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THE PROJECTION SCHEME FOR HANDLING LARGE-NUMBER CANCELLATION RELATED TO GAUGE INVARIANCE [1]

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Abstract

A scheme, the so-called "projection", for handling singularities in the processes such as that in the process $e^+e^- \to t\bar{b}e^-\bar{\nu}$ (or $e^+e^- \to u\bar{d}e^-\bar{\nu}$), is proposed. In the scheme, with the help of the gauge invariance, the large power quantities $(\frac{s}{m_e^2})^n$ ($n \geq 1; s \to \infty$) are removed from the calculation totally, while in usual schemes the large quantities appear and only will be cancelled at last only. The advantages of the scheme in numerical calculations are obvious, thus we focus our discussions mainly on the advantages of the scheme in the special case, where the absorptive part for some propagators relevant to the process could not be ignored, and a not-satisfactory but widely adopted approximation is made i.e. a finite constant "width" is introduced to approximate the absorptive part of the propagators phenomenologically even though the QED gauge invariance is violated.

I. Introduction.

Since the top quark mass has been measured in Fermilab [1][2] and the Higgs mass seems to possess some constraints [3], at present there is better ground to precisely testify the validity of the Standard Model. The LEP II of 200 GeV and NLC of about 500 GeV probably can do the job further in the near future.

Of the possible reactions, $e^+e^- \to t\bar{b}e^-\bar{\nu}$ is an interesting one. It is asserted that of the Feynman diagrams, the four associated with a t-channel photon exchange shown in Fig. 1, being gauge invariant themselves, are dominant over others as $\sqrt{s} > 250$ GeV [4]. The propagator of the photon is proportional to $1/k^2$ where $k^2 \equiv (p_1 - p)^2$ and p_1 and p are the momenta of the outgoing and incoming electrons. The kinematics tells us

$$
\begin{aligned}
k^2 &= (p_1 - p)^2 = 2m_e^2 - 2E_1E + 2|\vec{p}_1||\vec{p}|\cos\theta_e \\
&\sim (2 - \frac{|\vec{p}_1|}{|\vec{p}|} - \frac{|\vec{p}|}{|\vec{p}_1|})m_e^2 - 2|\vec{p}_1||\vec{p}|(1 - \cos\theta_e),
\end{aligned}
\tag{1}
$$

as θ_e, the angle between $\vec{p}_1$ and $\vec{p}$ approaches to zero, $k^2 \to 0$, if $m_e \to 0$ i.e. the photon approaches to mass shell, i.e. the propagator becomes singular. This singularity is essential, because even after the final state phase space integration it still survives. One can divide the integration over θ_e into two parts:

$$
\int_0^{(\theta_e)_{cut}} \sin\theta_e d\theta_e f(\theta_e, m_e) + \int_{(\theta_e)_{cut}}^{\infty} \sin\theta_e d\theta_e f(\theta_e, m_e)
\tag{2}
$$

where $(\theta_e)_{cut}$ is a small angle, and may be related to experimental measurements as one like. For the second integral, being regular m_e can be safely set zero approximately, whereas in general up to the first order of $(\theta_e)_{cut}$, the first integration may be written as

$$
[a(\frac{s}{m_e^2})^n + b(ln\frac{s}{m_e^2}) + \cdots] \cdot \Delta\theta_e
\tag{3}
$$

where $n \geq 1$ and $\Delta\theta_e$ is a small angle equivalent to $(\theta_e)_{cut}$.

Since the small-angle electron which finally scatters into the beam tube (i.e. θ_e being very small), cannot be detected, the phase space integration for final state should always start from the small angle instead of zero if the electron is 'exclusively' measured, so the $(\theta_e)_{cut}$ in eq.(2) has physical meaning. However if the electron in the reaction $e^+e^- \to t\bar{b}e^-\bar{\nu}$ is detected inclusively, the small θ_e contribution cannot be negligible.

