Vainsteyn optical resonator model [12, 13], a model that has been shown to work well, for example, for the SLC [9] and the NLC [10] structures. The real part of the impedance (assuming t/L is small) becomes [10]

$$R_z(\omega) = \sum_{n=1}^N \pi k_n \delta(\omega - \omega_n) + \frac{2Z_0 j_{01}^2}{\pi L \zeta^2} \frac{\sqrt{\nu} + 1}{(\nu + 2\sqrt{\nu} + 2)^2} \Theta(\omega - \omega_N) \quad \omega > 0 \quad , \quad (2)$$

with k_n the loss factor (in units of V/pC/m) and ω_n the frequency of the n^{th} mode, $Z_0 \cong 377 \Omega$, $j_{01} \cong 2.41$ the first zero of the Bessel function J_0 , $\zeta \cong 0.824$, $\nu = 4a^2\omega/(c\bar{L}\zeta^2)$, with c the speed of light and $\bar{L} = \sqrt{L(L-t)}$; $\Theta(x) = 0$ for $x < 0$, 1 for $x > 0$. Fourier transforming $R_z(\omega)$ we obtain the longitudinal wakefield:

$$W_z(s) = \sum_{n=1}^N 2k_n \cos \frac{\omega_n s}{c} + \frac{Z_0 c j_{01}^2 \bar{L}}{\pi^2 a^2 L} \int_{\nu_N}^{\infty} \frac{\sqrt{\nu} + 1}{(\nu + 2\sqrt{\nu} + 2)^2} \cos\left(\frac{\zeta^2 \bar{L} s \nu}{4a^2}\right) d\nu \quad , \quad (3)$$

with $\nu_N = 4a^2\omega_N/(c\bar{L}\zeta^2)$.

The transverse (dipole) wakefield is obtained in the analogous manner, but using the computer program TRANSVRS [14] to obtain the dipole mode frequencies and loss factors. We first obtain the longitudinal *dipole* wakefield $W_z^{(1)}(s)$ using an equation very similar to Eq. 3, except using the transverse

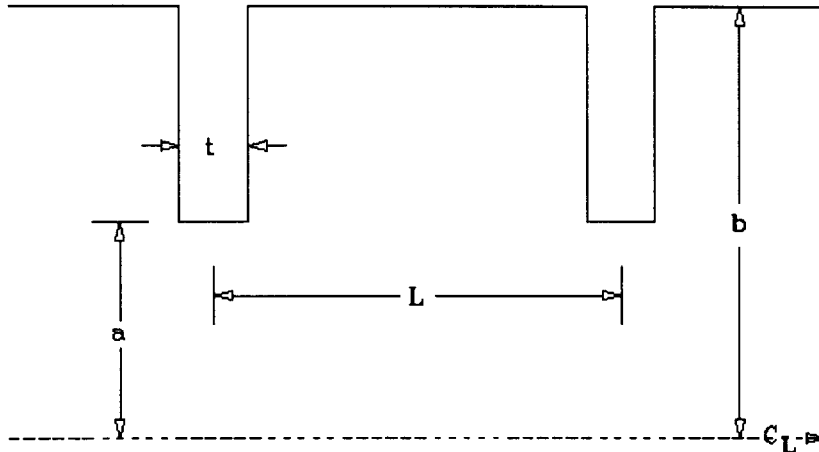


FIGURE 1: The model of one cell of linac structure that is used in the calculations. For the average cell in the SBLC structure (cell 90) we take $a = 1.35$ cm, $b = 4.00$ cm, $t = .59$ cm, and $L = 3.50$ cm.

mode frequencies and loss factors, and replacing j_{01} by $j_{11} \cong 3.83$, the first zero of the first order Bessel function J_1 . Then, following the Panofsky-Wenzel theorem [15] we obtain the transverse dipole wakefield: $W_x(s) = \int_0^s W_z^{(1)}(s') ds'$.

