ON THE S-MATRIX FOR THE FIELD THEORY WITH LAGRANGIANS DEPENDING ON DERIVATIVES

M.A. Braun and Yu.A. Evlashev

Leningrad State University

Teoreticheskaya i matematicheskaya fizika 6, 318-27 (1971)

Translated at CERN by B. Hodge
(Original: Russian)
Not revised by the Translation Service

(CERN Trans. Int. 71-6)

Geneva
June 1971

An examination is made of the quantum scalar field theory with the interaction depending on derivatives. A covariant expression is obtained for the S-matrix in the form of the interaction Lagrangian. It is shown that the theory is invariant in respect of local transformation of the field variables. The changes in the results after regularisation are investigated.

1. INTRODUCTION

The expression for the S-matrix of scattering in the quantum scalar field theory, with the density of the interaction Lagrangian L_{int} depending on derivatives, was, as we know, first investigated in the classical papers by Umezawa and Takahashi (1). They showed that if the density of the self-action Lagrangian of one scalar neutral field is $L_{int}(\varphi, \varphi_\mu)$ where $\varphi_\mu = \partial\varphi/\partial x_\mu$, the S-matrix of scattering can be determined by the formula

$$S = T^* \left\{ \exp i \int L_{int}(x) d^4x \right\}. \tag{1}$$

Here, T* is the chronological derivative after discarding the non-covariant additions occurring as a result of temporal regularisation of the field operator φ_μ (T* is sometimes called the "Wick" derivative to distinguish it from the "Dyson" T-derivative (2)).

In the present paper we draw attention to the fact that the expression (1) for the S-matrix of scattering is, strictly speaking, true only when L_{int} is linearly dependent on φ_μ. Even when there is a quadratic dependence of the density of the interaction Lagrangian on the derivatives, the correct determination of the scattering matrix does not coincide with (1) but is obtained by the substitution of L_{int} in (1) by a certain effective Lagrangian density L_{eff}, which differs from L_{int} by a term proportional to the space volume of the momenta:

$$L_{eff} = L_{int} - i\delta^4(0) \ln \sqrt{1 + c(\varphi)}, \tag{2}$$

where $c(\varphi)$ is the coefficient of proportionality in the dependence of L_{int} on $\varphi_\mu \varphi^\mu$. As we show, the terms added, which are, as it were, of a non-Hermitian character, serve to compensate the specific divergences in the theory with the derivatives and must definitely be taken into account. In the case of a more complex dependence of L_{int} on the derivative it is not possible to find L_{eff} ; however, calculations based on the perturbation theory reveal that the terms which are added to L_{int} are not only non-Hermitian as in (2) but also non-covariant. Let us note that for the special case of the interaction of charged vector particles with an electromagnetic field, Lee and Yang found a formula similar to (2) in which, however, a non-covariant expression appeared under the sign of a logarithm (3).

The correct determination of the S-matrix of scattering for the density of a Lagrangian depending quadratically on the derivatives has a direct relationship to the study of non-linear transformations of the field functions. As we know, first Chisholm (4) and then, more strictly, Kamefuchi, O'Raifeartaigh and Salam (5) showed that the physical substance of the theory, i.e. the canonical equation of motion, the commutation relations and the energy-momentum tensor do not change during arbitrary local transformation of the field functions. From this also follows the invariance of the scattering matrix, provided that in the transformation theory, in which the Lagrangian density is inevitably quadratically dependent on the derivatives, use is made of the correct determination of the S-matrix of scattering in terms of L_{eff} (2).

However, Kamefuchi et al. in their paper (5) started from the assumption that in the original theory the density of the Lagrangian interaction did not contain any terms which were quadratically dependent on the derivatives. In the present paper, we show that this restriction is superfluous for determining the equivalence of the theories. This result may be of interest, for example, for phenomenological non-linear Lagrangians in the theory of chiral symmetry, where the presence of derivatives in the interaction is inevitable in any parametrization.