Note that in eq.(3) the power terms $(s/m_e^2)^n \times \Delta\theta$ being very large even for very small $\Delta\theta$ and the logarithmic terms $ln(s/m_e^2) \times \Delta\theta$ being of much milder divergent behavior at $\theta \sim 0$, all of the terms may be suppressed if there is some symmetry e.g. the gauge symmetry for the concerned process. Many authors have investigated this process. Raidal et al. [5], Panella et al. [6] had obtained quite different conclusions, then Boos et al. [4] pointed out that due to an extravagant destructive interference among the four diagrams, the unexpected large contributions disappear after summing up the contributions with the large number cancellation at last.

It is proved that due to the gauge invariance playing roles in the process the troublesome power terms $(s/m_e^2)^n$ do not exist at the final cross section but only the well-known logarithmic term as $ln(s/m_e^2)$, that also forms the basis of the Weiszäker-Williams approximation [7]. In principle, a straightforward calculation can give the correct final result as done in literature [4] as long as the numerical calculation keeps all the large numbers being accurate enough. Even though the troublesome power terms $(s/m_e^2)^n$, generating large quantities, would be killed the large quantities greatly and retain much smaller ones are retained due to the destructive interference nature, in practice, such a cancellations may cause 6 to 8 magnitude orders reduction (see ref.[4]), so it very often causes problem i.e. it can lead to totally wrong result for numerical computation, at least, it becomes very difficult to estimate the errors of the computation.

Formulating the above statement, one can write the amplitude contributed by each individual of the four diagrams as

$$M_i = a_i + b_i \cos\theta \tag{4}$$

where θ is the angle between the 3-momenta of the incoming and outgoing electrons. The troublesome photon propagator contributes a factor $1/[\alpha + \beta\sin^2(\theta/2)]^2$ to the differential cross section. α is proportional to m_e^2 and β is related to the energy, so as $\theta \to 0$ and $m_e \to 0$, this is a singular term. The final state phase space integration includes a part over the solid angle $d\sin^2(\theta/2)$, therefore it alleviate the singular degree. When we take the integration of $|\sum M_i|^2$ over $d\sin^2(\theta/2)$, we have

$$\int d\sin^2(\frac{\theta}{2})|\sum_{i=1}^{4}(a_i + b_i\cos\theta)|^2 \frac{1}{(\alpha + \beta\sin^2(\frac{\theta}{2}))^2}$$

$$= \int d\sin^2(\frac{\theta}{2})\sum_{i,j}[a_i^*a_j + (a_i^*b_j + a_jb_i^*)\cos\theta + b_i^*b_j\cos^2\theta]\frac{1}{(\alpha + \beta\sin^2(\frac{\theta}{2}))^2}$$

$$= \int d\sin^2(\frac{\theta}{2})[|\sum_i(a_i + b_i)|^2 - \sum_{i,j}(4b_i^*b_j + 2(a_i^*b_j + a_jb_i^*))\sin^2(\frac{\theta}{2})$$

$$+4\sum_{i,j}b_i^*b_j\sin^4(\frac{\theta}{2})]\frac{1}{(\alpha + \beta\sin^2(\frac{\theta}{2}))^2}. \tag{5}$$

It is easy to be rewritten as

$$\int_0^1 dx\frac{(A + Bx + Cx^2)}{(\alpha + \beta x)^2}$$

$$= \int_0^1 dx[\frac{C}{\beta^2}(\alpha + \beta x)^2 + \frac{1}{\beta}(\frac{-2\alpha C}{\beta} + B)(\alpha + \beta x) + (A + \frac{\alpha^2 C}{\beta^2} - \frac{\alpha B}{\beta})]\frac{1}{(\alpha + \beta x)^2} \tag{6}$$

where $x = \sin^2\theta/2$.