The Wakefields

The SBLC linac operates at a frequency of 3.0 GHz and at a phase advance of $2\pi/3$ per cell; therefore, the cell length is $L = 3.33$ cm. The irises, which are rounded, have a thickness $t = 5.3$ mm. The minimum iris radius varies linearly from $a = 1.60$ cm near the beginning of the structure to $a = 1.10$ cm near the end; the outer cavity radius b varies from 4.06 cm to 3.93 cm, from the beginning to the end of the structure. For the average cell $a = 1.35$ cm and $b = 4.00$ cm. (Note that the dimensions are similar to those of the SLAC linac structure, an 84 cell structure that is also constant gradient, but which operates at 2.856 GHz, and for which the average cell dimensions are $a = 1.16$ cm, $b = 4.13$ cm, $L = 3.50$ cm, $t = 5.9$ mm.) For our 5 representative, periodic models we have taken as cell dimensions those of cells 1, 45, 90, 135, and 180 of the SBLC structure, with a set to the minimum corresponding iris radius, *i.e.* set to, respectively, 1.1 cm, 1.225 cm, 1.35 cm, 1.475 cm, and 1.6 cm.

In the longitudinal case we have found, for each of the 5 representative structures, ω_n and k_n for all modes up to 75 GHz (about 250 modes) using the computer program KN7C. Comparing, at the higher frequencies, the binned modal contribution to the Sessler-Vainsteyn part of Eq. 2 we find good agree-

ment, to within 10%. Then substituting into Eq. 3 we obtain the wakefields shown in Fig. 2 (the solid curves). The values at the origin, which should equal [5] $W_z(0) = Z_0 c / (\pi a^2)$ ($= 198$ V/pC/m for cell 90 dimensions), are 4–5% low, indicating some calculation error. Note that if we compare the modal sum contribution to the wake, say, of cell 90 to the total of modal sum plus analytic extension we find that at the $s = 0$ the modal sum is 64% of the wakefield, at $s = 150$ μm 84%, and at $s = 300$ μm 93%. We see that, had we wanted to obtain $W_z(0)$ to 95% accuracy from the modal sum alone, we would have needed to find the modes over $(0.36/0.05)^2 = 52$ times the frequency range that we did, since at high frequencies $R_z \sim \omega^{3/2}$; this is something that we could not accurately have done. The average of the 5 representative wakes (with the wakes of cell 1 and 180 weighted by half) is given by the dashed curve in Fig. 2. Finally, a fit to the average wake, given by

$$W_z = 200.(\text{V/pC/m}) \cdot \exp \left[-0.77(s/\text{mm})^{\frac{1}{2}} \right] , \quad (4)$$

is shown by the dots in Fig. 2.

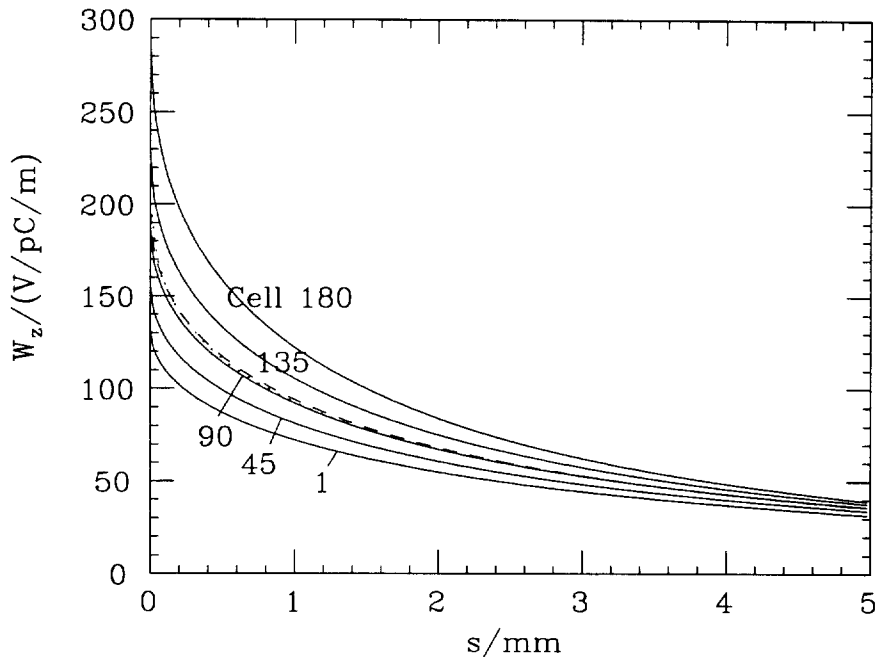


FIGURE 2: The longitudinal wakefield of representative cells in the SBLC structure (solid curves). The dashed curve represents the average, and the dotted curve the model fit, Eq. 4.

