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Relativistic corrections to the central $q\bar{q}$ potential from pure $SU(3)$ lattice gauge theory

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Abstract

We present a comprehensive high statistics analysis of spin- and velocity dependent corrections to the static (central) $q\bar{q}$ potential in pure $SU(3)$ lattice gauge theory. Simulations have been performed at $\beta = 6.0$ and $\beta = 6.2$ which corresponds to lattice spacings of 0.1 and 0.07 fm.

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We present a comprehensive high statistics analysis of spin- and velocity dependent corrections to the static (central) $q\bar{q}$ potential in pure $SU(3)$ lattice gauge theory. Simulations have been performed at $\beta = 6.0$ and $\beta = 6.2$ which corresponds to lattice spacings of 0.1 and 0.07 fm.

1. Introduction

Potential models have been proven to describe meson spectra remarkably well, even at rather small quark masses. To derive the semi-relativistic Hamiltonian one usually starts from a Fouldy-Wouthuysen transformation of the one-particle propagator and its corresponding path integral representation and expands this in inverse powers of the quark mass around the static limit ($m \to \infty$). The result up to order $\frac{1}{m^2}$ is a generalization of the familiar Breit-Fermi Hamiltonian, containing the central potential $V_0(r)$, and spin and velocity dependent parts $V_{sd}(r), V_{vd}(r)$ that are related to coefficient functions (potentials) $V_1 - V_4$ and $V_a - V_e$, respectively. Explicit expectation values have been associated with the sd and vd potentials that can be calculated on the lattice [1-3]. The sd potentials are related to scalar (S), vector (V) and pseudoscalar (P) exchange contributions in the following way:

$$V_0(r) = S(r) + V(r) \quad (1)$$

$$V_3(r) = \frac{V'(r) - P'(r)}{r} - \left(V''(r) - P''(r)\right) \quad (2)$$

$$V_4(r) = 2\nabla^2 V(r) - \Delta P(r)\,. \quad (3)$$

Relativistic invariance yields the relations

$$V_2'(r) - V_1'(r) = V_0'(r) \text{ (Gromes relation)} \quad (4)$$

$$\frac{1}{2}V_b(r) + V_d(r) = \frac{r}{12}V_0'(r) - \frac{1}{4}V_0(r) \quad (5)$$

$$\frac{1}{2}V_c(r) + V_e(r) = -\frac{r}{4}V_0'(r)\,. \quad (6)$$

2. Lattice Techniques

In table 1 we display the parameters of our lattice simulations.

Table 1
Simulation parameter. The physical values are obtained by using $\sqrt{\sigma} = 0.44$ GeV.

	$\beta = 6.0$	$\beta = 6.2$	
$V = L_S^3 \cdot L_T$	16^4	16^4	32^4
a/fm	0.098(2)	0.0710(4)	
a^{-1}/GeV	2.02(3)	2.79(2)	
$a \cdot L_S$/fm	1.57(3)	1.136(6)	2.27(1)
# conf	370	700	116

In order to reduce statistical fluctuations temporal links have been integrated out analytically wherever possible.

To determine the sd and vd potentials on the lattice one has to integrate correlation functions $C(T)$ consisting of two chromoelectric or magnetic insertions ("ears") into the temporal transporters of the Wilson loop divided by the Wilson loop without ears (T denotes the temporal ear-to-ear distance). The minimal distance Δ of one ear to a Wilson loop end is the time the gluon field needs – after creation – to decay into its ground state and therefore governs contaminations of excited states. To suppress such pollutions all spatial links have been smeared, allowing us to re-

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duce Δ. For all correlators we observed a plateau for $\Delta \geq 2$. In this simulation we keep the ears fixed at $\Delta = 2$ with respect to the Wilson loop ends and vary the integration time by enlarging the temporal Wilson loop extent. This leads to smaller statistical fluctuations in comparison to the usual method [1] where the Wilson loop is kept fixed and the positions of the ears are varied.

Correlation functions are numerically interpolated and integrated up to cut-off values $T_{\max}$ where saturation sets in. Some $< EE >$ correlators converge towards non-zero constants in the limit $T \to \infty$, in agreement with their spectral decompositions. These constants are determined from fits to the last few data points of $C(T)$ ($T \geq T_{\max}$) and subsequently subtracted.