It should be stressed that the conclusions we express in this work, are, like indeed the results obtained by the authors cited, obtained in the non-regularised theory and are of a somewhat formal nature on account of ultra-violet divergence. All theories with Lagrangians dependent on derivatives (with the exception of the linear dependence) relate to the re-normalised type. We have therefore carried out a special discussion of the changes which occur when regularisation is effected. As it happens, the additional terms in (2) disappear, and formula (1) is correct, for a density of the interaction Lagrangian $L_{int}(\varphi, \varphi_{\mu})$ which depends arbitrarily on the derivatives. However, for the regularised theory there is no simple theorem for Chisholm's equivalence, since regularisation in the transformation theory should be effected according to a strictly defined non-trivial formula which is determined by the regularisation method of the initial theory.

2. SCATTERING MATRIX FOR LAGRANGIANS DEPENDING ON DERIVATIVES

We shall start from the assumption that in any field theory the correct determination of the S-matrix of scattering derived from the formal scattering theory is its determination by the density of the Hamiltonian

$$S = T \exp\left\{-i\int H_{int}(x)\, d^4x\right\}, \tag{3}$$

where T is the usual temporal regularisation, whilst the operator of the density of the interaction Hamiltonian H_{int} is selected in the representation of the interaction. As we know, formula (3) can be obtained from an adiabatic hypothesis or, more strictly, from the weak convergence of the field operators according to Lehman-Symanzik-Zimmermann (with the required number of additional factors $Z^{-1/2}$).

Let us examine the theory of one scalar neutral field $\varphi(x)$ with the density of the interaction Lagrangian depending both on φ and on $\partial_{\mu}\varphi$. In this case, the canonically coupled momentum

$\pi(x)$ does not coincide with $\partial_0 \varphi$, $H_{int} \neq -L_{int}$ and the integral which figures in (3) is not relativistically invariant. This is, however, not the only non-invariance in (3). It is well-known that the average of the chronological derivation, for the vacuum, of the derivatives φ_μ contains a non-covariant local addition:

$$\langle T\{\partial_\mu\varphi\,.\,\partial_\nu\varphi\}\rangle_0 = \langle T^*\{\partial_\mu\varphi\,.\,\partial_\nu\varphi\}\rangle_0 - i g_{\mu 0} g_{\nu 0} \delta^4(x-y), \tag{4}$$

where $\langle T^* \rangle_0$ is a covariant expression also for the free fields

$$\langle T^*\{\partial_\mu\varphi\,.\,\partial_\nu\varphi\}\rangle_0 = i\partial_\mu\partial_\nu\Delta_c(x-y).$$

Taking the δ-form term (4) into account will cause a change in the interaction shown in the exponent of (1). The formula for the scattering matrix may therefore be rewritten in the form

$$S = T^* \exp\left\{ i \int L_{\text{эфф}}(x)\, d^4x \right\}. \tag{5}$$

Expression (5) is also a determination of the density of the effective Lagrangian L_{eff}.*) If L_{int} does not contain any derivatives then, of course, $L_{eff} = L_{int}$.

Let us find an explicit expression for L_{eff} for the case when L_{int} depends on the derivatives in a fairly simple manner - such as a quadratic polynomial:

$$L_{int} = a(\varphi) + b_\mu(\varphi)\varphi^\mu + \frac{1}{2} c(\varphi)\varphi_\mu\varphi^\mu. \tag{6}$$

Then, the canonically coupled pulse $\pi = (1+c)\dot{\varphi} + b_0$ and the density of the interaction Hamiltonian is of the form:

$$H_{int}(\pi, \varphi_s, \varphi) = -a + \frac{b_0^2}{2(1+c)} + b_s\varphi_s + \frac{1}{2} c\varphi_s^2 - \frac{b_0}{1+c}\pi - \frac{c}{2(1+c)}\pi^2. \tag{7}$$

In the representation of the interaction, π must be replaced by $\dot{\varphi}$. It will then be clearly seen that the density of the interaction

*) Translators Note : $L_{\text{эфф}}$ in the formulae means L_{eff}.

