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J.C. HARDY, E. HAGBERG, V.T. KOSLOWSKY, G. SAVARD AND I.S. TOWNER

AECL, Chalk River Laboratories, Chalk River, Ontario, Canada, K0J 1J0

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Superallowed $0^+ \rightarrow 0^+$ nuclear beta decay provides a direct measure of the weak vector coupling constant, G_v. The nine most accurately determined transitions, ranging from the decay of ^{10}C to that of ^{54}Co, are surveyed, and the results used to test the conservation of the weak vector current (CVC) and the unitarity of the Cabibbo-Kobayashi-Maskawa matrix.

1. Introduction

Nuclear beta decay between $(J^\pi,T) = (0^+,1)$ analogue states provides a straightforward, yet demanding probe of the weak vector current. Because the axial current cannot contribute between spin-0 states, the experimental *ft* value is directly related to the vector coupling constant, G_v, via [1]

$$ft(1+\delta_R) = \frac{K}{G_V^2(1+\Delta_R^V)\langle M_V\rangle^2} \qquad (1)$$

with

$$\langle M_V\rangle^2 = 2(1-\delta_C) \text{ for } T = 1$$
$$K/(\hbar c)^6 = 2\pi^3 \ln 2\hbar/(m_e c^2)^5 = (8120.271 \pm 0.012) \times 10^{-10}\,\text{GeV}^{-4}\text{s},$$

where *f* is the statistical rate function, *t* is the partial half-life for the transition, and $\langle M_v\rangle$ is the Fermi matrix element. The calculated radiative corrections, δ_R and Δ_R^V, and charge correction δ_c, are each of order 0.01, with δ_R and particularly δ_c being dependent on nuclear structure. These correction terms introduce theoretical uncertainties but they are at least an order of magnitude smaller, so eq. 1 offers an experimental method for determining G_v to about one part in a thousand if the *ft* value can be measured to that level of precision or better.

To date, nine $0^+ \rightarrow 0^+$ "superallowed" transitions have been measured with sufficient precision: the decays of ^{10}C, ^{14}O, ^{26m}Al, ^{34}Cl, ^{38m}K, ^{42}Sc, ^{46}V, ^{50}Mn, and ^{54}Co. In analyzing these results, we find it convenient to define a "corrected" $\mathcal{F}t$ value, which is derived from the conventional *ft* value by the removal of all known effects that depend on nuclear structure:

$$\mathcal{F}t = ft(1+\delta_R)(1-\delta_c). \qquad (2)$$

Since, now

$$\mathfrak{I}t = \frac{K}{2G_V^2(1+\Delta_R^V)}, \qquad (3)$$

it is evident that the $\mathfrak{I}t$ value must be the same for all transitions if indeed G_v is a constant, as required by the Conserved Vector Current (CVC) hypothesis. The nine measured transitions span a wide mass range, $10 \leq A \leq 54$, and so constitute a demanding test of CVC.

Their significance also extends to a second fundamental test: the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. In the Standard Model, the quark mass eigenstates are related to the weak-interaction eigenstates through the CKM matrix:

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix} \qquad (4)$$

If the model is correct, the nine CKM matrix elements, which can in principal be determined experimentally from weak decays of the relevant quarks[2], should satisfy the unitarity conditions.

The value of V_{ud} can be obtained directly from the best value of G_v, which in turn follows from the experimental $\mathfrak{I}t$ values (as long as these show consistency with CVC):

$$\begin{aligned} V_{ud}^2 &= G_V^2 / G_F^2 \\ &= \frac{K}{2G_F^2(1+\Delta_R^V)\overline{\mathfrak{I}t}}, \end{aligned} \qquad (5)$$

where G_F is the Fermi coupling constant derived from the pure-leptonic muon decay and $\overline{\mathfrak{I}t}$ is the average result from the nine superallowed decays. This leads to the most stringent unitarity test of the CKM matrix currently possible, in which

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \qquad (6)$$

must be satisfied.

2. Experiments

Three measured quantities are required to determine the $\mathfrak{I}t$ value of a particular β-transition: the decay energy, Q_{EC}, which is used in calculating the statistical rate function, f; the half-life, $t_{1/2}$, of the β-emitter and the branching ratio, R, for the transition of interest, which together with the calculated electron-capture probability yield the partial half-life, t. To ensure that the experimental contribution to the overall uncertainty in $\mathfrak{I}t$ is smaller than that from the theoretical corrections, Q_{EC} measurements must be accurate to better than 100 ppm, and half-lives and branching ratios to better than about 500 ppm. These requirements are at the limits of what is currently possible and have led to highly refined experimental techniques.