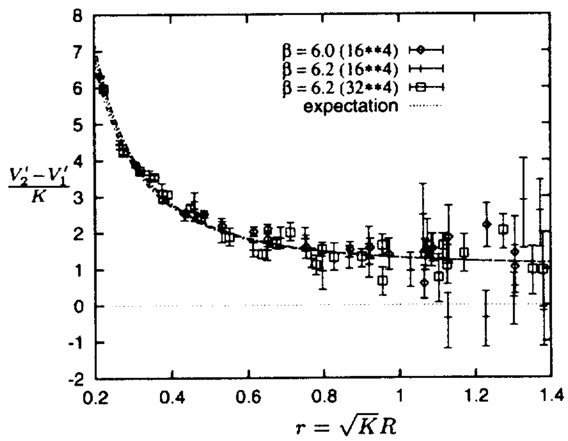

Figure 1. Test of the Gromes relation.

The lattice *sd* and *vd* potentials undergo multiplicative renormalization in respect to their continuum counterparts while $V_0(r)$ does not. We use the nonperturbative renormalization procedure of ref. [4]. Fig. 1 shows a comparison of the Gromes combination $V_2' - V_1'$ (in units of the stringtension) with the central force (solid line), derived from a fit to the central potential

$$V_0(R) = V_0 + KR - \frac{e}{R} + \frac{f}{R^2} . \tag{7}$$

We find reasonable scaling and consistency of the data with the Gromes relation. This supports the renormalization procedure.

3. Results

From eq. (7) and tree level perturbation theory one expects for the *sd* potentials:

$$V_1'(R) = -K \quad , \quad V_2'(R) = \frac{e}{R^2} - \frac{2f}{R^3} \tag{8}$$

$$V_3(R) = \frac{3e}{R^3} - \frac{8f}{R^4} \quad , \quad V_4(R) = 8\pi e\delta(\vec{R}) + \frac{2f}{R^4} . \tag{9}$$

For the *vd* potentials we just quote [2]:

$$V_a(R) = 0 \quad , \quad V_c(R) = -\frac{e}{2R} - \frac{KR}{6} . \tag{10}$$

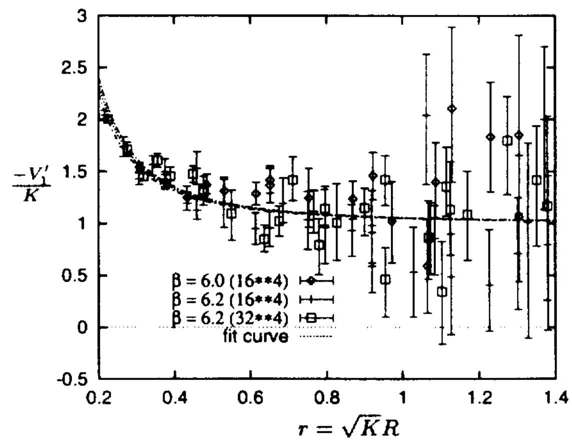

Figure 2. V_1' together with a fit curve $K + \frac{a}{r^2}$.

The first *sd* potential V_1' is displayed in fig. 2. Besides the long range confining term we observe an additional short range contribution, that can be well fitted to $\frac{a}{R^2}$. Assuming P interactions to be negligible and V_1 to be pure scalar (which induces the equalities $V_2 = V$ and $V_1 = -S$) one expects relations similar to eqs. (8),(9) with e replaced by $e - a$. In fig. 3 we find good agreement between V_3 and this expectation. The *sd* potential V_4 shows oscillatory behaviour that can largely be understood as a (lattice) δ-contribution. Fig. 4 shows $\delta V_4(R) = V_4(R) - 8\pi(e-a)\delta_L(\vec{R})$ where $\delta_L(\vec{R})$ is a lattice δ-function from lattice perturbation theory. The data points show qualitative agreement with the expectation from eq. (9). As can be seen from fig. 5 $\nabla^2 V_a$ deviates from zero. The data can be fitted to the parametrization $-\frac{d}{r}$ with d=0.7 GeV^2.