It is easy to see that the first term is completely benign as $m_e \to 0$, and the second term gives a logarithmic term lnm_e^2 while the last one produces an extravagantly singular term proportional to $1/m_e^2$ at $\theta \to 0$. Therefore as we discussed above, the gauge invariance demands vanishing of the last term, namely one should expect

$$A + \frac{\alpha^2 C}{\beta^2} - \frac{\alpha B}{\beta} = 0 \tag{7}$$

where

$$A = |\sum_i(a_i + b_i)|^2 \tag{8}$$

$$B = -\sum_{i,j}(4b_i^*b_j + 2a_i^*b_j + 2a_jb_i^*) \tag{9}$$

$$C = 4\sum_{i,j}b_i^*b_j. \tag{10}$$

Furthermore, a very interesting and important issue is addressed recently i.e. some authors [8] [9][10][12] pointed out that to get rid of the singularity at the W-boson propagator $\frac{1}{q^2 - M_W^2}$ for higher energies, a regular Breit-Wigner form $\frac{1}{q^2 - M_W^2 + i\Gamma_W M_W}$ where Γ_W is the measured decay width of W-boson is introduced, whereas the gauge invariance of QED is violated. The power divergent terms may appear again. It is well known that the QED gauge invariance is a fundamental principle and cannot be upset at any case. Here the apparent violation is artificial or due to an inappropriate approximation and misapplication of the Breit-Wigner form. They suggested many

approaches to restore the gauge invariance. However since most of the treatments possess certain arbitrariness depending on the way to restore the gauge invariance as long as it is not based on a solid stone of the quantum field theory, even the gauge invariance is respected, some "unphysical" contributions emerge into the final results. Aeppli [10] and Papavassiliou, Pilaftsis [11] provide very elegant ways to set more solid foundations to deal with the width of the W-propagator for the processes, namely loop contributions for its self-energy and vertex are considered, thus the effective width Γ in the W-propagator is a function of momentum. But this procedure involves complicated loop calculations so the intuitive meaning of the width is lost and is the convenience for cross section evaluation. Argyres et al. [12] also suggested that taking into account the absorptive part of the triangular loop correction to the $WW\gamma$−vertex, one can regain the gauge invariance, the way they provided is practically efficient for real processes and the results obtained in various schemes which restore gauge invariance coincide with each other (see table I of ref.[12]).

Alternatively, we propose a different scheme to approach the problem, namely "project out" the large component from each piece of the amplitude (corresponds to each diagram), by means of choosing a special gauge based on the gauge invariance of the processes, thus the power terms i.e. the large numbers do not appear in the calculations completely. Furthermore, even the gauge invariance is artificially violated, in this scheme, the additional contribution related to the violation is also suppressed. For instance, even though the naive Breit-Wigner formulation of the propagator for describing the unstable nature of the particle is adopted, the large terms are eliminated and only the terms as $\frac{\Gamma_W}{M_W} ln(s/m_e^2) \times \Delta\theta$ survive. A more precise discussion and comparisons of the results obtained in this scheme with the others, especially that of the authors of ref.[12] will be given below. Reasonable consistence with others is found.

II. The projection scheme.

(i) The scheme.

The amplitude corresponding to the Feynman diagrams shown in Fig.1 (a) through (d) characterized by possessing a common electron line, can be written

as

$$
\begin{aligned}
M &= \sum_{i=1}^{4} M_i = l^\mu T_\mu \frac{-i}{k^2} \\
&= \overline{u}_e(p_1, s_1)\gamma^\mu u_e(p, s)\frac{-i}{k^2}(T_1 + T_2 + T_3 + T_4)_\mu
\end{aligned}
\tag{11}
$$

where l_μ is the lepton current, $u_e's$ are the incoming and outgoing electrons and $T_i's$ are the effective currents determined by the weak interaction and $k^2 = (p_1 - p)^2$ is the squared momentum carried by the photon. Due to the gauge invariance for the four diagrams themselves, we have

$$
k \cdot T \equiv k^\mu \cdot \sum_{i=1}^{4} T_{i\mu} = 0.
\tag{12}
$$

As aforementioned, through a straightforward calculation one may show that each of $T_i's, i = 1 \cdots 4$ itself will contributes power terms s/m_e^2 to the cross section at vicinity of $\theta_e = 0$. The "total cross section" σ (here only the four Feynman diagrams in Fig.1 are included, and it is introduced only for the discussion convenience):