Hamiltonian is not a Lorentz-invariant expression. Moreover, if the a, b_μ and c coefficients depend on φ in a polynomial manner, then H_{int} will depend on φ in a non-polynomial manner.

Let us remove from the operator of the density of the Hamiltonian (7) the part of H'_{int} which is not dependent on the temporal derivatives

$$H = H'_{int} + H''_{int}, \tag{8}$$

where $H''_{int} = -\alpha\dot{\varphi} - \beta\dot{\varphi}^2$ and the designations $a_0 = b_0/(1+c)$ and $\beta = c/2(1+c)$ have been introduced.

To find L_{eff} we shall use the following obvious formula. For an arbitrary functional

$$T\{\Phi\} = T^*\left\{\exp\left(-\frac{i}{2}\int d^4x \frac{\delta^2}{\delta\dot{\varphi}^2}\right)\Phi\right\}. \tag{9}$$

In our case, $\Phi = \exp\left(-i\int H_{int}(x)\, d^4x\right)$ and H_{int} is given by formula (8).

Let us change over to discrete space, and transform that part of the functional Φ which depends on $\dot{\varphi}$ in the Fourier manner in respect of $\dot{\varphi}$

$$\Phi(\dot{\varphi}) = \int \prod_i \frac{d\xi_i}{\sqrt{2\pi}} e^{-i\sum_i \xi_i \dot{\varphi}_i}\, \tilde{\Phi}(\xi_i), \tag{10}$$

where, as can easily be verified

$$\tilde{\Phi}(\xi_i) = \sqrt{\frac{1}{-\det(2i\beta_{ik})}}\exp\left\{-\frac{i}{4}(\alpha+\xi)_i\,\beta_{ik}\,(\alpha+\xi)_k\right\} e^{-i\int H_{int}(x)\,d^4x}. \tag{11}$$

For a correct correspondence of the normal and variational derivative terms it was necessary to replace $\beta\dot{\varphi}^2$ by the limit of non-local interaction $\beta_{ik}\dot{\varphi}_i\dot{\varphi}_k$ when $\beta_{ik} \to \beta_i \delta^4(x_i - x_k)$. Now there is no difficulty in computing the variational derivative in (9), and after reverting to continuous space we find from (5)

$$L_{eff} = L_{int} - i\delta^4(0)\ln\sqrt{1+c(\varphi)}. \tag{12}$$

In discussing the result obtained let us note, in particular, that expression (12) obtained for the density of the effective Lagrangian is clearly relativistically invariant. In this way the two non-covariances, - one in the density of the Hamiltonian and the other in the T-derivation, fully compensate each other.

If in (12) we assume that $c \equiv 0$, i.e. that the density of the interaction Lagrangian depends in only a linear manner on φ_μ, we obtain the intuitively anticipated result $L_{eff} = L_{int}$. If, however, $c \neq 0$, then L_{eff} differs from L_{int} by a term which, at first sight, is quite strange, and even meaningless. On a closer examination, however, the sense of the additional term in L_{eff} is fairly apparent. The fact is that after expansion of the T^*-derivation in expression (5) closed rings appear from the coupling of $\partial_\mu \varphi$ with $\partial_\mu \varphi$. Any such ring diverges as the fourth power of the momentum. It can easily be checked that this divergent term is, in accuracy, of equal magnitude, and is opposite in sign to the corresponding term in the scattering matrix resulting from expansion of the logarithm.