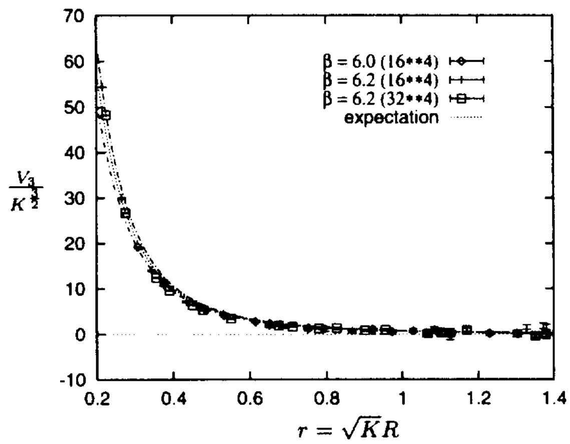

Figure 3. V_3 with $\frac{V'}{r} - V''$.

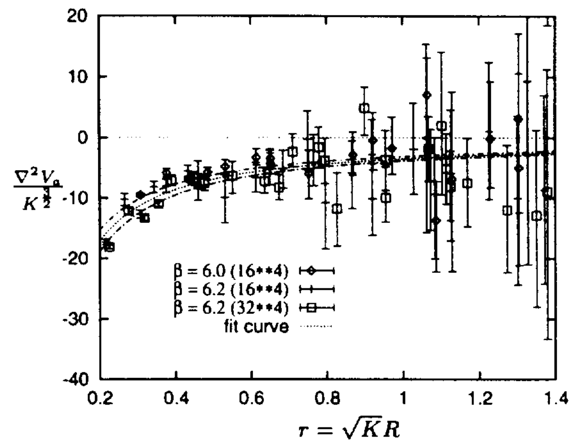

Figure 5. $\nabla^2 V_a$ together with a fit curve $\frac{-d}{r}$.

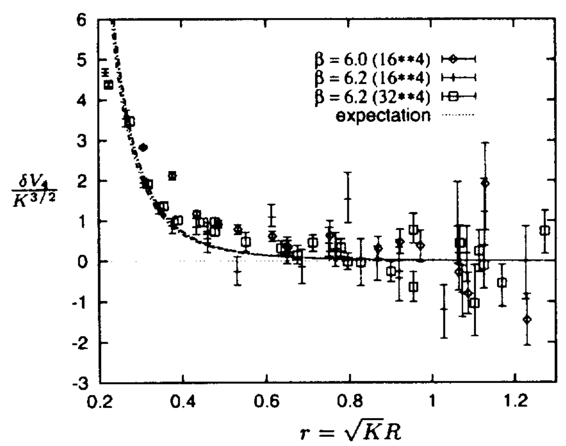

Figure 4. δV_4 compared to its expectation.

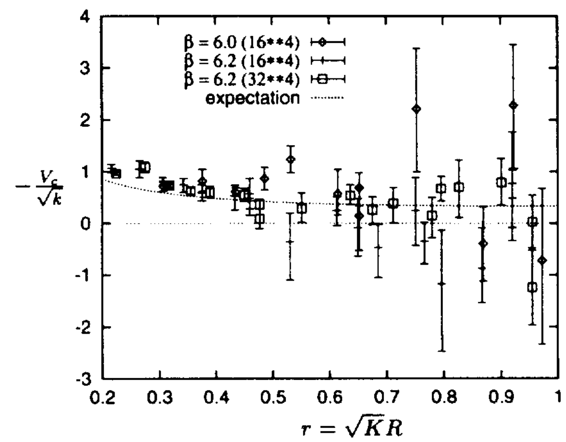

Figure 6. V_c compared to its expectation.

The other vd potentials are found to agree with naive expectations. This is illustrated for V_c in fig. 6.

4. Conclusions

Apart from the linear large distant part we observe an additional Coulomb like scalar contribution in V_1'. This leads to a scalar/vector splitting of the central Coulomb part inbetween $\frac{1}{7} - \frac{1}{5}$. All other sd potentials are short ranged and in perfect agreement with lattice perturbation theory. The vd potentials V_b to V_e follow naive expectations. V_a contains a $\frac{d}{r}$-contribution which is important for spectroscopy since it modifies the Coulomb part of the central potential (Factor 2 of charmquark mass). A detailed investigation of the various potentials can be found in ref. [5].

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