$$
\begin{aligned}
\sigma &= \frac{1}{4s^2} \int \prod_{j=1}^{4} \frac{d^3 p_j}{(2\pi)^3 2E_j} \sum_{all\ spins} |M|^2 \\
&= \frac{1}{4s^2} \int \prod_{j=1}^{4} \frac{d^3 p_j}{(2\pi)^3 2E_j} \sum_{all\ spins} |\sum_{i=1}^{4} l^\mu T_{i\mu}|^2 \frac{1}{k^4},
\end{aligned}
\tag{13}
$$

where $p_j's$ are the momenta of the outgoing $t, \bar{b}, e^-$ and $\bar{\nu}$. Note once more: it has been proved [4] that the final state integration only results in a $ln(s/m_e^2)$ term, but not any power term $(s/m_e^2)^n$ $(n \geq 1)$ at all. The disappearance of the troublesome power terms finally is due to the gauge invariance of QED.

Nevertheless, there is still a problem in numerical calculation that the "cross section" involves subtraction among large quantities with a very small quantity remaining as shown in Fig. (3) of ref.[4]. To solve the problem, we propose the so-called project scheme by choosing a very special gauge. One can add an arbitrary term proportional to k^μ to the lepton current l^μ such as

$$
l'_\mu = l_\mu - ck_\mu,
\tag{14}
$$

due to the gauge invariance, here c may be any variable or constant. The idea about the projection scheme is to subtract a suitable quantity from every amplitude

(corresponding to each Feynman diagram) by choosing a proper gauge (here the quantity c) , so as to project out a fraction which results in the large power divergent term in the final cross section. Indeed the idea may be carried through successfully as follows. Simply to make each components of the 4-vector l'_μ to be minimal, we choose a condition

$$\frac{\delta}{\delta c} max(|l'_0|, |l'_1|, |l'_2|, |l'_3|) = 0. \tag{15}$$

To be symmetric, alternatively we adopt the following condition instead,

$$\frac{\delta}{\delta c}(l_0^* l_0 + l_1^* l_1 + l_2^* l_2 + l_3^* l_3) = 0. \tag{16}$$

Note here that the summation $\sum l_i^* l_i$ is defined in "Euclidean space measure" but not Minkovsky one, thus it means to minimize the squared radius of the Euclidean four-sphere. Then we obtain

$$c = \frac{k_0 l_0 + k_1 l_1 + k_2 l_2 + k_3 l_3}{k_0^2 + k_1^2 + k_2^2 + k_3^2}, \tag{17}$$

under the condition. Because $k^\mu l_\mu = 0$ (with the metric $(1, -1, -1, -1)$ as convention), we have

$$c = \frac{2 k_0 l_0}{||k||^2}$$

where $||k||^2 \equiv k_0^2 + k_1^2 + k_2^2 + k_3^2$. Thus with the gauge c, the lepton current may be replaced by

$$l'_\mu = l_\mu - \frac{2 k_0 l_0}{||k||^2} k_\mu, \tag{18}$$

which indeed projects out the large term from l_μ. Due to smallness of m_e in the process, one can expand $k_0 l_0$ in m_e and only keep terms up to m_e^3 in the series.

To see the results, by comparing with eq.(4) under the limit of $m_e^2 \to 0$, i.e. α in eq.(5) is zero, if one calculate the amplitude for each diagram in terms of the projection scheme, one will find that each amplitude is proportional to $1/\sin(\theta/2)$ instead of $1/\sin^2(\theta/2)$ in usual scheme, thus the singularity becomes mild and after the integration over final phase space, only the logarithm term remains even for the contribution from each diagram individually.