2.1 ***Decay-energy measurements***. The nine well-measured superallowed β-emitters populate daughters with stable ground states. As a result, reaction Q-value measurements, which lend themselves to experimental precision, can be used to determine decay energies instead of direct measurements of positron end-point energies, which are fraught with precision-limiting difficulties. So far, three different experimental techniques have yielded the required accuracy. If a transition is designated $A(\beta^+)B$, these are: the measurement of the B(p,n)A threshold energy, with the energy of the proton beam related to a one-volt standard (for example, see ref [3]); the measurement of the reaction Q-values for two reactions C(p,γ)A and C(n,γ)B on a common target, with energies determined by comparison with known resonances and γ-ray standards (see ref [4]); and the in-beam comparison with a second superallowed transition, $A'(\beta^+)B'$, via the reactions $B(^3He,t)A$ and $B'(^3He,t)A'$, in which a composite target of B and B' is employed (see ref [5]). This last technique — the one developed at Chalk River — involved the application of a periodic voltage (up to 150 kV) to the reaction target so that two triton groups, one from each reaction, could alternately follow the same path through a magnetic spectrometer. The Q-value difference could thus be accurately established much the same way atomic mass differences are determined from a mass spectrometer.

2.2 ***Half-lives***. Though deceptively simple when precision is not an issue, half-life measurements do not easily yield high accuracy. Sample impurity, electronic count-rate dependence, and inappropriate statistical analyses are only a few of the difficult problems faced when uncertainties below 0.1% are sought, particularly for short half-lives. To obviate these problems, we have developed rigorous procedures at Chalk River[6]. Currently, all emitters except ^{14}O and ^{10}C have been measured with an on-line isotope separator. Typically, thousands of multi-scaled spectra are acquired from successive samples observed in a large-solid-angle, high-efficiency, low-background gas proportional counter; the counting electronics have a well-defined, non-extendible dead time. The analysis procedure uses modified Poisson variances and is based on a simultaneous fit of up to 500 decay curves with the same half-life, but with individual intensity parameters for each decay curve. This procedure is checked with simulated data, computer-generated to reproduce closely the experimental counting conditions, and has been proven correct at the 0.01% level.

2.3 ***Branching ratios.*** In all cases but for the decay of ^{10}C, the superallowed transition dominates ($\geq$99%) the decay of the parent nucleus and is best determined via measurement of the other decay branches, their branching ratios being subtracted from 100%. For these other branches, precision is not as important as completeness; therefore we have developed special techniques to observe β-delayed γ rays from branches as weak as a few parts per million[7]. Since the main difficulty in observing these branches results from the intense background generated by the prolific and energetic positrons from the superallowed branch, we use thin plastic scintillators to detect the direction of outgoing positrons, thus rejecting events in which positrons have entered the HPGe γ-ray detector.

For the only case in which the superallowed transition is itself weak — the decay of ^{10}C — a recent experiment at Chalk River[8] has, for the first time, used a large γ-ray array to measure a β-decay branching ratio with high precision: the intensities of β-delayed γ rays in ^{10}B, produced via the reaction $^{10}B(p,n)^{10}C(\beta^+)^{10}B^*$, were compared with the same γ rays produced via $^{10}B(p,p')^{10}B$ in alternating off-line, on-line measurements with the 8π spectrometer.

3. Theoretical Corrections

In a nucleus, the charged particles involved in β decay interact with the electromagnet field from the nucleus, which leads to effects of order $(Z\alpha)^m$, m=1,2..., where Z is the atomic number of the daughter nucleus and α is the fine-structure constant. For the electron (or positron), these effects are incorporated in the calculation of f through the use of exact solutions to the Dirac equation for electrons in the field of a modelled nuclear charge distribution. For the proton, these effects are reflected in the breaking of nuclear isospin symmetry and a resulting correction, δ_c, to the Fermi matrix element.

There are also effects that arise from interactions between the charged particles themselves and their associated bremsstrahlung emissions. These are of order $Z^m\alpha^n$, with m<n, and lead to radiative corrections to the bare β-decay process. By convention these corrections are grouped into two terms, δ_R and Δ_R^V, the former incorporating corrections that are nuclear-structure dependent, the latter, those that are not.