In fact, let us first examine a ring of n couplings of $\langle T^*\{\varphi_\mu, \varphi_\nu\}\rangle_0$ with uncoupled operators $C(\varphi)$. If there is a large virtual momentum of the ring we can disregard, in comparison with it, external momenta. Then, all the internal lines have the same momentum k, and in each vertex the factor k^2 will appear, which for large values of k, cancels out one of the denominators $m^2 - k^2$. There appears, therefore, a divergence of the fourth order, of the form $\delta^4(0)(1/2n)(-1)^n$, where the factor $1/2n$ appears owing to the equivalence of all the external lines of the ring and the two φ_μ in each vertex. Let us compare this contribution with that from the first term of the expansion of the scattering matrix (5) in powers of the logarithm in (12), proportional to $\delta^4(0)$. If we expand the logarithm into a series, we obtain in the n^{th} order of the perturbation theory exactly the same expression with the opposite sign. In this way, the divergence concerned disappears. Diagrams with the additional couplings can be obtained by replacing, as above, the pair of free operators by their coupling. This replacement can be effected at the same time also in the corresponding term arising from the logarithm. It is clear that the result will differ only in sign, and consequently such divergences will also disappear. Finally, in the diagrams with two or more rings from the couplings $\langle T^*\{\varphi_\mu, \varphi_\nu\}\rangle_c$ will appear divergences proportional to $(\delta^4(0))^\ell$, where ℓ is the number of such rings which in precisely this manner will be cancelled with the contribution from the ℓ^{th} term of the expansion of the scattering matrix in powers of the logarithm.

In this way, the additional term in (12) serves to remove the divergence in the closed circles due to the singular nature of T^*.

3. EQUIVALENCE OF THE THEORY WITH REGARD TO LOCAL NON-LINEAR FIELD TRANSFORMATIONS

The result obtained enables a correct approach to be used when examining the following problem, which in recent years has become popular owing to the study of the non-linear interaction Lagrangians. Let us pose an interaction Lagrangian $L_{int}(\varphi)$ which contains no derivatives. Let us effect an arbitrary local replacement of the field function $\varphi = f(\psi)$; the total Lagrangian $L'(\psi) = L(\varphi)$ can then be broken down into a free part $L_{free}(\psi)$ and the field interaction $L'_{int}(\psi, \psi_\mu)$ which, however, will now depend on the derivatives. As can easily be found, L'_{int} will have the form of (6), while

$$a = L_{int} + \frac{1}{2} m^2 (\psi^2 - f^2),$$
$$b_\mu = 0,$$
$$c = f'^2 - 1.$$

The question is raised as to whether it is permissible to write in new variables the S-matrix in the standard form (5) with $L_{eff} = L_{int}$. A positive answer to this question is indeed given by the contents of the above-mentioned theory of Chisholm, Kamefuchi, Salam and O'Raifeartaigh. It follows from our reasoning that this is absolutely justified, provided that when the S-matrix is calculated from formula (1) the integrals containing divergences of the type $(\delta^4(0))^n$ are discarded.

The correct proof of the theory of equivalence in paper (5) was based on the assumption that the initial interaction Lagrangian does not depend on the derivatives. We shall now show that this restriction is too strong and that the theorem of equivalence is correct also in the particularly important case when the interaction is quadratically dependent on the derivatives. Proof of the equivalence of the theory consists in establishing the invariance of the commutation relations, the canonical equations of motion and the energy-momentum tensor. In contrast to the classical theory it is necessary to ensure the correct arrangement of the non-commutative values.