To show the advantages more precisely, we recalculate the process numerically. The numerical results are shown in Fig.2. We plot the dependence of the cross sections on the $(\theta_e)_{cut}$ which is the lower limit of the angle integration. In order to

compare with the results of ref.[4], we recalculate the contributions from the four diagrams of Fig.1 (without the interference among them) in terms of the usual way, where we deliberately choose $m_t = 140$ GeV and $\sqrt{s} = 190$ GeV precisely as given in ref.[4] the individual curves are the upper four in the figure and they are exactly consistent with those of ref.[4]. It is noticed that the total cross section is lower than them by 8 magnitude orders as pointed out by Boos et al. The lower four curves correspond to the contributions from the four diagrams individually too but are calculated in the projection scheme and the total cross section exactly coincides with that obtained in the usual scheme where high accuracy computation technique is employed. It is interesting to note that the "individual" curves obtained by the projection scheme have the same order as the total cross section.

(ii) Under the approximation of a finite W-boson width Γ_W.

As the concerned energy is relatively low that all intermediate W-bosons are far away from its mass shell, the propagator can be written as

$$\frac{-i}{q^2 - M_W^2 + i\epsilon}(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_W^2}).$$

As showed by Kurihara [8], with propagators corresponding to zero-width, it is easy to check $k^\mu T_\mu = 0$, i.e. the gauge invariance holds. However, when the other s-channel diagrams for the process $e^+ e^- \to t\bar{b}e^-\bar{\nu}$ are concerned and as the energy is increasing ($\sqrt{s} \geq m_t + m_b + M_W$), q^2 of the process may cross the mass shell of W-boson and the singularity of the propagator would result in a new singularity. In fact this divergence is caused by a unsuitable approximation. Since the intermediate boson is not a stable particle, the propagator should be modified, for instance,

$$\frac{-i}{q^2 - M_W^2 + i\Gamma_W M_W}(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_W^2}).$$

However, it still is a problem what Γ_W in the propagator is. Generally it is an "effective" width, corresponding to the absorptive part of the self-energy of the particle, therefore only s-channel W-boson, being timelike, can be nonzero, whereas the t-channel is space-like, so always be zero. Therefore Γ_W in the propagator should be a function of momentum behaving as

$$\Gamma_W(q^2) = \Gamma_1(q^2)\theta(q^2 - \Lambda_1^2) + \Gamma_2(q^2)\theta(q^2 - \Lambda_2^2) + \cdots, \tag{19}$$

where Λ_i^2 is the threshold of a corresponding channel (i). In practice, the Breit-Wigner formulation is adopted widely[13] i.e. one has

$$\Gamma_W(q^2) = \begin{cases} \Gamma_W & q^2 > 0 \quad (s-\text{channel}) \\ 0 & q^2 < 0 \quad (t-\text{channel}) \end{cases} \qquad (20)$$

where $\bar{\Gamma}_W$, being constant, is taken as the measured width of real W-boson. This brings in an inconsistency, i.e. violating the gauge invariance of QED artificially. To amend the fake violation, many authors proposed various ways [8][9] and Aeppli [10] summarized them and indicated that all schemes may possess some unphysical additions artificially imposed to the results.

In our scheme to take the special gauge (eq.(17)), thanks to the projection in the gauge, the large terms $(\frac{s}{m_e^2})^n \times \Delta\theta$ does not occur at all whereas in usual schemes they appear in the intermediate stage of the calculation. when gauge invariance is violated, such dangerous power terms still do not appear, namely even the gauge invariance being violated, in the final result only the term proportional to $ln(s/overm_e 62)$ appears in terms of the the projection scheme. In summary, when the gauge invariance is artificially violated, in this scheme there could be only additional terms:

$$a(\frac{\Gamma_W}{M_W})^m(\frac{s}{m_e^2})^n \times \Delta\theta + b(\frac{\Gamma_W}{M_W})^{m'} ln\frac{s}{m_e^2}\Delta\theta, \qquad (21)$$

which vanishes as $\Gamma_W \to 0$. In the Breit-Wigner formulation, both a and b terms in eq.(21) are non-zero and any procedure to restore the gauge invariance would make the power term disappear, i.e. impose a to be zero, and bring in some change to the logarithmic term. The change, in fact, must involve unphysical components due to violating the gauge invariance.