The most controversial of these correction terms is δ_c. Both Coulomb and charge-dependent nuclear forces destroy isospin symmetry between the initial and final states in superallowed β decay. Consequently, there are different degrees of configuration mixing in these two states, and, because the odd proton in the initial state is less bound than the odd neutron in the final state, their radial wave functions differ as well. Calculations accommodate both effects by splitting δ_c

into two components, δ_{c1} and δ_{c2}, to account for configuration mixing and radial mismatch, respectively. Shell-model calculations[7,9,10] have been used to determine δ_{c1} and have been tested experimentally[7] by measurements, in a few of the same nuclei, of weak non-analogue $0^+ \rightarrow 0^+$ transitions, which are also predicted by the calculations and are sensitive to the same effects as δ_{c1}. The δ_{c2} correction has been calculated by full parentage expansions in terms of Woods-Saxon wave functions suitably matched to the experimentally known separation energies[11]; also by a self-consistent Hartree-Fock calculation, in which the central potential was adjusted to match the single-particle eigenvalues with the experimental separation energies[10]; and, most recently, by Hartree-Fock calculations that incorporate charge-symmetry and charge-independence breaking forces in the mean-field potential to take account of isospin impurity of the core[12]. The various calculations for δ_c are displayed in figure 1 where it can be seen that they agree remarkably well with one another, except for ^{34}Cl and ^{38m}K, where the RPA calculations[12] are inadequate to deal with the details of the nuclear structure.

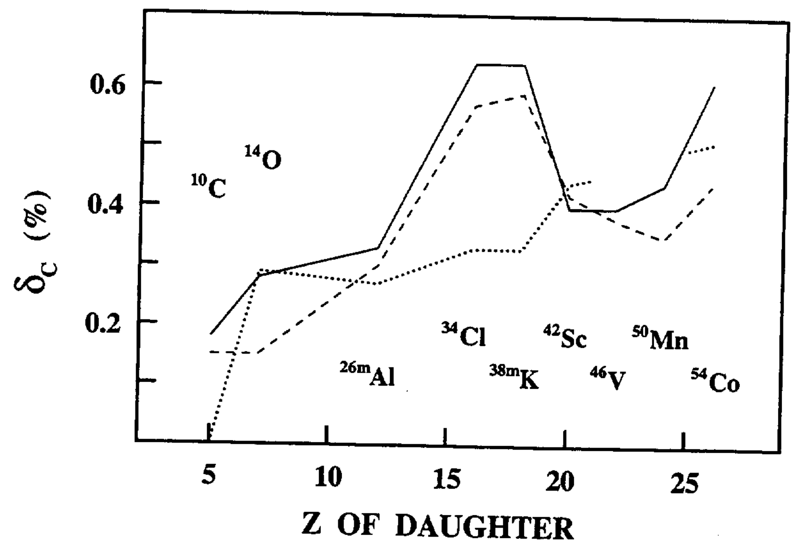

Figure 1 The results of various calculations of δ_c. The solid line joins calculations[7,9,11] that use the shell model for δ_{c2}; the dashed line is for the combined shell model and Hartree-Fock matched to separation energies[10]; and the dotted line is for RPA and Hartree-Fock with core effects included[12].

The radiative corrections, δ_R and Δ_R^V, have been calculated[13] up to order $Z^2\alpha^3$, and recently have been refined to include detailed calculations of order-α axial-vector photonic contributions from both one and two nucleons, the latter contribution being dependent on nuclear structure[14].

Table 1. Experimental results (Q_{EC}, $t_{1/2}$ and branching ratio, R) and calculated corrections (δ_c and δ_R) for $0^+ \rightarrow 0^+$ transitions.

	Q_{EC} (keV)	$t_{1/2}$ (ms)	R (%)	ft (s)	δ_c (%)	δ_R (%)	$\mathcal{F}t$ (s)
^{10}C	1907.77(9)	19290(12)	1.4638(22)	3040.1(51)	0.16(3)	1.30(4)	3074.4(54)
^{14}O	2830.51(22)	70603(18)	99.336(10)	3038.1(18)	0.22(3)	1.26(5)	3069.7(26)
^{26m}Al	4232.42(35)	6344.9(19)	≥99.97	3035.8(17)	0.31(3)	1.45(2)	3070.0(21)
^{34}Cl	5491.71(22)	1525.76(88)	≥99.988	3048.4(19)	0.61(3)	1.33(3)	3070.1(24)
^{38m}K	6043.76(56)	923.95(64)	≥99.998	3047.9(26)	0.62(3)	1.33(4)	3069.4(31)
^{42}Sc	6425.58(28)	680.72(26)	99.9941(14)	3045.1(14)	0.41(3)	1.47(5)	3077.3(24)
^{46}V	7050.63(69)	422.51(11)	99.9848(13)	3044.6(18)	0.41(3)	1.40(6)	3074.4(27)
^{50}Mn	7632.39(28)	283.25(14)	99.942(3)	3043.7(16)	0.41(3)	1.40(7)	3073.8(27)
^{54}Co	8242.56(28)	193.270(63)	99.9955(6)	3045.8(11)	0.52(3)	1.39(7)	3072.2(27)
					Average $\overline{\mathcal{F}t}$		3072.3(10)
					$X^2/(N-1)$		1.1

4. Current status of CVC and unitarity tests

Relevant world data were thoroughly surveyed[9] in 1989 and again last year[1]. Table 1 contains all requisite experimental and calculated information for the determination of $\mathcal{F}t$ values, the results for which appear in the last column together with their weighted average, $\overline{\mathcal{F}t}$, and scaled uncertainty. The δ_c values shown are the unweighted average of the results of ref[10] and ref[9] updated by ref[7], with a "statistical" uncertainty representing the scatter of the two calculations about their mean. (The results of ref[12] only cover seven of the nine cases and are less reliable for two more, as discussed in section 3; they do, however, corroborate the behaviour of the two complete calculations.)