Let us pose the Lagrangian

$$L = \frac{1}{2} c(\varphi)\dot{\varphi}^2 + G, \tag{13}$$

where G depends on φ and on the space derivatives from φ. It is clear from physical considerations that π, and, incidentally, the operator $\dot{\varphi}$, must be Hermitian, and for this they must be expressed by the anticommutators from the commutative values. We shall therefore write the density of the interaction Lagrangian in the form

$$L = \frac{1}{4} \{c(\varphi), \dot{\varphi}^2\} + G, \tag{14}$$

where $\{\ \}$ denotes an anticommutator, and we shall follow the rules of differentiation defined in (5). To this density of the interaction Lagrangian corresponds the canonically coupled field momentum

$$\pi = \frac{1}{2}\{c(\varphi), \dot{\varphi}\}. \tag{15}$$

By inverting (15) it can easily be found that

$$\dot{\varphi} = \frac{1}{2}\left\{\frac{1}{c(\varphi)}, \pi\right\}. \tag{15a}$$

We shall now effect transformation of the field variables, so that the new variable φ_2 depends only on the initial variable φ, but not on its derivative φ_μ.

$$\varphi \rightarrow \varphi_1 = \varphi_1(\varphi), \tag{16}$$

then in the transformation theory

$$L = \frac{1}{4}\{c_1(\varphi), \dot{\varphi}_1^2\} + c_1. \tag{17}$$

We will note that for the arbitrary value $\Phi(\varphi)$ expanded into a power series

$$\dot{\Phi} = \frac{1}{2}\{\Phi', \dot{\varphi}\} \tag{18}$$

is fulfilled, in which the prime denotes the derivative for φ. From (17) it follows, taking into account (18), that $c_1\varphi_1'^2 = c$.

We shall now calculate directly the commutator of the new canonical variables π and φ and find proof that it coincides with the commutator π_1 and φ_1. It follows from the density of the Lagrangian of (17) that $\pi_1 = \frac{1}{2}\{c_1, \dot{\varphi}_1\}$, or, if (18) is taken into account, that

$$\pi_1 = \frac{1}{4}\{c_1\{\varphi_1', \dot{\varphi}\}\}.$$

Then, the commutator

$$[\pi_1, \varphi_1] = \frac{1}{4} 4c_1\varphi_1'[\dot{\varphi}, \varphi_1] = c_1\varphi_1'^2(\dot{\varphi}, \varphi). \tag{19}$$

Whence, taking into account the coupling between c and c_1 and (15a)

$$[\pi_1, \varphi_1] = [\pi, \varphi].$$

It is now necessary to check the invariance of the equations of motion. The equation of motion following from the initial Lagrangian is of the form

$$\{c, \ddot{\varphi}\} = 2G' - \dot{\varphi}c'\dot{\varphi}, \tag{20}$$

where

$$G' = \partial G / \partial\varphi - \partial_s(\partial G / \partial\varphi_s), \qquad s = 1, 2, 3.$$

After transformation of the field variables we have

$$\partial_0(\partial L/\partial\dot{\varphi}_1) = \partial_0 \frac{1}{4}\{c\{\partial\varphi/\partial\dot{\varphi}_1, \dot{\varphi}\}\},$$

which, when using the identities proved by Kamefuchi et al. (5), $\partial\dot{\varphi}/\partial\dot{\varphi}_1 = \partial\varphi/\partial\varphi_1$ and $\partial_0(\partial\varphi/\partial\varphi_1) = \partial\dot{\varphi}/d\varphi_1$ and also that the equation of motion (20), gives the desired result $\partial_0(\partial L/\partial\dot{\varphi}_1) = \partial L/\partial\varphi_1$.

The invariance of the energy-momentum tensor $T_{\mu\nu}$ need not be proved, since the method proposed in (5) for proving its invariance does not make use of the restriction $c(\varphi)$= c-number.