For an explicit comparison with the literatures [12], we have calculated the cross section of $e^+e^- \to u\bar{d}e^-\bar{\nu}_e$ for $\sqrt{s} = 175$ GeV. As the collision energy is so high, we have adopted a little bigger fine-structure constant:

$$\alpha_e(s = 175^2\,GeV^2) = 1/125.0$$

in our computations. The numerical values for the cross section we obtained are listed in Table I. When calculating the values in the table, the parameters as following:

$M_W = 80.22$ GeV;
$m_e = 0.511 \times 10^{-3}$ GeV;
$\alpha_e(0) = 1/127.034$;
$\sin\theta_W = 0.232$;
$50 < \sqrt{P_+^2} < 110$ GeV

are taken and with the definition $P_+ = P_l + P_\nu$. The coupling constant α_e is mainly based on the formulae [15]. In the calculations $m_t = 176$ GeV is used.

σ (pb)	0.08977 ($\pm$ 0.000200)	with fixed W-width
σ (pb)	0.08983 ($\pm$ 0.000200)	with running W-width

Table I. The cross sections of $e^+e^- \to u\bar{d}e^-\bar{\nu}_e$ obtained in the projection scheme (the definition of the W-width appearing in the table may be found in ref.[12].

To compare with the results of ref.[12], theirs for the cross section is 0.08887(8)pb for fixed width ($\theta_{min} = 0$), while from Table I one may see ours is about 1% larger only. The result with running width is only 0.07% larger than that for the fixed width. We should note here that a comparatively larger value for α_e is adopted in our calculation, hence a slight larger number should be expected.

III. Discussions and Conclusion.

To solve the problem of large number cancellations around the singularity such as that in the forward direction for the process $e^+e^- \to t\bar{b}e^-\bar{\nu}$, we propose a "projection" scheme so as to priori project out the large quantity in each amplitude where a t-channel photon-propagator is involved.

The figures in Fig.2 show that in usual schemes, the curves corresponding to the contribution of the individual diagrams of Fig.1 rise very fast as $(\theta_e)_{cut}$ approaches to zero, but the total cross section does not. It is a result of the gauge invariance as discussed above. In contrast, in the new scheme (a specially gauge is applied) the contribution presents a smooth behavior at zero-$(\theta_e)_{cut}$.

Indeed, this intriguing problem was conceived by some authors long time ago and they employed a special scheme[14]. The squared amplitude is

$$\frac{1}{4}\sum |T|^2 = L_0^{\mu\nu}(p, p_1)H_{\mu\nu}(p_t, p_b, q) \qquad (22)$$

8

9

where $L_0^{\mu\nu}$ is the lepton current part, $q = p - p_1$ and $q^\mu H_{\mu\nu} = 0$ by gauge invariance. In general, $L_0^{\mu\nu}$ may be written as follows:

$$L_0^{\mu\nu} = (\epsilon_1^\mu \epsilon_1^\nu + q^2 g^{\mu\nu}) L(q^2) \tag{23}$$

where $L(q^2)$ is a scalar function; and the polarization may be written as

$$\epsilon_1^\mu = p^\mu + p_1^\mu + Z_1 q^\mu,$$

where Z_1 stands for an arbitrary parameter. The authors proved that if a special choice

$$Z_1 = -(p^0 + p_1^0)/q^0,$$

is taken, and further to demand $\epsilon_1^0 = 0$ and

$$\epsilon^2 = -|\vec{\epsilon}_1|^2 = 4m_e^2 + (Z_1^2 - 1)q^2 + 0(m_e^2),$$

the power terms $(s/m_e^2)^n$ can be effectively eliminated. Our projection scheme is along a way parallel to the treatment. Our scheme systematically handles the power singularity at the collinear limit. We adopt the projection at the amplitude level while the authors of the ref.[14] dealt with it at the amplitude-square level.