As demonstrated in table 1 and illustrated in figure 2, there is no statistically significant evidence of inconsistencies in the data, thus verifying the expectation of CVC at the level of 4 x 10^{-4}, the fractional uncertainty quoted on the average $\mathcal{F}t$ value.

Proceeding next to test CKM unitarity, we must go beyond the "statistical" uncertainties in δ_c and incorporate a "systematic" uncertainty that arises from the small systematic difference between the two independent model calculations of δ_c (ref[10] vs refs[7,9]); our approach is elaborated in ref[9]. This results in $\overline{\mathcal{F}t} = 3072.3 \pm 2.0$s, which, together with the value of G_F derived from muon decay[2] and the calculated[15] $\Delta_R^V = 2.40 \pm 0.08)$ %, yields via eq (5) the result:

$$V_{ud} = 0.9740 \pm 0.0005 \qquad (7)$$

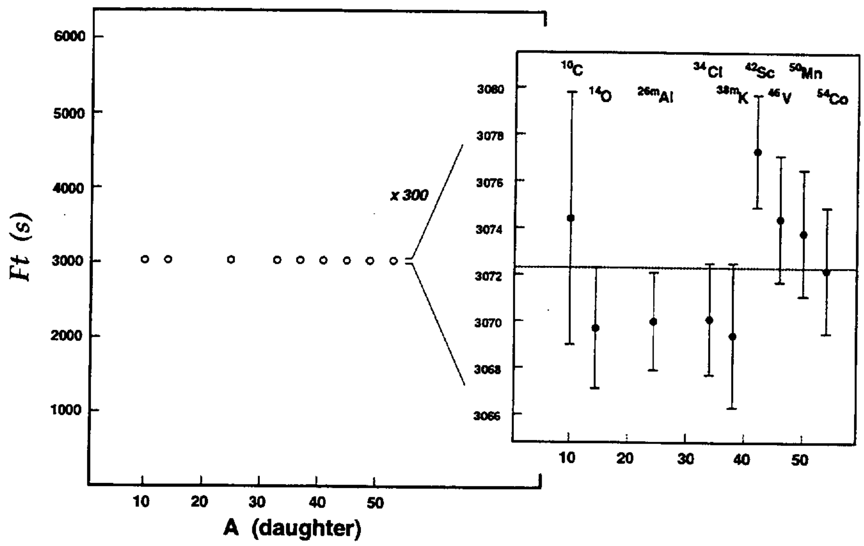

Figure 2: $\mathcal{F}t$ values for the nine best-known superallowed decays and the least-squares one-parameter fit.

The quoted uncertainty is dominated by uncertainties in the theoretical corrections, particularly Δ_R^V; experimental errors on the input data for the $\mathcal{F}t$ values contribute less than ±0.0002 to the uncertainty in eq (7).

Now, we take the value[2] $|V_{us}| = 0.2205 \pm 0.0018$, which is the average of two results, one from the analysis of K_{e3} decay, the other from hyperon decays. The small value[2] of $|V_{ub}| = 0.0032 \pm 0.0009$, which is derived from the semi-leptonic decay of B mesons, makes it negligible in the unitarity test. The result

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9972 \pm 0.0013 \qquad (8)$$

differs from unitarity at the 98% confidence level.

5. Conclusions

It is too early to assess the significance of the apparent non-unitarity of the top row of the CKM matrix. Taken at face value, it indicates the need for some extension of the three-generation Standard Model — perhaps in the form of right-hand currents or additional gauge bosons. However, less profoundly, it may instead reflect a flaw in the evaluation of V_{us} (as already suggested for other reasons in ref[16]). It seems unlikely, though, that inadequacies in δ_c can explain the failure of the test as has also been suggested[17]: the $\mathcal{F}t$ data do not support any residual Z-dependence, nor does the most recent δ_c calculation[12] offer any support to there being significant undiscovered effects from the nuclear core.

Finally, it is interesting to note that the world data for neutron decay (as surveyed in ref[1] with the addition of ref[18]) yields a value of $V_{ud} = 0.9767 \pm 0.0026$. This is consistent with the result from superallowed β decay, given in eq (7), but is five times less precise. In the neutron case, the main sources of uncertainty are experimental.

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