To obtain the transverse wakefield of the 5 representative, periodic models we have calculated modes up to 68 GHz (about 350 modes), using the computer program TRANSVRS. Comparing, at the higher frequencies, the binned modal

contribution to the Sessler-Vainsteyn part of the impedance we again find good agreement. The transverse wakefields for the 5 geometries are shown in Fig. 3 (the solid curves). In this case the slope at the origin should equal [10] $W'_x(0) = 2Z_0c/(\pi a^4)$ ($= 2.17 \text{ V/pC/mm}^2/\text{m}$ for cell 90); our numerical results agree to within 2% in all cases. Note that for cell 90 dimensions the modal sum is 67% of the wakefield at $s = 150 \mu\text{m}$ and 74% at $s = 300 \mu\text{m}$. The average wake is given by the dashed curve in Fig. 3. A fit to the average wake, given by

$$W_x = 4.10(\text{V/pC/mm/m}) \cdot \left[1 - \left(1 + 1.15[s/\text{mm}]^{\frac{1}{2}} \right) \exp \left(-1.15[s/\text{mm}]^{\frac{1}{2}} \right) \right], \quad (5)$$

is indicated by the dots.

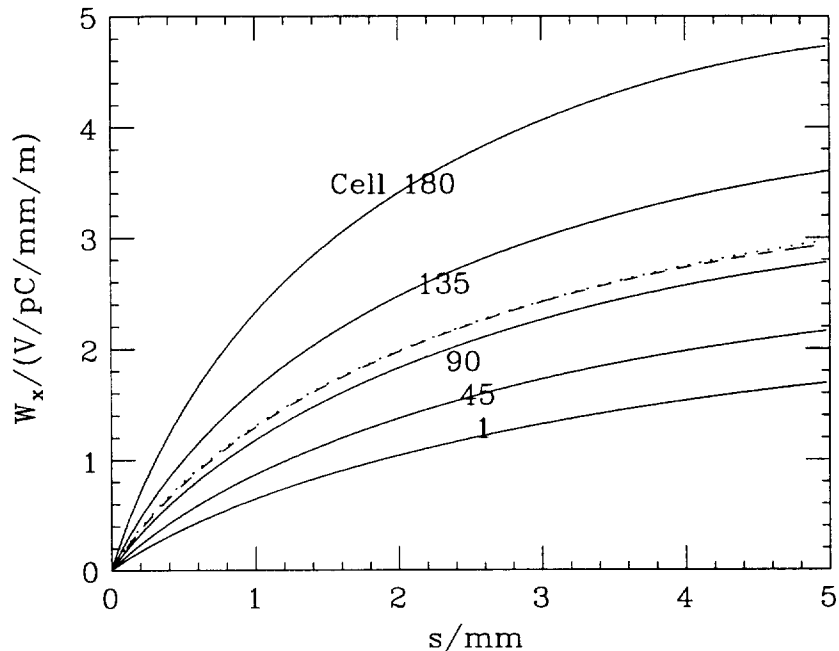


FIGURE 3: The transverse (dipole) wakefield of representative cells in the SBLC structure (solid curves). The dashed curve gives the average, the dotted curve gives the model fit of Eq. 5.

Determining N_{crit}

Consider first the wakefields obtained for the periodic models in the previous section (the solid curves in Figs. 2 and 3). These are asymptotic wakefields; for a finite structure they do not apply until the beam has traversed a sufficient number of cells N_{crit} . To test the validity of Eq. 1 for N_{crit} and to find

α , at least for the longitudinal case, we have performed a series of MAFIA time domain calculations to obtain the wakefield of short Gaussian bunches of various lengths in structures consisting of a finite number of cells with infinitely long beam tubes. Although the SBLC structure is a constant gradient structure, for simplicity, we use only identical cells, with the dimensions of cell 144 in the real structure, for which $a = 1.20$ cm. The bunch lengths vary from $\sigma_z = 0.1$ mm, 0.3 mm, 0.5 mm, and then in 0.25 mm steps up to 4 mm; the number of cells vary from $N_{cell} = 1$ to 10. For the short bunch lengths of interest it is difficult to obtain accurate results since the number of time steps and mesh points that need to be traversed becomes very large. For example, for $\sigma_z = 0.1$ mm and $N_{cell} = 10$ we used a mesh size of $\sigma_z/13$ and a total number of 65×10^6 mesh points for good accuracy.