4. APPLICATION OF REGULARISATION

As has already been stated in the Introduction, all of our arguments have concerned the non-regularisation theory. The application of regularisation will profoundly change the results obtained and, in particular, all of the non-covariant terms will disappear from the theory. Let us apply a regularised field χ with a negative metric and mass μ. To removal of regularisation corresponds $\mu \to \infty$. The density of the total Lagrangian of the system of the interacting φ and χ fields

$$L = L_\varphi - L_\chi + L_{int}(\varphi + \chi), \tag{21}$$

where L_φ and L_χ are free Lagrangians of the fields φ and χ respectively. This method of regularisation ensures the covariance of the contraction. It is convenient to change over from the variables φ and χ to the new variables $\varphi + \chi = \psi_+$ and $\varphi - \chi = \psi_-$. The intensity of the interaction Lagrangian, which depends on the derivatives in an arbitrary manner, can then be written

$$L_{int} = L_{int}(\psi_{-,\mu};\ \psi_+),$$

and the density of the free Lagrangian will be

$$L_{free} = \frac{1}{2}\psi_{+\mu}\psi_{-\mu} - \frac{1}{8}(m^2 - \mu^2)(\psi_+^2 + \psi_-^2) - \frac{1}{4}(m^2 + \mu^2)\psi_+\psi_-.$$

The canonically coupled field pulses are of the form

$$\pi_+ = \partial L/\partial\dot{\psi}_+ = \frac{1}{2}\dot{\psi}_- + \partial L_{int}/\partial\dot{\psi}_+, \qquad (22)$$

$$\pi_- = \partial L/\partial\dot{\psi}_- = \frac{1}{2}\dot{\psi}_+.$$

The density of the total Hamiltonian of the system

$$H = \pi_+\dot{\psi}_+ + \pi_-\dot{\psi}_- - L,$$

where all the temporal derivatives $\dot{\psi}_+$ and $\dot{\psi}_-$ must be expressed by the canonically coupled momenta π_+ and π_- from system (22). By cancelling such terms, we find that

$$H = 2\pi_+\pi_- - L_{free}' - L_{int}(\psi_{+,\mu};\ \psi_+), \qquad (23)$$

where $\dot{\psi}_+$ must be substituted by $2\pi_-$, and L'_{free} denotes the part of the free Lagrangian which contains no temporal derivatives. If we subtract from (23) the density of the free Hamiltonian, we find that the density of the interaction Hamiltonian

$$H_{int} = -L_{int}(\psi_{+,\mu};\ \psi_+)\,|_{\dot{\psi}_+\to 2\pi_-}.$$

It is necessary subsequently to change over to a representation of the interaction to which the substitution $\pi_- \to \frac{1}{2}\dot{\psi}_+$ corresponds. In this way we finally find that the density of the interaction Hamiltonian in the interaction representation

$$\bar{H}_{int} = -L_{int}(\psi_{+,\mu};\ \psi_+),$$

i.e. it differs from the density of the interaction Lagrangian only in sign and contains no non-covariants.

It is remarkable that the coupling for ψ_{+} also contains no non-covariant terms

$$\langle T\{\partial_\mu\psi_+, \partial_\nu\psi_+\}\rangle_0 = \langle T\{\partial_\mu\varphi, \partial_\nu\varphi\}\rangle_0 + \langle T\{\partial_\mu\chi, \partial_\nu\chi\}\rangle_0 =$$
$$= i\partial_\mu\partial_\nu(\Delta_c(m, x) - \Delta_c(\mu, x)).$$

This being taken into account, the scattering matrix is determined by the usual formula (1) and now all of the chronological derivatives of T^* and T coincide. It must be stressed that when regularisation is removed, i.e. during the limit transition $\mu \to \infty$, there appear in the above theory, among other divergences, also the already-mentioned divergencies, which are proportional to $(\delta^4(0))^n$ and were previously completely removed by the logarithmic addition in L_{eff}. Consequently, the removal of the regularisation does not change our theory into a non-regularised one. It denotes, of course, that the limit transition depends substantially on the manner in which regularisation is effected.