As pointed out above, so far there is no very satisfactory (simple, intuitive and not breaking the existent symmetries etc.) way to dictate the absorptive part of the propagator when the "finite width" effects could not be ignored. Usually when the finite width is introduced phenomenologically the gauge invariance is artificially violated. Large power singular terms generally emerge. Therefore one would try some methods to restore the gauge invariance, but so far the most of the treatments (a few exceptions e.g. ref.[12]) 'planting in' gauge invariance by hand, may get rid of the unphysical power singular terms, but at the same time would bring in other new unphysical and un-wished changes. Whereas in our scheme, the unphysical power singularity is eliminated from very beginning, and so is even with the artificial gauge invariance violation. Even though an unphysical logarithmic term due to the violation of the gauge invariance, indeed, emerges and is added to the final result, compared to other schemes this additional unphysical contribution is much suppressed and the influence to the total cross section is within 1%, a tolerable error at least for the tree level. It is because for all known unstable particles so far we

are treating, we always have its width being much smaller than its mass, e.g. for W-boson we have $\Gamma_W \ll M_W$, the extra term, behaving as eq.(21), does not make substantial contribution at the highest energy in the foreseen future.

For $e^+ e^- \rightarrow t\bar{b}e^-\bar{\nu}$, since in the four t-channel photon exchange diagrams of Fig.1, the W-boson cannot go onto its mass shell no matter at s or t channels, so the finite Γ_W should not give rise to any substantial change in that case. Our results with the W-propagator having a finite width only at the time-like region are numerically consistent with that of null Γ_W, and the error is within 1%, so it confirms our aforementioned discussions that the gauge invariance violation can only cause a term proportional to $(\frac{\Gamma_W}{M_W})^2 ln\frac{s}{m_t^2}$. Whereas in the process $e^+ e^- \rightarrow u\bar{d}e^-\bar{\nu}$, the effects of a finite width Γ_W become important even for the four photon-exchange diagrams in Fig.1. With our scheme, the troublesome unphysical power term does not appear either and the unphysical term corresponding to artificial gauge invariance is also suppressed to $(\Gamma_W/M_W)^2$ order, so they are negligible up to a sufficient accuracy, for example as $\sqrt{s} = 200$ GeV, $(\frac{\Gamma_W}{M_W})^2 ln\frac{s}{m_t^2} \sim 0.016$, and $0.016 \times \Delta\theta$ must be much less than 1. The results, shown in Table I, indicate that the effect of violating gauge invariance caused by the running W-width does not affect the final conclusion within a range of 1%. In contrary, without the projection, the power term caused by the artificial gauge invariance violation is too large to tolerable, in fact, it blows up the numerical results (see Table of ref.[12]).

It is certain that the scheme of ref. [12] is more solid from theoretical point of view which is based on more solid ground such as quantum field theory where through loops one can connect the vertex to the self-energy diagrams to restore the gauge invariance when finite width effects are concerned. Even though, we still should note that if one restricts himself to work in an exact perturbative theory, surely the gauge invariance will be kept just order by order, however the W-propagator cannot be simply written in the compact form $-i/(q^2 - m_W^2 + i\Gamma_W M_W)$, which is a result of resummation of chain diagrams, thus it is not easy to mend the singularity problem at a given order. In fact, what we need is to eliminate the dangerous power divergence caused by the artificial violation of gauge invariance so to reach a reliable result in requested accuracy. To serve the goal, one either restores the gauge invariance to automatically remove the dangerous power divergence or gets rid of the troublesome term directly as we do in this work. For restoration of

the gauge invariance, an appropriate scheme is to include all loop corrections of the self-energy and vertex whose absorptive parts would consistently result in the imaginary part in the full propagator of W-boson, Z-boson and fermions and retains the gauge invariance [11][12]. In ref.[12], the authors proved that the absorptive part of the triangular loop compensates the unbalance at s- and t-channels due to introducing finite width to the unstable W-boson, so the gauge invariance is regained in the calculations. The authors showed that deviations for various schemes which are adopted to retain gauge invariance and eliminate the power divergence s/m_e^2, are reasonably small. In our scheme, we just simply avoid the trouble of power divergence and suppress the gauge invariance violation effects. Indeed, in principle and in practice our scheme may let the calculations escape from the trouble due to violation of the gauge invariance caused by phenomenologically introducing a finite width in the propagator(s).