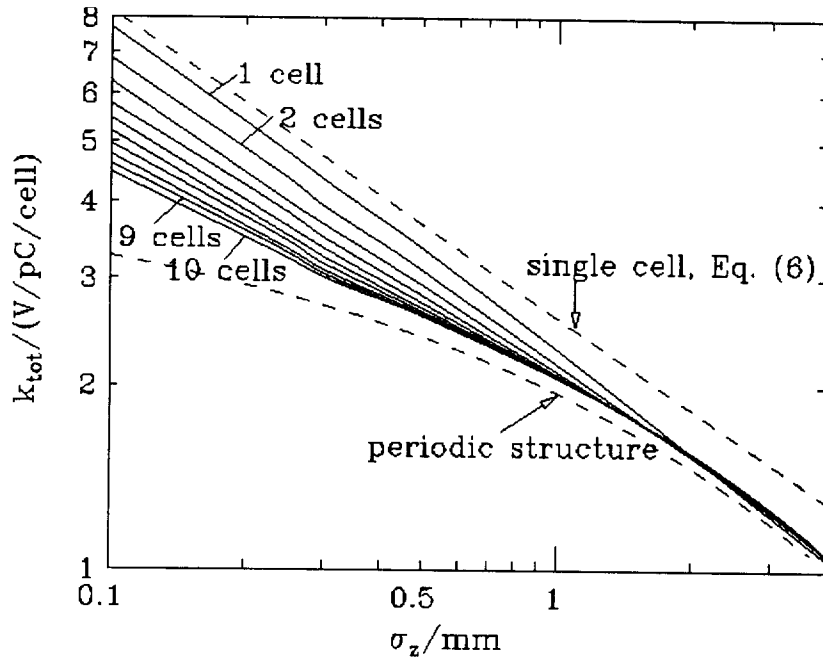


FIGURE 4: The loss factor obtained by MAFIA for a Gaussian bunch in a structure consisting of a finite number of cells connected to infinite beam tubes, as function of bunch length. Curves are given for structures having from 1 to 10 cells. Also shown are the loss factor according to the diffraction model, Eq. 6, and that for the periodic structure (the dashes).

Fig. 4 shows the results. Plotted are the voltage loss per charge per cell, k_{tot} , as function of bunch length, for various length structures. Also shown by the dashes, for comparison, are the loss for a very short Gaussian bunch in a

single cell cavity, given by the Lawson diffraction model [3]:

$$k_{tot} = \frac{\Gamma(1/4)Z_0c}{4\pi^{5/2}a} \sqrt{\frac{g}{\sigma_z}} \quad , \quad (6)$$

with $\Gamma(1/4) \cong 3.63$ and g the cavity gap (*i.e.* $L - t$), and for that using the wake of a periodic structure (calculated by the modal method) in

$$k_{tot} = \frac{L}{2\sqrt{\pi}\sigma_z} \int_0^\infty ds W_z(s) e^{-\frac{s^2}{4\sigma_z^2}} \quad . \quad (7)$$

From Fig. 4 we see that the MAFIA results are bounded by the two asymptotic curves. For bunch lengths larger than about 2 mm the loss per cell is independent of the number of cells. For shorter bunches we note that the single cell results approach the single cell asymptotes, as the bunch length decreases. Also, for a given bunch length, as the number of cells increases, the loss per cell asymptotically approaches the periodic structure curve, however, with a slight [8% at $\sigma_z = 0.5$ mm, 6% at $\sigma_z = 1.0$ mm] systematic offset. The agreement is quite good. We believe that the discrepancy is partly due to the slight ($\sim 5\%$) error in the modal calculation (discussed above), and, for shorter bunch lengths, partly due to the fact that the MAFIA results slightly overestimate the asymptotic solution since they represent the average of all cells in the finite structure (see below). Note that for bunch lengths $\sigma_z < 0.3$ mm, 10 cells do not seem to be sufficient for the loss to reach the asymptotic value. Also note that for a finite number of cells, as the bunch length becomes ever shorter the loss again varies as $\sigma_z^{-1/2}$.

To determine N_{crit} we first calculate the differential loss factor for each cell from the MAFIA data, κ_n , defined as the total loss factor for an N_{cell} cell structure minus that of an $N_{cell} - 1$ cell structure, which we then convert into a function of cell number through cubic spline interpolation (see Fig. 5). In Fig. 5 we note that for $\sigma_z = 0.1$ mm the curve decreases throughout the range; for the longer bunch lengths it is rather constant throughout the range. For the intermediate bunch lengths, however, κ_n begins by decreasing gradually, and then levels off for high cell number. Upon careful inspection one notices that for these curves there is also a slight ($\sim 3\%$) dip before the curve reaches the final, asymptotic value. Note that, as mentioned before, for the shorter bunch lengths (0.1 – 0.5 mm) and long structures, due to the great number of mesh points and time steps needed, it is difficult to obtain accurate values of k_{tot} ; and it is even more difficult to obtain accurate values of κ_n . For example, if we want κ_n for cell 10 to be accurate to 2% we need the k_{tot} calculations of the 9- and the 10-cell structures to be accurate to .2%. In Fig. 6 numerical errors probably account for the slight anomalies that we see in the high end of the curves for short bunches; nevertheless, the general trend of these curves is still correct.