In itself, this occurrence should not cause surprise since we are dealing with theories relating to the un-renormalised type when the limit $\mu \to \infty$, as a rule, does not exist. Nevertheless, there is, among our theories, a certain class in which the limit $\mu \to \infty$ is well-defined. They are the theories obtained as a result of renormalisation by a non-linear transformation of the field function. In this case, the scattering matrix determined in accordance with the rules of the present paragraph would, in the limits of $\mu \to \infty$, lead to irremovable divergences proportional to $(\delta^4(0))^n$, which certainly do not exist in the original theory. This immediately places some doubt over the correctness of the theorem of Chisholm et al. in the regularised theory. The fact is, of course, that in a local transformation of the field functions in the regularised theory it is necessary simultaneously to transform also the regularising field. Consequently, in the transformation theory, the dependence on the regularised field cannot be chosen arbitrarily and, in particular, in such a simple manner as in (21) but is fairly complex and does not correspond to a usual simple regularisation.

Let us, for example, apply to the original theory a regularisation in accordance with (21). Let us transform the fields ψ_+ and ψ_- in an arbitrary local manner

$$\psi_+ = \psi_+(\eta_+), \qquad \psi_- = \psi_-(\eta_+, \eta_-), \tag{24}$$

then, as we can easily check, it is not possible to make the density of the interaction Lagrangian depend only on η_+ also in the transformation theory. Generally speaking, there appears a dependence both on η_- and on $\eta_{-\mu}$. The dependence on $\eta_{-\mu}$ can be excepted, but the dependence on η_- definitely remains. In fact, in the terms of the new field variables η_+ and η_-

$$L'_{int} = \frac{1}{2}\frac{\partial\psi_-}{\partial\eta_+}\eta_{+\mu}\frac{\partial\psi_-}{\partial\eta_+}\eta_{+\mu} + \frac{1}{2}\left(\frac{\partial\psi_+}{\partial\eta_+}\frac{\partial\psi_-}{\partial\eta_-} - 1\right)\eta_{+\mu}\eta_{-\mu} - \tag{25}$$
$$-\frac{1}{8}(m^2-\mu^2)[\psi_+^2(\eta_+) + \psi_-^2(\eta_+,\eta_-) - \eta_+^2 - \eta_-^2] -$$
$$-\frac{1}{4}(m^2+\mu^2)[\psi_+(\eta_+)\psi_-(\eta_+,\eta_-) - \eta_+\eta_-] + L_{int}\left(\frac{\partial\psi_+}{\partial\eta_+}\eta_{+\mu};\ \psi_+(\eta_+)\right).$$

The dependence on $\eta_{-\mu}$ disappears if it is required that

$$\frac{\partial\psi_+}{\partial\eta_+}\frac{\partial\psi_-}{\partial\eta_-} = 1, \qquad \text{т. е.}\ \ \psi_-(\eta_+,\eta_-) = \frac{\partial\eta_+}{\partial\psi_+}\eta_- + F(\eta_+).$$

Then, however, a term occurs from the first component of (25) which depends linearly on η_-, is proportional to $\partial^2\eta_+/\partial\psi_+^2$ and therefore disapears only in the trivial case when η_+ depends linearly on ψ_+.

The authors wish to express their sincere thanks to Yu.V. Novozhilov and V.A. Franke for their useful discussions. The authors are particularly grateful to L.V. Prokhorov for the constructive discussion of the terms proportional to $\delta^4(0)$ in the method of continuous integration, and to A.A. Grib for his valuable comments.

References

(1) H. Umezawa, Y. Takahashi. Progr. Theor. Phys. 9, 14, 501, 1953.

(2) A.D. Sukhanov. Zh. Eksper. Teor. Fiz. 41, 1915, 1961;
A.D. Sukhanov. Zh. Eksper. Teor. Fiz. 47, 1303, 1964;
B.V. Medvedev. Zh. Eksper. Teor. Fiz. 49, 1518, 1965.

(3) T.D. Lee, C.N. Yang. Phys. Rev. 129, 885, 1962

(4) J.S.R. Chisholm. Nucl. Phys. 26, 469, 1961.

(5) S. Kamefuchi, L. O'Raifeartaigh, A. Salam. Nucl. Phys. 28, 529, 1961.