The advantages of the scheme are obvious. At least many large number cancellations due to internal gauge invariance in a concerned process is avoided. Those advantages are crucial sometimes for numerical calculations. Futhermore, a simple but rough numerical computation indicates that the final results for the cross sections of $e^+e^- \to e^-\bar{\nu}_e u\bar{d}$ in the projection scheme only deviates from that in the schemes which restore the gauge invariance by considering a loop correction to the $WW\gamma-$vertex by less than 1%. A more careful calculation is in progress and the results will be published some time later. [16].

Since the process $e^+e^- \to u\bar{d}e^-\bar{\nu}_e$ attracts much attentions due to its significance for better understanding top physics and precise test of the standard model, further studies are going on [17]. Indeed, a convenient method which greatly simplifies analysis of data and at the same time obtains results deviating from the 'accurate' values obtained by other more solid, but much more complicated ways only by a small fraction within the experimentally allowed tolerance, should be helpful and probably preferable. This projection scheme may be one of the appropriate and desired ones for both experimentalists and theoreticians.

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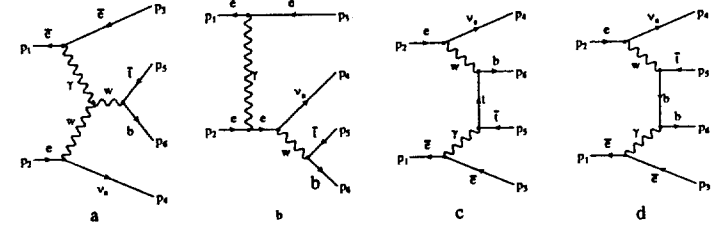Preprint-95-26/390.

Fig. 1

Figure Captions

Fig.1, (a) through (d), the Feynman diagrams where a t-channel photon propagator
is involved.

Fig.2. The dependence of the cross sections on the angle cut $(\theta_e)_{cut}$. The upper
four curves show a rapid rise near $(\theta_e)_{cut} \to 0$, which correspond to the cross sections
of the individual diagrams of Fig.1 calculated in the regular scheme. Whereas
the lower four solid lines also correspond to the individual diagrams of Fig.1, but
calculated with the projection scheme. The dashed line is the total cross section.
To compare with the previous calculations, we take $m_t = 140$ GeV and $\sqrt{s} = 190$
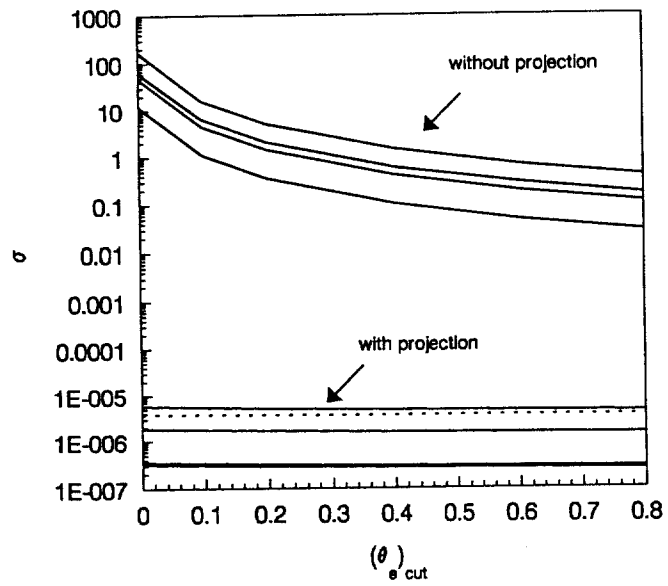GeV.

Fig.2