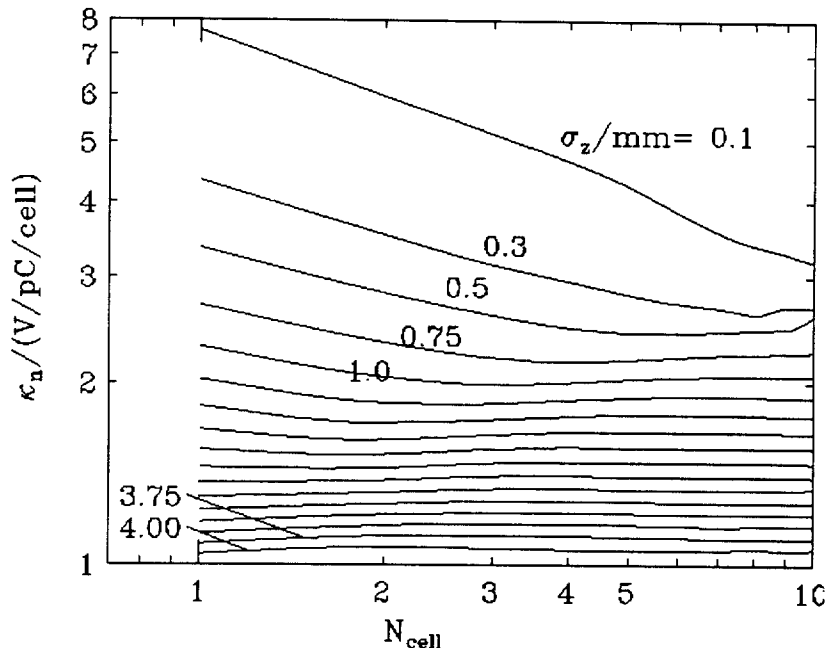


FIGURE 5: The differential loss factor of cell N_{cell} , κ_n , for various bunch lengths, as obtained by MAFIA (the points at integer values of N_{cell}), made into continuous functions by splining.

From Fig. 5 we see that κ_n as function of N_{cell} approaches the asymptote only gradually; therefore, what we take as N_{crit} , *i.e.* the number of cells needed for reaching the asymptote, will depend strongly on the criterion that we use. Let us consider 3 different criteria that result in a κ_n that is within a few percent of the asymptotic value: (1) the point in the curve, before the dip, where κ_n reaches the same value as the asymptote, (2) the dip position, and (3) the position (after the dip) where the curve reaches to 98% of its asymptotic value. The $\sigma_z = 0.1$ and 0.3 curves, as well as those for which $\sigma_z \geq 2.0$ mm, are not included; as asymptote we take the value of κ_n at $N_{cell} = 8$. The results are shown in Fig. 6, with case 1 given by the x's, case 2 by the diamonds, and case 3 by the +'s. Fitting the data to a power law (N_{crit} as a function of σ_z) we obtain as exponent -1.06 in the first case and -0.85 in the others. However, given the accuracy of the calculation, we can say that the results are consistent with the -1 power dependence of Eq. 1. As for the coefficient α , when fitting to Eq. 1 we obtain 0.5 , 0.7 , and 1.0 for cases 1, 2, and 3, respectively. In summary, to obtain an average loss at the end of a finite length, repeating structure that is within a few percent of the average loss in the truly periodic structure the critical number of cells needed appear to be given by Eq. 1, with $\alpha \sim 0.5 - 1.0$.

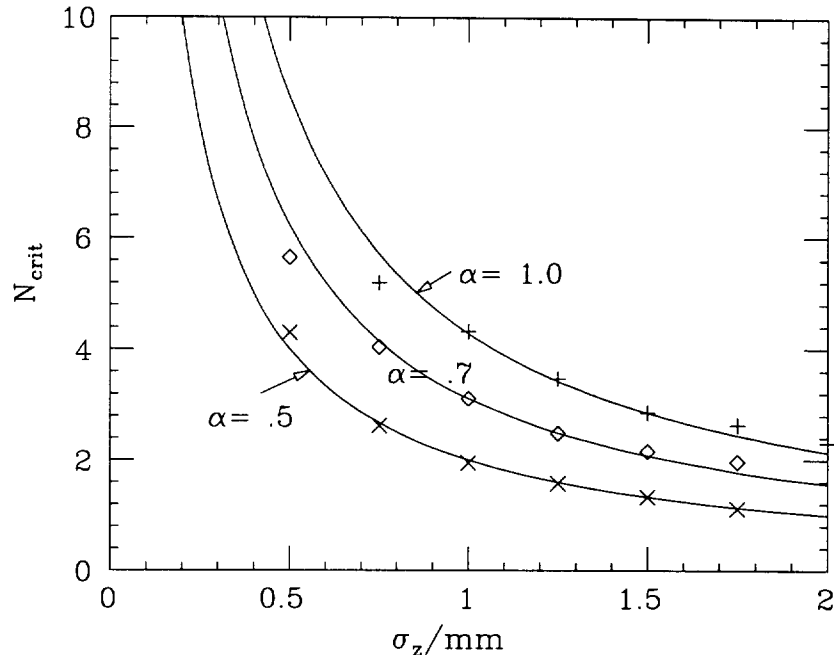


FIGURE 6: The critical number of cells as function of bunch length (the plotting symbols). N_{crit} is obtained from Fig. 5 following 3 different criteria: (1) the point in the curves, before the dip, where κ_n reaches the same value as the asymptotic value (x's), (2) the dip position (diamonds), and (3) the position (after the dip) where the curves reach to 98% of their asymptotic value (+'s). Also shown are the fits to Eq. 1, taking, respectively, $\alpha = 0.5, 0.7, 1.0$ (the curves).

Comparisons

Let us first consider a Gaussian bunch with $\sigma_z = 500 \mu\text{m}$. For our 10-cell example structure (remember $a = 1.2 \text{ cm}$) this bunch needs to traverse $N_{crit} \sim 4 - 8$ cells to reach to within a few percent of the asymptotic loss factor. Therefore, the initial transient effect should be small. According to the modal calculation the asymptotic value is 2.4 V/pC/cell , which is 9% less than the average loss factor, and 1.5% less than the differential loss factor, as obtained by MAFIA. In Ref. [7] Drevlak obtains the wakefields of Gaussian bunches in 15-cell models of the SBLC structure (in this case the irises are rounded) by direct time domain integration, also using MAFIA. For a constant impedance structure with the dimensions of cell 90 ($a = 1.35 \text{ cm}$) he finds that $k_{tot} = 2.23 \text{ V/pC/cell}$; the result using the modal wake of the cell 90 model is 12% less. For a constant gradient model of the SBLC linac cavity Drevlak obtains $k_{tot} = 2.30 \text{ V/pC/cell}$; the modal result, using the wake of Eq. 4, gives 13% less.

Consider now the nominal SBLC bunch length $\sigma_z = 300 \mu\text{m}$. In this case (since $a \approx 1.6 \text{ cm}$ at the beginning of the structure) we estimate that $N_{crit} \approx 13 - 26$ cells, which is still small compared to the total number of cells in the SBLC linac cavity. For his constant gradient model Drevlak gets $k_{tot} = 2.63 \text{ V/pC/cell}$, and the modal result is 15% less. Finally, for the dipole case, Drevlak obtains an average dipole kick of $0.0135 \text{ V/pC/mm/cell}$, and the modal result, using the wake of Eq. 5, is 32% less.

Although there appears to be reasonable agreement between our results and those obtained by MAFIA for our 10-cell, $a = 1.2 \text{ cm}$, model, the agreement is not so good when comparing with Drevlak's results, particularly in the transverse case. It is not clear where this disagreement comes from. It is probably not all explained by the initial transients. Other possible sources of error in our modal wakes: (i) As mentioned before, we expect the wakes for our 5 periodic models to each contain errors, but none much larger than about 5%. (ii) The effect of using flat instead of rounded irises in our model will introduce an error; however, since the minimum iris radius was kept the same this will give a larger (not smaller) wakefield, by a few percent. (iii) We approximate a constant gradient structure locally by a periodic structure; however, with the very gradual variation in iris radius of the real structure (only 0.2% variation per cell) we expect this effect also to be small. More study is needed.

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